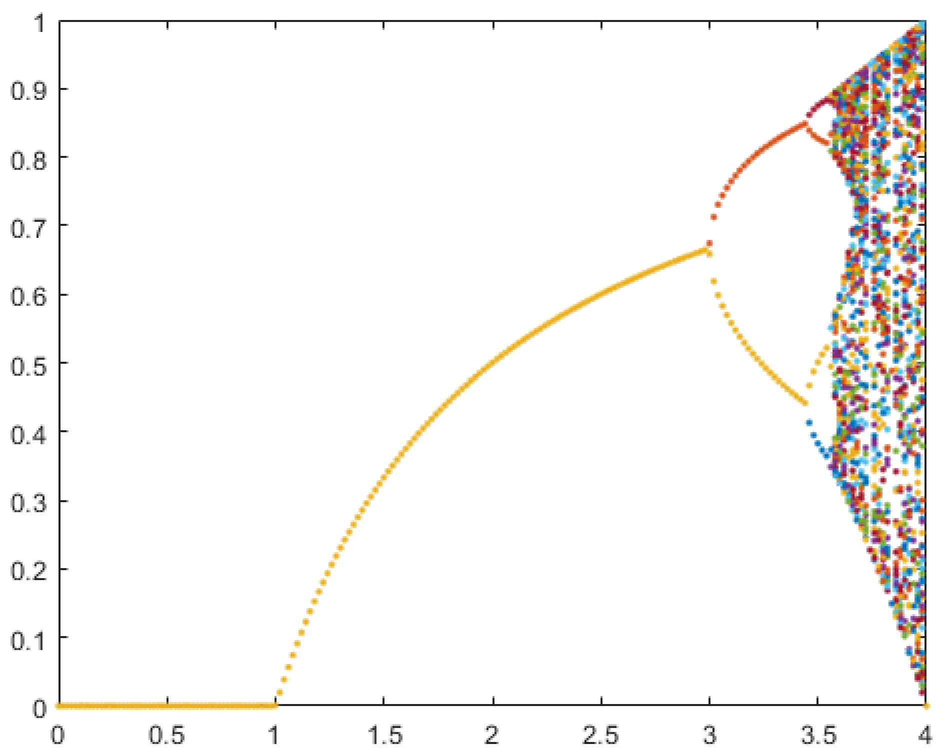
Table of Contents

Problem 1

```
M = [];
max = 1000;
xo = 0.5;
probs = 0:0.02:4;
for p = 0:0.02:4
    M = [M, logistic_map(p, xo, max)];
end
figure(1);
plot(probs,M(end-100:end,:),'.');

% 0 <= p < 1, system converges to 0
% 1 <= p < 3, system has single amplitude that varies with p
% 3 <= p < 3.54, system demonstrates period doubling
% 3.54 <= p, system exhibits chaos
```

