10.6 VELOCITY RATIO OF SIMPLE GEAR DRIVE

The velocity ratio of a gear drive is defined as the ratio of the angular velocity of the driver gear to the driven gear or follower. Consider two gears A and B which are in mesh as shown in Fig 10.9. The driving gear A rotates with an angular velocity ω_1 rad/s in anti clockwise direction and the driven gear B rotates with an angular velocity ω_2 rad/s in clockwise direction.

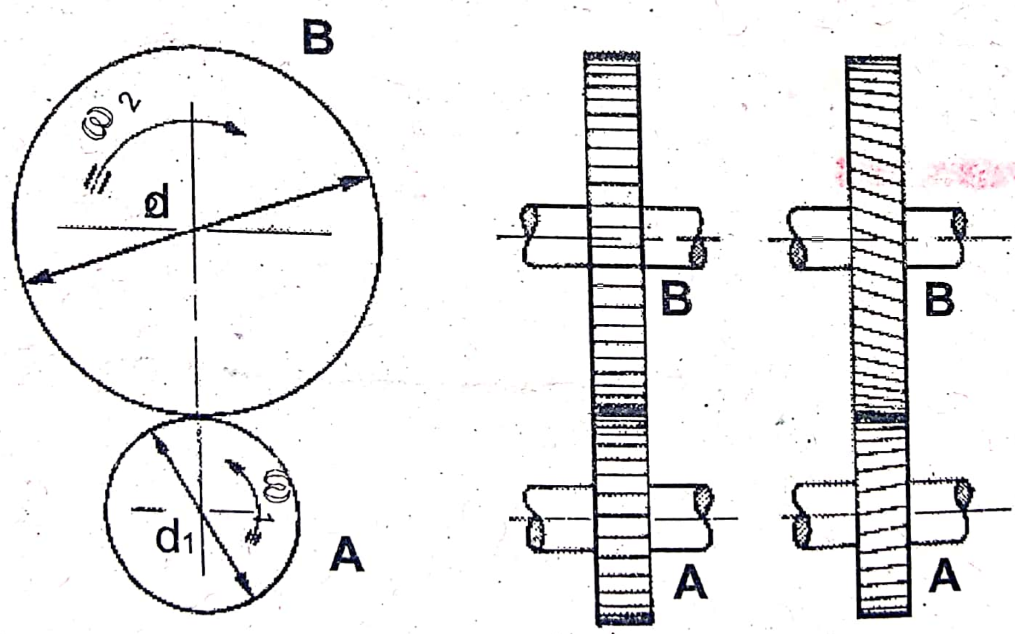

Fig. 10.9 Simple gear drive

Scanned with CamScanner

Let

d_1 = Pitch circle diameter of gear A in mm.

d_2 = Pitch circle diameter of gear B in mm.

T_1 = Number of teeths on driver gear A .

T_2 = Number of teeths on driven gear B.

$$\boxed{\text{Velocity ratio} = \frac{\text{Angular velocity of driver}}{\text{Angular velocity of driven or follower}}}$$

$$\textbf{VR} = \frac{\omega_1}{\omega_2} \quad \text{...(10.8a)}$$

$$= \frac{2\pi N_1}{2\pi N_2} \quad (\because \omega = 2\pi N)$$

$$\textbf{VR} = \frac{N_1}{N_2} \quad \text{...(10.8b)}$$

Since the gear drive is positive, the peripheral velocity of driven gear will be equal to the peripheral velocity of the driver gear, Therefore, we have

$$\omega_1 \frac{d_1}{2} = \omega_2 \frac{d_2}{2} \quad (\because v = \omega r)$$

$$\frac{\omega_1}{\omega_2} = \frac{d_2}{d_1}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{d_2}{d_1} \quad \text{................(i)}$$

Since the circular pitch of each gear must be same for correct meshing,
Therefore, we have

$$p_{c1} = p_{c2}$$

$$\frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2}$$

$$\therefore \quad \frac{d_2}{d_1} = \frac{T_2}{T_1} \qquad \text{..............(ii)}$$

Using above equations (i) and (ii), we have

$$\text{Velocity ratio, } \boxed{VR = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}} \qquad \text{...(10.8c)}$$

The gear with smaller number of teeth is called *pinion* and the other with larger number of teeth is called *wheel*. The ratio of larger number of teeth to smaller number of teeth is called *gear ratio* .

