Asymptotics 2025/2026 Sheet 2[1]

Local approximation to linear ODEs

Please hand in questions 1(a) and 2(a) on Thursday 10 February 2025 at 12pm

1. *Find the leading behaviours as $x \to 0^+$ of the following equations*

   (a) $x^4 y''' = y$

   (b) $y'' = (\cot x)^4 y$ *(hint:* $\cot x \sim 1/x - x/3 + \ldots$ *as* $x \to 0$*)*

   (c) $x^4 y''' - 3x^2 y' + 2y = 0$

   (d) $y'' = \sqrt{x} y$

   (e) $x^5 y''' - 2xy' + y = 0$

2. *Find the leading behaviours as $x \to +\infty$ of the following equations*

   (a) $xy''' = y'$

   (b) $y'' = \sqrt{x} y$