PARCIAL N° 3

Tema 1

Ejercicio N° 1

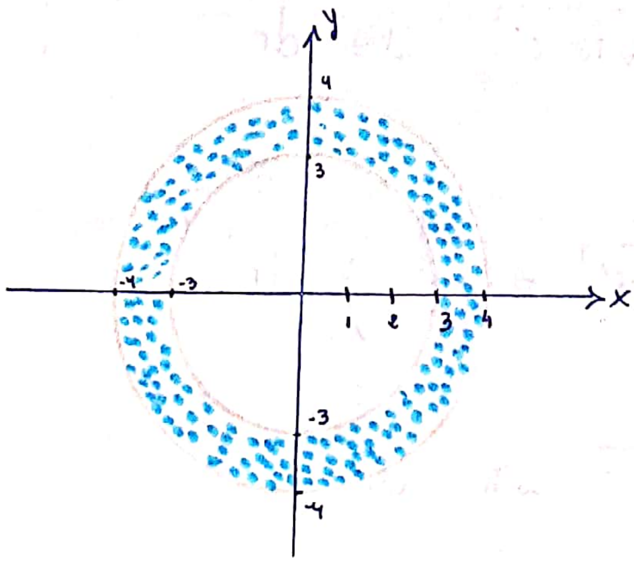
Analizamos la proyección del sólido al plano $\overrightarrow{xy}$.

$$\begin{cases} z^2 = 16 - x^2 - y^2 \\ z = 0 \end{cases} \Rightarrow \quad 0 = 16 - (x^2 + y^2)$$

$$x^2 + y^2 = 16.$$

$$\begin{cases} x^2 + y^2 = 9 \\ z = 0 \end{cases} \Rightarrow \quad x^2 + y^2 = 9.$$

Por lo tanto:



Entonces:

PARCIAL Nº 3

Tema 1

$3 < r < 4$

$0 < \theta < 2\pi$

$0 < z < \sqrt{16-(x^2+y^2)} \Rightarrow 0 < z < \sqrt{16-r^2}$

Luego:

$$V = \int_3^4 \int_0^{2\pi} \int_0^{\sqrt{16-r^2}} r \; dz \, d\theta \, dr$$

$$= \int_3^4 \int_0^{2\pi} \left(rz \Big|_0^{\sqrt{16-r^2}} \right) d\theta \, dr$$

$$= \int_3^4 \int_0^{2\pi} r \cdot \sqrt{16-r^2} \; d\theta \, dr$$

$$= \int_3^4 \left(r\sqrt{16-r^2} \cdot \theta \Big|_0^{2\pi} \right) . dr$$

$$= \int_3^4 r\sqrt{16-r^2} \cdot 2\pi \; dr$$

$$= 2\pi \int_3^4 r\sqrt{16-r^2} \; dr$$

Por sustitución, llamamos $u = 16 - r^2$

$$\frac{du}{dr} = -2r$$

$$du = -2r\, dr$$

$$-\frac{1}{2}\, du = r\, dr$$

Entonces

$$= 2\pi \int_a^b \sqrt{u} \cdot \left(-\frac{1}{2}\right) du$$

$$= \cancel{2}\pi \cdot \left(-\frac{1}{\cancel{2}}\right) \int_a^b \sqrt{u}\ du$$