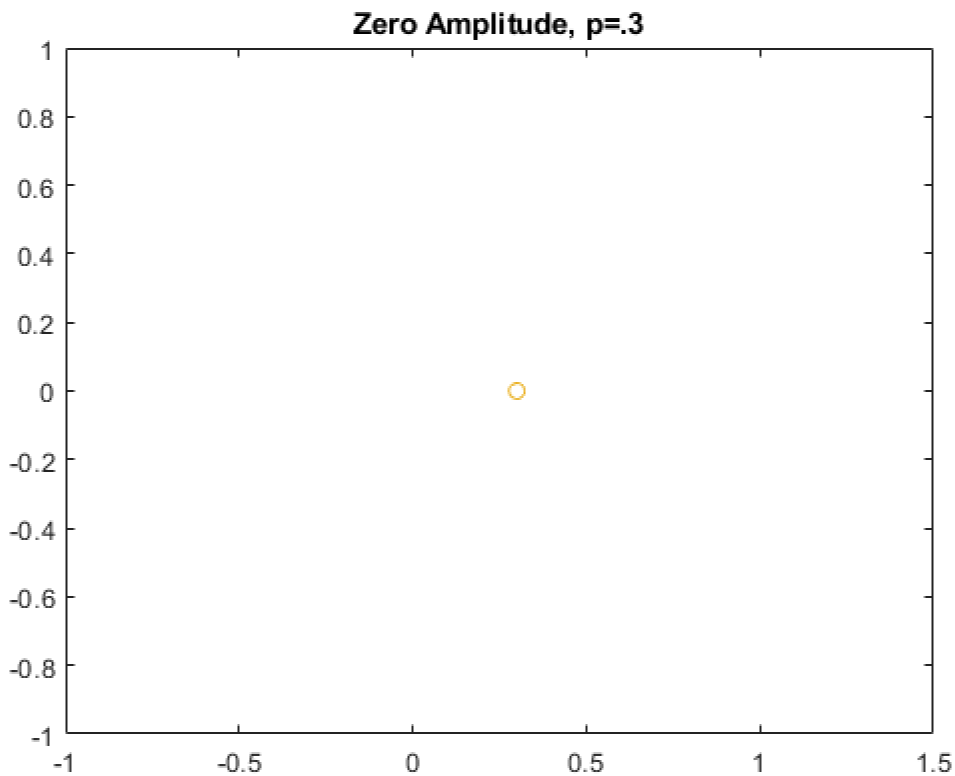


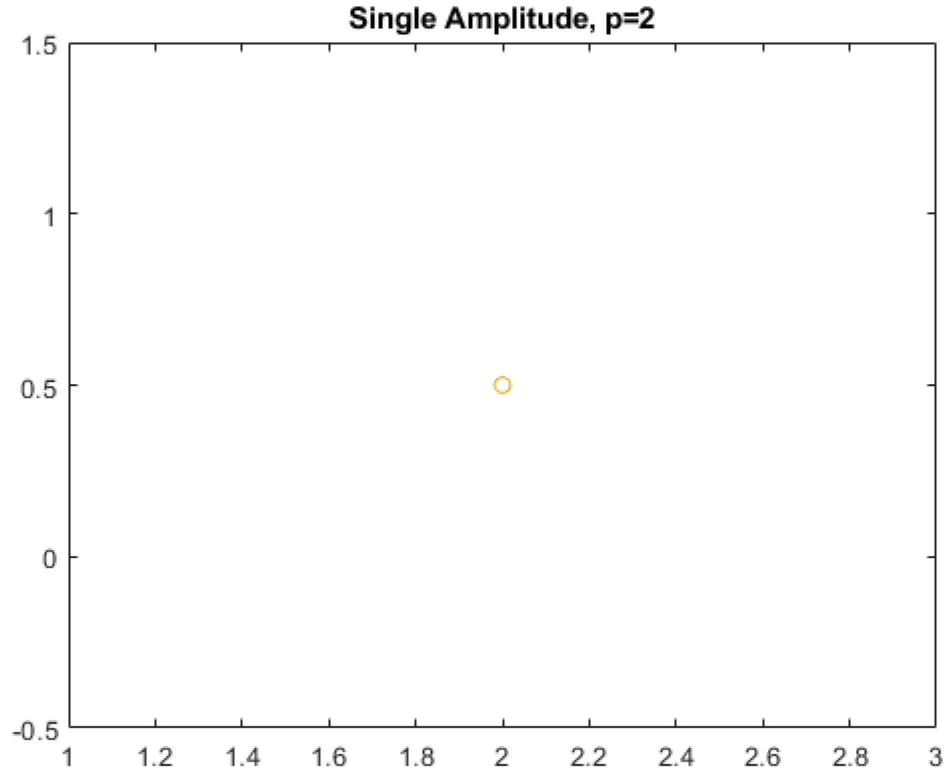
Problem 2

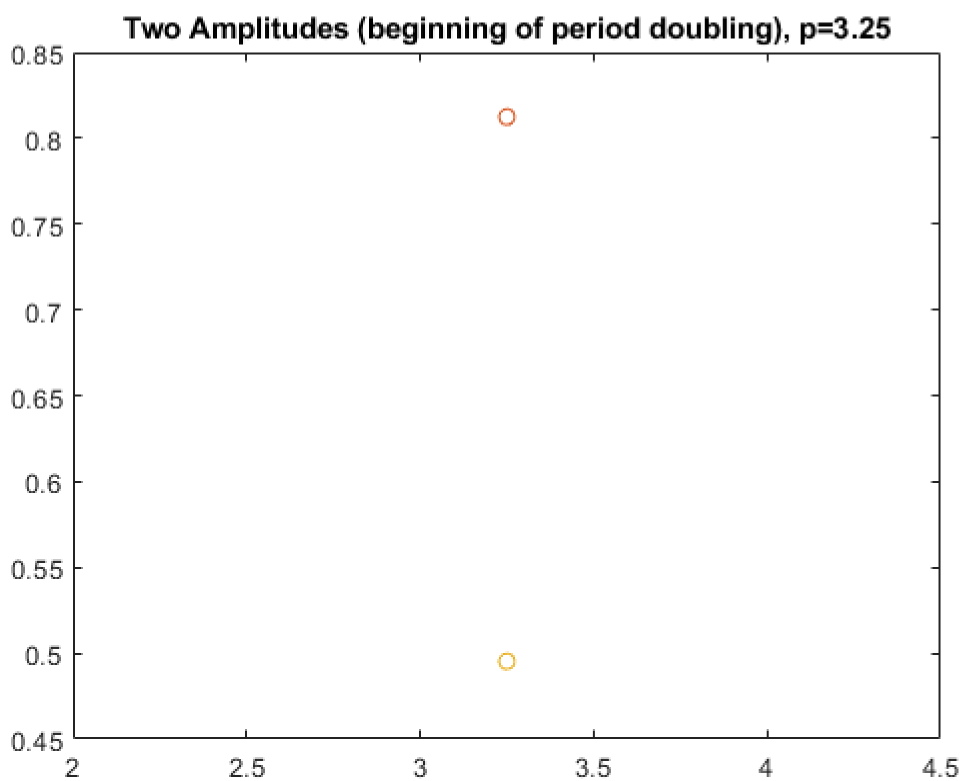
a) using p

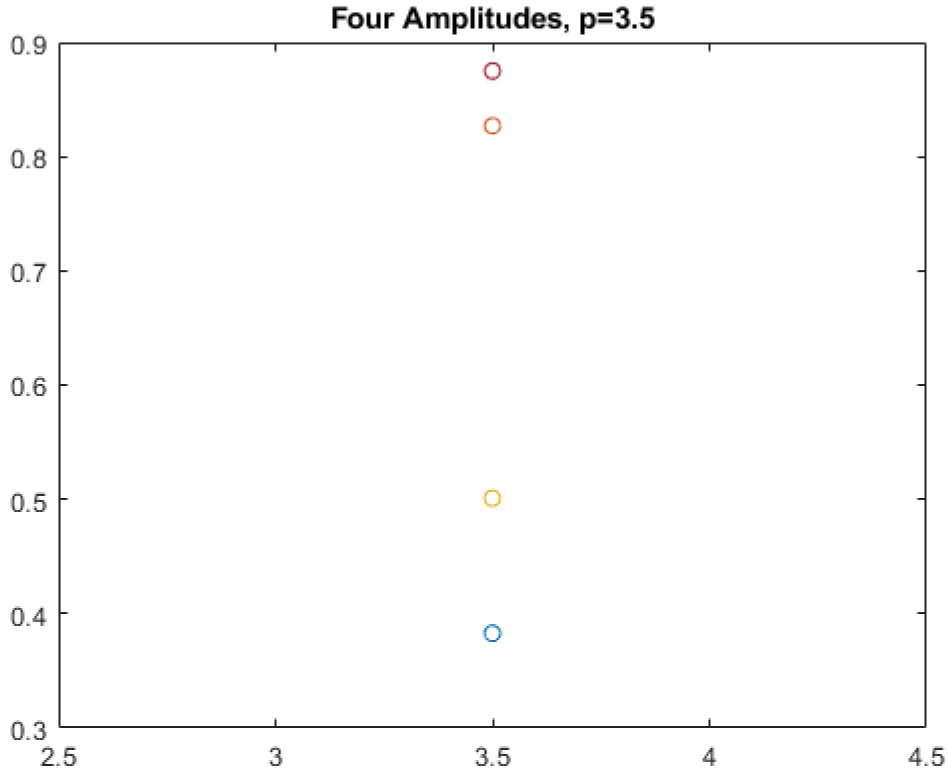
```
%constants
max = 1000;
xo = 0.5;

f1 = logistic_map(.3, xo, max);
figure();
plot(.3 ,f1(end-100:end,:),'o')
title('Zero Amplitude, p=.3')

f2 = logistic_map(2, xo, max);
figure();
plot(2,f2(end-100:end,:),'o')
title('Single Amplitude, p=2')

f3 = logistic_map(3.25, xo, max);
figure();
plot(3.25,f3(end-100:end,:),'o')
title('Two Amplitudes (beginning of period doubling), p=3.25')

f4 = logistic_map(3.5, xo, max);
figure();
plot(3.5,f4(end-100:end,:),'o')
title('Four Amplitudes, p=3.5')

f5 = logistic_map(3.8, xo, max);
figure();
plot(3.8,f5(end-100:end,:),'o')
title('Chaos, p=3.8')

f6 = logistic_map(4.5, xo, max);
figure();
plot(4.5,f6(end-100:end,:),'o')
title('Equilibria diverge, p=4.5')

% a) using k
k = 0:1000;

f1 = logistic_map(.3, xo, max);
figure();
plot(k ,f1(end-1000:end,:))
title('Zero Amplitude, p=.3')

f2 = logistic_map(2, xo, max);
figure();
plot(k,f2(end-1000:end,:))
title('Single Amplitude, p=2')

f3 = logistic_map(3.25, xo, max);
figure();
```

```
plot(k,f3(end-1000:end,:))
title('Two Amplitudes (beginning of period doubling), p=3.25')

f4 = logistic_map(3.5, xo, max);
figure();
plot(k,f4(end-1000:end,:))
title('Four Amplitudes, p=3.5')

f5 = logistic_map(3.8, xo, max);
figure();
plot(k,f5(end-1000:end,:))
title('Chaos, p=3.8')

f6 = logistic_map(4.5, xo, max);
figure();
plot(k,f6(end-1000:end,:))
title('Equilibria diverge, p=4.5')
```

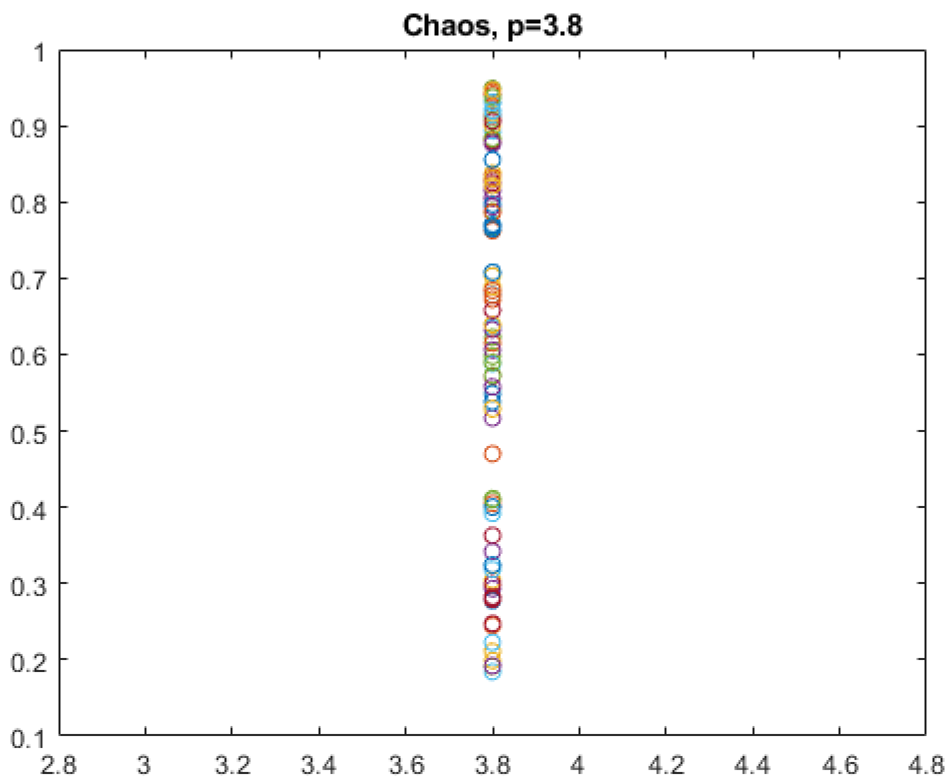



Single Amplitude, p=2
1.5
1
0.5
0
-0.5
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

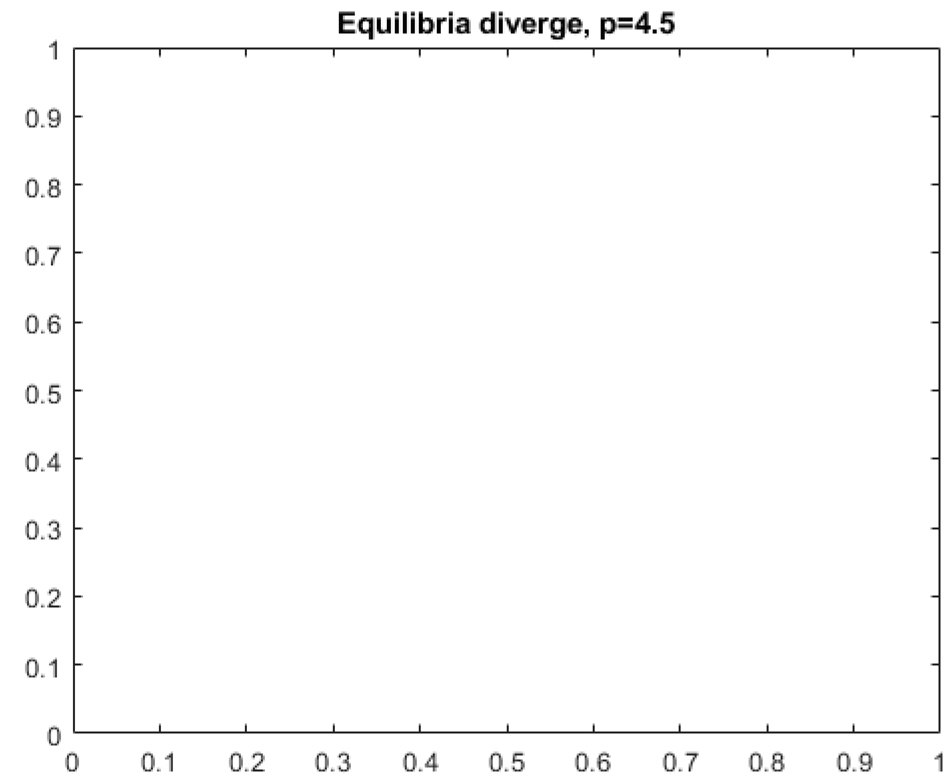

Two Amplitudes (beginning of period doubling), p=3.25
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
2
2.5
3
3.5
4
4.5

