CSIE 2344, Spring 2019 — Homework 1

Due March 11 (Monday) at Noon

There are 60 points in total. Points will be deducted if no appropriate intermediate step is provided.

When you submit your homework on Gradescope, please select the corresponding page(s) of each problem.

1 Base-3 to Base-9 Conversion (6pts)

Devise a scheme for converting base 3 numbers directly to based 9. Use your method to convert the following number to base 9: 1110212.20211_3.

Answer: Consider the following table:

Base 3	Base 9
00	0
01	1
02	2
10	3
11	4
12	5
20	6
21	7
22	8

Start at the point, divide digits into groups of two, add 0's if necessary, and follow the table to complete the conversion: $1110212.20211_3 = 01\ 11\ 02\ 12.20\ 21\ 10_3 = 1425.673_9$.

2 Base Determination (6pts)

Assume three digits are used to represent positive integers and also assume the following operation $024 + 043 + 013 + 033 = 201$ is correct. Determine all possible bases of the numbers.

Answer: There is a digit 4, so the base is at least 5. $4 + 3 + 3 + 3 = 13_{10} = 23_5 = 21_6 = 16_7 = 15_8 = 14_9 = 12_{11} = 11_{12} = 10_{13}$. The following bases have no carry, so the last digit is impossible to be 1. The remaining possible bases are 6 and 12. Try base 6: $2 + 2 + 4 + 1 + 3 = 20$, so base 6 is correct. Try base 12: $1 + 2 + 4 + 1 + 3 = B$ in base 12, so base 12 does not work. As a result, the base is 6.

3 8-4-(-2)-(-1) Code (6pts)

It is possible to have negative weights in a weighted code for the decimal digits, *e.g.*, 8, 4, -2, and -1 can be used. Construct a table for this weighted code.

Answer: The table is as follows:

	8	4	−2	−1
0	0	0	0	0
1	0	1	1	1
2	0	1	1	0
3	0	1	0	1
4	0	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	1	1	1	1

4 Logic Simplification (6pts)

Use only DeMorgan's relationships and Involution to find the complement of the following function: $f(A,B,C,D) = [A + (BCD)'][(AD)' + B(C' + A)]$. The complements in the final answer should only come with literals, *e.g.*, A', B', C', or D'.

Answer:

$$\begin{aligned} f' &= \{[A + (BCD)'][(AD)' + B(C' + A)]\}' \\ &= [A + (BCD)']' + [(AD)' + B(C' + A)]' \\ &= A'(BCD)'' + (AD)''[B(C' + A)]' \\ &= A'BCD + AD[B' + (C' + A)'] \\ &= A'BCD + AD[B' + (C''A')] \\ &= A'BCD + AD(B' + A'C) \\ &= A'BCD + AB'D + AA'CD \\ &= A'BCD + AB'D. \end{aligned}$$

5 Switch Circuit (18pts)

Consider the switch circuit in Figure 1.

1. Derive the switching algebra expression that corresponds one to one with the switch circuit.
2. Derive an equivalent switch circuit with a structure consisting of a parallel connection of groups of switches connected in series (hint: use 9 switches).
3. Derive an equivalent switch circuit with a structure consisting of a series connection of groups of switches connected in parallel (hint: use 6 switches).

Answer:

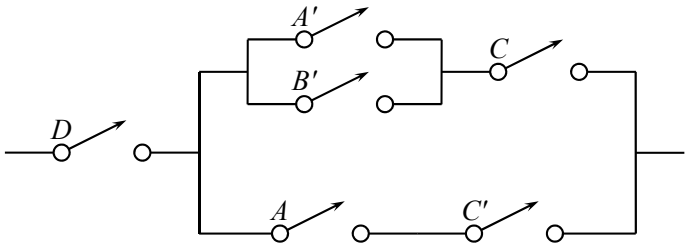

Figure 1: The given switch circuit.

1. $F = D[(A' + B')C + AC']$.
2. $F = D[(A' + B')C + AC'] = AC'D + A'CD + B'CD$. The switch circuit is in Figure 2.

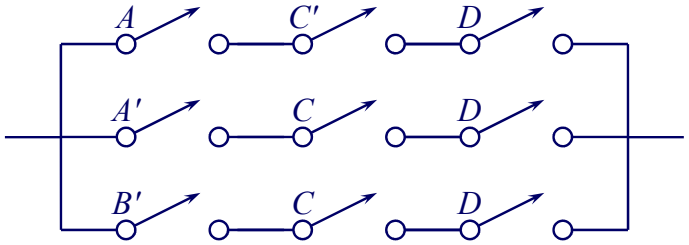

Figure 2: The switch circuit with a structure consistent of a parallel connection of groups of switches connected in series.

3. $F = D[(A' + B')C + AC'] = D(A' + B' + AC')(C + AC') = D(A' + B' + C')(A + C)$. The switch circuit is in Figure 3.

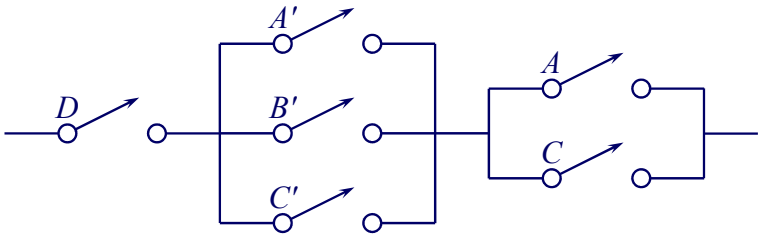

Figure 3: The switch circuit with a structure consistent of a series connection of groups of switches connected in parallel.

6 Sum of Products (6pts)

Multiply out to obtain a sum of products: $(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)$ (simplify where possible).

Answer:

$$(A + B + C + D)(A' + B' + C + D')(A' + C)(A + D)(B + C + D)$$

$$
\begin{aligned}
&= [(A+B+C+D)(B+C+D)][(A'+B'+C+D')(A'+C)](A+D) \\
&= [(B+C+D)][(A'+C)](A+D) \\
&= (B+C+D)(A'+C)(A+D) \\
&= (B+C+D)[(A'+C)(A+D)] \\
&= (B+C+D)[(AC+A'D)] \\
&= (B+C+D)(AC+A'D) \\
&= ABC+AC+ACD+A'BD+A'CD+A'D \\
&= (ABC+AC+ACD)+(A'BD+A'CD+A'D) \\
&= AC+A'D.
\end{aligned}
$$

7 Product of Sums (6pts)

Factor to obtain a product of sums: $BCD + C'D' + B'C'D + CD$ (simplify where possible).

Answer:

$$
\begin{aligned}
& BCD + C'D' + B'C'D + CD \\
= \; & (BCD + CD) + (C'D' + B'C'D) \\
= \; & CD + C'(D' + B'D) \\
= \; & (C' + D)[C + (D' + B'D)] \\
= \; & (C' + D)[C + (D' + B')] \\
= \; & (C' + D)(B' + C + D').
\end{aligned}
$$

8 Majority Circuit (6pts)

The output of a majority circuit is 1 if a majority (more than half) of its inputs are equal to 1, and the output is 0 otherwise. Construct a truth table for a three-input majority circuit and derive a simplified sum-of-products expression for its output.

Answer: Assume that the three inputs are A, B, and C, and the output is F. The truth table is as follows:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

The SOP expression is $F = BC + AC + AB + ABC = AB + AC + BC$.