Asymptotics 2025/2026 Sheet 3[1]

Laplace-type integrals

Please hand in questions 2(d), 2(e), 3 and 4 Thursday 16 October 2025 at 12pm

In all of the below, the relevant asymptotic limit is $X \to \infty$.

1. Use integration by parts to obtain the first two terms in the asymptotic expansion of

$$I(X) = \int_1^\infty e^{-X(t^2+1)}\,dt.$$

2. Obtain the leading order asymptotic behaviour of the following integrals

   (a) $\int_X^\infty e^{-t^3}\,dt$.

   (b) $\int_3^6 e^{-Xt^2}\sqrt{1+t^2}\,dt$.

   (c) $\int_0^{\pi/2} e^{X(\sin t+\cos t)}\sqrt{t}\,dt$.

   (d) $\int_0^\infty e^{X(2t-t^2)}\log(1+t^2)\,dt$.

   (e) $\int_{-1}^1 e^{-X(\cosh t+1)}e^t\,dt$.

3. Use Watson's lemma to find an infinite asymptotic expansion of

$$I(X) = \int_1^\infty e^{-X(t^2+1)}\,dt\ .$$

4. Use Watson's lemma to find an infinite asymptotic expansion of

$$I(X) = \int_0^\pi e^{-Xt}t^{-1/3}\cos t\,dt\ .$$

5. Show that

$$\int_0^\infty \left(1+\frac{u}{X}\right)^{-X} e^{-u}\,du \sim \frac{1}{2}+\frac{1}{8X}-\frac{1}{32X^2}\ .$$