Methods of Applied Mathematics - Part 1 [SEMT30006]

Exercise Sheet 0 – Revision

These are basic exercises you should be able to do, so give them a quick look through, anything you can't do you'll need to revise from previous years.

1. **Matrices**
   Find the eigenvalues of the following matrices.
   (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
   (you should be able to spot one of the eigenvalues just by looking at it; hint: what is the rank of this matrix?)
   (b) $\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ (you should be able to spot both eigenvalues just by looking at it)

2. **Complex numbers**
   (a) Expand $(1+2i)\,\mathrm{e}^{2it} + (1-2i)\,\mathrm{e}^{-2it}$ in terms of sin and cos
   (b) Find the solutions of $u^3 = 2$

3. **Ordinary Differential Equations**
   Solve the following ordinary differential equations (ODEs).
   (a) $\frac{dx}{dt} = \frac{1}{2}(1-x)$
   (b) $\frac{d^2x}{dt^2} + \frac{dx}{dt} + 4x = 0$
   (c) $\frac{dx}{dt} = -3x + y$, $\frac{dy}{dt} = x - 3y$ (hint: use the trial solution of $x = \mathrm{e}^{\lambda t}$, $y = a\,\mathrm{e}^{\lambda t}$ and the principle of linear superposition)

4. **Taylors series**
   Estimate the value of $\sin(0.1)$ *by hand*. How quickly does the Taylors series expansion approach the actual value?