$$= -\pi \cdot \left(\frac{2}{3} u^{3/2} \Big|_a^b \right)$$

$$= -\pi \cdot \left(\frac{2}{3} \cdot (16 - r^2)^{3/2} \Big|_3^4 \right)$$

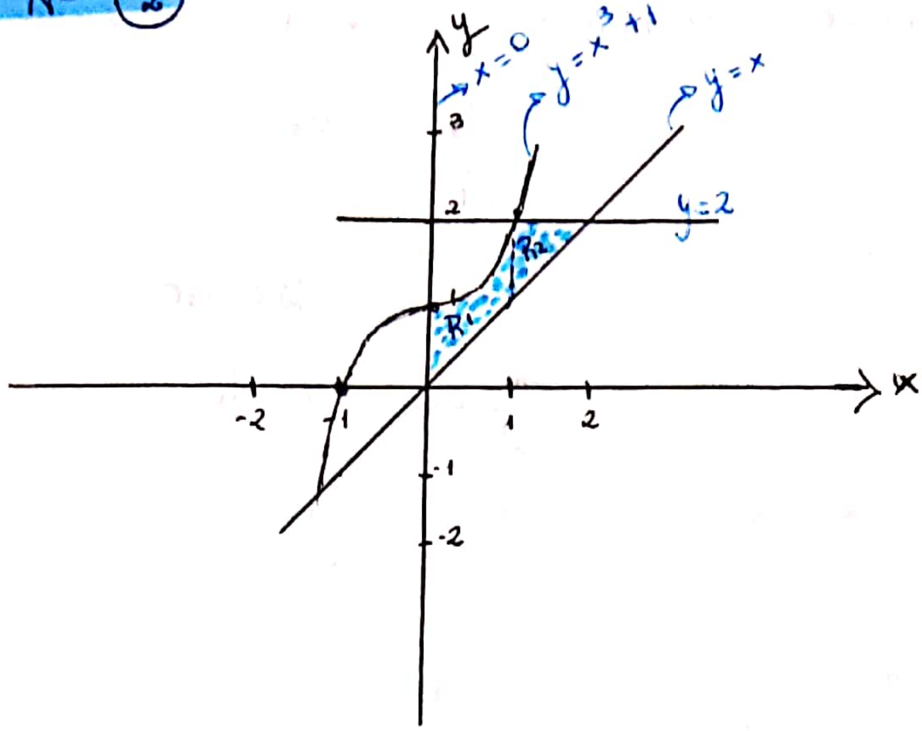
$$= -\pi \cdot \left(\frac{2}{3} (16 - 4^2)^{3/2} - \frac{2}{3} (16 - 3^2)^{3/2} \right)$$

$$= -\pi \cdot \left(0 - \frac{2}{3} \cdot 7\sqrt{7} \right)$$

$$= \boxed{\frac{14\sqrt{7}}{3} \pi \quad \text{u.v}} \approx 38{,}78.$$

Ejercicio N° 2

a)



b) $A(R) = A(R_1) + A(R_2)$

$$= \int_0^1 \int_x^{x^3+1} dy\, dx + \int_1^2 \int_x^2 dy\, dx$$

$$= \int_0^1 \left(y \Big|_x^{x^3+1} \right) dx + \int_1^2 \left(y \Big|_x^2 \right) dx$$

$$= \int_0^1 x^3 + 1 - x \; dx + \int_1^2 2 - x \; dx$$

$$= \left(\frac{1}{4}x^4 + x - \frac{1}{2}x^2 \Big|_0^1 \right) + \left(2x - \frac{1}{2}x^2 \Big|_1^2 \right)$$

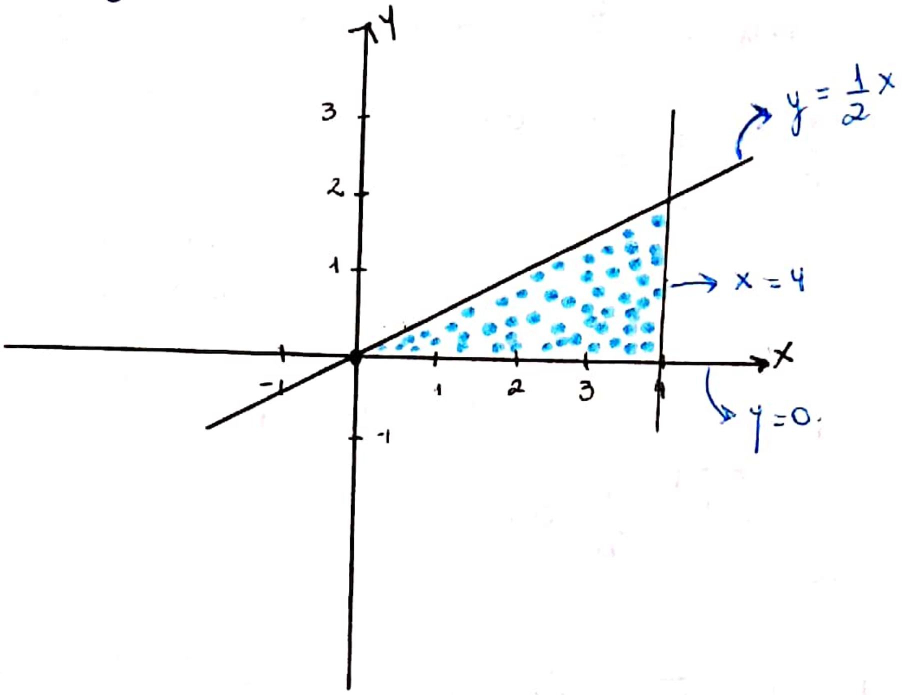
$$= \frac{1}{4}.1^4 + 1 - \frac{1}{2}1^2 - 0 + 2.2 - \frac{1}{2}.2^2 - \left(2.1 - \frac{1}{2}.1^2 \right)$$