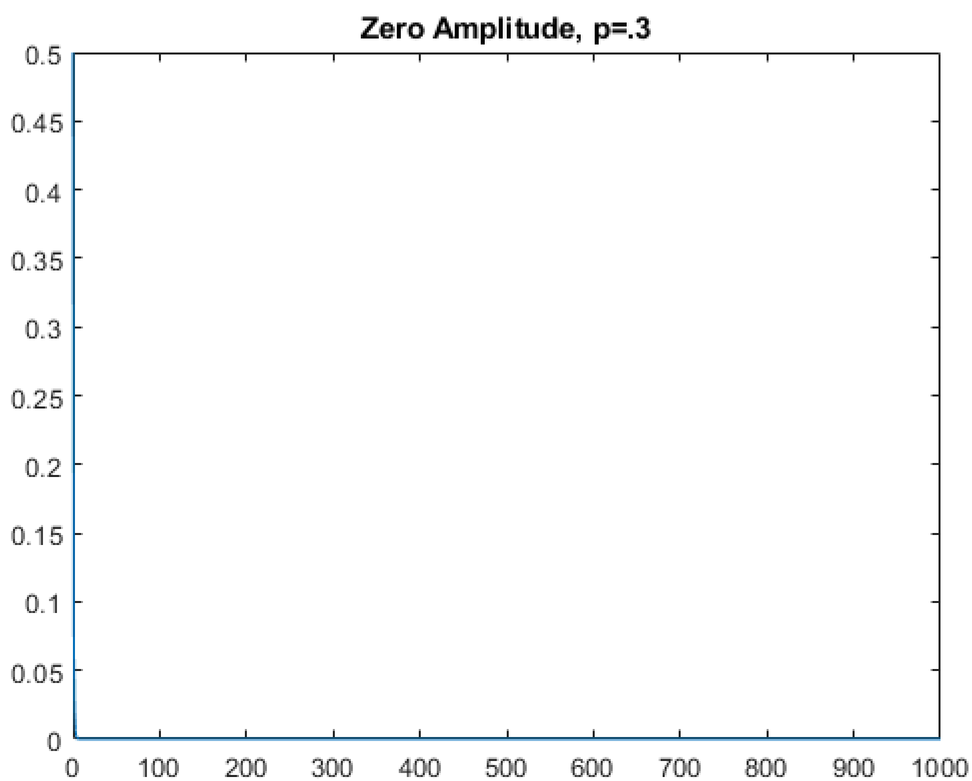


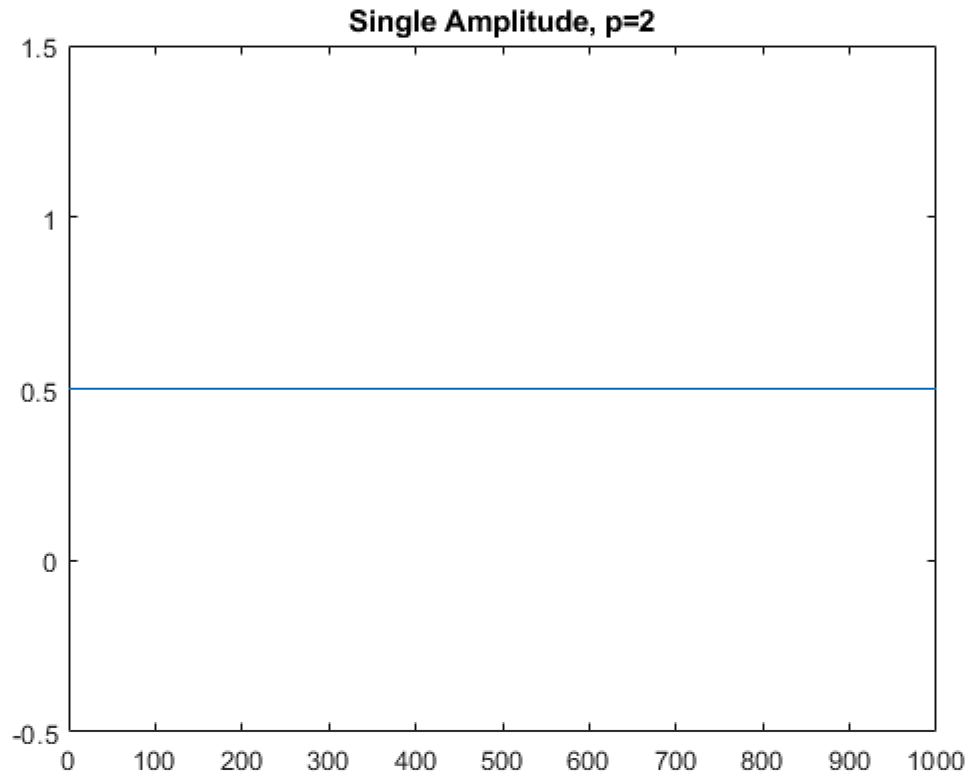
Four Amplitudes, p=3.5
0.9
0.8
0.7
0.6
0.5
0.4
0.3
2.5
3
3.5
4
4.5

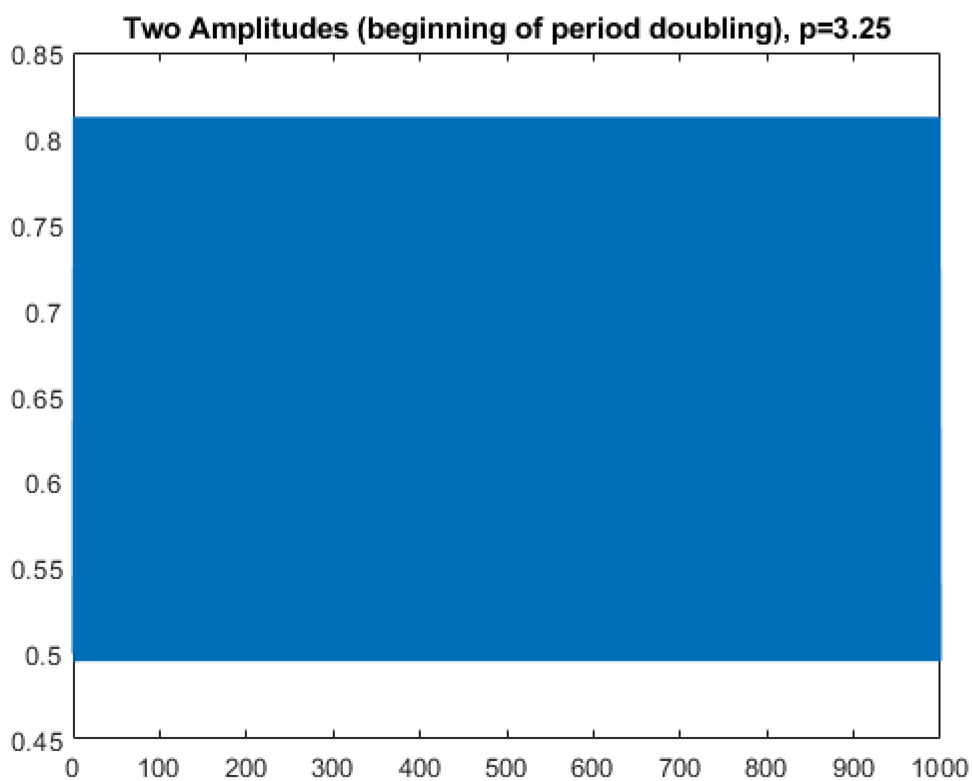


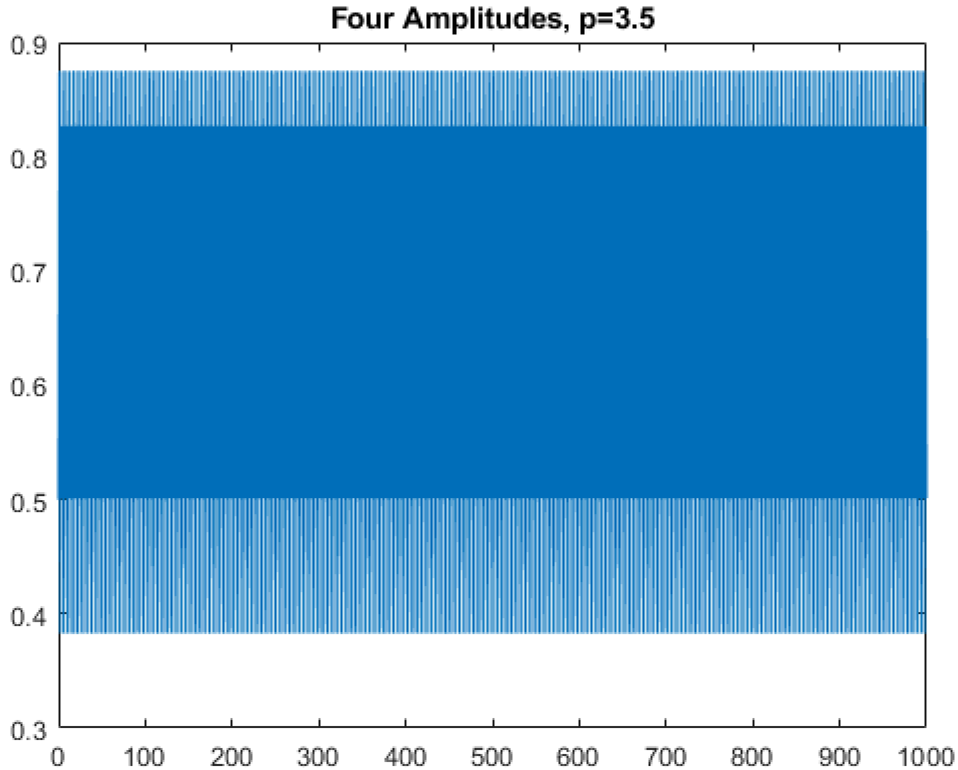
Chaos, p=3.8
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2.8
3
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
4.8

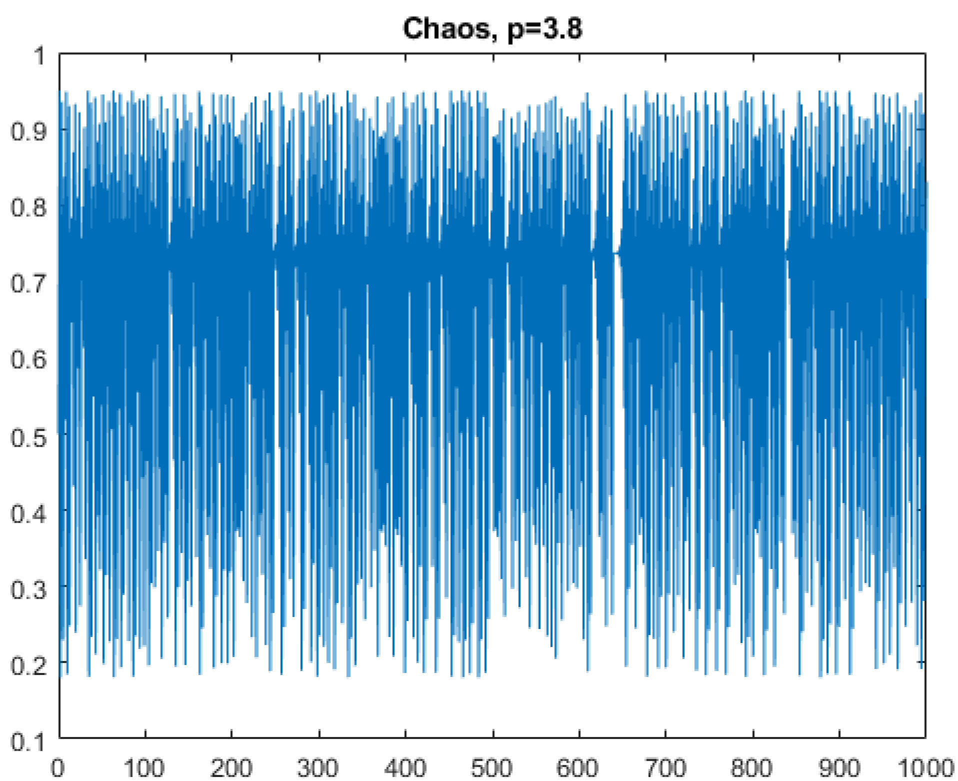

Equilibria diverge, p=4.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

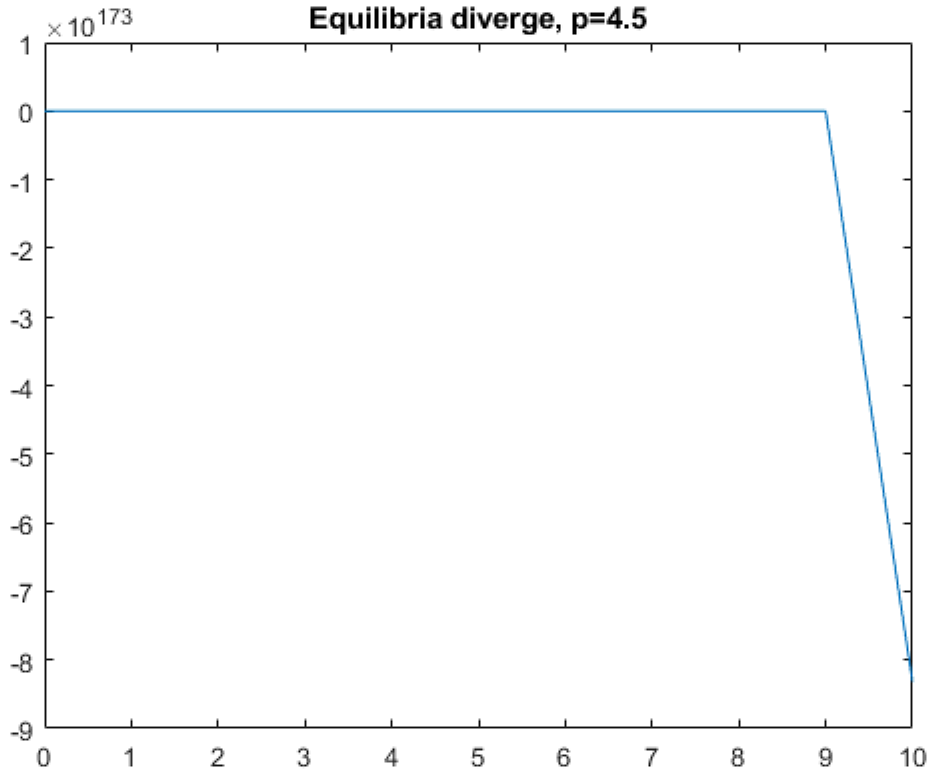

Zero Amplitude, p=.3
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
100
200
300
400
500
600
700
800
900
1000


