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1、$S \to 00S \mid 0A \mid 1A$, $A \to 11A \mid \varepsilon$

Step 1: Make a new starting variable

$S_0 \to S$, $S \to 00S \mid 0A \mid 1A$, $A \to 11A \mid \varepsilon$

Step 2: Remove (var $\to \varepsilon$)

$S_0 \to S$, $S \to 00S \mid 0A \mid 1A$, $A \to 11A \mid 11$

Step 3: Remove (var $\to$ var)

$S_0 \to 00S \mid 0A \mid 1A$, $S \to 00S \mid 0A \mid 1A$, $A \to 11A \mid 11$

Step 4: Add variable for alphabets, $X=0$, $Y=1$

$S_0 \to XXS \mid XA \mid YA$, $S \to XXS \mid XA \mid YA$, $A \to YYA \mid YY$, $X \to 0$, $Y \to 1$

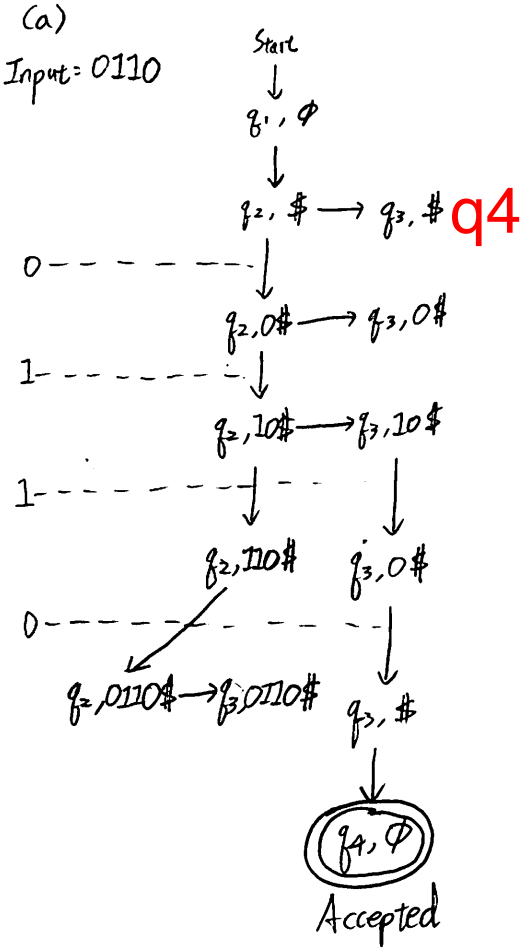
Step 5: Remove (var $\to$ (>2 var)), $I=XX$, $J=YY$

$S_0 \to IS \mid XA \mid YA$, $S \to IS \mid XA \mid YA$, $A \to JA \mid YY$, $I \to XX$, $J \to YY$, $X \to 0$, $Y \to 1$

2、$L=\{ww^R \mid w \in \{0,1\}^*\}$

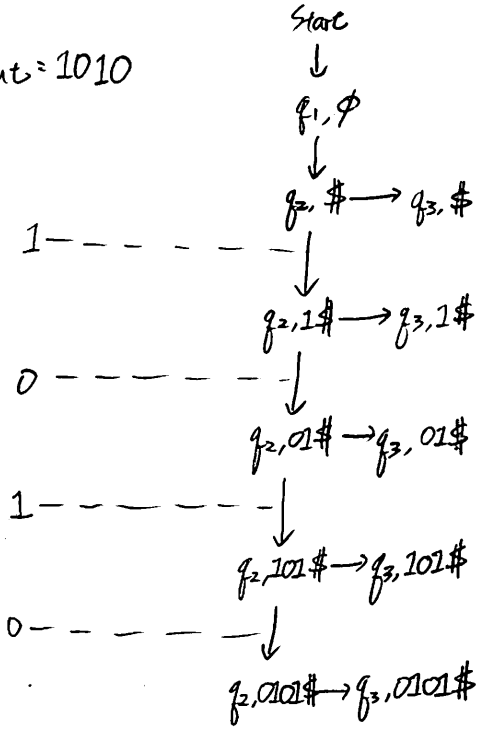
(a)

Input: 0110




(a)

Input: 1010




6

q4

4

2. (b) Give CFG and 3 rules for L. Explain why it has the smallest number of rules.