$$= \frac{1}{4} + 1 - \cancel{\frac{1}{2}} + \cancel{4} - \cancel{2} - \cancel{2} + \cancel{\frac{1}{2}}$$

$$= \boxed{\frac{5}{4}}$$

Ejercicio N° 3

Graficamos la región.



$$A(S) = \int_0^4 \int_0^{\frac{1}{2}x} \sqrt{1 + (2x)^2 + 2^2}\; dy\, dx$$

$$= \int_0^4 \int_0^{\frac{1}{2}x} \sqrt{5 + 4x^2}\; dy\, dx$$

$$= \int_0^4 \left(\sqrt{5 + 4x^2} \cdot y \Big|_0^{\frac{1}{2}x} \right) dx$$

$$= \int_0^4 \sqrt{5 + 4x^2} \cdot \frac{1}{2}x \; dx$$

$$= \frac{1}{2} \cdot \int_0^4 \sqrt{5 + 4x^2} \cdot x \; dx$$

Por sustitución, llamamos $u = 5 + 4x^2$

Entonces $\frac{du}{dx} = 8x$

$$du = 8x\ dx$$

$$\frac{1}{8} du = x\ dx$$

Luego:

$$= \frac{1}{2} \int_a^b \sqrt{u}\ \frac{1}{8} du$$

$$= \frac{1}{2} \cdot \frac{1}{8} \int_a^b \sqrt{u}\ du$$

$$= \frac{1}{16} \int_a^b \sqrt{u}\ du$$

$$= \frac{1}{16} \cdot \left(\frac{2}{3} u^{3/2} \Big|_a^b \right)$$

$$= \frac{1}{16} \cdot \left(\frac{2}{3} (5 + 4x^2)^{3/2} \Big|_0^4 \right)$$

$$= \frac{1}{16} \left(\frac{2}{3} \cdot (5 + 4 . 4^2)^{3/2} - \frac{2}{3} \cdot (5 + 4 . 0^2)^{3/2} \right)$$

$$= \frac{1}{16} \cdot \left(\frac{2}{3} . 573,15 - \frac{2}{3} . 11,18 \right)$$

$$\approx \boxed{23,41}$$

Ejercicio 4

a)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{2\pi}\cdot\left(\int_{-\pi}^{0} 0\, dx + \int_{0}^{\pi} -1\, dx\right)$$

$$= \frac{1}{2\pi}\left(0 + \left(-x\Big|_{0}^{\pi}\right)\right)$$

$$= \frac{1}{2\cancel{\pi}}\cdot(-\cancel{\pi})$$

$$= \boxed{-\frac{1}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos(nx)\, dx$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} 0\cdot\cos(nx)\, dx + \int_{0}^{\pi} -1\cdot\cos(nx)\, dx\right)$$

$$= \frac{1}{\pi}\cdot\left(0 - \int_{0}^{\pi}\cos(nx)\, dx\right)$$

$$= \frac{1}{\pi}\cdot\left(-\left(\frac{\operatorname{sen}(nx)}{n}\Big|_{0}^{\pi}\right)\right)$$

