

First assessed homework

Hand in all the questions by 23 October 2025 at 12pm

1. Obtain the leading order asymptotic of the following integrals as $X \rightarrow \infty$,

- (a) (5 marks) $\int_X^\infty e^{-t^3} dt.$
- (b) (5 marks) $\int_3^6 e^{-Xt^2} \sqrt{1+t^2} dt.$
- (c) (5 marks) $\int_0^{\pi/2} e^{X(\sin t + \cos t)} \sqrt{t} dt.$

2. (10 marks) Compute the two-term expansions as $\epsilon \rightarrow 0$ of the solutions of the equation

$$(1 - \epsilon)x^3 + (\epsilon - 3)x^2 + 3x - 1 = 0.$$

3. Consider the differential equation

$$\frac{d^2y}{dx^2} - \left(1 + \frac{1}{x}\right)y = 0$$

- (a) (5 marks)

Show that it has an irregular singular point at $x = \infty$.

- (b) (10 marks)

Compute the two linearly independent solutions at leading order as $x \rightarrow \infty$.

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