# CSIE 2344, Spring 2019 — Homework

Due April 1 (Monday) at Noon

There are 60 points in total. Points will be deducted if no appropriate intermediate step is provided.

When you submit your homework on Gradescope, please select the corresponding page(s) of each problem.

## 1 Minterm and Maxterm Expansions (6pts)

A combinational circuit has three inputs $(A, B, C)$ and two outputs $(X, Y)$. $XY$ represents a binary number whose value equals the number of 1's at the input. For example, if $ABC = 101$, then $XY = 10$. Find the minterm and maxterm expansions for $X$ and $Y$, respectively.

**Answer:** Construct the following truth table:

| $A$ | $B$ | $C$ | $X$ | $Y$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$X = A'BC + AB'C + ABC' + ABC = m_3 + m_5 + m_6 + m_7 = \sum m(3, 5, 6, 7) = \prod M(0, 1, 2, 4)$.

$Y = A'B'C + A'BC' + AB'C' + ABC = m_1 + m_2 + m_4 + m_7 = \sum m(1, 2, 4, 7) = \prod M(0, 3, 5, 6)$.

## 2 BCD Multiplication (12pts)

A combinational circuit has four inputs $(A, B, C, D)$, which represent a binary-coded-decimal (BCD) digit. The circuit has two groups of four outputs—$S, T, U, V$ and $W, X, Y, Z$. Each group represents a BCD digit. The output digits represent a decimal number which is five times the input number. For example, if $ABCD = 0111$, then the outputs are 0011 0101. Assume that invalid BCD digits do not occur as inputs.

1. Construct the truth table.

2. Write down the minimum expressions for the outputs by inspection of the truth table.

**Answer:**

1. Construct the following truth table, where the rows 1010 through 1111 have don't case outputs:

| $A$ | $B$ | $C$ | $D$ | | $S$ | $T$ | $U$ | $V$ | $W$ | $X$ | $Y$ | $Z$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $0 \times 5 = 00$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | $1 \times 5 = 05$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | $2 \times 5 = 10$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | $3 \times 5 = 15$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | $4 \times 5 = 20$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | $5 \times 5 = 25$ | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | $6 \times 5 = 30$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | $7 \times 5 = 35$ | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | $8 \times 5 = 40$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | $9 \times 5 = 45$ | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |

2. By inspection of the truth table, $S = 0$, $T = A$, $U = B$, $V = C$, $W = 0$, $X = D$, $Y = 0$, $Z = D$.

## 3 Karnaugh Maps — Part 1 (6pts)

Use Karnaugh maps to find a minimum sum-of-products expressions for $F(A, B, C) = \sum m(1, 4, 6) + \sum d(0, 2, 7)$.

**Answer:** Using the Karnaugh map in Figure 1, the minimum sum-of-products expression is $F = C' + A'B'$.

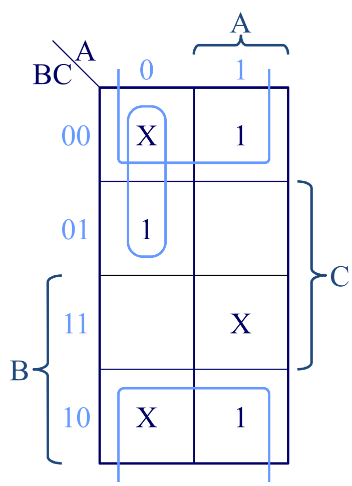


Figure 1: The Karnaugh map.

## 4 Karnaugh Maps — Part 2 (12pts)

1. Find a minimum sum-of-products expression for $F(A, B, C, D) = \prod M(0, 2, 10, 11, 12, 14, 15) \cdot \prod D(5, 7)$.
2. Find a minimum product-of-sums expression for $F(A, B, C, D) = \prod M(0, 2, 10, 11, 12, 14, 15) \cdot \prod D(5, 7)$.

**Answer:**

1. Using the left Karnaugh map in Figure 2, the minimum sum-of-products expression is $F = A'B + A'D + C'D + AB'C'$.

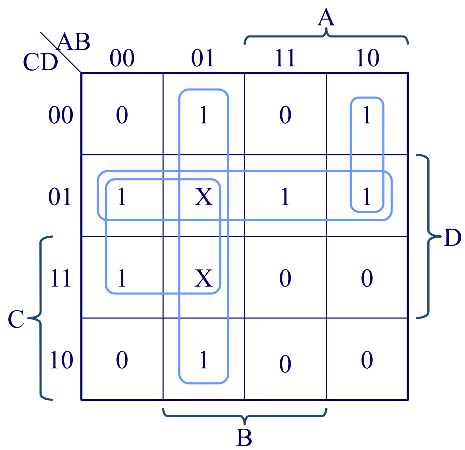


Figure 2: The Karnaugh maps.

2. Using the right Karnaugh map in Figure 2, the minimum sum-of-products expression for $F'$ is $AC + ABD' + A'B'D'$, and thus the minimum product-of-sums expression for $F$ is $(A' + C')(A' + B' + D)(A + B + D)$.

# 5 Karnaugh Maps — Part 3 (6pts)

A logic circuit realizes the function $F(A, B, C, D) = A'B' + A'CD + AC'D + AB'D'$. Assuming that $A = C$ never occurs when $B = D = 1$, find a simplified expression for $F$.

**Answer:** $A = C$ never occurs when $B = D = 1$, implying that $ABCD = 0101$ and $1111$ never occur. Using the Karnaugh map in Figure 3, the minimum sum-of-products expression is $F = A'D + B'D' + C'D$.

