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Subject: 自動機與形式語言 ID: B04705001 Name: 陳約廷 Page: 1

Problem 1

$f(n) = e^{-n}$, $g(n) = \sin n + 2$

(a) Is $f(n) = o(g(n))$?

By definition of small-o, $\forall c > 0, \exists n_0$ s.t. $f(n) < cg(n)$ $\forall n \geq n_0$.

If $c < 1$, we can have $n_0 = \lceil \frac{1}{c^2} \rceil$,

If $c > 1$, we can have $n_0 = c^2$,

$f(n) \leq e^{-c^2} < c \leq cg(n)$ $\forall$ $n \geq n_0$, $c > 1$

$f(n) \leq e^{-\frac{1}{c^2}} < c \leq cg(n)$ $\forall$ $n \geq n_0$, $c \leq 1$

We have $f(n) < cg(n)$ $\forall n \geq n_0$, therefore $f(n) = o(g(n))$

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(b) Is $f(n) = O(g(n))$?

By definition of Big-O, if $f(n) = O(g(n))$, $\exists$ $c, n_0 > 0$ s.t. $f(n) \leq cg(n)$ $\forall n \geq n_0$.

We can have $c = 1, n = 10$, $f(n) = e^{-n} \leq 1 \leq \sin n + 2$ $\forall n \geq 10$.

Therefore $f(n) = O(g(n))$.

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Problem 2

$f(n) = 2^{O(n^3)}$, Is $f(n)^3 = 2^{O(n^3)}$?

Since $f(n) = 2^{O(n^3)}$, we can find c_0, n_0 s.t. $f(n) \le 2^{c_0 n^3}$ $\forall\, n \ge n_0$.

So $f(n)^3 \le 2^{3c_0 n^3}$ $\forall\, n \ge n_0$.

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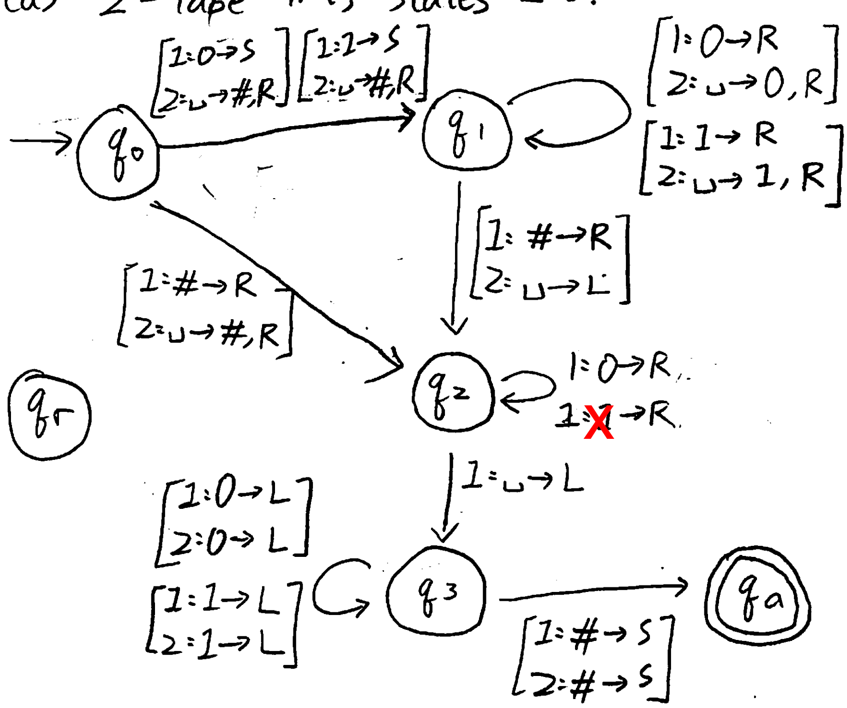
We can have $c_1 = 3c_0$, $n_1 = n_0$, $f(n^3) \le 2^{c_1 n^3}$ $\forall\, n \ge n_1$.

Therefore $f(n)^3 = 2^{O(n^3)}$.

Problem 3

$\{w\#w \mid w \in \{0,1\}^*\}$, $\Gamma = \{0, 1, \#, \sqcup\}$, no "$\sqcup$" before the input.

