Digital Systems Design and Laboratory [6. Quine-McCluskey Method]

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Spring 2019

Two-Level Logic Minimization

- ☐ Karnaugh map vs. Quine-McCluskey method
 - > Same input
 - Minterm expansion
 - > Same output
 - A minimum SOP
 - > Same high-level procedure
 - Find all Prime Implicants (PIs)
 - Find a minimum SOP

Finding All Prime Implicants (1/5)

- ☐ Start with all minterms
- ☐ Group pairs of adjacent minterms
 - > Mark and remove all covered terms
- Repeat grouping until no more grouping possible

CD AE	00	01	11	10
00	1 m ₀	m_4	m ₁₂	1 m ₈
01	1 m ₁	1 m ₅	m ₁₃	1 m ₉
11	m_{3}	1 m ₇	m ₁₅	m ₁₁
10	1 m ₂	1 m ₆	1 m ₁₄	1 m ₁₀

#1s	Minterm	Binary
0	0	0000
	1	0001
1	2	0010
	8	1000
	5	0101
2	6	0110
Z	9	1001
	10	1010
3	7	0111
	14	1110

Finding All Prime Implicants (2/5)

- ☐ Start with all minterms
- ☐ Group pairs of adjacent minterms
 - Mark and remove all covered terms
- ☐ Repeat grouping until no more grouping possible

CDAE	3 00	01	11	10
00	1 m ₀	${\sf m_4}$	m ₁₂	1 m ₈
01	1 m ₁	1 m ₅	m ₁₃	1 m ₉
11	m_{3}	1 m ₇	m ₁₅	m ₁₁
10	1 m ₂	1 m ₆	1 m ₁₄	1 m ₁₀

-				
#1s	Minterm	Size 1	Minterm	Size 2
0	0	0000 -	→ 0, 1	000-
	1	0001 /	0, 2	00-0
1	2	0010	0, 8	-000
	8	1000	1, 5	0-01
	5	0101	1, 9	-001
2	6	0110	2, 6	0-10
2	9	1001	2, 10	-010
	10	1010	8, 9	100-
3	7	0111	8, 10	10-0
	14	1110	5, 7	01-1
		_	6, 7	011-
			6, 14	-110
			10,14	1-10

Finding All Prime Implicants (3/5)

- ☐ Start with all minterms
- ☐ Group pairs of adjacent minterms
 - **→** Mark and remove all covered terms
- ☐ Repeat grouping until no more grouping possible

CDAE	00	01	11	10
00	1 m ₀	m_4	m ₁₂	1 m ₈
01	1 m₁	1 m ₅	m ₁₃	1 m ₉
11	m ₃	1 m ₇	m ₁₅	m ₁₁
10	1 m ₂	1 m ₆	1 m ₁₄	1 m ₁₀

#1s	Minterm	Size 1
0	0	0000
	1	0001
1	2	0010
	8	1000
	5	0101
2	6	0110
Z	9	1001
	10	1010
3	7	0111
	14	1110
	·	

Minterm	Size 2
0, 1	000-
0, 2	00-0
0,8	-000
1, 5	0-01
1, 9	-001
2, 6	0-10
2, 10	-010
8, 9	100-
8, 10	10-0
5, 7	01-1
6, 7	011-
6, 14	-110
10,14	1-10

Finding All Prime Implicants (4/5)

- ☐ Start with all minterms
- ☐ Group pairs of adjacent minterms
 - > Mark and remove all covered terms
- ☐ Repeat grouping until no more grouping possible

CD AB	00	01	11	10
00	1			1
	\overline{m}_0	m_4	m ₁₂	m ₈
01	1 m₁	1 m ₅	m ₁₃	1 m ₉
11	m ₃	1 m ₇	m ₁₅	m ₁₁
10	1 m ₂	1 m ₆	1 m ₁₄	1 m ₁₀

#1s	Minterm	Size 1
0	0	0000
	1	0001
1	2	0010
	8	1000
	5	0101
2	6	0110
2	9	1001
	10	1010
2	7	0111
3	14	1110

Minte	rm Size 2
0, 1	000-
0, 2	00-0
0, 8	-000
1, 5	0-01
1, 9	-001
2, 6	0-10
2, 10	0 -010
8, 9	100-
8, 10	0 10-0
5, 7	01-1
6, 7	011-
6, 14	4 -110
10, 1	4 1-10

