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1. Base-3 to Base-9 Conversion (6 pts)

Since $9 = 3^2$, 2 digits of base-3 can be converted to 1 digit of base-9.

$xy_3 = z_9$ where x, y, z are digits, $z = 3x + y$.

So for 1110212.20211_3, we first pad 0s so we can pair up the digits $\Rightarrow \underline{01}\,\underline{11}\,\underline{02}\,\underline{12}.\underline{20}\,\underline{21}\,\underline{10}_3$. Converting the digit pairs.

$$\Rightarrow \left.\begin{array}{l} 0\times3+1=1 \\ 1\times3+1=4 \\ 0\times3+2=2 \\ 1\times3+2=5 \\ 2\times3+0=6 \\ 2\times3+1=7 \\ 1\times3+0=3 \end{array}\right\} \Rightarrow 1425.673_9 \ \#$$

2. Base Determination (6 pts)

$024 + 043 + 013 + 033 = 201$

We try to add up each digit,

$$4+3+3+3=13$$
$$2+4+1+3=10$$
$$0+0+0+0=0$$

Assume it is base x, the number shall be

$$0x^2 + 10x + 13 = 2x^2 + 0x + 1$$

$$\Rightarrow 2x^2 - 10x - 12 = 0$$

$$\Rightarrow 2(x^2 - 5x - 6) = 0 \Rightarrow 2(x-6)(x+1) = 0$$

$$\Rightarrow x = -1, 6 \ \#$$

Since negative base number is possible, both numbers (-1, 6) are possible.

3. 8-4-(-2)-(-1) Code (6pts)

We construct a table for the code.

0 = 0000 ; 4 = 0100 ; 8 = 1000

1 = 0111 ; 5 = 1011 ; 9 = 1111

2 = 0110 ; 6 = 1010

3 = 0101 ; 7 = 1001

This code is usable since it is able to represent 0~9.

4. Logic Simplification (6 pts)

$f(A,B,C,D) = [A+(BCD)'][(AD)'+B(C'+A)]$, find the complement.

$[f(A,B,C,D)]' = \{[A+(BCD)'][(AD')+B(C'+A)]\}'$

$= [A+(BCD)']' + [(AD')+B(C'+A)]'$

$= A'BCD + (AD')'[B(C'+A)]'$

$= A'BCD + (A'+D)(B+A'C)$

$= A'BCD + A'B + A'C + BD + A'CD$ #

5. Switch Circuit (18 pts)

1. Expression for the circuit.

$D[(A'+B')C+AC']$

2. Turn the expression into SOP

$D[(A'+B')C+AC'] = D[A'C+B'C+AC'] = A'CD+B'CD+AC'D$

3. Turn the expression into POS

$D[(A'+B')C+AC'] = D[(C+A)(C'+A'+B')]$

$= (A+C+D)(A'+B'+C'+D)$

6. Sum of Product (6pts)

1. $(X+Y)(X+Z) = X+YZ$

2. $(X+Y)(X'+Z) = XZ+X'Y$

$(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)$

$= (A+B+C+D)(A'+B'+C+D')(A+D) = (A+B+C+D)(A'+B'+C+D') = C+(A+B+D)(A'+B'+D')$

$= C + A(B'+D') + A'(B+D) = C + AB' + AD' + A'B + A'D$ #

7. Product of Sum (6 pts)

1. $X(Y+Z) = XY + XZ$
2. $(X+Y)(X'+Z) = XZ + X'Y$

$$\underbrace{BCD}_{1.} + \underbrace{C'D'}_{1.} + \underbrace{B'C'D}_{1.} + \underbrace{CD}_{1.}$$

$$= C(\underbrace{BD}_{1} + \underbrace{D}_{1.}) + C'(B'D + D') = C[(B+1)D] + C'(B'D + D') = CD + C'(B'D + D')$$

$$= CD + C'[(D+1)(D'+B')] = \underbrace{CD}_{2.} + \underbrace{C'}_{2.}(B'+D') = (C'+D)(C+B'+D')$$

$$= (C'+D)(B'+C+D') \#$$

8. Majority Circuit (6pts)

For three input, A, B, C, we construct the truth table

A	B	C	Output	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	√
1	0	0	0	
1	0	1	1	√
1	1	0	1	√
1	1	1	1	√

Simplify into SOP expression.

$$ABC + A'BC + AB'C + ABC'$$

$$= (ABC + A'BC) + (ABC + AB'C) + (ABC + ABC')$$

$$= (A+A')BC + (B+B')AC + (C+C')AB$$

$$= BC + AC + AB \#$$