Starting variable: S, $S \to 0S0 \mid 1S1 \mid \varepsilon$

To prove it to have the smallest number of rules, we explain that it is not enough to generate the language with 2 rules.

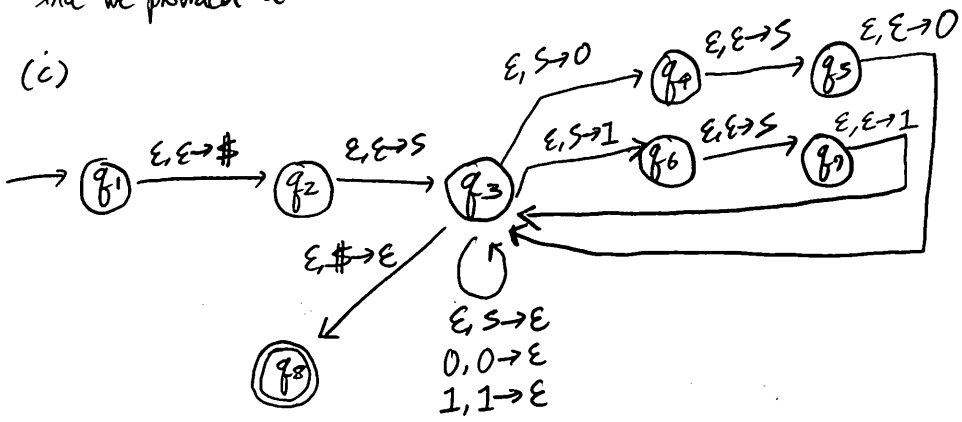
First we need $S \to \varepsilon$ to terminate the string. This gives us 1 rule left.

