Digital Systems Design and Laboratory
[8. Combinational Circuit Design]

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Spring 2019

We would like to thank Prof. Hui-Ru Jiang (NTU) for contributing the lecture material

Outline

- **Review of Combinational Circuit Design**
- Design of Circuits with Limited Gate Fan-In
- Gate Delays and Timing Diagrams
- Hazards in Combinational Logic
- Simulation and Testing of Logic Circuits

Combinational Circuit Design

- Generic combinational circuit design steps
  - Translate the word description into a switching function (Unit 4)
  - Simplify the function
    - Boolean algebra (Units 2 and 3)
    - Karnaugh map (Unit 5)
    - Quine-McCluskey (Unit 6)
    - Other methods
  - Realize it using available logic gates (Unit 7)
    - Hardware cost = (#terms, #literals)
    - Start from minimum SOP ≡ AND-OR → NAND-NAND/OR-NAND/NOR-OR
    - Start from minimum POS ≡ OR-AND → NOR-NOR/AND-NOR/NAND-AND
    - # of levels = maximum # of cascaded gates between I/Os
      - Change level by factoring or multiplying out

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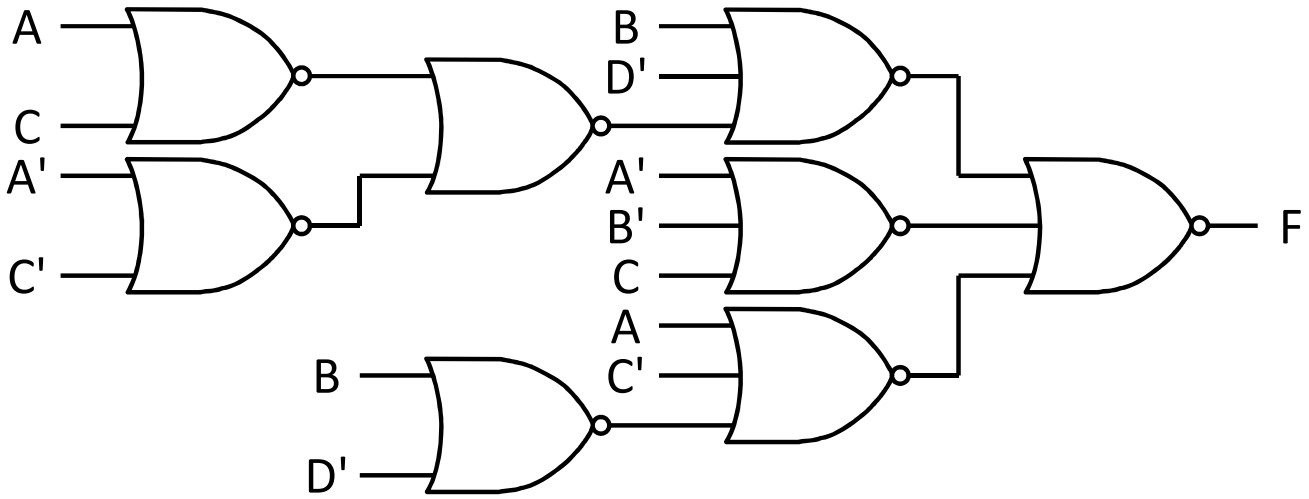
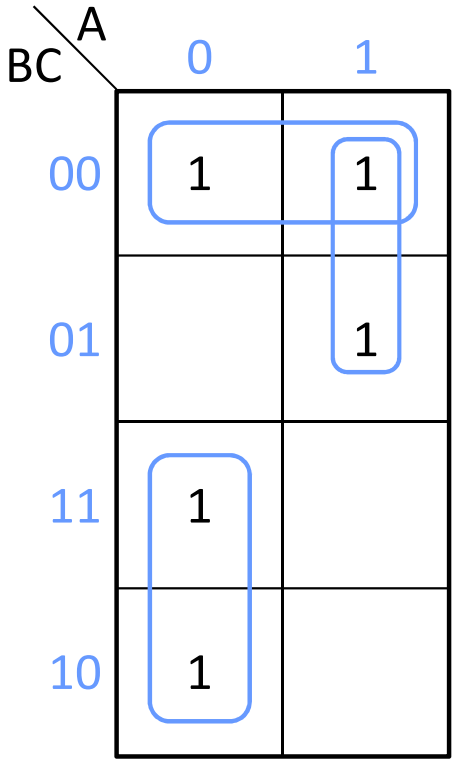
Example 1: Single Output

- Realize f(a, b, c, d) = ∑ m(0, 3, 4, 5, 8, 9, 10, 14, 15) using 3-input NOR gates
  - F' = A'B'C'D + AB'CD + ABC' + A'BC + A'CD' (from K-map)
  - F = (A + B + C + D')(A' + B + C' + D')(A' + B' + C)(A + B' + C')(A + C' + D)
    - Two 4-input OR gates (X) → factoring
    - Three 3-input OR gates (O)
    - One 5-input AND gate (X)
  - F = [B + D' + (A + C)(A' + C')] [A' + B' + C] [A + C' + B'D]

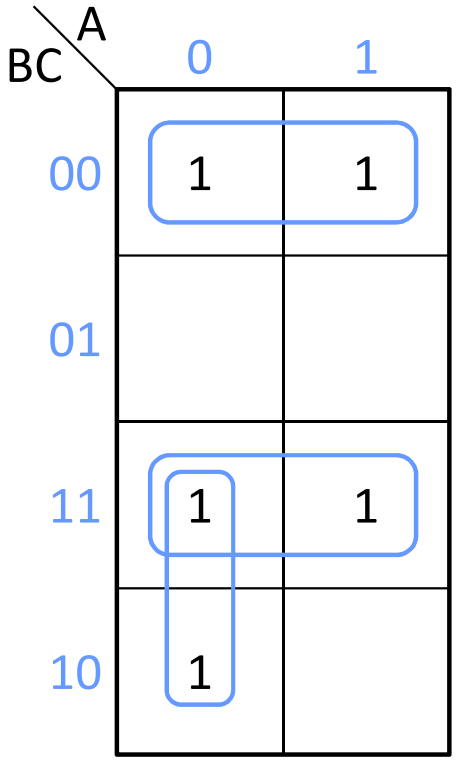


Example 2: Multiple Outputs (1/2)

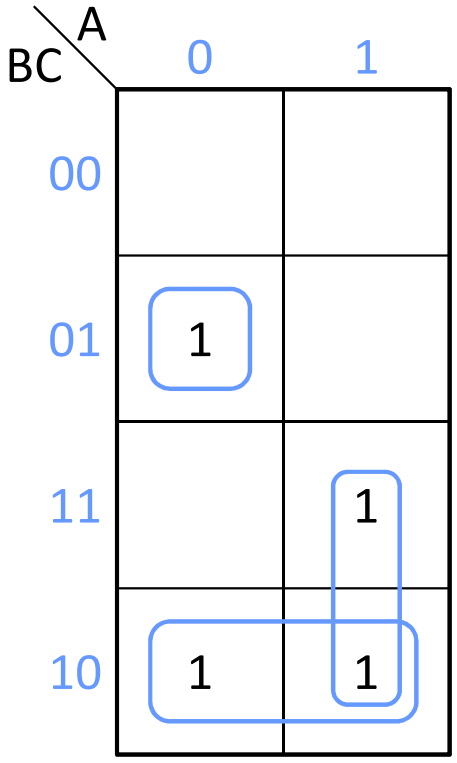
- Realize the functions using only 2-input NAND gates and inverters



