Lab Report 1: The Apollo Missions

I. Introduction:

This report discusses the physics and computational implementation of gravitational potential and force fields in the Earth-Moon system, along with the trajectory projection of the Saturn V Stage 1 rocket. The primary focus is on visualizing gravitational interactions and computing the velocity and altitude of a launched module. The analysis involves using Python for numerical calculations and plotting, with key libraries such as NumPy, Matplotlib, and SciPy.

II. The gravitational potential of the Earth-Moon system

The gravitational potential at a point due to a massive body is given by:

$$\Phi = -\frac{GM}{r}$$

where G is the gravitational constant, M is the mass of the body, and r is the radial distance from the body's center.

In this model, the function phi(G, M, xM, yM, x, y) computes the gravitational potential at a given point (x,y). The function is then used to evaluate the potential across a range of distances from Earth to the Moon, as well as in a 2D meshgrid representing space around Earth. A contour plot provides the 2D representation of the gravitational influence of the Earth.

For visualization, two plots were generated using Matplotlib:

1. A 1D plot of gravitational potential vs. distance from Earth to the Moon.

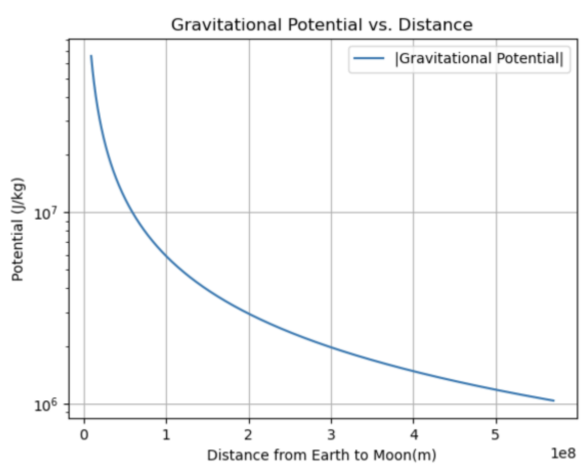

Figure 1. Variation of Gravitational Potential Magnitude of the Earth to Moon distance

2. A 2D color mesh plot of gravitational potential in the plane, representing the field in space.

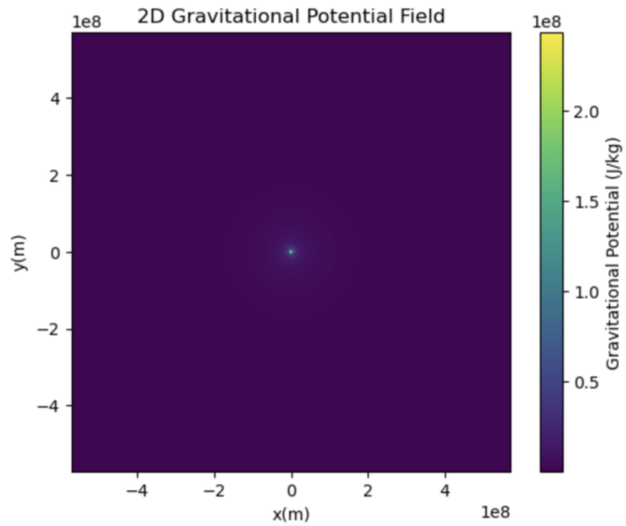

Figure 2. Two Dimensional Gravitational Potential Field of Earth

III. The gravitational force of the Earth-Moon system

The gravitational force exerted by a body of mass M on an object of mass m at a distance r is given by Newton's law of gravitation:

$$F = \frac{GMm}{r^2}$$

To simulate this, the Earth and the Moon were positioned at (0,0) and $(\frac{d}{\sqrt{2}}, \frac{d}{\sqrt{2}})$, where d is the distance between the Earth and the Moon. By taking the summation of the contribution of both the Earth and the Moon, the net gravitational potential was computed. The logarithmic color scale was applied using the colormesh() function and values were normalized using LogNorm() because the gravitational forces vary by several orders of magnitude. Gravitational force was calculated and visualized as a vector field by using equations for the force components in x and y as $Fx = F\frac{r_x}{r}$, and $Fy = F\frac{r_y}{r}$. Using NumPy, the meshgrid was created to compute the force vectors at multiple points in space. This was represented as$X, Y = np.meshgrid(x, y)$. Then Matplotlib was used to generate the streamplot, and the values for the force field magnitude were displayed in log scale to show the variations in the strength of the force in Newtons. For further visibility, the Earth and Moon are marked as blue and grey respectively.

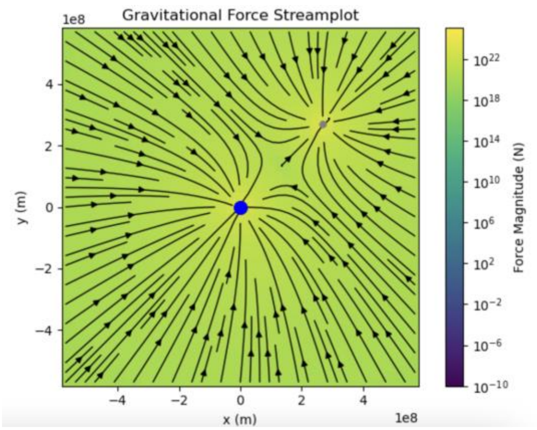

Figure 3. A streamplot of the gravitational forces between the Earth and the Moon

IV. Projected performance of the Saturn V Stage 1

The velocity change of a rocket as a function of time can be described by Tsiolkovsky's rocket equation:

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

Where v_e is the exhaust velocity, m_0 is the initial mass, and $m(t) = m_0 - m_{dot}t$ represents mass depletion over time, and g accounts for gravitational acceleration. Altitude at burnout can be calculated by integrating velocity over time: $h = \int_0^T \Delta v(t)dt$, where $T = \frac{m_0 - m_f}{m}$ is the total burn time. These calculations were performed using the quad function of the scipy.integrate module in Python.

V. Discussion and Further Work

The calculated results of the simulation yielded a burn time of 126.92 seconds and an altitude of 64,546.80 meters. Compared to the test values, the model underestimates the burn time and altitude, likely stemming from assumptions that disregard atmospheric drag, and a constant gravitational acceleration.

These limitations leave room for improvements in future work, where more detailed physical models would be more accurate and applicable to modern space mission plans.