Single Amplitude, p=2
1.5
1
0.5
0
-0.5
0
100
200
300
400
500
600
700
800
900
1000


Two Amplitudes (beginning of period doubling), p=3.25
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0
100
200
300
400
500
600
700
800
900
1000


Four Amplitudes, p=3.5
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
100
200
300
400
500
600
700
800
900
1000

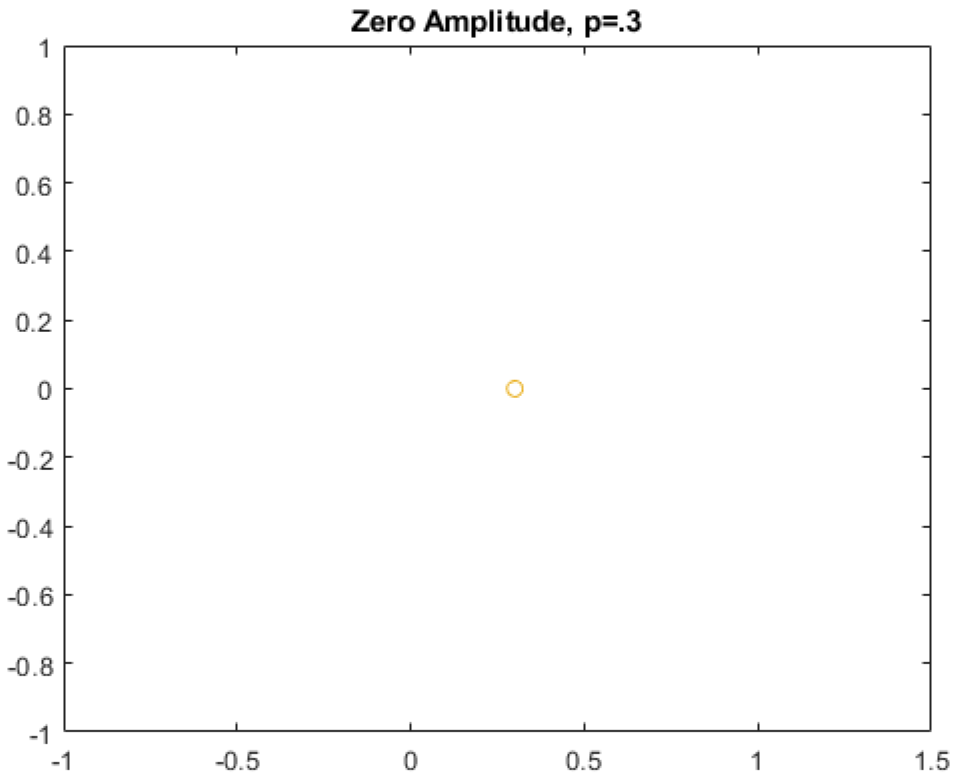

Chaos, p=3.8
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600
700
800
900
1000

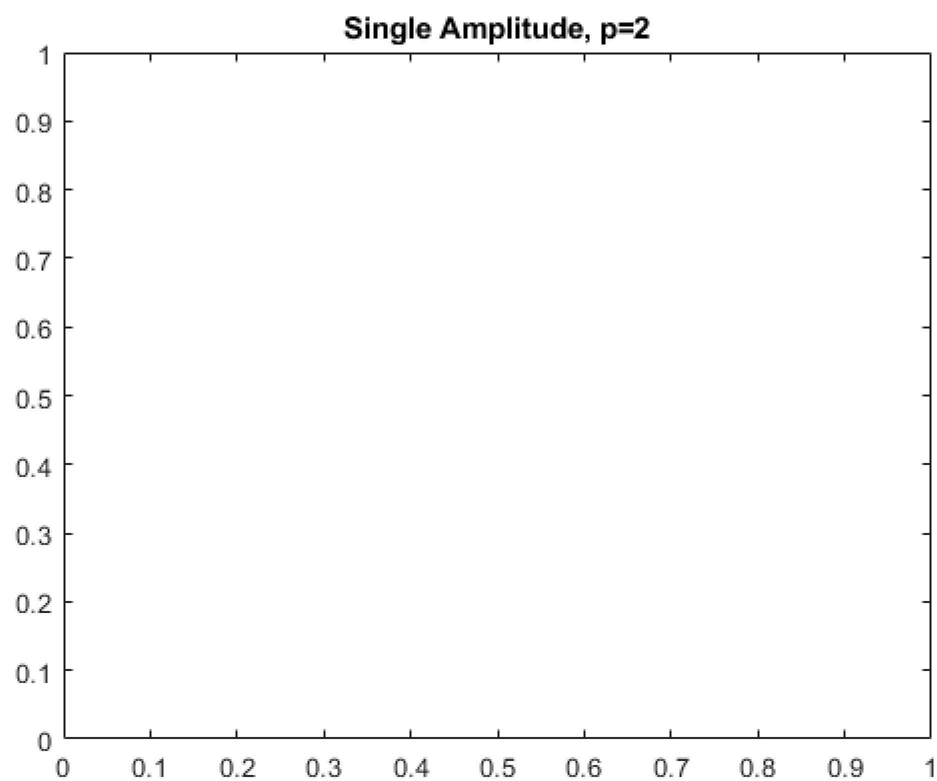


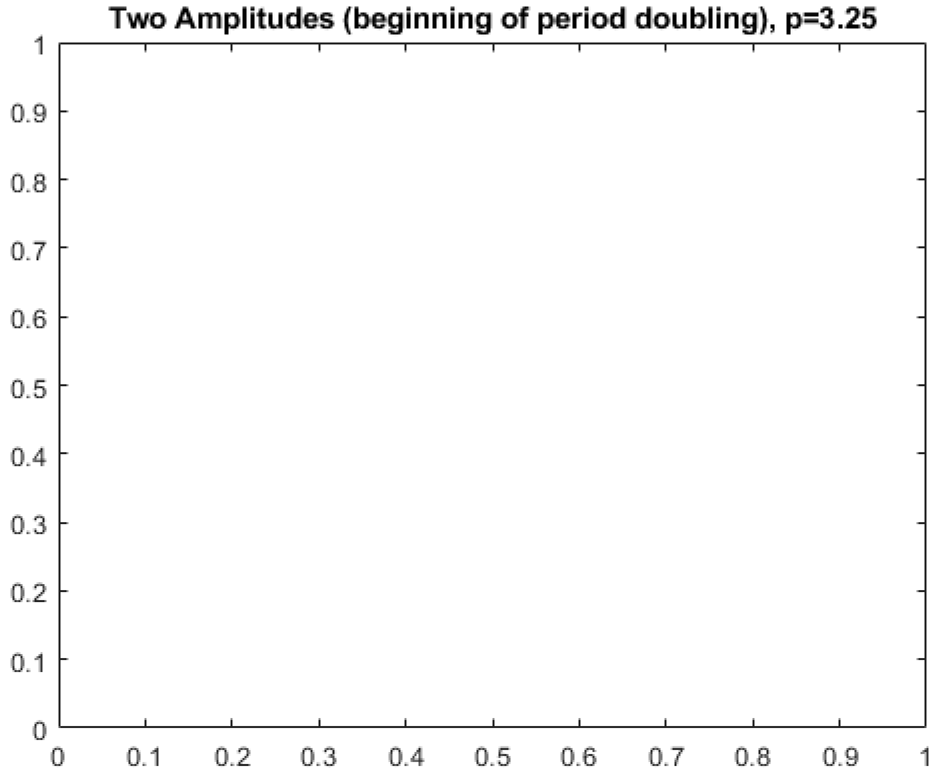
b) Initial condition 1.5

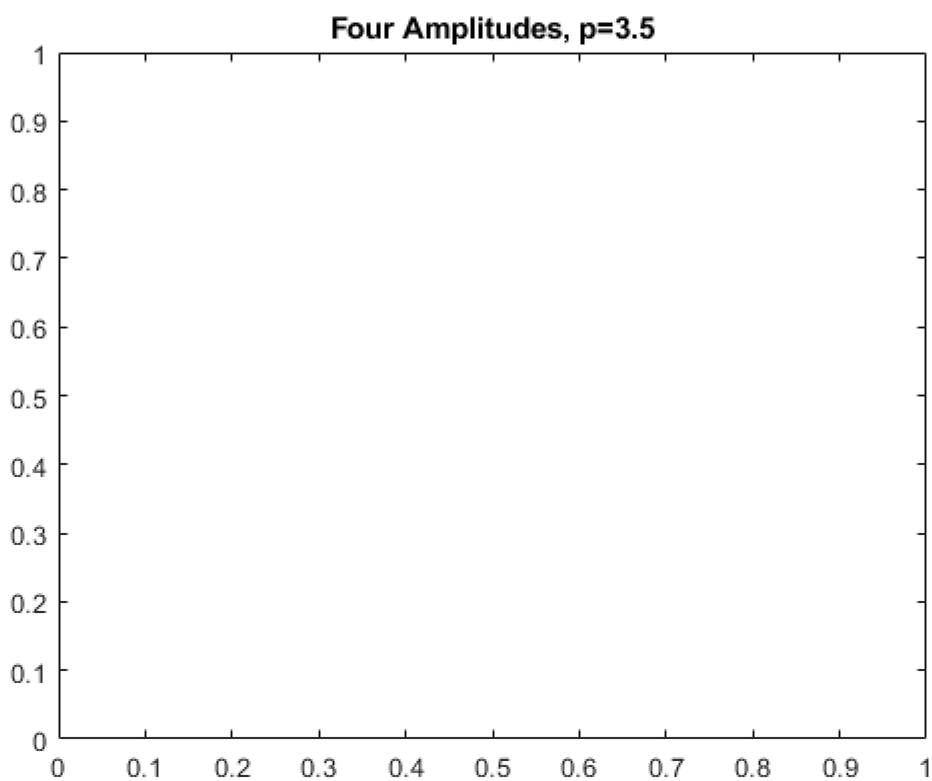
```
%constants
max = 1000;
xo = 1.5;

f1 = logistic_map(.3, xo, max);
figure();
plot(.3 ,f1(end-100:end,:),'o')
title('Zero Amplitude, p=.3')

f2 = logistic_map(2, xo, max);
figure();
plot(2,f2(end-100:end,:),'o')
title('Single Amplitude, p=2')

f3 = logistic_map(3.25, xo, max);
figure();
plot(3.25,f3(end-100:end,:),'o')
title('Two Amplitudes (beginning of period doubling), p=3.25')

f4 = logistic_map(3.5, xo, max);
figure();
plot(3.5,f4(end-100:end,:),'o')
title('Four Amplitudes, p=3.5')
```