$$= \frac{1}{\pi}\cdot\left(-\left(\frac{\operatorname{sen}(n\pi)}{n} - \frac{\operatorname{sen}(n.0)}{n}\right)\right)$$

$$= \frac{1}{\pi} \cdot \left(- \left(0 - 0 \right) \right)$$

$$= \boxed{0}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \operatorname{sen}(nx) \, dx$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} 0 \cdot \operatorname{sen}(nx) \, dx + \int_{0}^{\pi} -1 \cdot \operatorname{sen}(nx) \, dx \right)$$

$$= \frac{1}{\pi} \left(0 - \int_{0}^{\pi} \operatorname{sen}(nx) \, dx \right)$$

$$= \frac{1}{\pi} \left(- \left(- \frac{\cos(nx)}{n} \Big|_{0}^{\pi} \right) \right)$$

$$= \frac{1}{\pi} \cdot \left(- \left(- \frac{\cos(n\pi)}{n} + \frac{\cos(n.0)}{n} \right) \right)$$

$$= \frac{1}{\pi} \cdot \left(- \left(- \frac{(-1)^n}{n} + \frac{1}{n} \right) \right)$$

$$= \frac{1}{\pi} \cdot \left(- \frac{\left(1 - (-1)^n \right)}{n} \right)$$

$$= \boxed{\frac{(-1)^n - 1}{n\pi}}$$

Por lo tanto:

$$f(x) \approx -\frac{1}{2} + \sum_{n=1}^{\infty}\left(0 \cdot \cos(nx) + \frac{(-1)^n - 1}{n\pi}\operatorname{sen}(nx)\right)$$

$$\approx -\frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{(-1)^n - 1}{n\pi}\operatorname{sen}(nx)\right)$$

b

$$S_0 = -\frac{1}{2}$$

$$S_1 = -\frac{1}{2} + \frac{(-1)^1 - 1}{\pi}\operatorname{sen}(x)$$

$$= -\frac{1}{2} - \frac{2}{\pi}\operatorname{sen}(x)$$

$$S_2 = -\frac{1}{2} - \frac{2}{\pi}\operatorname{sen}(x) + \frac{(-1)^2 - 1}{2\pi}\operatorname{sen}(2x)$$

$$= -\frac{1}{2} - \frac{2}{\pi}\operatorname{sen}(x)$$

$$S_3 = -\frac{1}{2} - \frac{2}{\pi}\operatorname{sen}(x) + \frac{(-1)^3 - 1}{3\pi}\operatorname{sen}(3x)$$

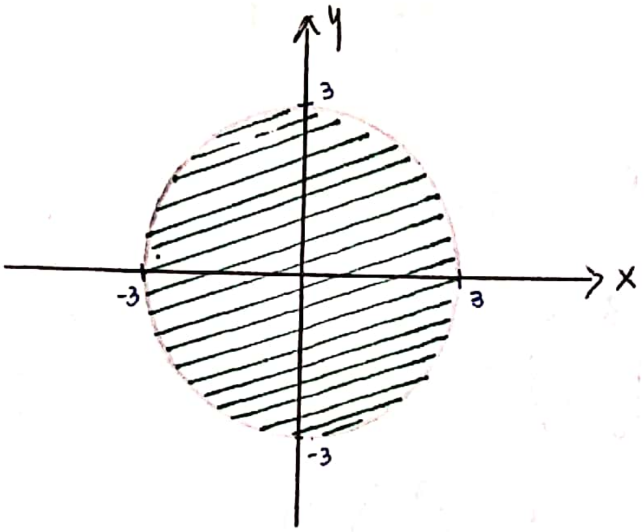
$$= -\frac{1}{2} - \frac{2}{\pi}\operatorname{sen}(x) - \frac{2}{3\pi}\operatorname{sen}(3x).$$

Tema 2

Ejercicio N° 1

Analizamos la proyección del sólido al plano $\vec{xy}$.

La cual está dada por $x^2 + y^2 = 9$.