(a) 2-tape TM, states ≤ 6.



Links not shown go to q_r.

2

(b) Simulate "01#01"

2-tape TM: $\begin{bmatrix} q_0 01\#01\sqcup \\ q_0 \sqcup\sqcup\sqcup\sqcup\sqcup \end{bmatrix} \to \begin{bmatrix} q_1 01\#01\sqcup \\ \#q_1 \sqcup\sqcup\sqcup \end{bmatrix} \to \begin{bmatrix} 0q_1 1\#01\sqcup \\ \#0q_1 \sqcup\sqcup \end{bmatrix} \to \begin{bmatrix} 01q_1\#01\sqcup \\ \#01q_1\sqcup \end{bmatrix} \to \begin{bmatrix} 01\#q_2 01\sqcup \\ \#0q_2 1\sqcup \end{bmatrix}$

2

$\to \begin{bmatrix} 01\#0q_2 1\sqcup \\ \#0q_2 1\sqcup \end{bmatrix} \to \begin{bmatrix} 01\#01q_2\sqcup \\ \#0q_2 1\sqcup \end{bmatrix} \to \begin{bmatrix} 01\#0q_3 1\sqcup \\ \#0q_3 1\sqcup \end{bmatrix} \to \begin{bmatrix} 01\#q_3 01\sqcup \\ \#q_3 01\sqcup \end{bmatrix}$

$\to \begin{bmatrix} 01q_3\#01\sqcup \\ q_3\#01\sqcup \end{bmatrix} \to \begin{bmatrix} 01q_a\#01\sqcup \\ q_a\#01\sqcup \end{bmatrix}$, 11 steps

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Subject: 自動機與形式語言 ID: B04705001 Name: 陳約廷 Page: 3

Problem 3

(b)

Standard TM: $[q_1\,01\#01\sqcup] \to [x\,q_2\,1\#01\sqcup] \to [x1\,q_2\,\#01\sqcup] \to [x1\#\,q_4\,01\sqcup]$

$\to [x1\,q_6\,\#x1\sqcup] \to [x\,q_7\,1\#x1\sqcup] \to [q_7\,x1\#x1\sqcup] \to [x\,q_1\,1\#x1\sqcup]$

$\to [xx\,q_3\,\#x1\sqcup] \to [xx\#\,q_5\,x1\sqcup] \to [xx\#x\,q_5\,1\sqcup] \to [xx\#\,q_6\,xx\sqcup]$

$\to [xx\,q_6\,\#xx\sqcup] \to [x\,q_7\,x\#xx\sqcup] \to [xx\,q_1\,\#xx\sqcup] \to [xx\#\,q_8\,xx\sqcup]$

$\to [xx\#x\,q_8\,x\sqcup] \to [xx\#xx\,q_8\,\sqcup] \to [xx\#xx\sqcup\,q_{accept}\,\sqcup]$, 19 steps

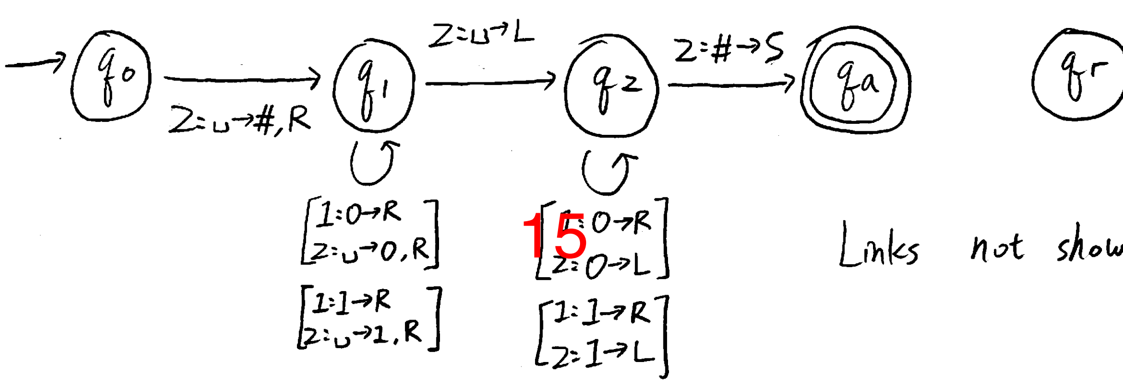
2-Tape TM: 11 steps, less than standard TM!

