# Asymptotics 2025/2026 Sheet 4[1]

**First assessed homework**

---

*Hand in all the questions by 23 October 2025 at 12pm*

1. Obtain the leading order asymptotic of the following integrals as $X \to \infty$,

   (a) **(5 marks)** $\int_X^\infty e^{-t^3}\,dt$.

   (b) **(5 marks)** $\int_3^6 e^{-Xt^2}\sqrt{1+t^2}\,dt$.

   (c) **(5 marks)** $\int_0^{\pi/2} e^{X(\sin t+\cos t)}\sqrt{t}\,dt$.

2. **(10 marks)** Compute the two-term expansions as $\epsilon \to 0$ of the solutions of the equation

$$(1-\epsilon)x^3 + (\epsilon-3)x^2 + 3x - 1 = 0.$$

3. Consider the differential equation

$$\frac{d^2y}{dx^2} - \left(1+\frac{1}{x}\right)y = 0$$

   (a) **(5 marks)**
   Show that it has an irregular singular point at $x = \infty$.

   (b) **(10 marks)**
   Compute the two linearly independent solutions at leading order as $x \to \infty$.