. 7

 $(1).4 \leftarrow$ 

(0).8

 $\frac{x}{(1).6}$ 

Χ

(1).2

x 2

 $(0).4 \leftarrow$ 

 $.7_{10} = .1 \ 0110 \ 0110 \dots$ 

1D.  $X \cdot 1 = X$ 

 $2D. X \cdot 0 = 0$ 

3D.  $X \cdot X = X$ 

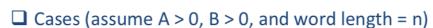
5D.  $X \cdot X' = 0$ 

6D. XY = YX

7D. (XY)Z = X(YZ) = XYZ

8D. X + YZ = (X + Y)(X + Z)

Repeating



- $\triangleright$  Case 1: A + B and  $|A + B| < 2^{n-1} \rightarrow$  Correct
- ➤ Case 2: A + B and  $|A + B| \ge 2^{n-1}$  → Wrong (overflow)
- ➤ Case 3: A B and A < B
- → Correct
- $\triangleright$  Case 4: -A + B and  $A \le B$
- → Correct (ignore the carry)
- ➤ Case 5: -A B and  $|A + B| \le 2^{n-1}$  → Correct (ignore the carry)
- $\triangleright$  Case 6: -A-B and  $|A+B| > 2^{n-1} \rightarrow$  Wrong (overflow)

Examples

➤ Q1: {AND, OR, NOT}

➤ A1: Of course, yes!

> A2: Yes (check figure)

> A3: Yes (check figure)

• XY = (XY)" = (X' + Y')"

• X + Y = (X + Y)'' = (X'Y')'

➤ Q2: {AND, NOT}

➤ Q3: {OR, NOT}

➤ Q4: {AND}

➤ Q5: {OR}

# Uniting

- > XY + XY' = X
- $\rightarrow$  (X + Y)(X + Y') = X

#### TABLE 2-3 Op

Laws of Boolean Algebra

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- Operations with 0 and 1:
- 1. X + 0 = X2. X + 1 = 1

Idempotent laws:

3. X + X = X

Involution law:

4. (X')' = X

Laws of complementarity:

5. X + X' = 1

Commutative laws: 6. X + Y = Y + X

Associative laws:

7. (X + Y) + Z = X + (Y + Z)= X + Y + Z

Distributive laws:

8. X(Y+Z) = XY + XZ

DeMorgan's laws: 9. (X + Y)' = X'Y'

1. X(Y + Z) = XY + XZ

2. (X + Y)(X + Z) = X + YZ

3. (X + Y)(X' + Z) = XZ + X''

9D. (XY)' = X' + Y'

# ☐ Elimination

Absorption

> X + XY = X

> X (X + Y) = X

- > X + X'Y = X + Y
- > X (X' + Y) = XY
- Consensus
  - $\triangleright$  XY + X'Z + YZ = XY + X'Z
  - (X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)

## ☐ From product-of-sums (POS) to sum-of-products (SOP)

## ■ Example

$$(\underline{A + B} + C')(\underline{A + B} + D)(A + B + E)(\underline{A} + D' + E)(\underline{A'} + C)$$
  
2.

- = (A + B + C'D)(A + B + E)[AC + A'(D' + E)]2.
- = (A + B + C'DE)(AC + A'D' + A'E)
- = AC + ABC + A'BD' + A'BE + A'C'DE
- = AC + A'BD' + A'BE + A'C'DE

#### Example

$$(A + B + C')(A + B + D')(B + C + D')$$

- $= (A + B + \underline{C'})(A + B + D')(B + \underline{C} + D')$
- = (A + B + C')(B + C + D')
- = ... can be further simplified
- = B + AC + C'D'

# Use the distributive laws to multiply out an expression to

## obtain a sum-of-products (SOP) form

- ➤ Ordinary distributive law: X(Y + Z) = XY + XZ
- $\triangleright$  Second distributive law: X + YZ = (X + Y)(X + Z)

### ☐ Example: multiply out (A+BC)(A+D+E)

- > Use the ordinary distributive law
  - (A + BC)(A + D + E) = A + AD + AE + ABC + BCD + BCE = A(1 + D + E + BC) + BCD + BCE = A + BCD + BCE
- Use the second distributive law

# ☐ Example (another ordering)

- $A'\underline{C'}D + A'BD + B\underline{C}D + ABC + ACD'$
- = A'C'D + BCD + ABC + ACD'
- $= A'C'D + BC\underline{D} + ABC + AC\underline{D'}$
- = A'C'D + BCD + ACD'

• (A + BC)(A + D + E) = A + BC(D + E) = A + BCD + BCE