Standard TM: 19 steps

Problem 4

$\{ww^R \mid w \in \{0,1\}^*\}$

(a) 2-tape non-deterministic TM, states ≤ 5



Links not shown go to q_r.

(b) Simulate the path that leads to acceptance of 0110.

Start $\begin{bmatrix} q_0 0110 \\ q_0 \sqcup\sqcup\sqcup\sqcup \end{bmatrix} \to \begin{bmatrix} q_1 0110 \\ \# q_1 \sqcup\sqcup\sqcup \end{bmatrix} \to \begin{bmatrix} 0 q_1 110 \\ \#0 q_1 \sqcup\sqcup \end{bmatrix} \to \begin{bmatrix} 01 q_1 10 \\ \#01 q_1 \sqcup \end{bmatrix} \to \begin{bmatrix} 01 q_2 10 \\ \#0 q_2 1 \sqcup \end{bmatrix} \to \begin{bmatrix} 011 q_2 0 \\ \# q_2 01 \sqcup \end{bmatrix}$

$\to \begin{bmatrix} 0110 q_2 \sqcup \\ q_2 \#01 \sqcup \end{bmatrix} \to \begin{bmatrix} 0110 q_a \sqcup \\ q_a \#01 \sqcup \end{bmatrix}$, 8 steps.

Problem 5

$I_{CNF} = \{ \langle G \rangle \mid G \text{ is a CNF and } |L(G)| = \infty \}$

(a) Show that any partial parse tree contains an odd number of nodes.

Fact: #nodes $\leq 2^{h+1} - 1$, if height $\leq h$.

For any partial parse tree, we have the start node and 2 subtrees that its mother node of the root of the subtree is the start node.

We can see that these 2 subtrees are both complete binary trees, we let their height be h_1 and h_2.

The total nodes of these 2 subtrees is $2^{h_1+1} - 1 + 2^{h_2+1} - 1 = 2^{h_1+1} + 2^{h_2+1} - 2$, which is even, and plus the start node the sum would be odd.

For CNF (V, Σ, R, S), when #nodes $\geq 2^{|V|+1} - 1$, where $|V|$ is the

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number of variables including the start variable, then some non-S variable will at least appear twice.

(b) We check to remove any non-deriving variables.

For each variable $\in V$, do the following procedure:

1. Make V as a start variable S'.
2. With rules with V on the LHS, mark variables on the RHS
3. Repeat until no new variables are marked:
   For rules with variable on LHS marked, mark the variables on the RHS,
4. Collect the rules, and with V as start variable, send it into decider D if it generates any string.

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5. If does not generate any string, then V is a non-deriving variable. Otherwise it is not a non-deriving variable.

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Subject: 自動機與形式語言 ID: B04705001 Name: 陳約廷 Page: 5

Problem 5

(d) 1. Remove non-deriving variables with procedure in (b), and obtain a new CNF

2. Check if there is looping part of non-S variable that can be repeated as many times as we want.

(We can do this by starting to mark from the start node, then for $S \to V_1V_2$, we mark V_1V_2.

For rules with variable on the LHS marked, mark the variables on the RHS.

If we mark a variable we previously marked (and not when S is on the LHS), then there is a looping non-S part. This procedure is promised to stop because we have eliminated the non-deriving variables. If no variable is repeatedly marked, then there is no looping part)

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3. If there is looping part of variable, accept.

Otherwise, reject

(e). Run on <G a>

Step 1: we do not remove any rules

Step 2: Start from S, mark A, B

A is marked, $A \to BB$, mark B

B is marked, $B \to AA$, mark A, it is marked by a non-S rule.

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So there is a looping non-S part

Step 3: Accept

Problem 5

(e) Run on $\langle G_b \rangle$

Step 1: Remove non-deriving variable, A.

Obtain new CNF, $S \to b$

Step 2: mark $S \to b$. No variables repeatly marked

Step 3: Reject.