UC SANTA CRUZ

Math 134: Cryptography

Lecture 3: substitution ciphers

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Ciphers so far:

Shift ciphers

- 1. Symmetric-key encryption.
- 2. Key is an integer k with $1 \le k \le 25$.
- 3. Shifts the alphabet by *k* modulo 26.

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Affine ciphers

- 1. Symmetric-key encryption.
- 2. Key is a pair of integers (α, β) where $1 \le \alpha \le 25$ is odd, $\alpha \ne 13$, and $0 \le \beta \le 25$.
- 3. Applies the affine transformation $x \mapsto \alpha \cdot x + \beta$ modulo 26 to the alphabet.

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- 2. To securely send a message $M=m_0m_1m_2m_3\ldots$, they first apply the permutation letterwise to get the ciphertext $C=f(m_0)f(m_1)f(m_2)\ldots$ Then, they send C.

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Example

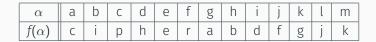
Using the permutation below, let's send the message **This is a message**.

α	a	b	С	d	е	f	g	h	i	j	k	l	m
$f(\alpha)$	С	i	р	h	е	r	a	b	d	f	g	j	k

 n	0	р	q	r	S	t	u	V	W	Х	У	Z
 l	m	n	0	q	S	t	u	V	W	Χ	У	Z

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Example (cont'd)

The plaintext This is a message converts to the ciphertext Tbds ds c kesscae.

Example

Using the same permutation, let's decrypt the message Pqyntmaqcnby ds gdlh mr pmmj.

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- We find that the α for which $f(\alpha) = p$ is $\alpha = c$.
- Next, f(r) = q.
- Next, f(y) = y.

Example

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Example (cont'd)

If we continue in this way, we should find that the plaintext message was **Cryptography** is kind of cool.

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 $26! = 403291461126605635584000000 \approx 4.03 \times 10^{26}$ many permutations.

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Question

So, are substitution ciphers secure?

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The answer is not always. With sufficiently many characters of ciphertext known, substitution ciphers become vulnerable to frequency attacks.

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Frequency attack

A frequency attack uses probability distributions innate to the language used to write plaintext messages in order to gain information about the message.

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Frequency attack

A frequency attack uses probability distributions innate to the language used to write plaintext messages in order to gain information about the message.

With enough information, frequency attacks can often yield entire messages.

Let's try to decrypt this message.

Ciphertext

Mitsfi ufvhdiafwhpy oe pswboly vwest ij kwdpskwdouwl dpsify wit uikhcdsf euosjus hfwudous; ufyhdigfwhpou wlgifodpke wfs tseogjst wficit uikhcdwdoijwl pwftjsee weeckhdoije, kwmoja ecup wlgifodpke pwft di vfswm oj wudcwl hfwudous vy wjy wtbsfewfy. Wpols od oe dpsifsdouwlly hieeovls di vfswm ojdi w asll-tseogjst eyedsk, od oe ojrsweovls oj wudcwl hfwudous di ti ei. Scup eupskse, or asll tseogist, wfs dpsfsrifs dsfkst "uikhcdwdoijwlly esucfs". Tpsifsdouwl wtbwjuse (s.g., okhfibsksjde oj ojdsgsf rwudifoxwdoij wlgifodpke) wit rwedsf uikhcdojg dsupjiligy fsgcofs dpses tseogie di vs uijdojcwlly fssbwlcwdst wjt, or jsuseewfy, wtwhdst. Ijrifkwdoij-dpsifsdouwlly esucfs eupskse dpwd hfibwyly uwijid vs vfimsj sbsj aodp cjlokodst uikhcdojg hiasf, ecup we dps ijs-doks hwt, wfs kcup kifs torroucld di ces oj hfwudous dpwj dps vsed dpsifsdouwlly vfswmwvls vcd uikhcdwdoijwlly esucfs eupskse.

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The character that appears most often in the above text is **s**. Since notation is also preserved, we see **s.q.** in the line **wtbwjuse** (**s.q.**, **okhfibsksjde** oj ojdsqsf **rwudifoxwdoij**.

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The letters \mathbf{d} and \mathbf{w} show up the next most often, with roughly the same frequency.

The most common adjacent pair of plaintext letters in English tends to be **th**. The most common triple of adjacent plaintext English letters tends to be **the**.

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Since dp shows up very frequently and dps shows up very frequently, we can most likely conclude d is the ciphertext corresponding to t and p is the ciphertext corresponding to h.

Continuing in this way (i.e. trial and error), we will probably recover the message:

Plaintext

Modern cryptography is heavily based on mathematical theory and computer science practice; cryptographic algorithms are designed around computational hardness assumptions, making such algorithms hard to break in actual practice by any adversary. While it is theoretically possible to break into a well-designed system, it is infeasible in actual practice to do so. Such schemes, if well designed, are therefore termed "computationally secure". Theoretical advances (e.g., improvements in integer factorization algorithms) and faster computing technology require these designs to be continually reevaluated and, if necessary, adapted. Information-theoretically secure schemes that provably cannot be broken even with unlimited computing power, such as the one-time pad, are much more difficult to use in practice than the best theoretically breakable but computationally secure schemes.