Por lo tanto:

$$0 < r < 3$$

$$0 < \theta < 2\pi$$

$$0 < z < \sqrt{x^2 + y^2} \rightarrow 0 < z < \sqrt{r^2}$$

$$0 < z < r$$

Entonces:

$$V = \int_0^3 \int_0^{2\pi} \int_0^r r \, dz \, d\theta \, dr$$

$$= \int_0^3 \int_0^{2\pi} \left(rz \Big|_0^r \right) d\theta \, dr$$

$$= \int_0^3 \int_0^{2\pi} r^2 \, d\theta \, dr$$

$$= \int_0^3 \left(r^2 \theta \Big|_0^{2\pi} \right) dr$$

$$= \int_0^3 r^2 . 2\pi \, dr$$

$$= 2\pi \int_0^3 r^2 \, dr$$

$$= 2\pi \left(\frac{1}{3} r^3 \Big|_0^3 \right)$$

$$= 2\pi \left(\frac{1}{3} . 3^3 - \frac{1}{3} . 0^3 \right)$$

$$= 2\pi . 9$$

$$= \boxed{18 \pi \text{ uv}}$$

Ejercicio N° 2

a)

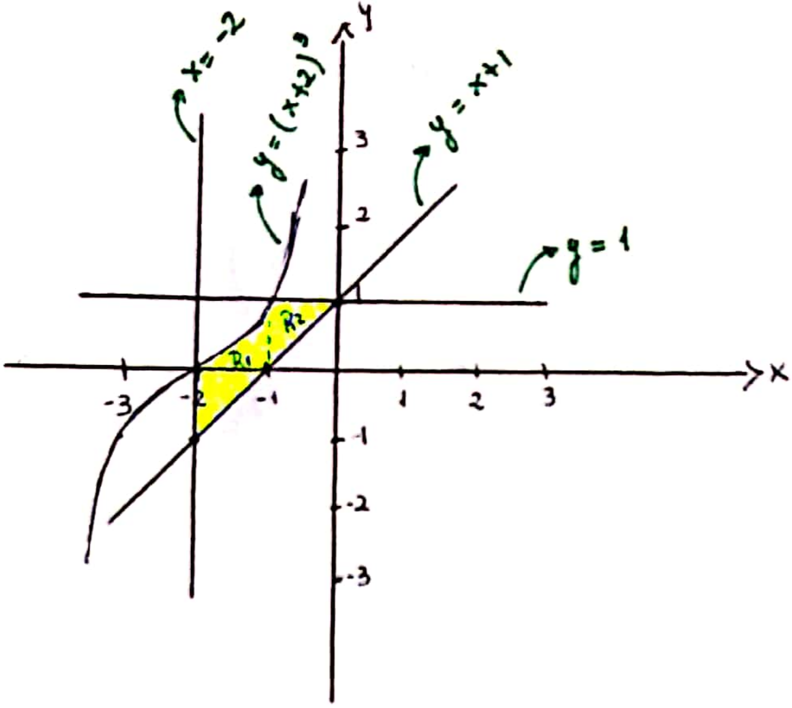
$$y = (x+2)^3$$

$$y = x+1$$

$$y = 1$$

$$x = -2.$$

7



(b) $A(R) = A(R_1) + A(R_2)$

$$= \int_{-2}^{-1}\int_{x+1}^{(x+2)^3} dy\,dx + \int_{-1}^{0}\int_{x+1}^{1} dy\,dx$$

$$= \int_{-2}^{-1}\left(y \Big|_{x+1}^{(x+2)^3} \right) dx + \int_{-1}^{0}\left(y \Big|_{x+1}^{1} \right) dx$$

$$= \int_{-2}^{-1} (x+2)^3 - (x+1)\, dx + \int_{-1}^{0} 1 - (x+1)\, dx$$

$$= \int_{-2}^{-1} (x^3 + 6x^2 + 12x + 8 - x - 1)\, dx + \int_{-1}^{0} -x\, dx$$

$$= \int_{-2}^{-1} x^3 + 6x^2 + 11x + 7\, dx - \int_{-1}^{0} x\, dx$$

$$= \left(\frac{1}{4}x^4 + 2x^3 + \frac{11}{2}x^2 + 7x \Big|_{-2}^{-1} \right) - \left(\frac{1}{2}x^2 \Big|_{-1}^{0} \right)$$

$$= \frac{1}{4}(-1)^4 + 2.(-1)^3 + \frac{11}{2}(-1)^2 + 7.(-1) - \left(\frac{1}{4}.(-2)^4 + \right.$$

