UC SANTA CRUZ

Math 134: Cryptography

Lecture 1

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University of California, Santa Cruz

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Syllabus

Course Overview

Structure: Lecture (MWF) and Discussion (T)

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Objectives:

1. learn how to apply concepts from number theory to cryptography,
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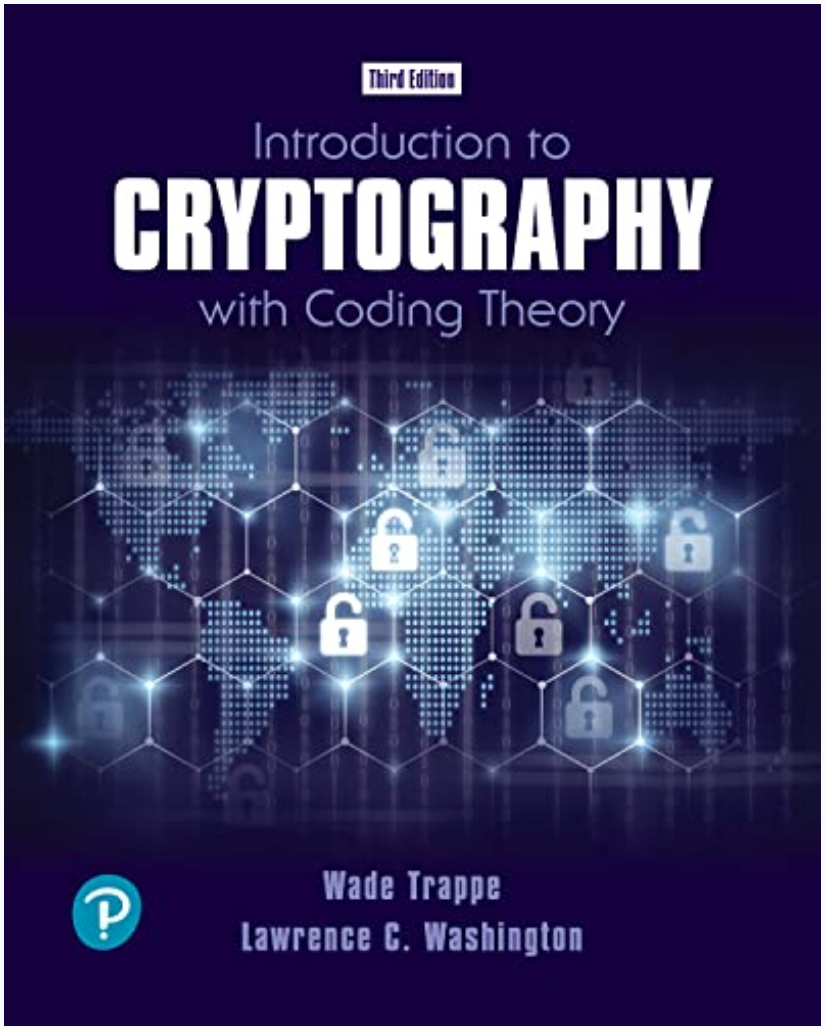
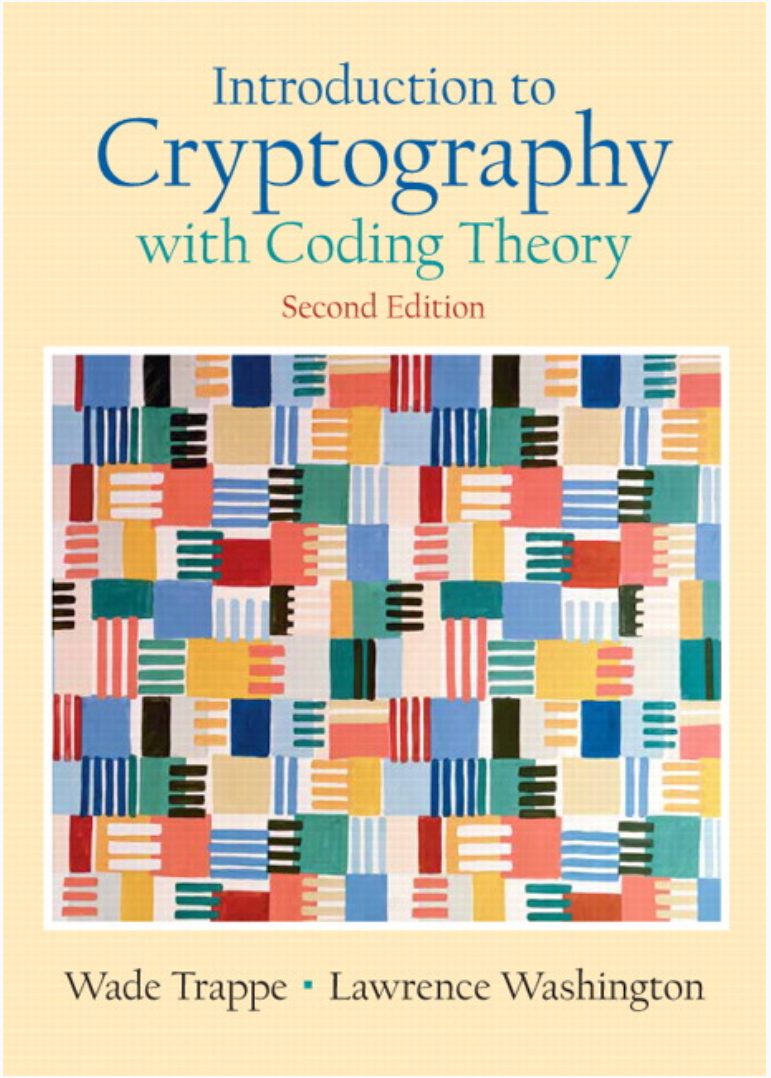
What do you get out of the course?

1. Abstractly: as much as you put in
2. Concretely: IBM cryptography certification [optional]

Textbook(s)


Third Edition
Introduction to
CRYPTOGRAPHY
with Coding Theory
Wade Trappe
Lawrence C. Washington


Introduction to
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5. **Final Project(1): 20%**
   Report on chosen topic from syllabus list.

Cryptography

Ciphers (set-up)

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- A *cipher* is a function which converts *plaintext* to *ciphertext*. E.g. a shift cipher on the english alphabet (shifting by 2) applied to the message above yields `Vjku ku c oguucig.`

Shift ciphers

Alice and Bob are sending messages using the English alphabet. They have the following idea:

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- Send the new ciphertext.

Shift ciphers

Example

We can implement a shift cipher with $k = 2$ by filling out the table below.

Plaintext	a	b	c	d	e	f	g	$\cdots$	x	y	z
j	0	1	2	3	4	5	6	$\cdots$	23	24	25
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Check

a goes to *c*, *e* goes to *g*, *m* goes to *o*, etc.

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The ciphertext message `Gcvrjv ivru kyv sffb` is decoded to recover the plaintext message `Please read the book`.

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- In our example, the space of all possible keys for the algorithm was the set of integers $1 \leq k \leq 25$. This is called the *keyspace*.

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- **Chosen plaintext**: If Eve is allowed to choose plaintext that will be converted to ciphertext, then any character can be sent to give the key.
- **Chosen ciphertext**: If Eve can choose one piece of ciphertext to decode, then the negative of the result can be used to deduce the encryption key.