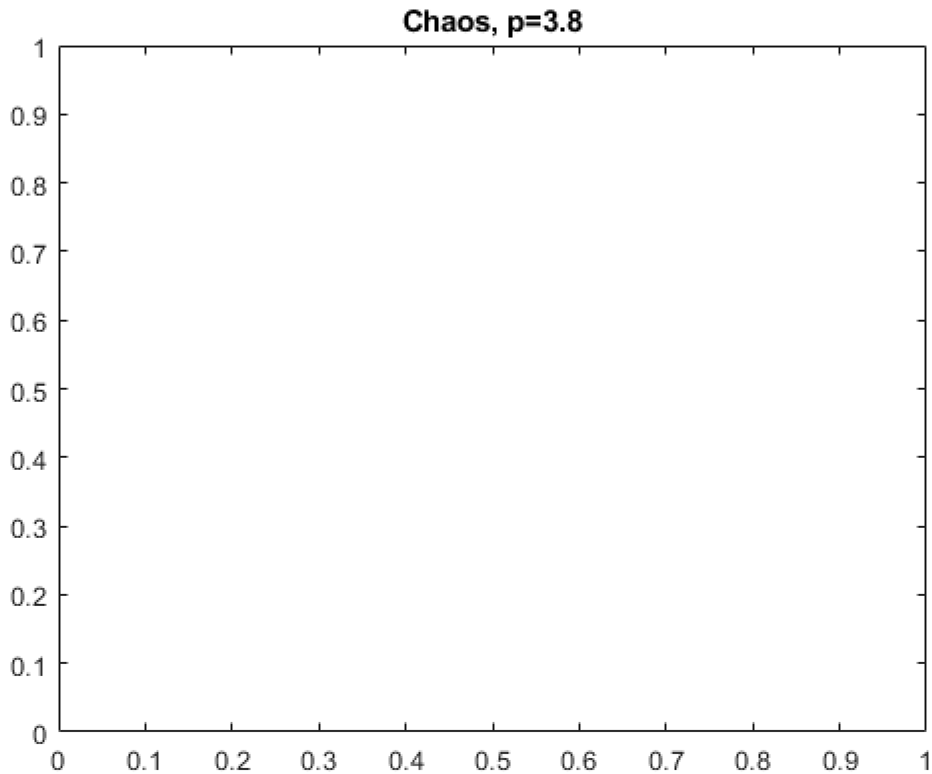
```
f5 = logistic_map(3.8, xo, max);
figure();
plot(3.8,f5(end-100:end,:),'o')
title('Chaos, p=3.8')

f6 = logistic_map(4.5, xo, max);
figure();
plot(4.5,f6(end-100:end,:),'o')
title('Equilibria diverge, p=4.5')

% b) using k
k = 0:1000;

f1 = logistic_map(.3, xo, max);
figure();
plot(k ,f1(end-1000:end,:))
title('Zero Amplitude, p=.3')

f2 = logistic_map(2, xo, max);
figure();
plot(k,f2(end-1000:end,:))
title('Single Amplitude, p=2')

f3 = logistic_map(3.25, xo, max);
figure();
plot(k,f3(end-1000:end,:))
title('Two Amplitudes (beginning of period doubling), p=3.25')

f4 = logistic_map(3.5, xo, max);
figure();
plot(k,f4(end-1000:end,:))
title('Four Amplitudes, p=3.5')

f5 = logistic_map(3.8, xo, max);
figure();
plot(k,f5(end-1000:end,:))
title('Chaos, p=3.8')

f6 = logistic_map(4.5, xo, max);
figure();
plot(k,f6(end-1000:end,:))
title('Equilibria diverge, p=4.5')
```

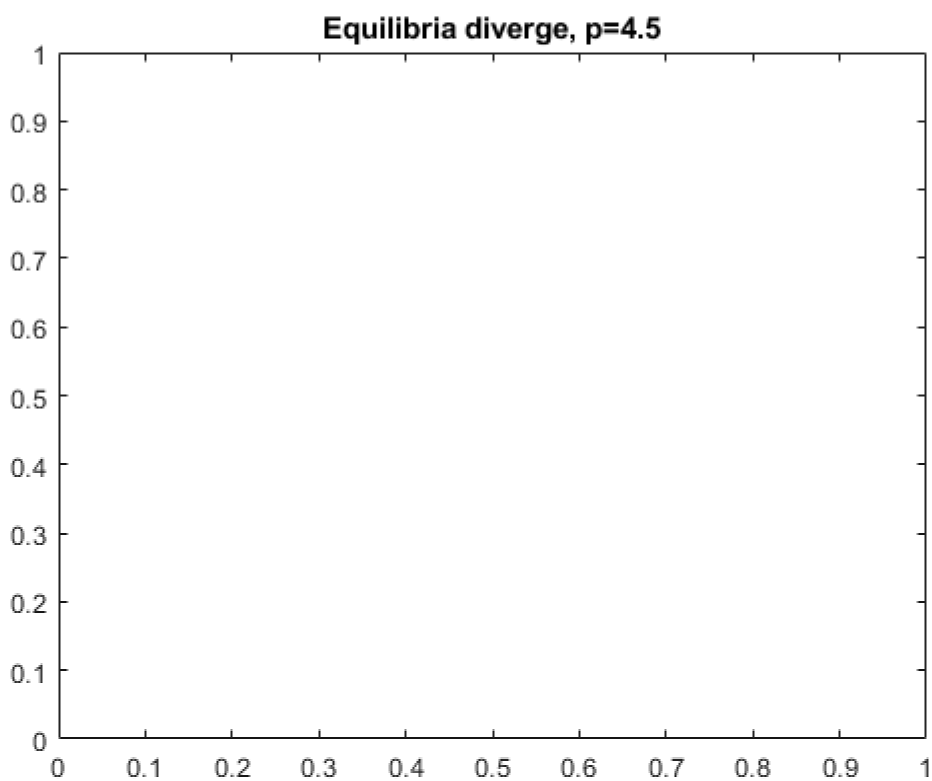

Zero Amplitude, p=.3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1
1.5

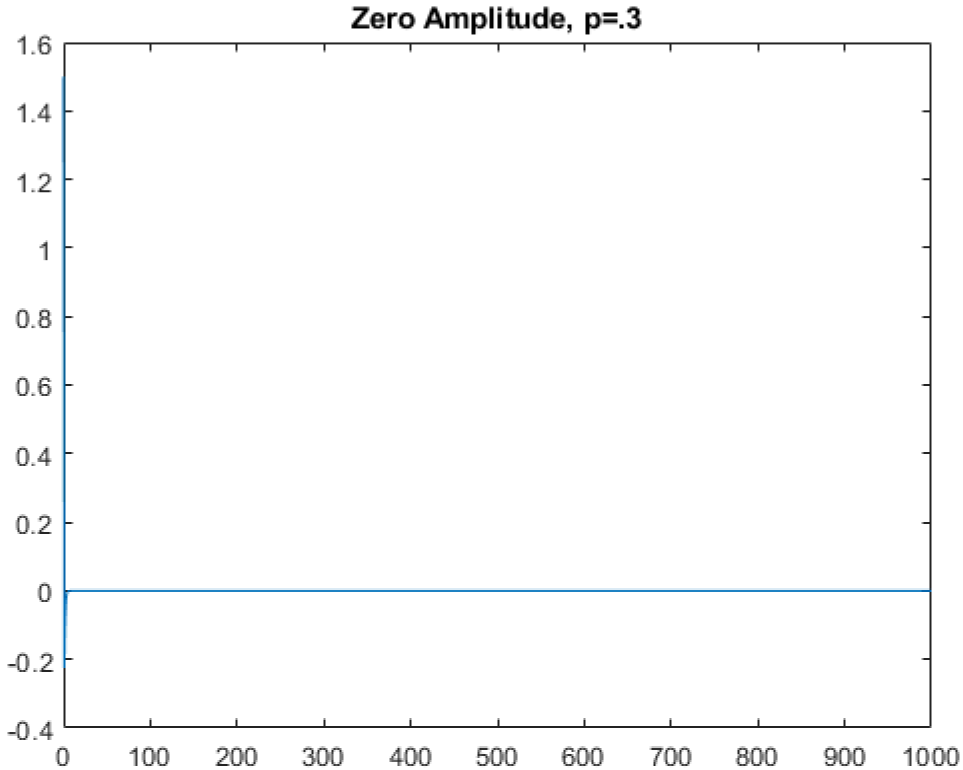

Single Amplitude, p=2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

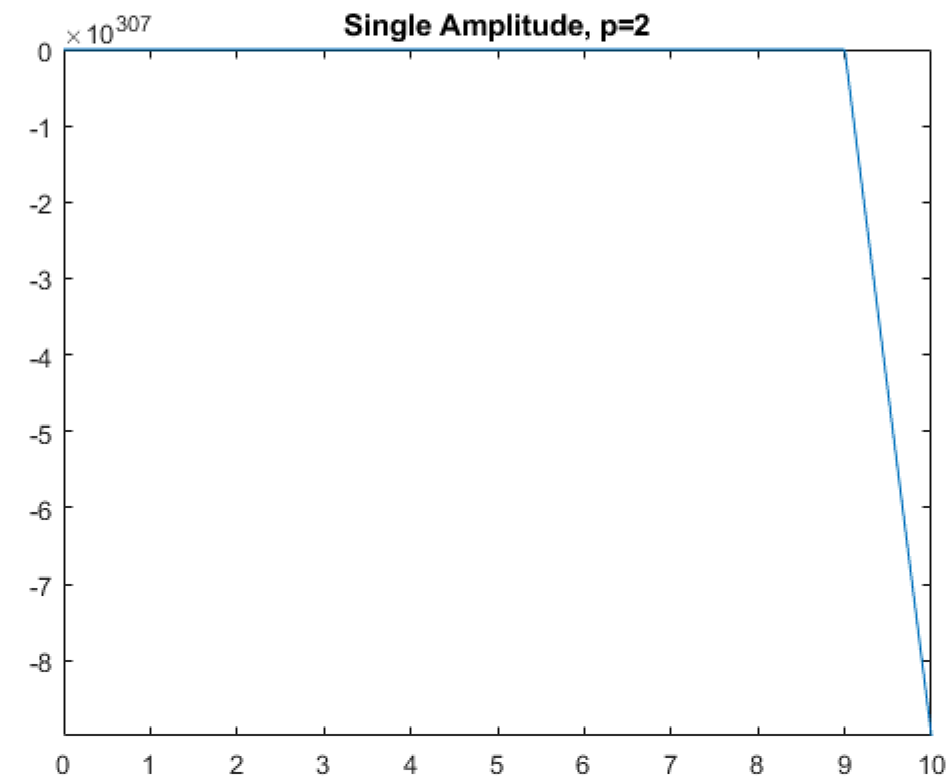

Two Amplitudes (beginning of period doubling), p=3.25
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