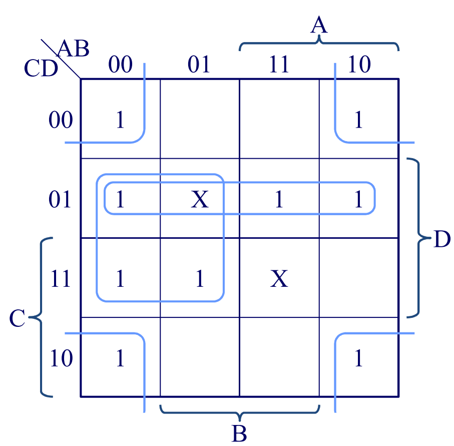


Figure 3: The Karnaugh map.

# 6 Two-Level Gate Circuits (12pts)

Find eight different minimum two-level gate circuits (draw them) to realize $F = A'BC' + AB'C' + BC'D$. You do not need to show the conversions between the eight forms, but you need to explain why they are minimum.

**Answer:** Using the left Karnaugh map in Figure 4, we can verify that $A'BC' + AB'C' + BC'D$ is the minimum sum-of-products expression. As a result, the corresponding two-level gate circuits are in Figure 5(a)–(d). Using the right Karnaugh map in Figure 4, the minimum product-of-sums expression is $F = C'(A+B)(A'+B'+D)$. As a result, the corresponding two-level gate circuits are in Figure 5(e)–(h).

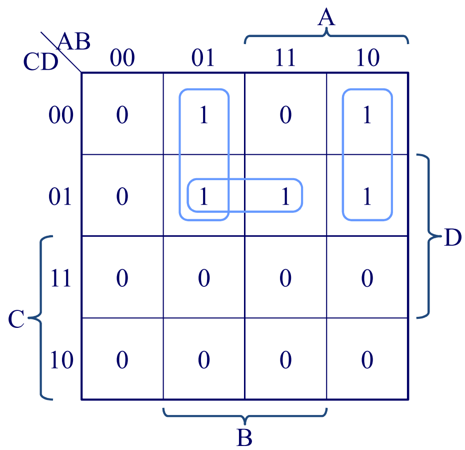


Figure 4: The Karnaugh maps.

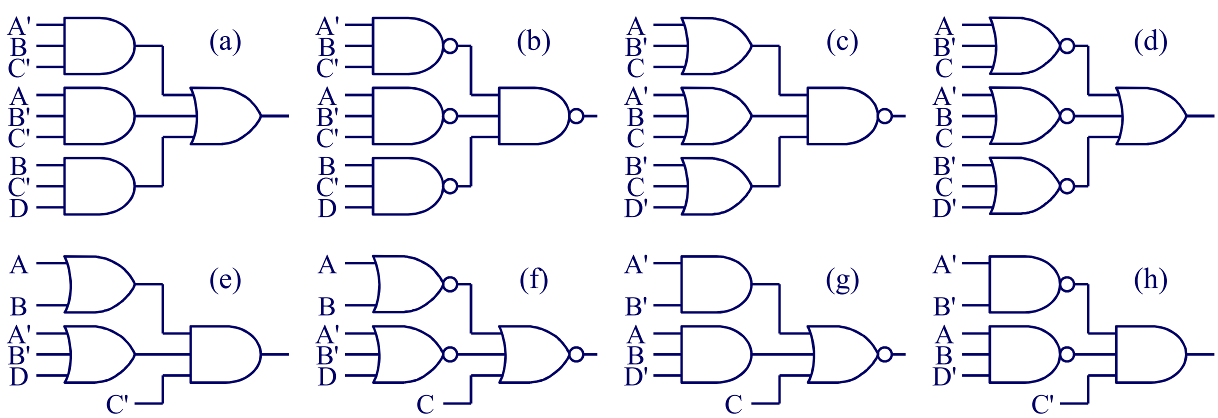


Figure 5: The two-level gate circuits.

## 7 Multi-Level Gate Circuits (6pts)

Using AND and OR gates, find a circuit (draw it) to realize $F(A,B,C,D) = \prod M(0,1,3,13,14,15)$. Try to minimize the number gates and gate inputs (fewer gates and gate inputs, more points). How many gates are there? How many gate inputs are there?

**Answer:** Using the Karnaugh map in Figure 6, the minimum sum-of-products expression for $F'$ is $A'B'C' + A'B'D + ABD + ABC$, and thus the minimum product-of-sums expression for $F$ is $(A+B+C)(A+B+D')(A'+B'+D')(A'+B'+C')$. Consider multi-level gate circuits, $F$ is also $(A+B+CD')(A'+B'+C'D')$, which has 5 gates and 12 gate inputs, as shown in Figure 7.

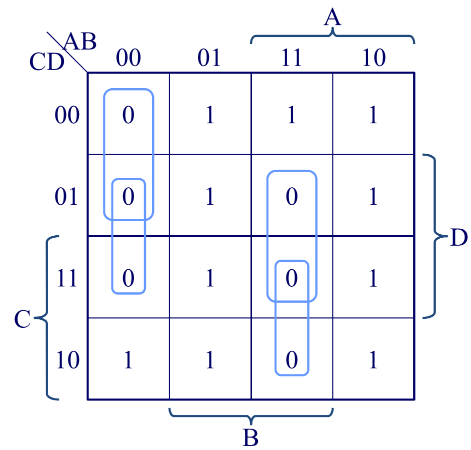


Figure 6: The Karnaugh map.

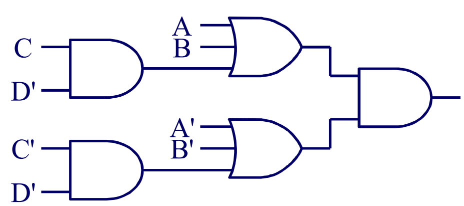


Figure 7: The three-level gate circuit.