Minterm	Size 4
0,1,8,9	-00-
0,2,8,10	-0-0
0,8,1,9	-00-
0,8,2,10	-0-0
2,6,10,14	10
2,10,6,14	10
0,8,1,9 0,8,2,10 2,6,10,14	-00- -0-0 10

Finding All Prime Implicants (5/5)

- ☐ Start with all minterms
- ☐ Group pairs of adjacent minterms
 - Mark and remove all covered terms
- ☐ Repeat grouping until no more grouping possible

CDAE	00	01	11	10
00	1			1
	m_0	m_4	m ₁₂	m ₈
01	1 m	1 m	m	1 m
	m ₁	m ₅	m ₁₃	m ₉
11		1		
	m_3	m ₇	m ₁₅	m ₁₁
10	1	1	1	1
	m_2	m_6	m ₁₄	m ₁₀

#1s	Minterm	Size 1
0	0	0000
	1	0001
1	2	0010
	8	1000
	5	0101
2	6	0110
2	9	1001
	10	1010
3	7	0111
<u> </u>	14	1110

Minterm	Size 2
0, 1	000-
0, 2	00-0
0, 8	-000
1, 5	0-01
1, 9	-001
2, 6	0-10
2, 10	-010
8, 9	100-
8, 10	10-0
5, 7	01-1
6, 7	011-
6, 14	-110
10, 14	1-10

Minterm	Size 4
0,1,8,9	-00-
0,2,8,10	-0-0
0,8,1,9	-00-
0,8,2,10	-0-0
2,6,10,14	10
2,10,6,14	10

Comparing with Karnaugh Map

☐ Try to find all prime implicants directly in the Karnaugh map

00	01	11	10
1			1
m_0	m ₄	m ₁₂	m ₈
1 m₁	1 m _s	m ₁₂	1 m ₉
-	1		m ₁₁
1 m ₂	1 m ₆	1 m ₁₄	1 m ₁₀
	1 m ₀ 1 m ₁	1 m ₀ m ₄ 1 1 m ₅ m ₁ 1 m ₇ 1 1	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

#1s	Minterm	Size 1
0	0	0000
	1	0001
1	2	0010
	8	1000
	5	0101
2	6	0110
Z	9	1001
	10	1010
3	7	0111
3	14	1110

Minterm	Size 2
0, 1	000-
0, 2	00-0
0, 8	-000
1, 5	0-01
1, 9	-001
2, 6	0-10
2, 10	-010
8, 9	100-
8, 10	10-0
5, 7	01-1
6, 7	011-
6, 14	-110
10, 14	1-10

Minterm	Size 4
0,1,8,9	-00-
0,2,8,10	-0-0
0,8,1,9	-00-
0,8,2,10	-0-0-
2,6,10,14	10
2,10,6,14	10

Finding a Minimum SOP (1/4)

- **□** Build the prime implicant chart (table)
- ☐ Simplify the table using essential
 - > Select an essential, delete covered minterms, and repeat
- Optimally select PIs by column covering

CD AE	3 00	01	11	10
00	1 m _o	${\sf m_4}$	m ₁₂	1 m ₈
01	1 m₁	1 m ₅	m ₁₃	1 m ₉
11	m_3	1 m ₇	m ₁₅	m ₁₁
10	1 m ₂	1 m ₆	1 m ₁₄	1 m ₁₀

					•	V 1 1 1 1 C		,			
PIs		0	1	2	5	6	7	8	9	10	14
0,1,8,9	-00-	Х	X					X	Х		
0,2,8,10	-0-0	Х		X				X		X	
2,6,10,14	10			X		X				X	Χ
1,5	0-01		X		X						
5,7	01-1				X		X				
6,7	011-					X	X				

Minterms

Finding a Minimum SOP (2/4)

- ☐ Build the prime implicant chart (table)
- ☐ Simplify the table using essential prime implicants
 - > <u>Select essential prime implicants</u> and delete covered minterms
- Optimally select PIs by column covering

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	CD AE	B 00	01	11	10
01 1 1 m ₅ m ₁₃ 11 1	00		m	m	1 m
11 1	01	1	1		1 m
1113 1117 11115	11		1		m ₉
10 1 1 1 m ₂ m ₆ m ₁₄	10	1	1	1	1 m ₁₀

						νιιιι	emi	5			
Pls		0	1	2	5	6	7	8	9	10	14
0,1,8,9	-00-	Х	Х					Х	Х		
0,2,8,10	-0-0	Х		Х				Χ		Χ	
2,6,10,14	10			Χ		Х				Χ	Х
1,5	0-01		Х		Χ						
5,7	01-1				X		X				
6,7	011-					X	X				