$F_1 = B'C' + AB' + A'B$



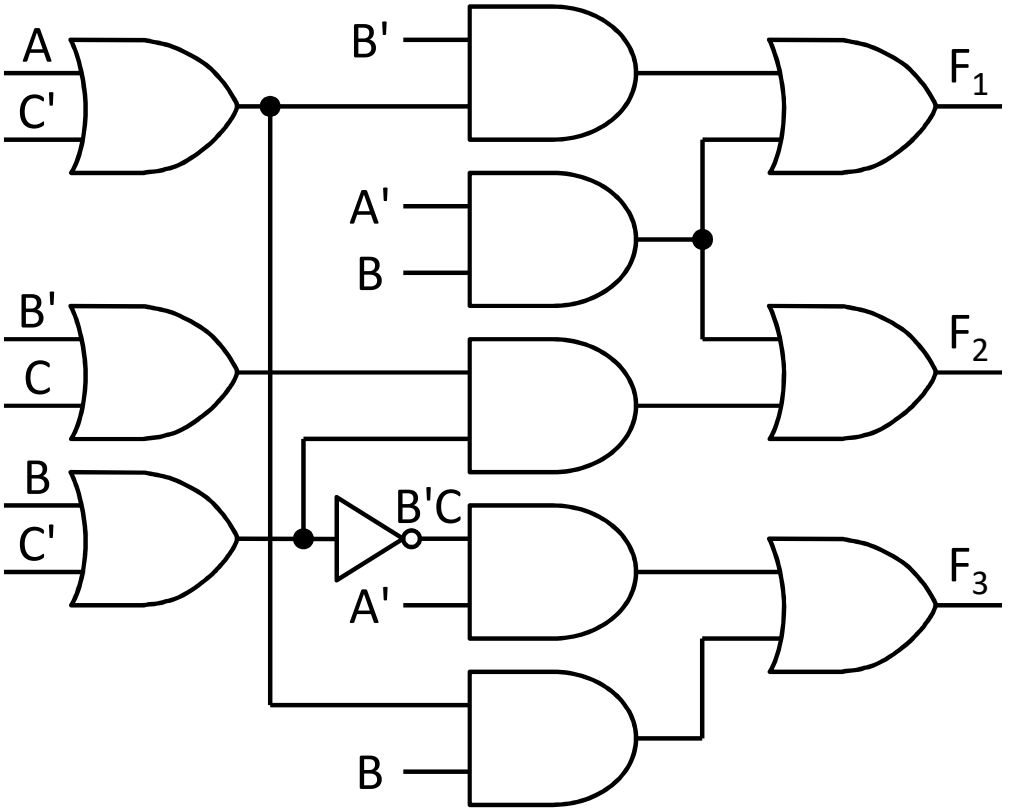
$F_2 = B'C' + BC + A'B$

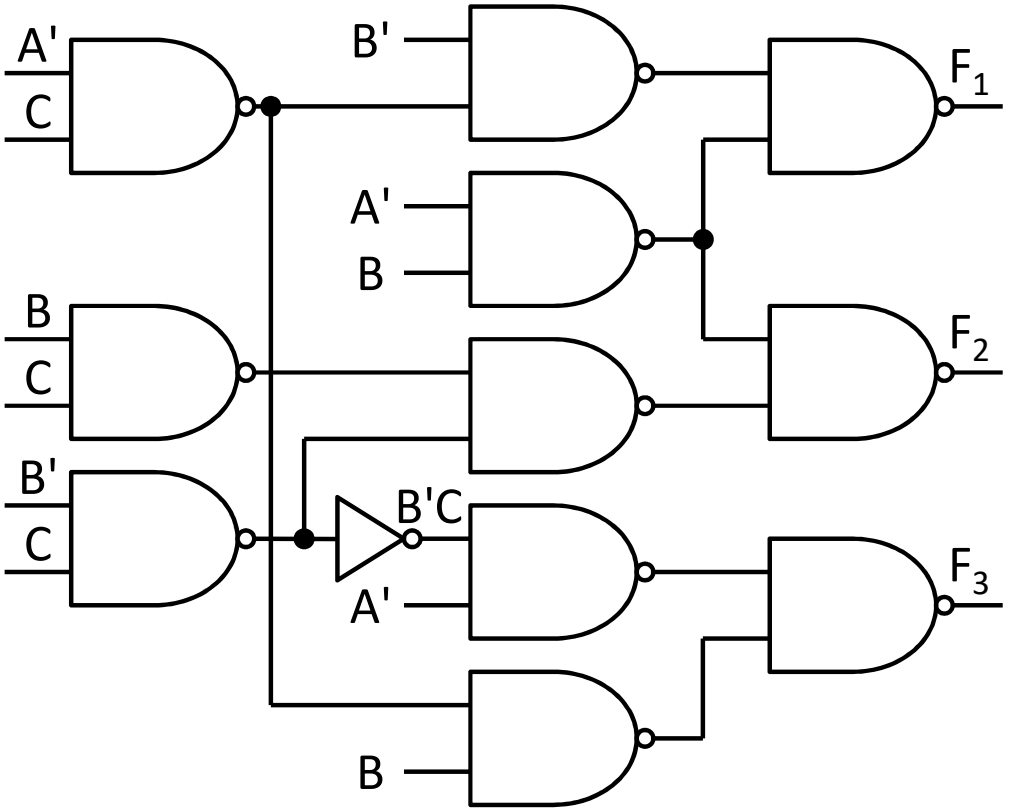


$F_3 = \underline{A'B'C} + AB + BC'$

Example 2: Multiple Outputs (2/2)

- Two-level
  - $F_1 = B'C' + AB' + A'B = B'(A + C') + A'B$
  - $F_2 = B'C' + BC + A'B = B'C' + B(A' + C)$
  - $F_3 = \underline{A'B'C} + AB + BC' = \underline{A'(B + C')'} + B(A + C')$