Example 10.1 : A toothed wheel has 108 teeth. Its module is

Example 10.3 : A gear wheel of 48 teeth has a pitch circle diameter of 367 mm . Find the module, diametral pitch and the circular pitch.

Solution :

Given :

Number of teeth, $T = 48$

Pitch circle diameter, $d = 367$ mm .

Analysis :

Using the relation for module of the gear wheel.

$$m = \frac{d}{T} = \frac{367}{48}$$

$$= 7.65 \text{ mm/tooth} \quad \text{or simply } 7.65 \text{ mm}$$

Using the relation for diametral pitch of the gear wheel

$$p_d = \frac{T}{d} = \frac{48}{367} = 0.13 \text{ tooth/mm}$$

Using the relation for circular pitch of the gear wheel

$$p_c = \frac{\pi d}{T} = \frac{\pi \times 367}{48} = 24.02 \text{ mm}$$

Result :

The module, diametral pitch and the circular pitch of the gear wheel are **7.65 mm** , **0.13 tooth/mm** and **24.02 mm** respectively.

Example 10.5 : A spur gear having 30 number of teeth rotates at 200 rpm . What will be its circular pitch and the pitch line velocity if it has a module of 2 mm ? (***April 2003***)

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Solution :

Given:

Number of teeth on spur gear, $T = 30$

Speed of spur gear, $N = 200$ rpm

Module of gear, $m = 2$ mm

***Analysis* :**

Using the relation for circular pitch

$$p_c = \pi m \quad = \pi \times 2 \quad = 6.28 \text{ mm}$$

Again ,Circular pitch, $p_c = \dfrac{\pi d}{T}$

Or, Diameter of pitch circle, $d = \dfrac{p_c T}{\pi} = \dfrac{6.28 \times 30}{\pi}$

$$= 59.97 \text{ mm}$$

Using the relation for peripheral speed or pitch line velocity of gear drive. i.e.,

$$\boxed{\text{Pitch line velocity, } v = \omega r = 2\pi N \times \frac{d}{2} \quad \text{or} \quad \pi d N}$$

$$= \pi \times 59.97 \times 200 \quad = 37680.26 \text{ mm / min}$$

$$= \frac{37680.26 \times 10^{-3}}{60} \quad \boxed{= 0.63 \text{ m/s}}$$

***Result* :**

The circular pitch and pitch line velocity of spur gear having **6.28 mm** and **0.63 m/s** respectively.

Example 10.12 : The axes of two parallel shafts are approximately 0.6 m apart. The motion is transmitted with the help of spur gears having module 8 mm. One shaft is to rotate 3 times as fast as the other. Find the number of teeth on the gear wheels and also the exact centre distance (***March 2009, November 2000***)

Solution :

Given :

Distance between axes of parallel shafts, $C = 0.6 \text{ m}$

$= 0.6 \times 10^3 \text{ mm}$

Module of the gears, $m = 8 \text{ mm}$

Speed relation of gears, $N_2 = 3N_1$

Analysis :

Using the relation for velocity ratio

$$\frac{d_2}{d_1} = \frac{N_1}{N_2} = \frac{N_1}{3N_1} = \frac{1}{3}$$

Or $$d_1 = 3d_2 \qquad \ldots\ldots\ldots\ldots\ldots (i)$$

Using the expression for centre distance between axes of shafts in gear drive.

$$C = \frac{d_1 + d_2}{2}$$

Or, $d_1 + d_2 = 2C = 2 \times 0.6 \times 10^3 = 1200$ ....(ii)

From equation (i) and (ii)

Pitch circle diameter of first gear, $d_1 = 900$ mm

Pitch circle diameter of second gear, $d_2 = 300$ mm

Now using relation for module of meshing gears

$$m = \frac{d_1}{T_1} = \frac{d_2}{T_2}$$

or, Number of teeth on first gear, $T_1 = \frac{d_1}{m} = \frac{900}{8} = 112.5$

Number of teeth on second gear, $T_2 = \frac{d_2}{m} = \frac{300}{8}$

$$= 37.5$$

Since the number of teeth on both the gears are to be incomplete numbers, therefore let us make the number of teeth on the second wheel as 38. Therefore, for a velocity ratio of 3, the number of teeth on the first gear should be $3 \times 38 = 114$