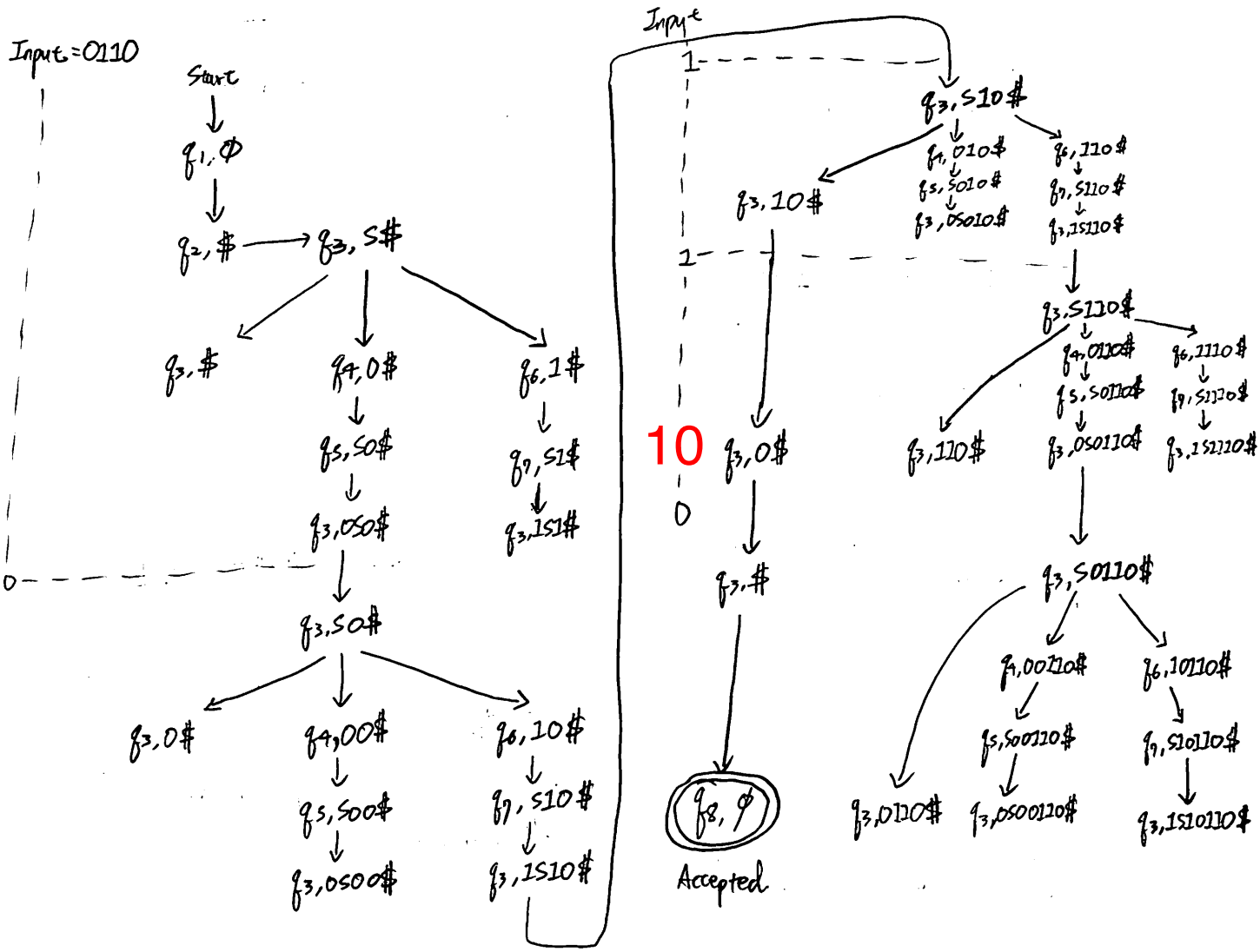
We have two basic strings of $ww^R$, 00 and 11, that belongs to L, but they have totally different alphabets.

So we can not include both of them in a single rule. So 2 rules is impossible to generate L.

Since we provided a 3 rule CFG, hence it is the smallest number of rules.

(c)



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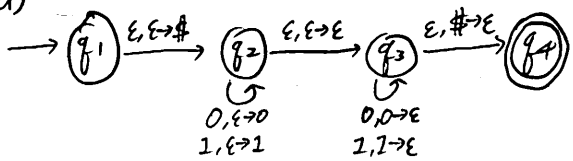
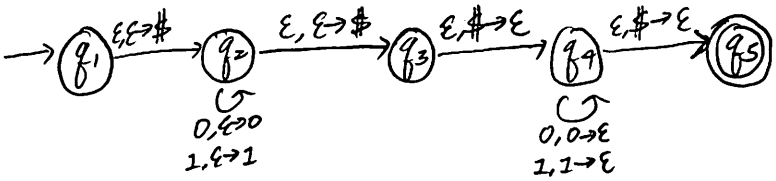
2. (d)




The original PDA does not satisfy (iii) because there is $\varepsilon,\varepsilon\to\varepsilon$ between $q_2$ and $q_3$. We need to modify it.




Now we construct CFG from this PDA. The CFG has 25 variables, $V=\{A_{ij} \mid 1\le i,j\le 5\}$

$\Sigma=\{0,1\}$, Start variable $S=A_{15}$. The rules R are:

$A_{ik}\to A_{ij}A_{jk}$ ($1\le i,j,k\le 5$, total of 125 rules)

$A_{ii}\to \varepsilon$ ($1\le i \le 5$, total of 5 rules)

$A_{15}\to A_{24}$

$A_{24}\to 0A_{24}0$

$A_{24}\to 1A_{24}1$

$A_{24}\to A_{33}$

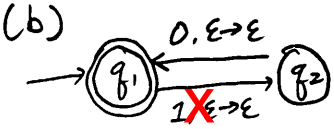
3. $M=(01)^*$

(a) Starting variable: S. $S\to 01S\mid\varepsilon$.