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- ❑ Hazards in Combinational Logic
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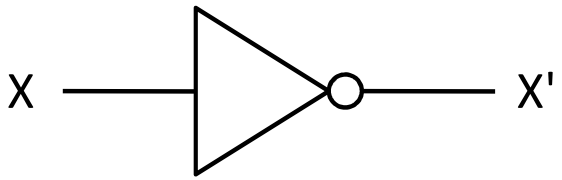
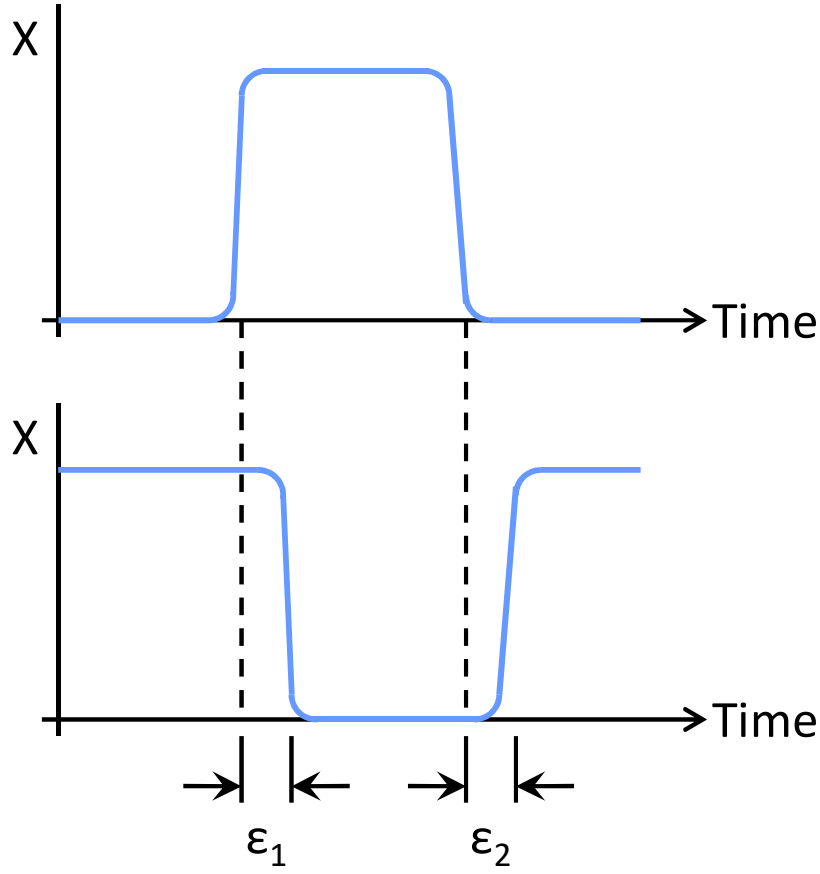
Gate Delay

Propagation delay

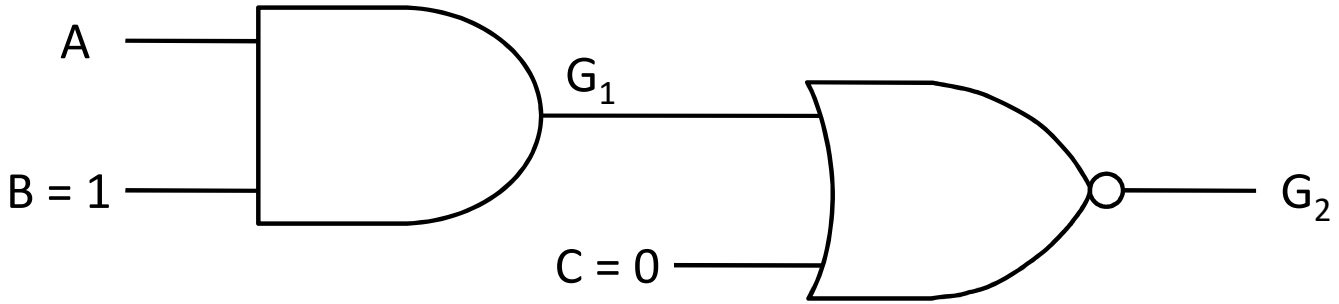
- When the input to a logic gate is changed, the output takes some time to change value
- Usually different for input rising 0 → 1 and falling 1 → 0

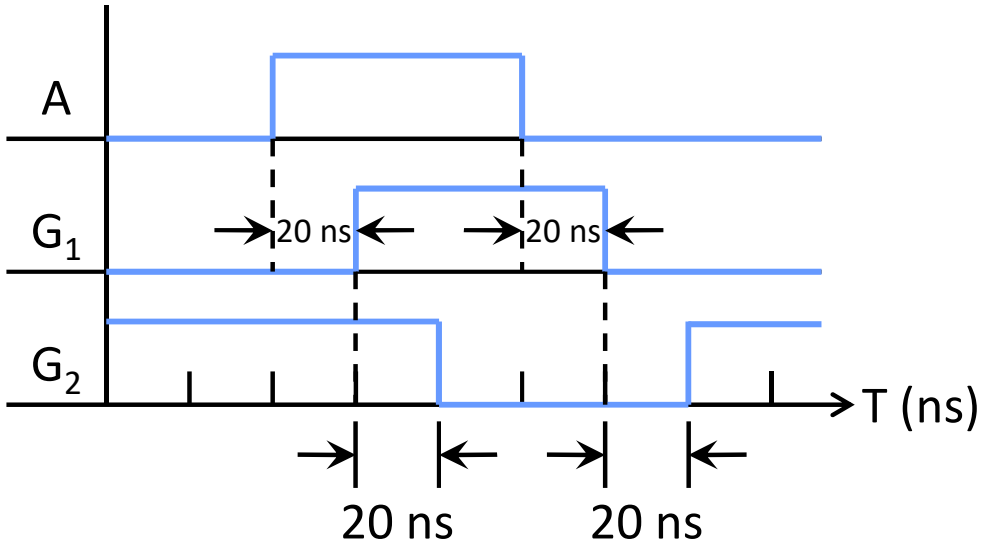




Sample Timing Diagram (1/2)

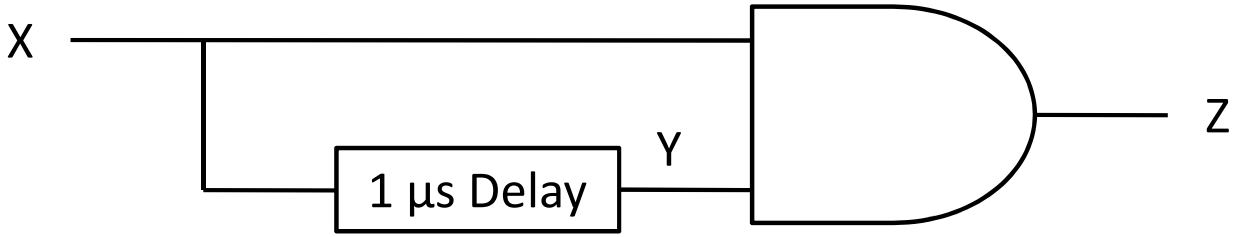
- Assume each gate has a propagation delay of 20 ns

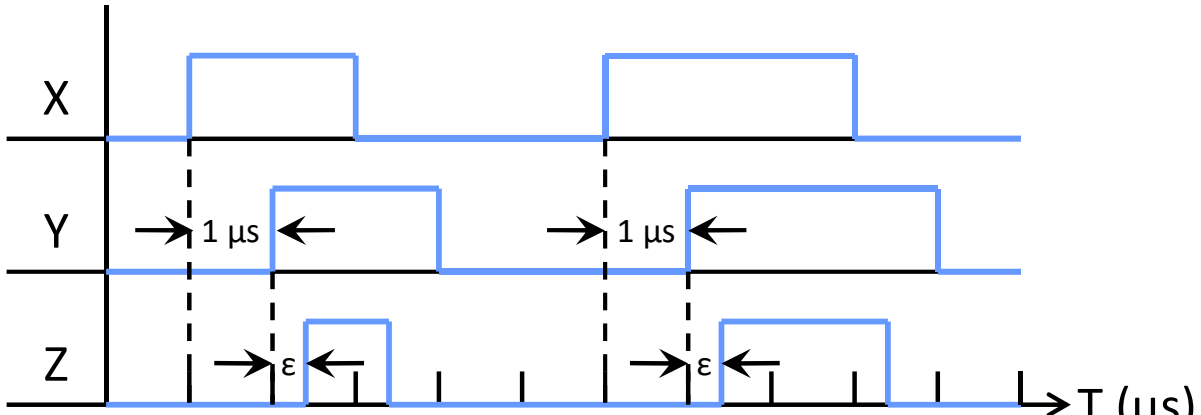




Sample Timing Diagram (2/2)