Now the exact diameter of first gear,

$$d_1' = mT_1 = 8 \times 114 = 912 \text{ mm}$$

Similarly,

$$d_2' = mT_2 = 8 \times 38 = 304 \text{ mm}$$

∴ The exact centre distance, $C' = \frac{d_1' + d_2'}{2} = \frac{912 + 304}{2}$

$$= 608 \text{ mm}$$

Result **:**

Number of teeth on the first and second gears must be **114**

Example 10.15 : A spur wheel of 300 mm pitch circle diameter and rotating at 210 rpm drives a machine using 3.5 kW. Neglecting friction calculate the tangential effort exerted at the teeth of the spur wheel.

***Solution*:**

Given :

Pitch circle diameter of wheel, $d = 300 \text{ mm} = 300 \times 10^{-3}$ m

Speed of wheel, $N = 210$ rpm

Power transmitted, $P = 3.5 \text{ kW} = 3.5 \times 10^{3}$ W

***Analysis*:**

Using the general equation for power transmitted

$$\boxed{P = \frac{2\pi NT}{60} = \frac{2\pi N}{60} \times F \times \frac{d}{2} \quad \ldots\ldots\left(\because T = F \times \frac{d}{2}\right)}$$

Or, Tangential effort, $F = \dfrac{P \times 60 \times 2}{2\pi N \times d} = \dfrac{3.5 \times 10^{3} \times 60 \times 2}{2 \times \pi \times 210 \times 300 \times 10^{-3}}$

$= 1061.03$ N

Example 10.17 : A set of spur wheels for the gearing of a machine are arranged as follows : *A* drives *B*, *C* drives *D* and *E* drives *F*. Gears *B* and C and gears *D* and *E* are the compounds wheels. When $T_A = 20, T_B = 50, T_C = 25, T_D = 75, T_E = 26$ and $T_F = 65$ teeth. If gear *A* rotates in clockwise direction at 975 rpm. Find the speed and direction of rotation of follower gear *F*.

Solution :

Given :

Teeth of driver gears, $T_A = 20, T_C = 25$ and $T_E = 26$.

Teeth on driven gears, $T_B = 50, T_D = 75$ and $T_F = 65$

Speed of first driver gear, $N_A = 975$ rpm(Clock wise)

Analysis :

The arrangement of gear wheels are shown in *Fig* 10.14

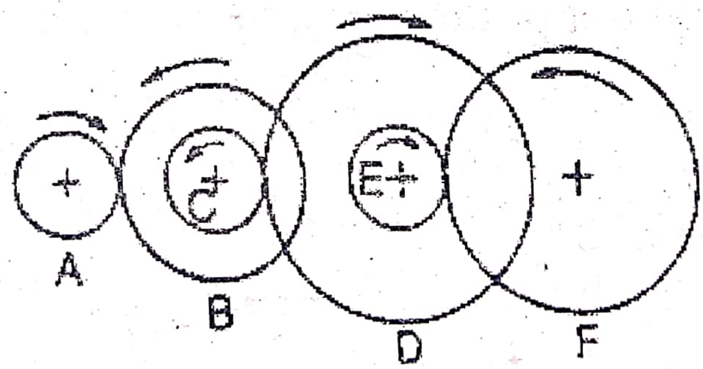

Fig 10.14 Arrangement of gear wheels

Using the relation , $\dfrac{N_F}{N_A} = \dfrac{T_A T_C T_E}{T_B T_D T_F}$

Or, Speed of follower or last driven, $N_F = \dfrac{T_A T_C T_E}{T_B T_D T_F} \times N_A$

$$= \frac{20 \times 25 \times 26}{50 \times 75 \times 65} \times 975 = 52 \text{ rpm}$$

Since the number of compound gears in the train is even (or the total number of shafts), therefore the first driver and the last driven are rotates in opposite directions.

$\therefore$ Direction of rotation of follower gear in *Anti-clock wise*

Result :

The speed of follower gear *F* is **52 rpm** and its rotation is in ***anti clockwise direction***.