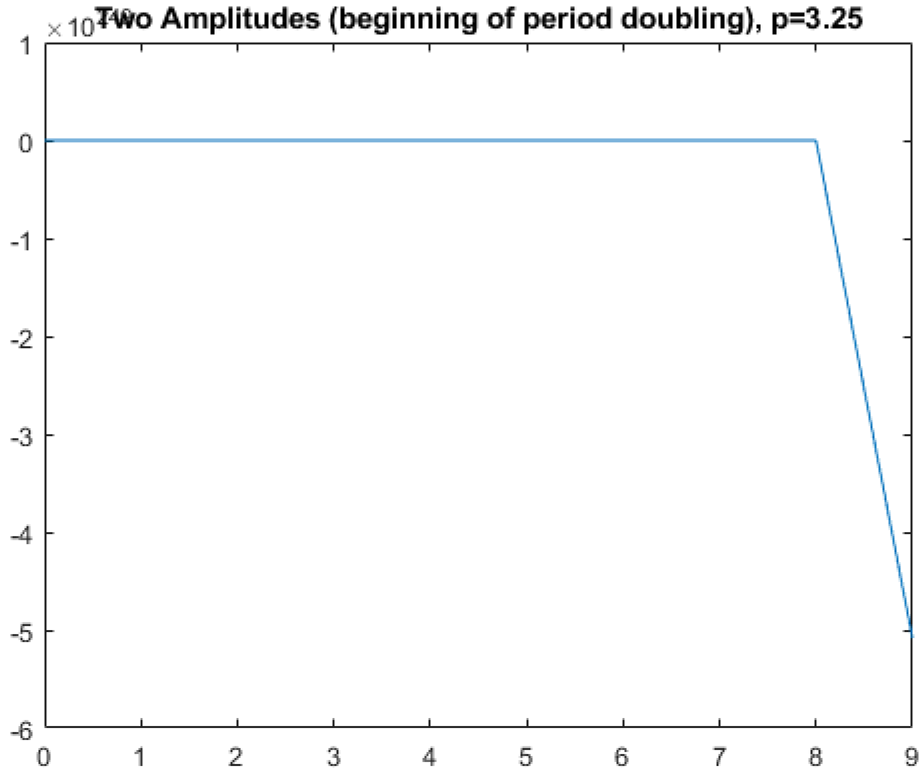

Four Amplitudes, p=3.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

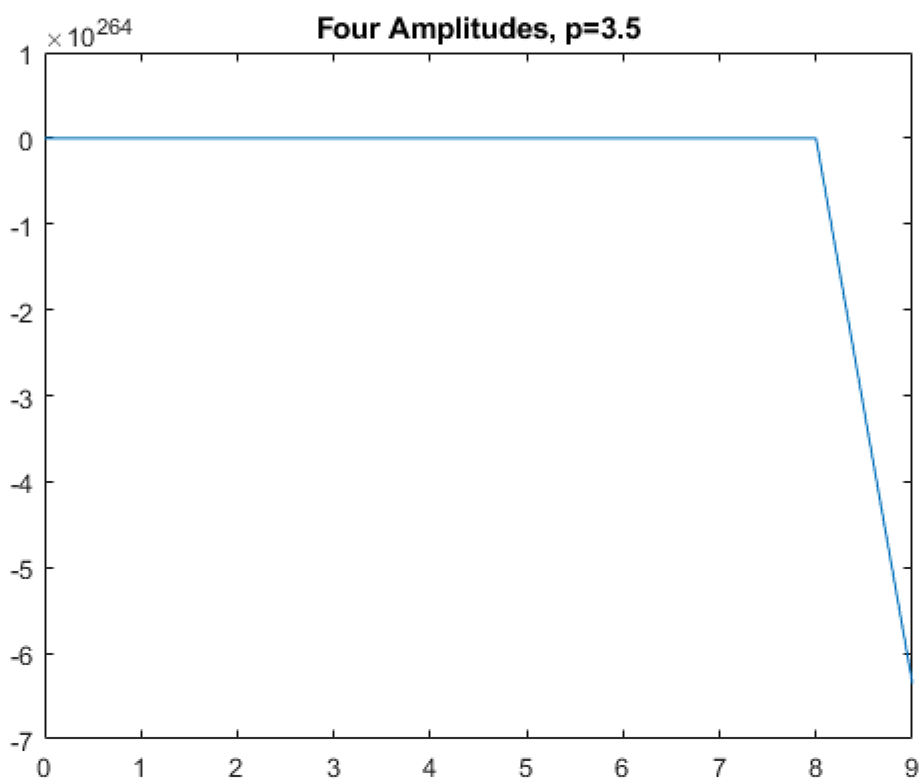

Chaos, p=3.8

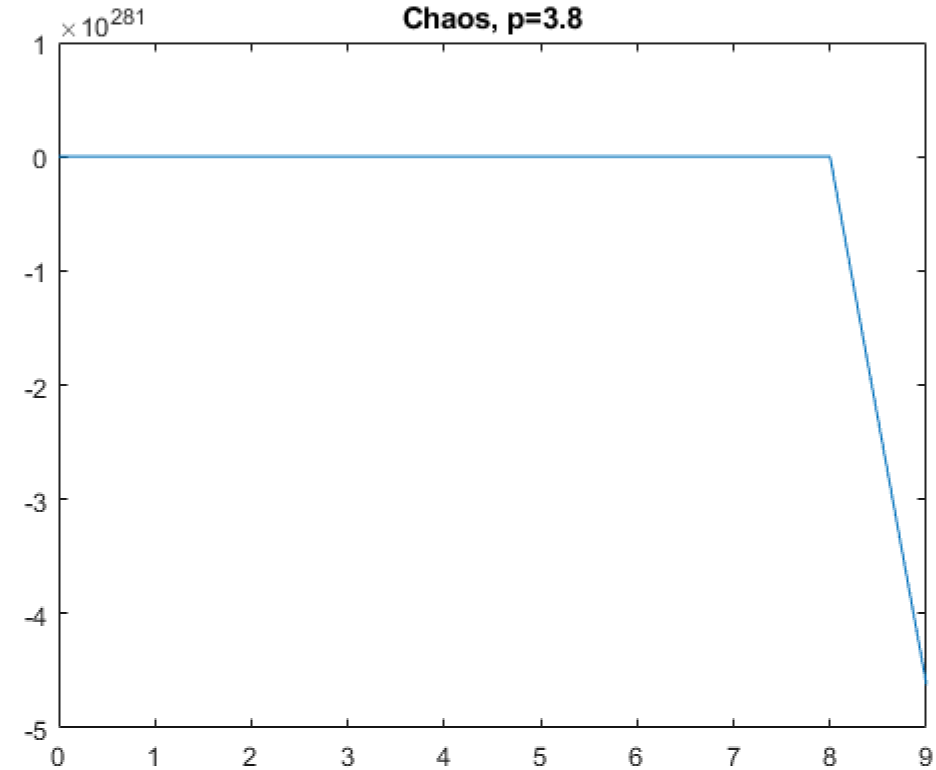

Equilibria diverge, p=4.5


Zero Amplitude, p=.3
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
100
200
300
400
500
600
700
800
900
1000

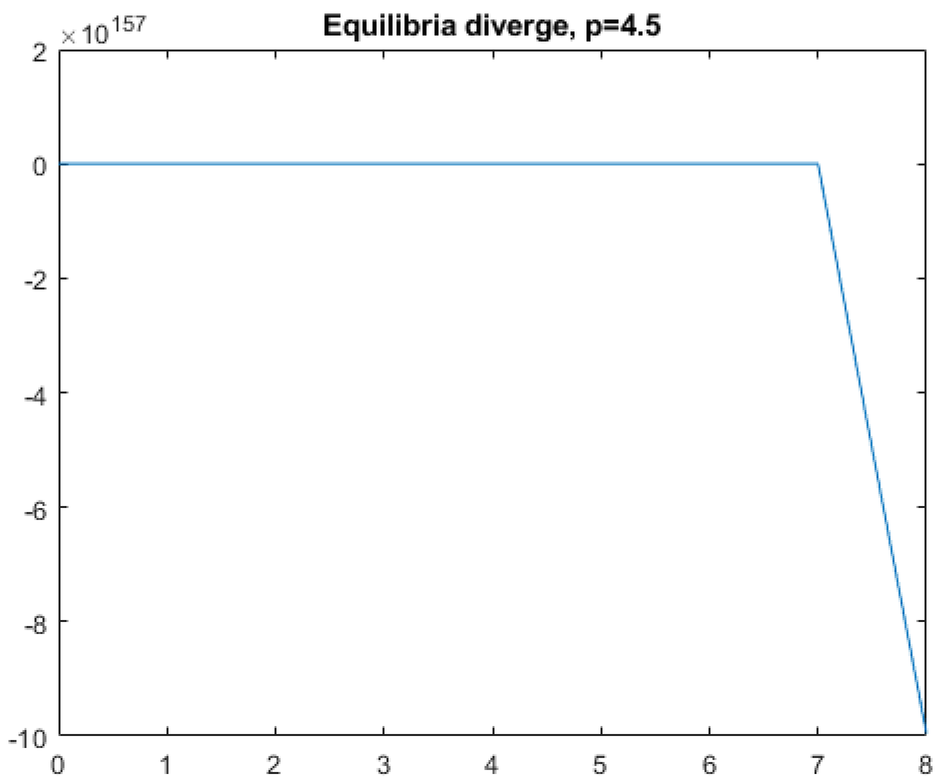

Single Amplitude, p=2
×10^307
0
-1
-2
-3
-4
-5
-6
-7
-8
0
1
2
3
4
5
6
7
8
9
10


×10
Two Amplitudes (beginning of period doubling), p=3.25
1
0
-1
-2
-3
-4
-5
-6
0
1
2
3
4
5
6
7
8
9