- A circuit with delay element





ε: Propagation Delay of AND

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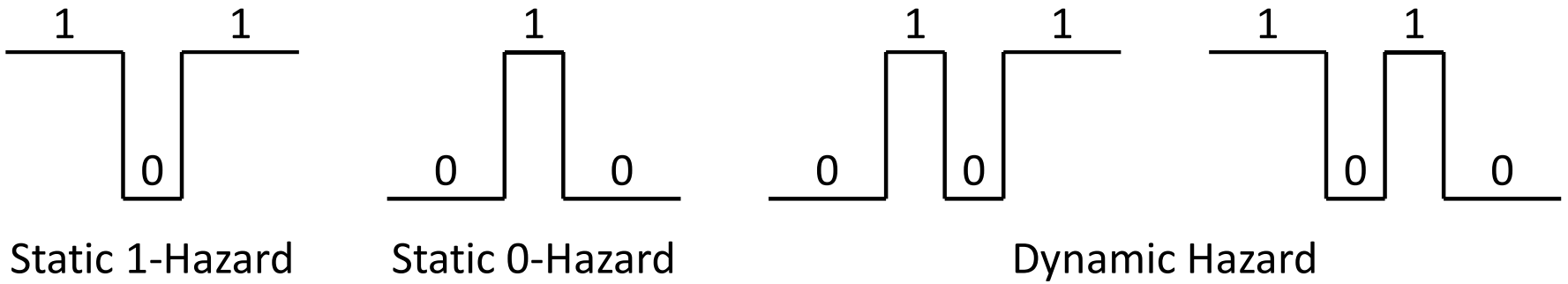
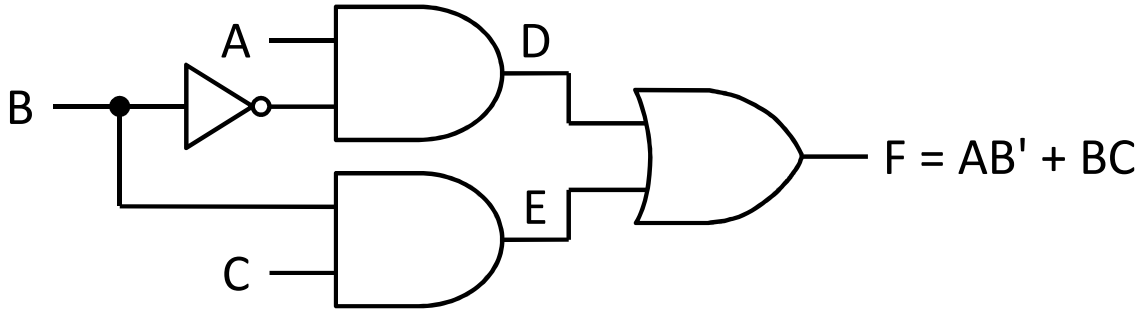
Hazards

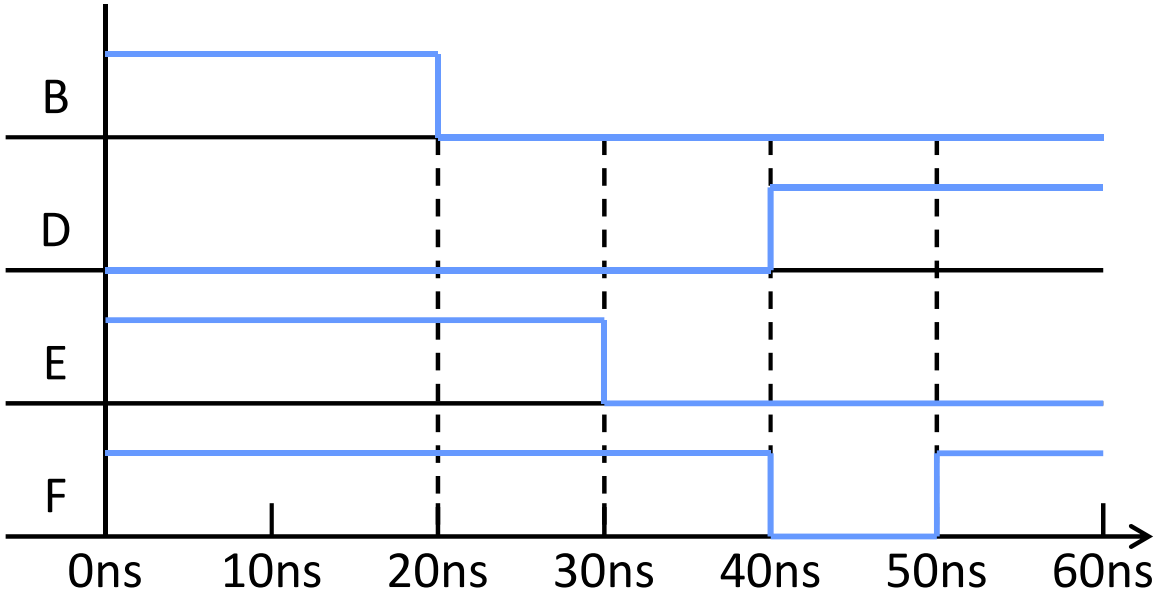
- Hazard
  - Unwanted switching transients appearing at the output when the input to a combinational circuit changes
    - Unequal propagation delays for different paths from inputs to outputs
  - Static 1-/0-hazard
    - Output momentarily goes to 0/1 when it should remain a constant 1/0
  - Dynamic hazard
    - Output change 3 or more times when the output changes from 0 to 1 (1 to 0)

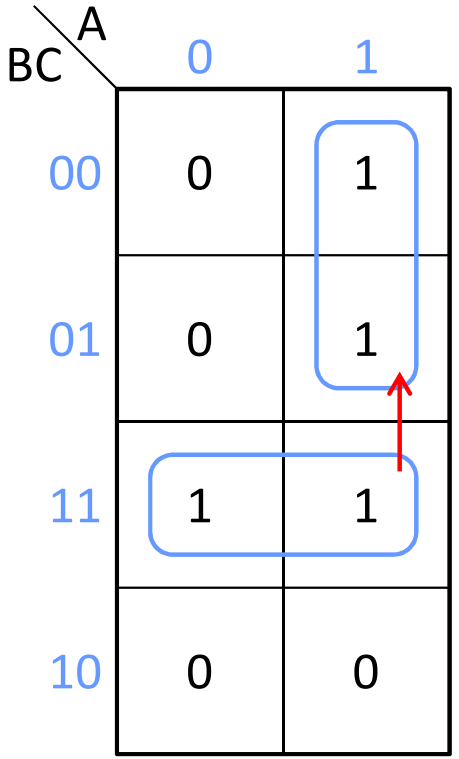


Static 1-Hazard Example

- Example: assume each gate has a propagation delay of 10 ns
  - If A = C = 1, static-1 hazard occurs when B changes from 1 to 0

