

Fourier-type integrals and steepest descent

Please hand in questions 1(a), 1(b) and 2(a) on Thursday 6 November 2025 at 12pm

In all of the below, the relevant asymptotic limit is $X \rightarrow \infty$.

1. Use the method of stationary phase to obtain the leading asymptotic behaviour of the following integrals

$$\begin{aligned} (a) \quad & \int_0^1 \tan(t) e^{iXt^4} dt . \\ (b) \quad & \int_{1/2}^2 (1+t) e^{iX(-t+t^{3/3})} dt . \\ (c) \quad & \int_0^\infty \frac{1}{1+t^2} e^{iXt} dt . \\ (d) \quad & \int_0^\infty e^{iX(2t-t^2)} \ln(1+t^2) dt . \\ (e) \quad & \int_0^\pi \sin(X \cos(t)) e^{-t^2} dt . \end{aligned}$$

2. Use the method of steepest descent to find the leading asymptotic behaviour of the following integrals

$$\begin{aligned} (a) \quad & \int_{-1}^\infty e^{X(t+it-t^2/2)} dt . \\ (b) \quad & \int_{-\infty}^\infty \frac{te^{iX(t^5/5+t)}}{1+t^2} dt . \\ (c) \quad & \int_0^\infty e^{iX(t^4/4+t^3/3)} e^{-t} dt . \\ (d) \quad & \int_0^1 \frac{e^{iXt^2}}{t^2-t+5/16} dt . \\ (e) \quad & \int_{-1}^\infty \sqrt{1+t} \cos(Xt^2) e^{X(t-t^3/3)} dt . \end{aligned}$$

3. Find the leading asymptotic behaviour of the Gamma function $\Gamma(X)$ using the integral representation

$$\frac{1}{\Gamma(X)} = \frac{1}{2\pi i} \int_\gamma e^t t^{-X} dt$$

where the path γ starts at $-\infty$ below the branch cut along the negative x-axis, encircles the origin and then goes off to $-\infty$ above the branch cut.

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