Example: convert $.375_{10}$ to binary

```
   .375
 x    2
(0).750  a-1 = 0  (MSB)
 x    2
(1).500  a-2 = 1
 x    2
(1).000  a-3 = 1  (LSB)
```

$.375_{10} = .011_2$

Example: convert $.7_{10}$ to binary

```
   .7
 x   2
(1).4
 x   2
(0).8
 x   2
(1).6
 x   2
(1).2
 x   2
(0).4   <- Repeating
```

$.7_{10} = .1\ 0110\ 0110\ \ldots_2$

Cases (assume A > 0, B > 0, and word length = n)

- Case 1: A + B and $|A + B| < 2^{n-1}$ → Correct
- Case 2: A + B and $|A + B| \geq 2^{n-1}$ → Wrong (overflow)
- Case 3: A – B and A < B → Correct
- Case 4: – A + B and A ≤ B → Correct (ignore the carry)
- Case 5: – A – B and $|A + B| \leq 2^{n-1}$ → Correct (ignore the carry)
- Case 6: – A – B and $|A + B| > 2^{n-1}$ → Wrong (overflow)

Uniting

- XY + XY' = X
- (X + Y)(X + Y') = X

Absorption

- X + XY = X
- X (X + Y) = X

Elimination

- X + X'Y = X + Y
- X (X' + Y) = XY

Consensus

- XY + X'Z + YZ = XY + X'Z
- (X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)

TABLE 2-3 Laws of Boolean Algebra

Operations with 0 and 1:	
1. $X + 0 = X$	1D. $X \cdot 1 = X$
2. $X + 1 = 1$	2D. $X \cdot 0 = 0$
Idempotent laws:	
3. $X + X = X$	3D. $X \cdot X = X$
Involution law:	
4. $(X')' = X$	
Laws of complementarity:	
5. $X + X' = 1$	5D. $X \cdot X' = 0$
Commutative laws:	
6. $X + Y = Y + X$	6D. $XY = YX$
Associative laws:	
7. $(X + Y) + Z = X + (Y + Z) = X + Y + Z$	7D. $(XY)Z = X(YZ) = XYZ$
Distributive laws:	
8. $X(Y + Z) = XY + XZ$	8D. $X + YZ = (X + Y)(X + Z)$
DeMorgan's laws:	
9. $(X + Y)' = X'Y'$	9D. $(XY)' = X' + Y'$

Examples

- Q1: {AND, OR, NOT}
- A1: Of course, yes!
- Q2: {AND, NOT}
- A2: Yes (check figure)
  - X + Y = (X + Y)'' = (X'Y')'
- Q3: {OR, NOT}
- A3: Yes (check figure)
  - XY = (XY)'' = (X' + Y')'
- Q4: {AND}
- Q5: {OR}

From product-of-sums (POS) to sum-of-products (SOP)

Example

(A + B + C')(A + B + D)(A + B + E)(A + D' + E)(A' + C)
(A + B + D): 2. (A + D' + E): 3.

= (A + B + C'D)(A + B + E)[AC + A'(D' + E)]
(A + B + C'D): 2.

= (A + B + C'DE)(AC + A'D' + A'E)

= AC + ABC + A'BD' + A'BE + A'C'DE

= AC + A'BD' + A'BE + A'C'DE

1. X(Y + Z) = XY + XZ
2. (X + Y)(X + Z) = X + YZ
3. (X + Y)(X' + Z) = XZ + X'Y

Example

(A + B + C')(A + B + D')(B + C + D')
= (A + B + C')(A + B + D')(B + C + D')
= (A + B + C')(B + C + D')
= ... can be further simplified
= B + AC + C'D'

Use the distributive laws to multiply out an expression to obtain a sum-of-products (SOP) form

- Ordinary distributive law: X(Y + Z) = XY + XZ
- Second distributive law: X + YZ = (X + Y)(X + Z)

Example: multiply out (A+BC)(A+D+E)

- Use the ordinary distributive law
  - (A + BC)(A + D + E) = A + AD + AE + ABC + BCD + BCE
    = A(1 + D + E + BC) + BCD + BCE
    = A + BCD + BCE
- Use the second distributive law
  - (A + BC)(A + D + E) = A + BC(D + E) = A + BCD + BCE

Example (another ordering)

A'C'D + A'BD + BCD + ABC + ACD'
= A'C'D + BCD + ABC + ACD'
= A'C'D + BCD + ABC + ACD'
= A'C'D + BCD + ACD'