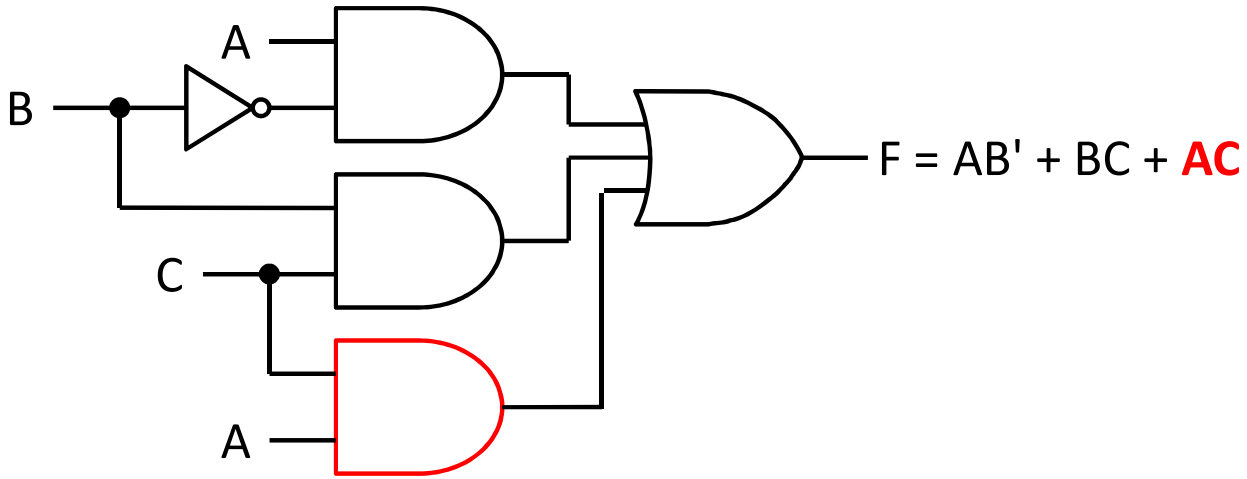






Static 1-Hazard Removal

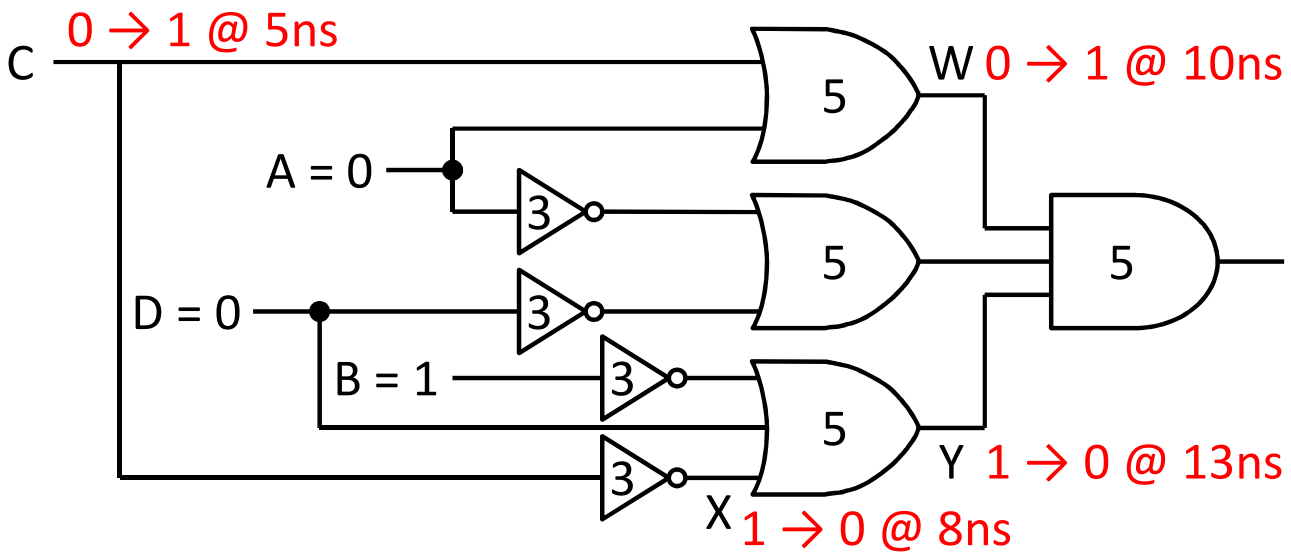
- When?
  - In Karnaugh maps, if any two adjacent 1's are not covered by the same loop, a 1-hazard exists for the transition between the two 1's
- How to remove hazards?
  - Add additional loops such that all adjacent 1's are covered by some loop