Mintormo

Finding a Minimum SOP (3/4)

- ☐ Build the prime implicant chart (table)
- ☐ Simplify the table using essential prime implicants
 - > Select essential prime implicants and delete covered minterms
- Optimally select PIs by column covering

CD AE	3 00	01	11	10
00	1 m ₀	${\sf m_4}$	m ₁₂	1 m ₈
01	1 m ₁	1 m ₅	m ₁₃	1 m ₉
11	m_{3}	1 m ₇	m ₁₅	m ₁₁
10	1 m ₂	1 m ₆	1 m ₁₄	1 m ₁₀

		iviliteiiiis										
Pls		0	1	2	5	6	7	8	9	10	14	
0,1,8,9	-00-	Х	Х					Х	Х			
0,2,8,10	-0-0	Х		Х				Х		Х		
2,6,10,14	10			Х		Х				Х	Ж	
1,5	0-01		ж		X							
5,7	01-1				X		X					
6,7	011-					\star	X					

Minterms

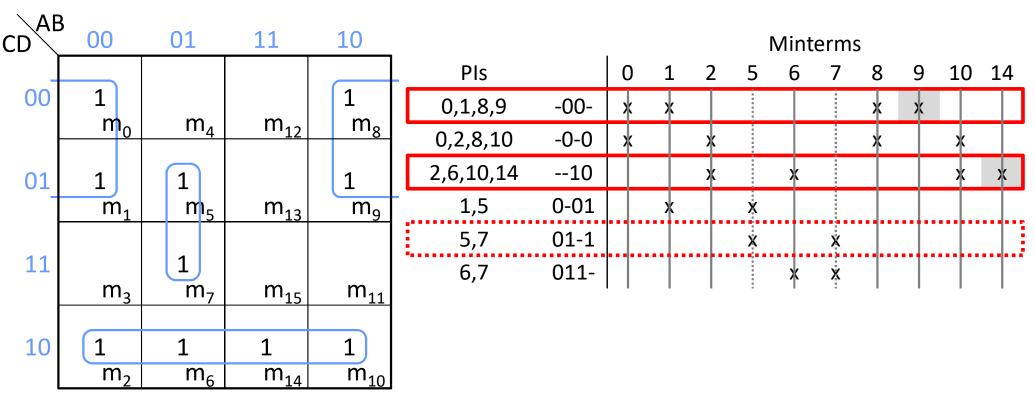
Finding a Minimum SOP (4/4)

- ☐ Build the prime implicant chart (table)
- ☐ Simplify the table using essential prime implicants
 - > Select essential prime implicants and delete covered minterms
- Optimally select PIs by column covering

CDAB	00	01	11	10						ſ	Mint	erms				
					Pls		0	1	2	5	6	7	8	9	10	14
00	1	m	100	1	0,1,8,9	-00-	×	ж					ж	Х		
	m_0	$m_0 m_4$	m ₁₂	m ₈	0,2,8,10	-0-0	ж		Х				Х		Х	\Box
01	1	1 1 1	1	2,6,10,14	10			Х		Х				Х	ж	
	\overline{m}_{1}	m_5	m ₁₃	m_{9}^{-}	1,5	0-01		ж		X						
					5,7	01-1				Х		X				
11	m	1	100		6,7	011-			[ж	X			[
_	m ₃	m ₇	m ₁₅	m ₁₁												
10	1	1	1	1												
	m_2	m_{6}	m ₁₄	m ₁₀												

Comparing with Karnaugh Map

☐ Try to find a minimum SOP directly in the Karnaugh map



Difficulty and Summary

- Column covering is hard
 - > NP-complete
 - Consider a Boolean expression with n variables, in general
 - ~2ⁿ minterms
 - ~3ⁿ/n prime implicants
 - The proof is optional: https://core.ac.uk/download/pdf/82016049.pdf

■ Summary

- Karnaugh map
 - Exact and effective as # of variables ≤ 5
- Quine-McCluskey method
 - Exact and Realizable for more variables
- > Espresso
 - Heuristic and faster than the Quine-McCluskey method

More Than Logic Design

- ☐ Are Integrated Circuit (IC) designers still doing this?
 - ➤ Maybe not...
 - > Existing semiconductor intellectual property (IP) core
 - ➤ Electronic Design Automation (EDA) tools
- ☐ How to decompose a complicated system to components which you can implement easily?

Q&A