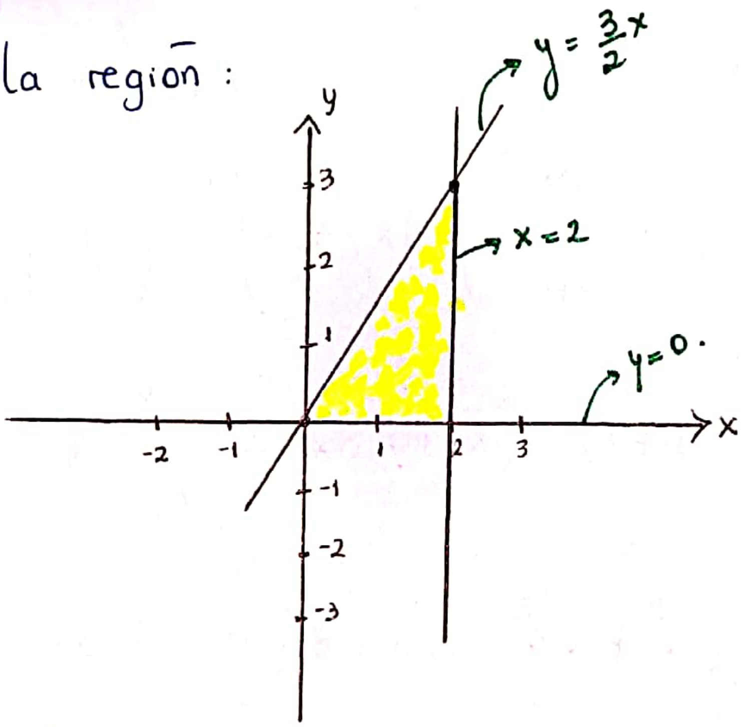
$$\left. 2.(-2)^3 + \frac{11}{2}.(-2)^2 + 7.(-2)\right) - \left(\frac{1}{2}.0^2 - \frac{1}{2}.(-1)^2\right)$$

$$= \frac{1}{4} - 2 + \frac{11}{2} - 7 - 4 + 16 - 22 + 14 + \frac{1}{2}$$

$$= \boxed{\frac{5}{4}}$$

Ejercicio N° 3

Graficamos la región :



$$A(S) = \int_0^2 \int_0^{\frac{3}{2}x} \sqrt{1 + (2x)^2 + 2^2}\; dy\, dx$$

$$= \int_0^2 \int_0^{\frac{3}{2}x} \sqrt{1 + 4x^2 + 4}\; dy\, dx$$

$$= \int_0^2 \int_0^{\frac{3}{2}x} \sqrt{5 + 4x^2}\; dy\, dx$$

$$= \int_0^2 \left(\sqrt{5+4x^2} \cdot y \Big|_0^{\frac{3}{2}x} \right) dx$$

$$= \int_0^2 \sqrt{5+4x^2} \cdot \frac{3}{2}x \, dx$$

$$= \frac{3}{2} \int_0^2 \sqrt{5+4x^2} \, x \, dx$$

Por sustitución, llamamos $u = 5 + 4x^2$.