×10^264
Four Amplitudes, p=3.5
1
0
-1
-2
-3
-4
-5
-6
-7
0
1
2
3
4
5
6
7
8
9



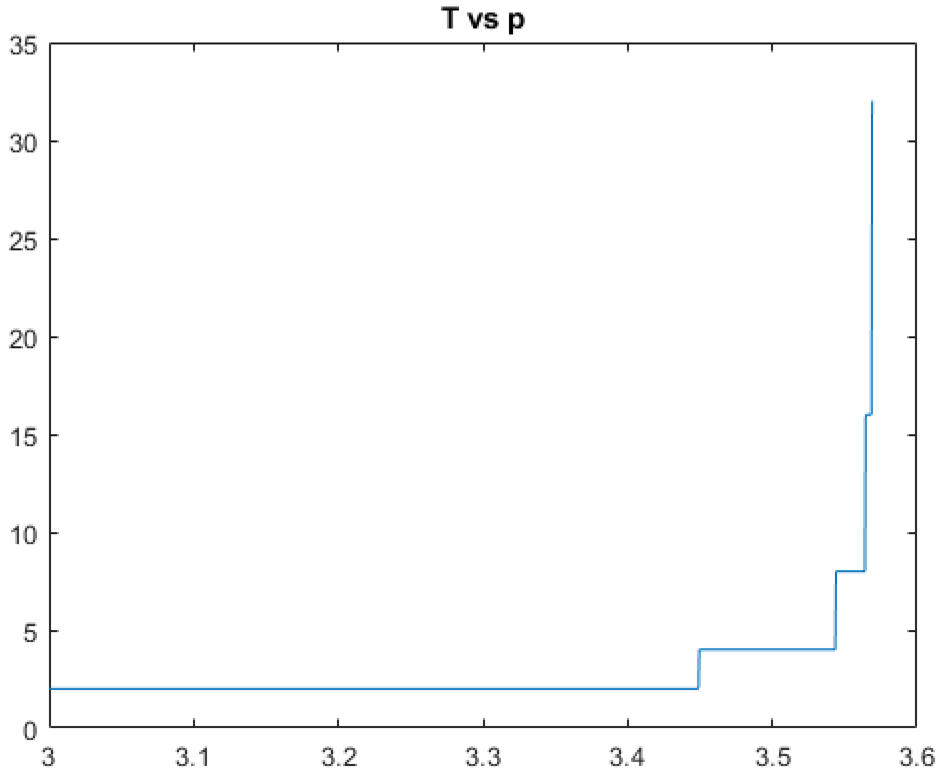
Chaos, p=3.8
×10^281
1
0
-1
-2
-3
-4
-5
0
1
2
3
4
5
6
7
8
9



Equilibria diverge, p=4.5
×10^157
2
0
-2
-4
-6
-8
-10
0
1
2
3
4
5
6
7
8

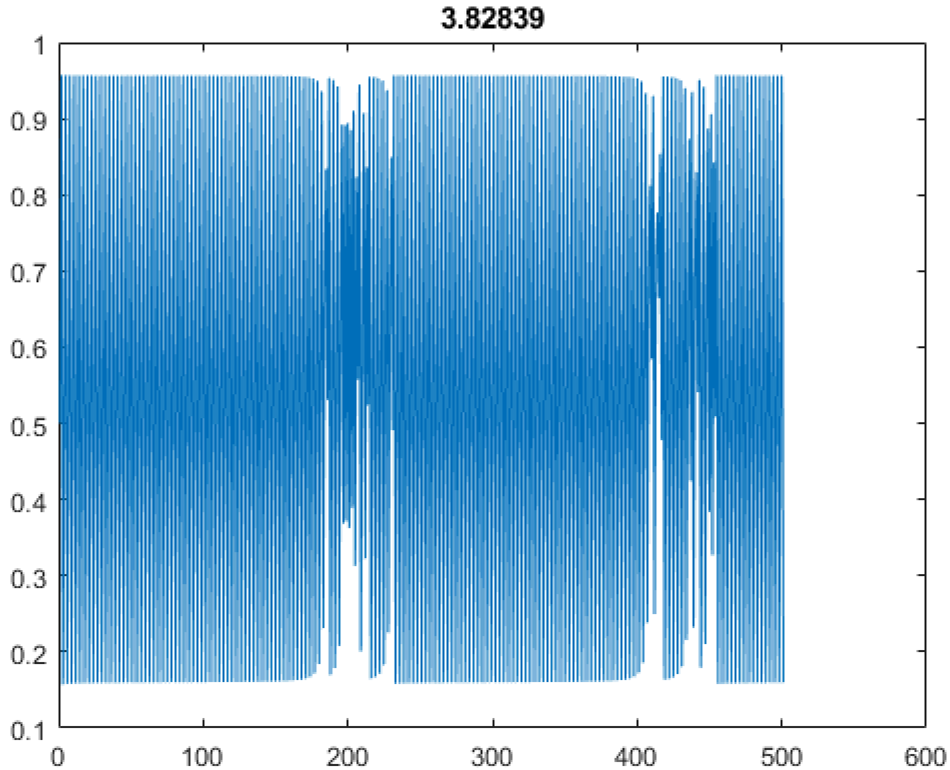
Problem 3

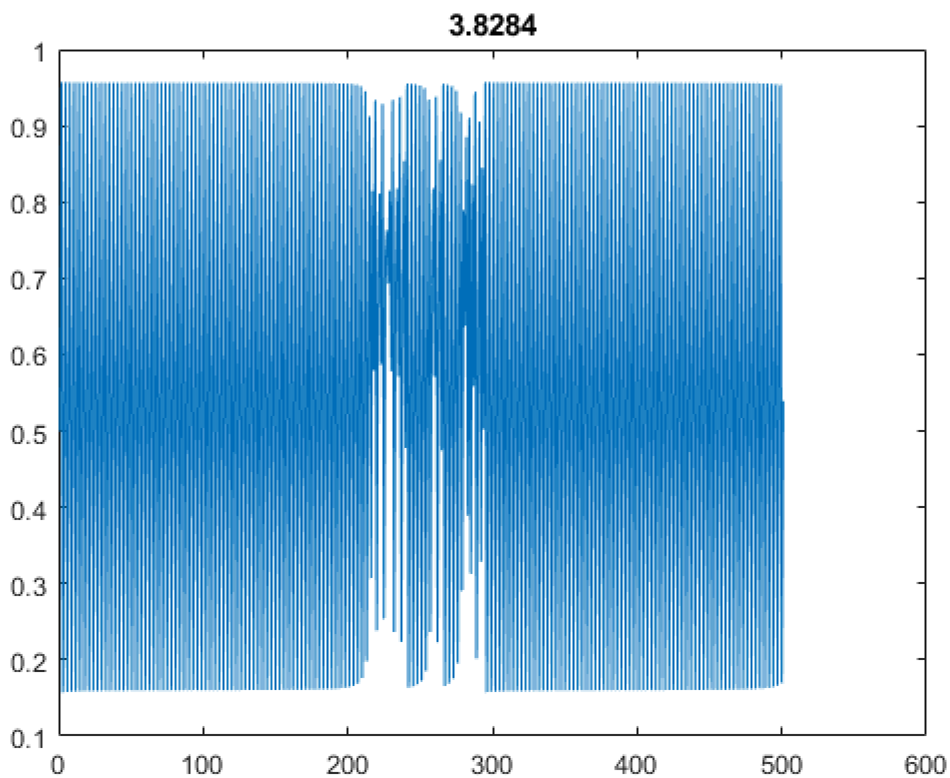
```
clear
M = [];
kmax = 5000;
init = 0.5;
probs = 3:0.0005:3.5697;
for p = 3:0.0005:3.5697
 res = logistic_map(p, init, kmax);
 M = [M, res];
end
M_slice = M(end-1000:end,:); % the last 100 or so rows of M
figure;
plot(probs,M_slice,'.');
title('Bifurcation Diagram (Greater Resolution): Period Doubling
 Region');
[row_num_M,col_num_M] = size(M_slice);
Ts = [];
deltas = [];
for i=1:col_num_M
 [T,delta] = compute_delta(M_slice(:,i));
 Ts = [Ts, T];
end
figure
plot(probs,Ts)
title('T vs p')
periods = [2, 4, 8, 16, 32];
ws = [];
for i=[2,4,8,16,32]
 indices = find(Ts==i);
 min_index = min(indices);
 max_index = max(indices);
 p_min = probs(min_index);
 p_max = probs(max_index);
 w = p_max-p_min;
 ws = [ws w];
 fprintf('T = %.4f \t p_min = %.4f \t p_max = %.4f \t w = %.4f\n', i,
 p_min, p_max, w)
end
F = [];
for i = 1:length(ws)-1
 ratio = ws(i)/ws(i+1);
 F = [F ratio];
end
% List of ratios: F1, F2, F3, and F4
disp(F);
% All ratios are between 4.6 and 4.8, and converging to about 4.7 or
 so. We
% know that the actual Feigenbaum constant is 4.669 so this checks
 out.
```

