

Laplace-type integrals

Please hand in questions 2(d), 2(e), 3 and 4 Thursday 16 October 2025 at 12pm

In all of the below, the relevant asymptotic limit is $X \rightarrow \infty$.

1. Use integration by parts to obtain the first two terms in the asymptotic expansion of

$$I(X) = \int_1^\infty e^{-X(t^2+1)} dt.$$

2. Obtain the leading order asymptotic behaviour of the following integrals

$$\begin{aligned} (a) \quad & \int_X^\infty e^{-t^3} dt. \\ (b) \quad & \int_3^6 e^{-Xt^2} \sqrt{1+t^2} dt. \\ (c) \quad & \int_0^{\pi/2} e^{X(\sin t + \cos t)} \sqrt{t} dt. \\ (d) \quad & \int_0^\infty e^{X(2t-t^2)} \log(1+t^2) dt. \\ (e) \quad & \int_{-1}^1 e^{-X(\cosh t+1)} e^t dt. \end{aligned}$$

3. Use Watson's lemma to find an infinite asymptotic expansion of

$$I(X) = \int_1^\infty e^{-X(t^2+1)} dt .$$

4. Use Watson's lemma to find an infinite asymptotic expansion of

$$I(X) = \int_0^\pi e^{-Xt} t^{-1/3} \cos t dt .$$

5. Show that

$$\int_0^\infty \left(1 + \frac{u}{X}\right)^{-X} e^{-u} du \sim \frac{1}{2} + \frac{1}{8X} - \frac{1}{32X^2} .$$

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