Entonces: $\dfrac{du}{dx} = 8x$

$$du = 8x \, dx$$

$$\frac{1}{8} du = x \, dx$$

Por lo tanto:

$$= \frac{3}{2} \int_a^b \sqrt{u} \cdot \frac{1}{8} du$$

$$= \frac{3}{2} \cdot \frac{1}{8} \int_a^b \sqrt{u} \, du$$

$$= \frac{3}{16} \left(\frac{2}{3} u^{3/2} \Big|_a^b \right)$$

$$= \frac{3}{16} \cdot \left(\frac{2}{3} (5 + 4x^2)^{3/2} \Big|_0^2 \right)$$

$$= \frac{3}{16} \cdot \left(\frac{2}{3} \cdot \left(5 + 4 \cdot 2^2 \right)^{3/2} - \frac{2}{3} \cdot \left(5 + 4 \cdot 0^2 \right)^{3/2} \right)$$

$$= \frac{3}{16} \cdot \left(\frac{2}{3} \cdot 96{,}23 - \frac{2}{3} \cdot 11{,}18 \right)$$

$$\approx \boxed{10{,}63}$$

Ejercicio N° 4

a

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{2\pi} \left(\int_{-\pi}^{0} -1\, dx + \int_{0}^{\pi} 0\, dx \right)$$

$$= \frac{1}{2\pi} \left(-x \Big|_{-\pi}^{0} + 0 \right)$$

$$= \frac{1}{2\pi} \left(-0 - (-(-\pi)) \right)$$

$$= \frac{1}{2\pi} (-\pi)$$

$$= \boxed{-\frac{1}{2}}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(nx)\, dx$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} -1 \cdot \cos(nx)\, dx + \int_{0}^{\pi} 0 \cdot \cos(nx)\, dx \right)$$

$$= \frac{1}{\pi}\left(-\int_{-\pi}^{0} \cos(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left(-\left(\frac{\operatorname{sen}(nx)}{n}\Big|_{-\pi}^{0}\right)\right)$$

$$= \frac{1}{\pi}\left(-\left(\frac{\operatorname{sen}(0)}{n} - \frac{\operatorname{sen}(-n\pi)}{n}\right)\right)$$

$$= \frac{1}{\pi}\cdot\left(-(0-0)\right)$$

$$= \boxed{0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,.\,\operatorname{sen}(nx)\,dx$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} -1\,.\,\operatorname{sen}(nx)\,dx + \int_{0}^{\pi} 0\,.\,\operatorname{sen}(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left(-\int_{-\pi}^{0} \operatorname{sen}(nx)\,dx\right)$$

$$= -\frac{1}{\pi}\left(-\frac{\cos(nx)}{n}\Big|_{-\pi}^{0}\right)$$

$$= -\frac{1}{\pi}\left(-\frac{\cos(n.0)}{n} + \frac{\cos(-n\pi)}{n}\right)$$

$$= -\frac{1}{\pi}\left(-\frac{1}{n} + \frac{(-1)^n}{n}\right)$$

$$= -\frac{1}{\pi}\left(\frac{(-1)^n - 1}{n}\right)$$

$$= \boxed{\frac{1-(-1)^n}{n\pi}}$$

Por lo tanto:

$$f(x) \approx -\frac{1}{2} + \sum_{n=1}^{\infty} \left(0 \cdot \cos(nx) + \frac{1-(-1)^n}{n\pi} \operatorname{sen}(nx) \right)$$

$$\approx -\frac{1}{2} + \sum_{n=1}^{\infty} \left(\frac{1-(-1)^n}{n\pi} \operatorname{sen}(nx) \right)$$

(b) $S_0 = -\frac{1}{2}$

$$S_1 = -\frac{1}{2} + \frac{1-(-1)^1}{\pi} \operatorname{sen}(x)$$

$$= -\frac{1}{2} + \frac{2}{\pi} \operatorname{sen}(x)$$

$$S_2 = -\frac{1}{2} + \frac{2}{\pi} \operatorname{sen}(x) + \frac{1-(-1)^2}{2\pi} \operatorname{sen}(2x)$$

$$= -\frac{1}{2} + \frac{2}{\pi} \operatorname{sen}(x)$$

$$S_3 = -\frac{1}{2} + \frac{2}{\pi} \operatorname{sen}(x) + \frac{1-(-1)^3}{3\pi} \operatorname{sen}(3x)$$

$$= -\frac{1}{2} + \frac{2}{\pi} \operatorname{sen}(x) + \frac{2}{3\pi} \operatorname{sen}(3x).$$