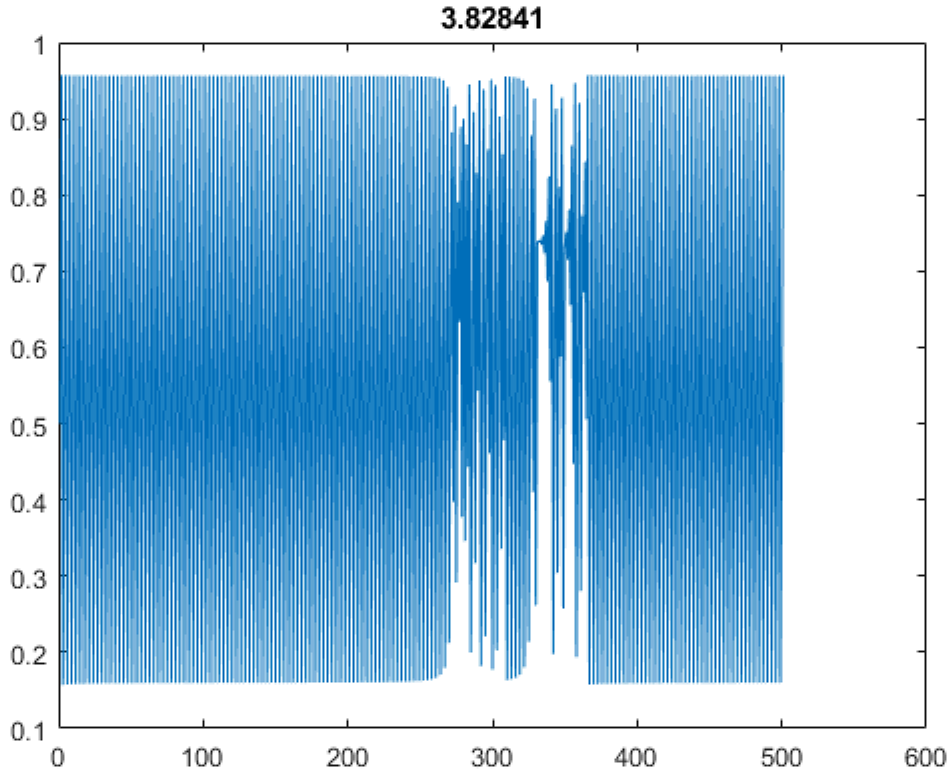


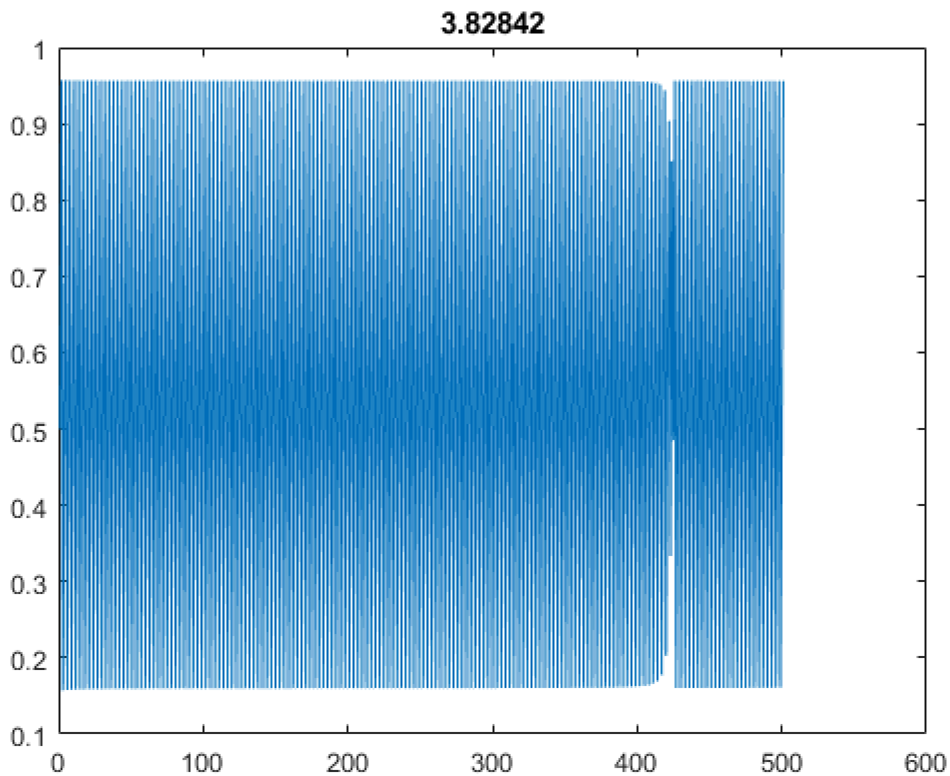
Problem 4

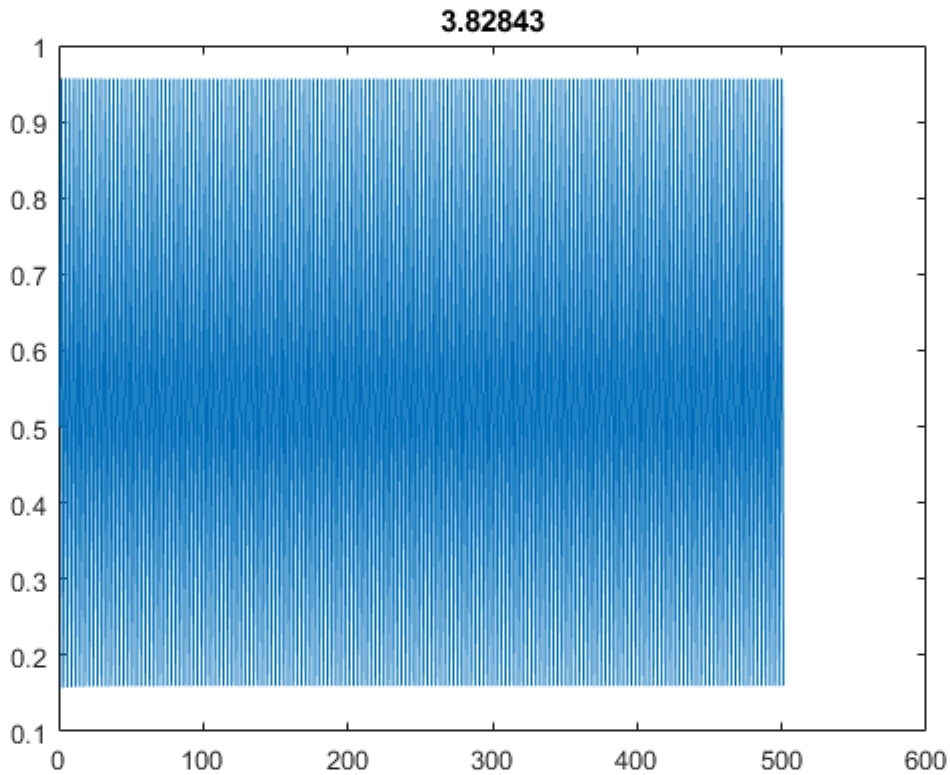
```
max = 500;
xo = 0.5;
for p = 3.82839:0.00001:3.82843
    figure
    plot(logistic_map(p, xo, max))
    title(p)
end

% find  brief "disruption" at 3.8284
```


3.82839
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600


3.8284
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600

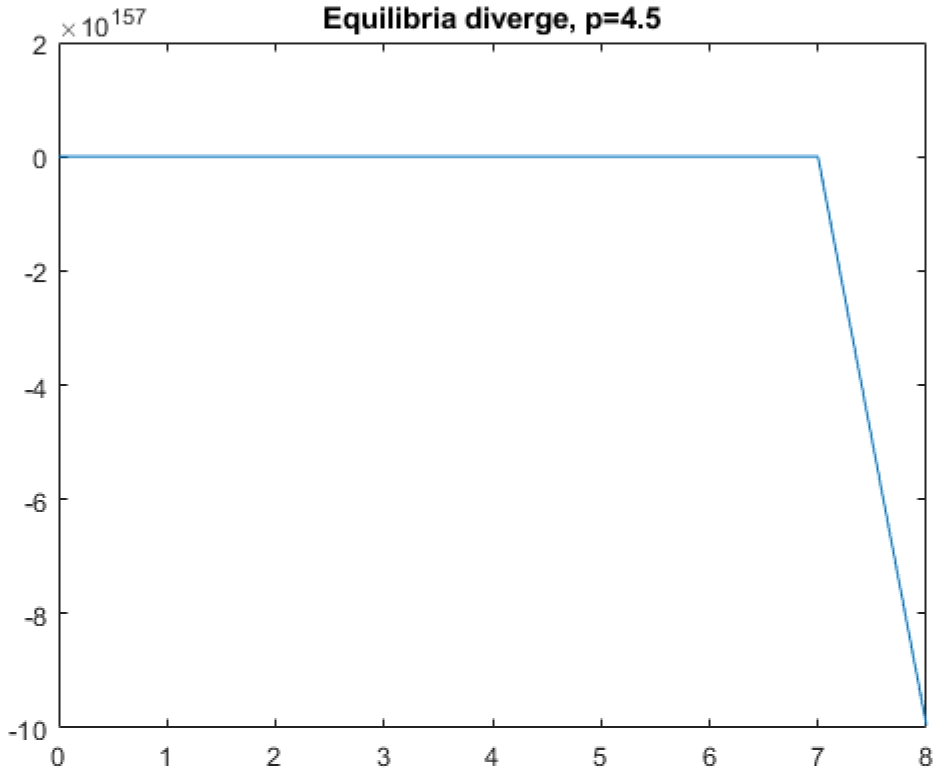

3.82841
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600


3.82842
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600

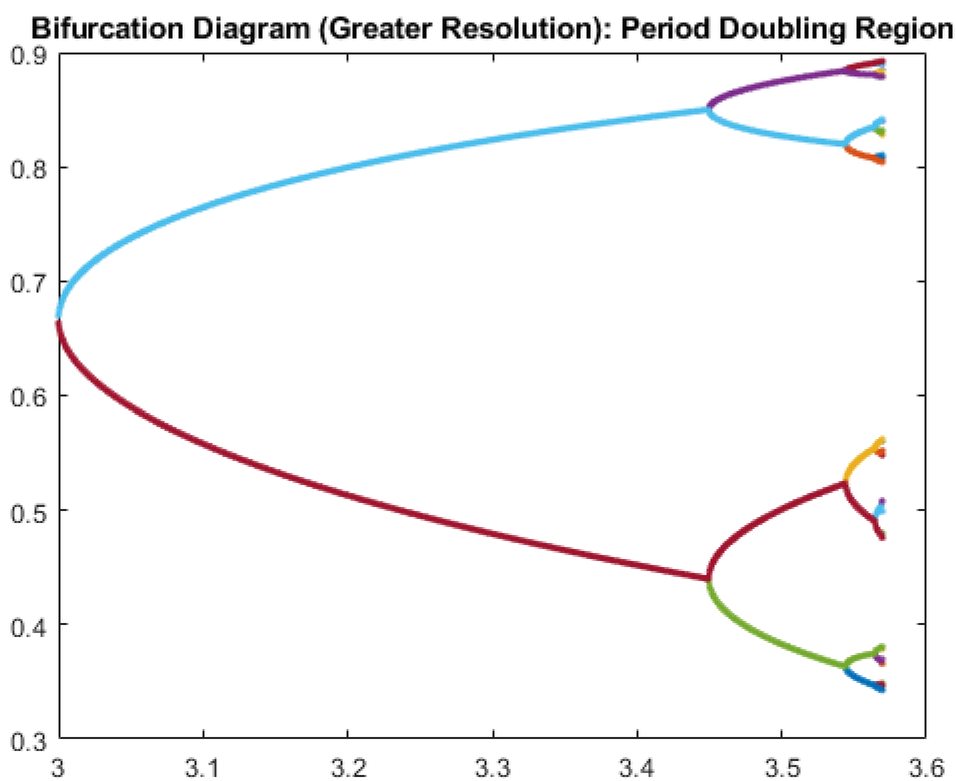


Problem 5

```
max = 500;
p = 3.9;

f1 = logistic_map(p, .5, max);
f2 = logistic_map(p, 0.5 + 10e-8, max);

fDiff = f2 - f1;
figure();
plot(fDiff)
% At around k = 43, the two solutions become visibly distinct

function x = logistic_map(p, xo, N)
    if (N < 1)
        return;
    end
    x = [xo];
    temp = xo;
    for i=1:N
        new = p*temp*(1-temp);
        x = [x;new];
        temp = new;
    end
end
```



Equilibria diverge, p=4.5
$\times 10^{157}$
2
0
-2
-4
-6
-8
-10
0
1
2
3
4
5
6
7
8



Bifurcation Diagram (Greater Resolution): Period Doubling Region
0.9
0.8
0.7
0.6
0.5
0.4
0.3
3
3.1
3.2
3.3
3.4
3.5
3.6

```
function [T,delta] = compute_delta(col_vec)
    possible_T = [2, 4, 8, 16, 32];
    for i = 1:length(possible_T)
        T = possible_T(i);
        a1 = col_vec(end-T+1:end);
        a2 = col_vec(end-2*T+1:end-T);
        delta = a1-a2;
        if norm(delta,Inf) < 10^-4
            return
        end
    end
end
```

```
T = 2.0000   p_min = 3.0000   p_max = 3.4490   w = 0.4490
T = 4.0000   p_min = 3.4495   p_max = 3.5435   w = 0.0940
T = 8.0000   p_min = 3.5440   p_max = 3.5640   w = 0.0200
T = 16.0000   p_min = 3.5645   p_max = 3.5685   w = 0.0040
T = 32.0000   p_min = 3.5690   p_max = 3.5695   w = 0.0005
    4.7766    4.7000    5.0000    8.0000
```

Published with MATLAB® R2020a