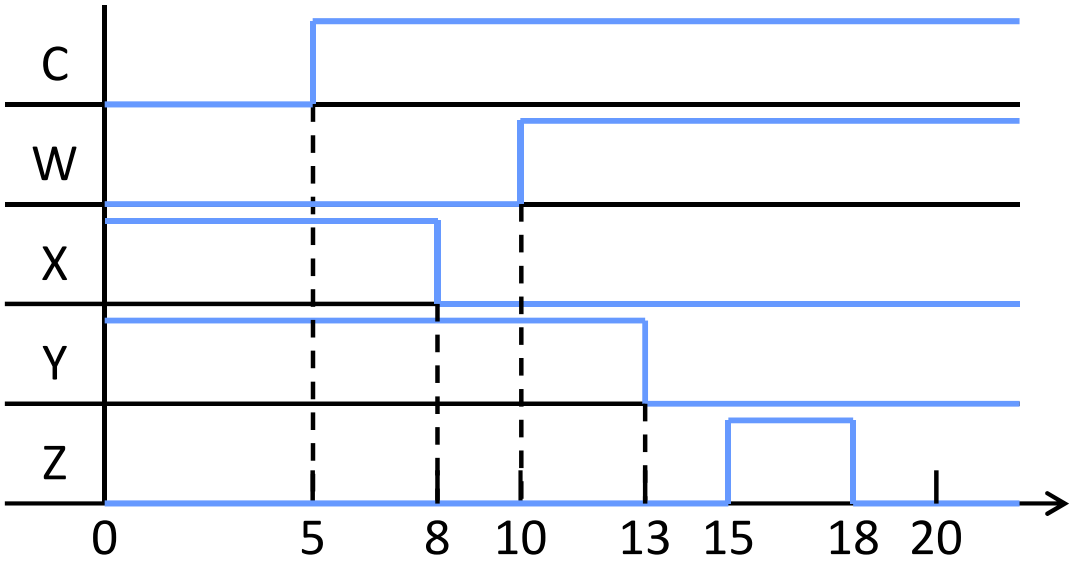
BC \ A	0	1
00	0	1
01	0	1
11	1	1
10	0	0

Static 0-Hazard Example

- Example: A = 0, B = 1, D = 0, and C changes from 0 to 1

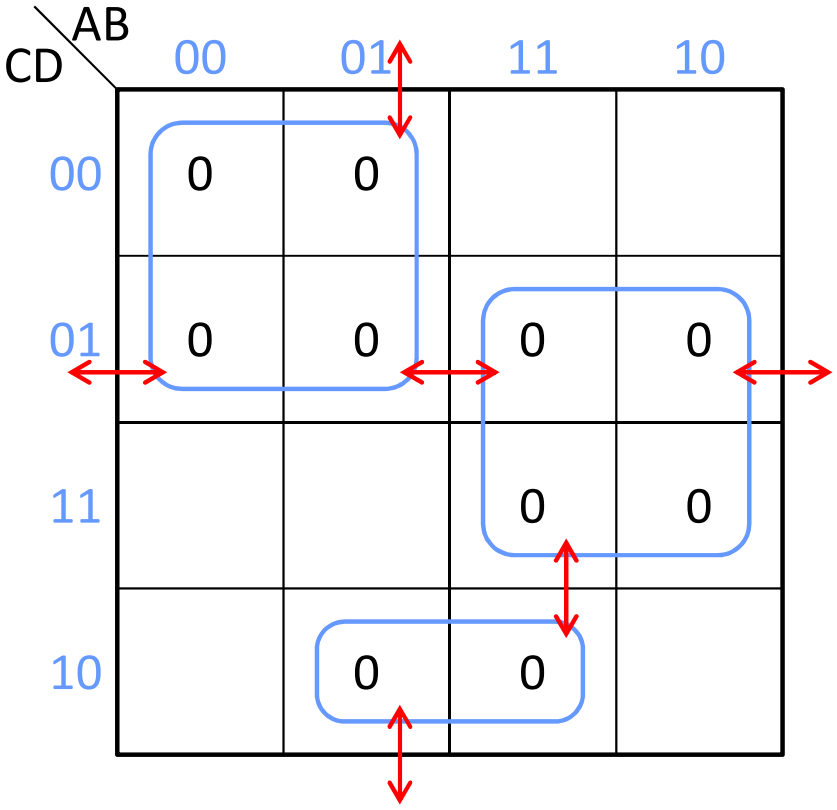


Propagation Delay
Inverter: 3 ns
AND/OR: 5 ns

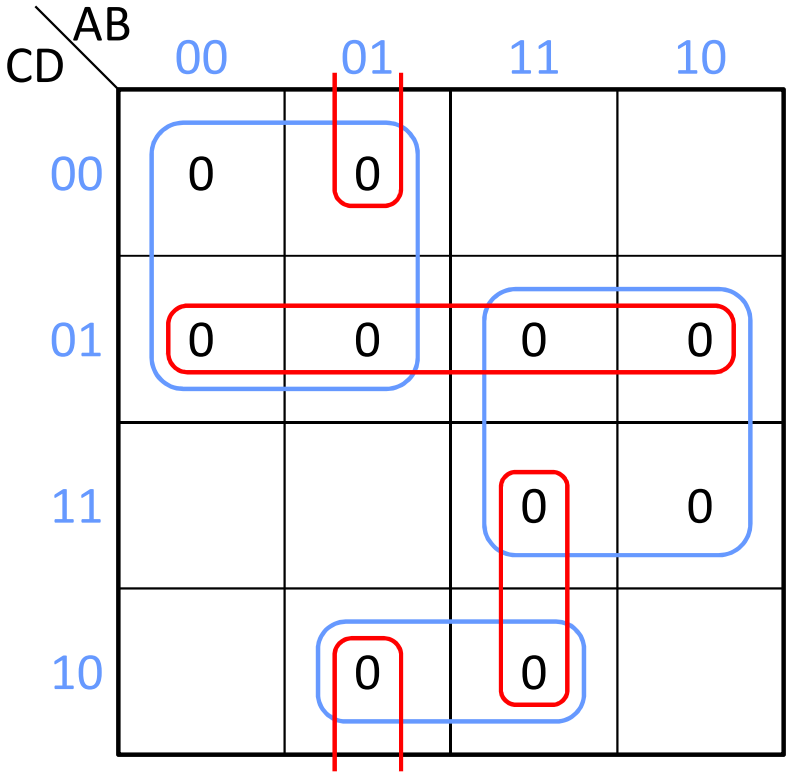


Static 0-Hazard Removal

- Similarly, add additional loops for adjacent 0's



Z = (A + C)(A' + D')(B' + C' + D)



Z = (A + C)(A' + D')(B' + C' + D)

(C + D')(A + B' + D)(A' + B' + C')

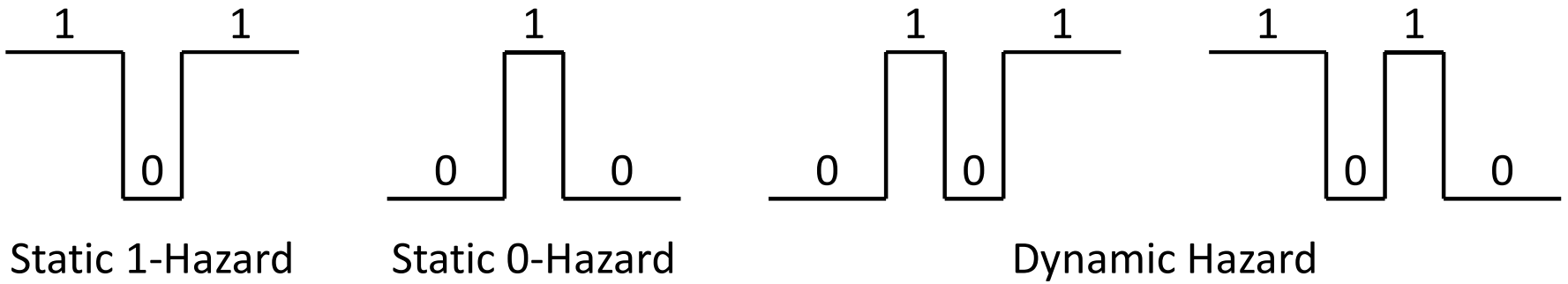
Stop Here (20190325)

Hazards in Multilevel Logic (1/2)

- Derive a SOP/POS for a multilevel circuit, but the complementation laws (XX' = 0, X + X' = 1) are NOT used
  - Some redundant terms exist in the SOP (POS)
    - XX'α
      - α is a product of literals or it may be null
      - XX'α represents a pseudo gate that may temporarily have the output value 1 as X changes and if α = 1
    - X + X' + β
      - β is a sum of literals or it may be empty
      - X + X' + β represents a pseudo gate that may temporarily have the output value 0 as X changes and if β = 0
- For SOP, ensure every pair of adjacent 1's is covered together
  - The sum of all prime implicants is safe!
- Treat each X and X' as independent variables to prevent introduction of hazards

Hazards in Multilevel Logic (2/2)

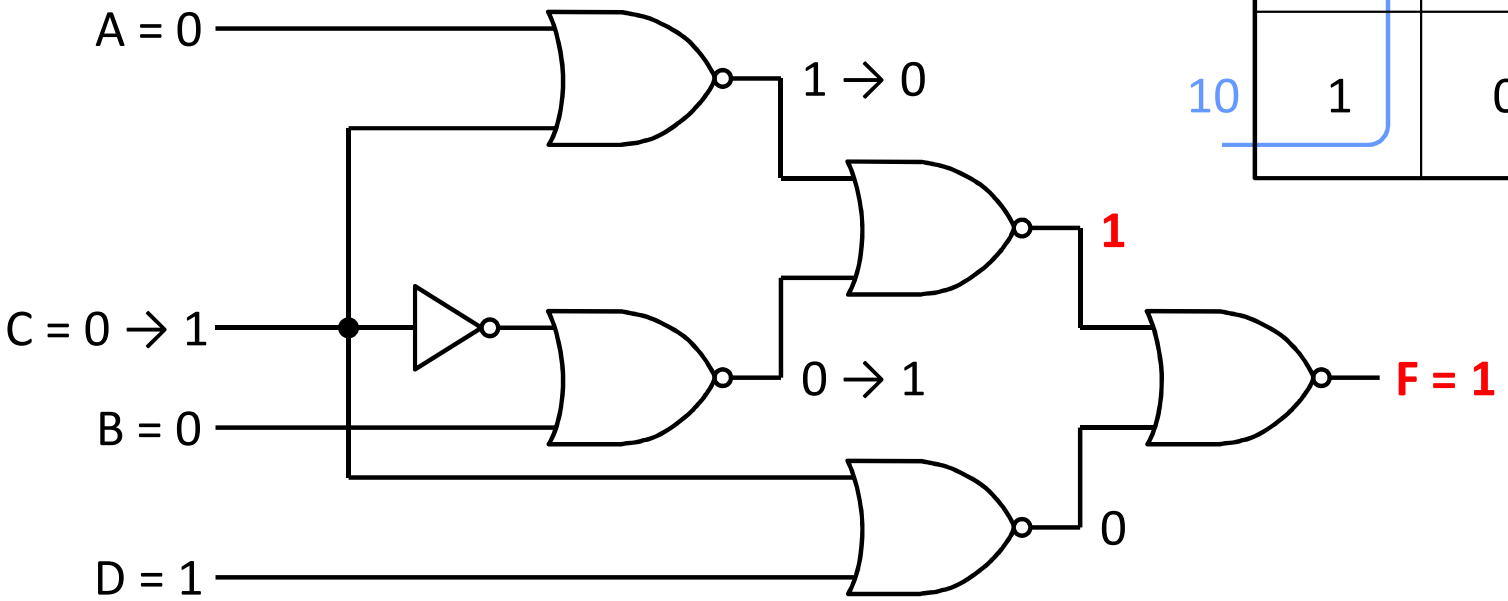
- Static 1-hazards: analyze the same as for 2-level
- Static 0-hazards: XX'α in SOP
  - Adjacent 0's differ in the value of X with α = 1
- Dynamic hazards: XX'α in SOP
  - Adjacent input combinations on K-map differ in the value of X with α = 1 and with opposite function values, and
  - For these combinations, the change in X propagates over ≥ 3 different paths



Example (1/3)

- F = (A'C' + B'C)(C + D)
  = A'C'C + A'C'D + B'C + B'CD
  = A'C'C + A'C'D + B'C
- Static 1-hazard
  - 0001 ↔ 0011

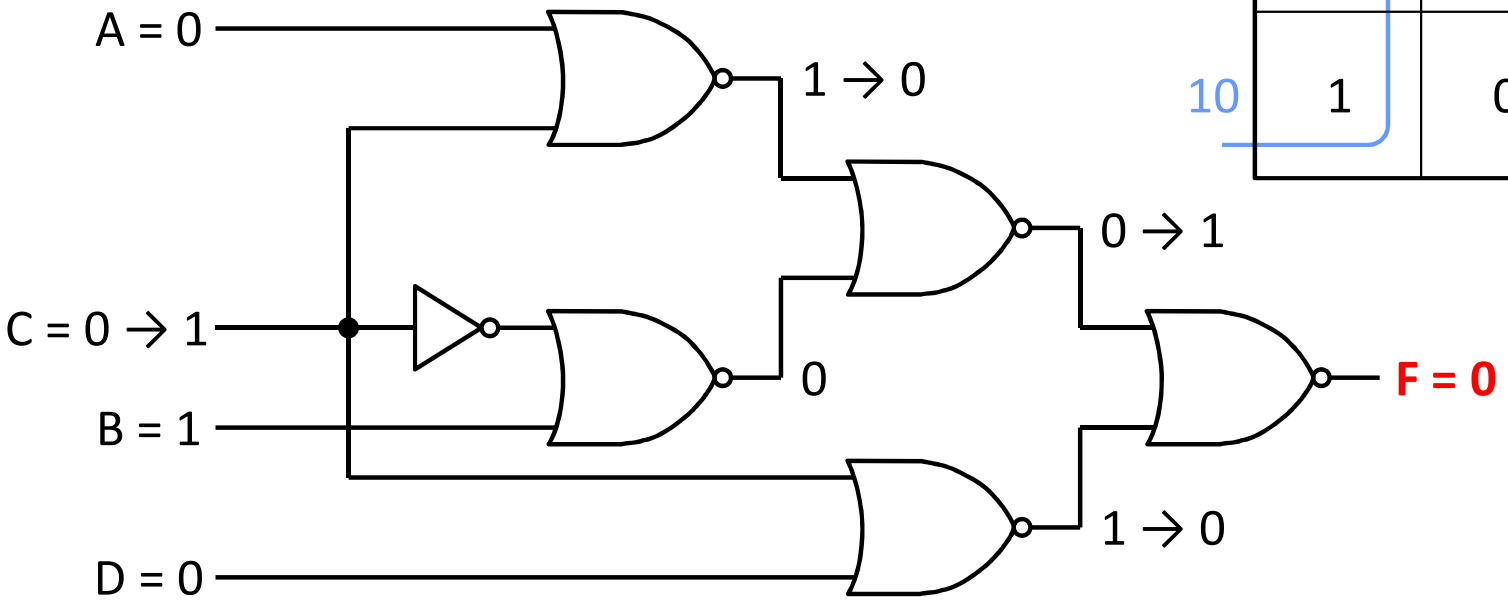
CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	1	0	0	1
10	1	0	0	1



Example (2/3)

- F = A'C'C + A'C'D + B'C
- Static 0-hazard
  - Adjacent 0's differ in the value of C with A' = 1
  - 0100 ↔ 0110

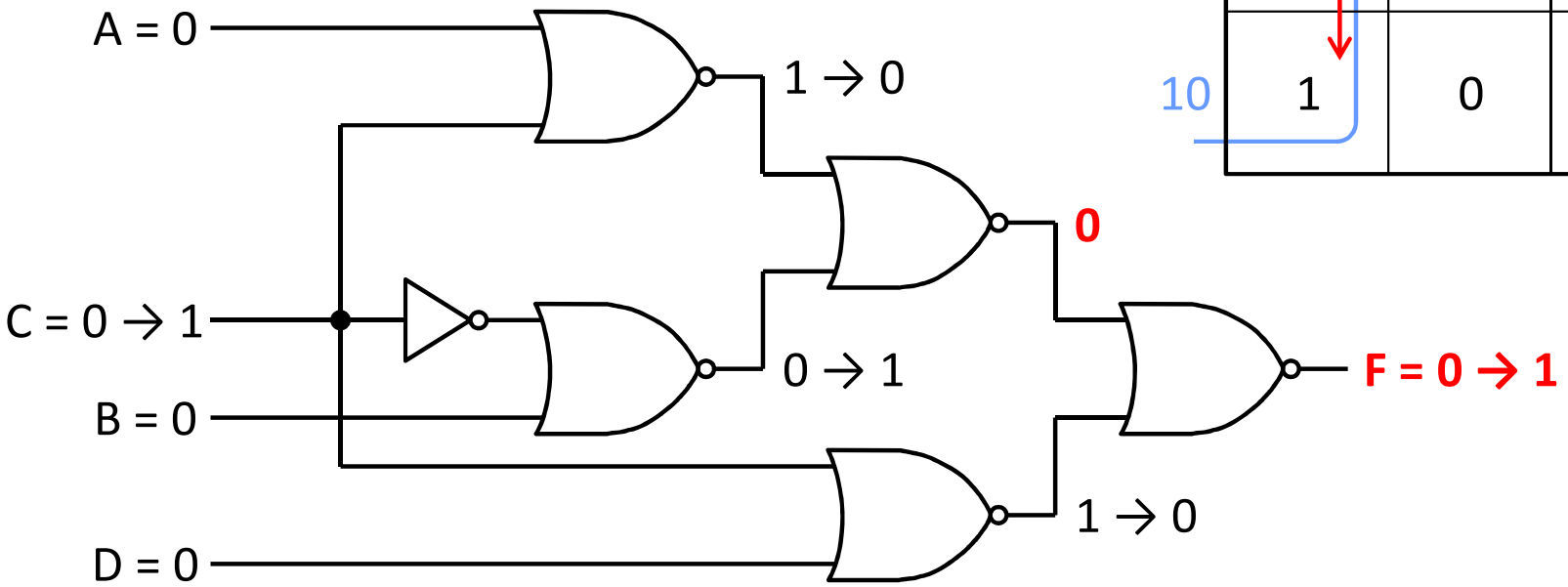
CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	1	0	0	1
10	1	0	0	1



Example (3/3)

- F = A'C'C + A'C'D + B'C
  - Adjacent input combinations on K-map differ in the value of C with A' = 1 and with opposite function values, and
  - For these combinations, the change in C propagates over ≥ 3 different paths
  - 0000 ↔ 0010 (how about 0101 ↔ 0111?)

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	1	0	0	1
10	1	0	0	1



Designing a Hazard-Free Circuit

- Hazard-free AND-OR circuit
  - Find a SOP expression for the output where each pair of adjacent 1's is covered by a 1-term
    - The sum of all prime implicants always satisfies this condition
    - A two-level AND-OR circuit based on this will be free of 1-, 0-, and dynamic hazards
  - Transform the SOP expression into the desired form by simple factoring, DeMorgan's laws, etc.
    - Treat each variable X and X' as independent variables
- Hazard-free OR-AND circuit
  - Alternatively, start with a POS form where every pair of adjacent 0's is covered by a 0-term
  - Dual procedure

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Simulation

- Verify logic circuits by
  - Actually building them
  - Simulating them on a computer
- 4 logic values
  - 0 (low), 1 (high), X (unknown), Z (high-impedance/open circuit)
- AND and OR functions for 4-valued simulation

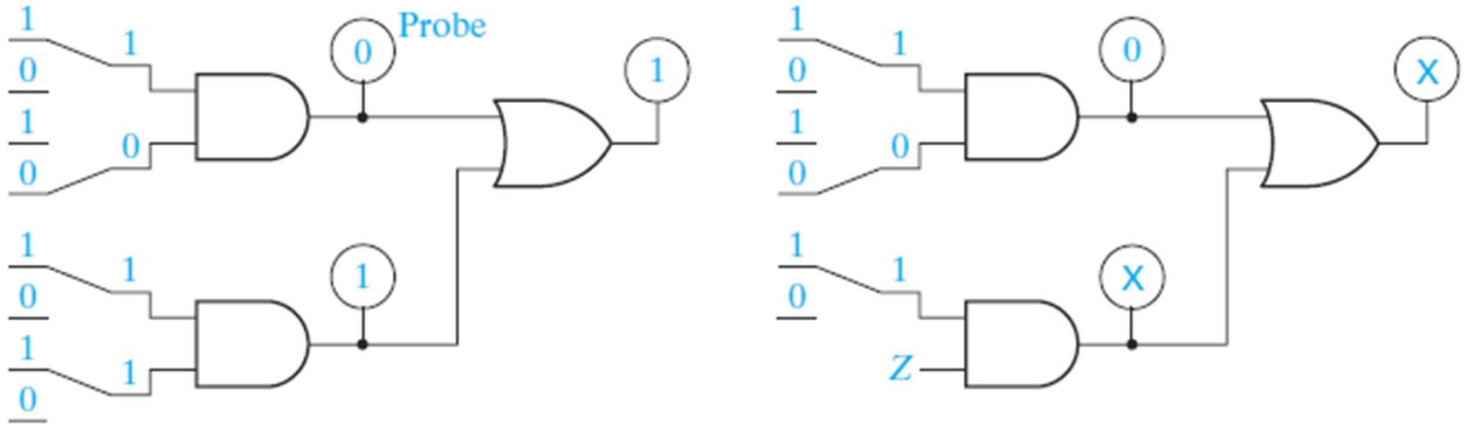
AND	0	1	X	Z
0	0	0	0	0
1	0	1	X	X
X	0	X	X	X
Z	0	X	X	X

OR	0	1	X	Z
0	0	1	X	X
1	1	1	1	1
X	X	1	X	X
Z	X	1	X	X

4-Valued Logic Simulator (1/2)

- Simulate a circuit
  - Computations: level-by-level from inputs to outputs
  - Evaluations: once inputs change

FIGURE 8-13



(a) Simulation screen showing switches

(b) Simulation screen with missing gate input

4-Valued Logic Simulator (2/2)

- Possible errors in designs
  - In simulation
    - Incorrect design
    - Gates with wrong connection
    - Wrong input signals to the circuits
  - In real circuits (testing)
    - Defective gates
    - Defective connecting wires
- Debug
  - Start from output, work back until the trouble is located

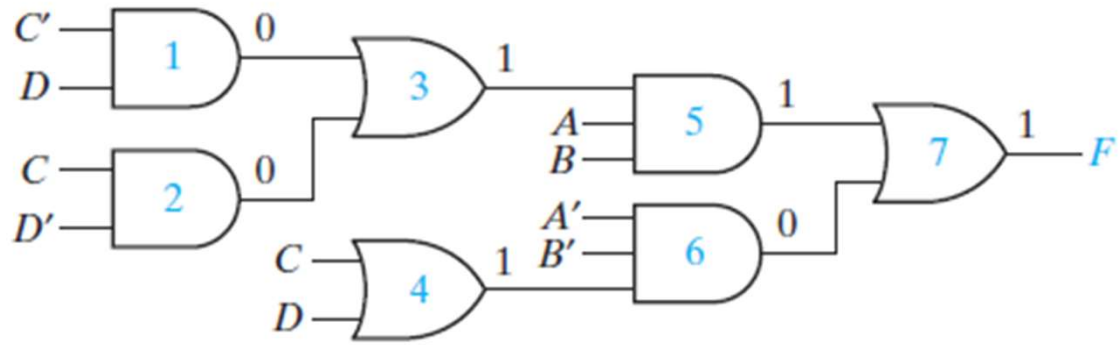


FIGURE 8-14
Logic Circuit with Incorrect Output

Q&A