First we explain why it is the smallest number of rules for M, because we cannot build M with only one rule. If we have $S\to 01S$, then we cannot accept an empty string, or if $S\to\varepsilon$, we cannot accept non-empty ones. Hence two rules shall be the minimum.

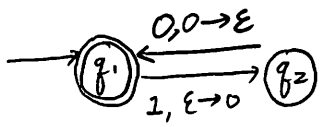
Also the rules are in minimum length. For $S\to\varepsilon$, it is clear that we cannot make it shorter. For $S\to 01S$, we cannot further reduce it because each time we recursively go into $S\to 01S$, the length of the string increases, so all strings generated are therefore unique. And we cannot replace one of the alphabet with another. So this rule is also minimum.

3. (b)




No it does not satisfy because each transition needs to either push or pop a symbol.

So we modify it, with $T=\{0\}$.




Now we construct CFG from this PDA. The CFG has 4 variables, $V=\{A_{ij} \mid 1\le i,j\le 2\}$. $\Sigma=\{0,1\}$, Start variable $S=A_{12}$, The rules R are:

$A_{ik}\to A_{ij}A_{jk}$ ($1\le i,j,k\le 2$, total of 8 rules)

$A_{ii}\to \varepsilon$ ($1\le i \le 2$, total of 2 rules)

$A_{12}\to 1A_{12}0$

~~$A_{12}\to \varepsilon$~~

(c) First we know that both (a) and (b) generates M.

We have the rules in (b)...

① $A_{ik}\to A_{ij}A_{jk}$ ($1\le i,j,k\le 2$, 8 rules)

② $A_{ii}\to \varepsilon$ ($1\le i \le 2$, 2 rules)

③ $A_{12}\to 1A_{22}0$, $A_{12}\to \varepsilon$

Lets first extract ② and ③, and show that ① is redundant.

Because only $A_{12}, A_{11}, A_{22}$ appears on the left-hand side of ② and ③, we only need to show that for these $A_{ij}$, $A_{ij}\to A_{i?}A_{?j}$ is redundant.

First we consider $A_{ii}\to A_{ij}A_{ji}$, $1\le i,j\le 2$.

For $j=1$, we have $A_{ii}\to A_{i2}A_{2i}$, and because $A_{12}$ is the only $A_{i1}$ that appears in ② and ③,

$A_{ii}\to A_{1x}\cdots A_{y1}A_{2i}$

$\to$ a string.

implies that $y=1$. Then eventually $x=1$, but $A_{i1}, i\neq 1$ does not generate any string by ② and ③. Therefore the only possibility is $i=1$ and

$A_{11}\to A_{12}A_{22}\cdots A_{12}$.

For $j=2$, $A_{22}\to A_{02}A_{22}\cdots A_{22}$ because $A_{22}\to\varepsilon$ is the only rule we have in ② and ③.

For $A_{12}$, we have

④ $A_{12}\to A_{1x}A_{x2}$
$\to A_{12}A_{12}$
$\to A_{12}A_{1x}\cdots A_{12}$

or

⑤ $A_{12}\to A_{1x}A_{x2}$
$\to A_{12}A_{22}$
$\to A_{12}A_{2x}\cdots A_{22}$.

We derived this because $A_{11}$ and $A_{22}$ are those of $A_{1x}$ and $A_{x2}$ that appears in ② and ③. For ④, x must be 1 or 2, for ⑤, x must be 2, because that is the only $A_{x2}$ in ② and ③. In the end, right-hand side of ④ and ⑤ is strings of $A_{22}$ and $A_{12}$, or $A_{12}$ and $A_{22}$. From ② and ③, they are redundant.

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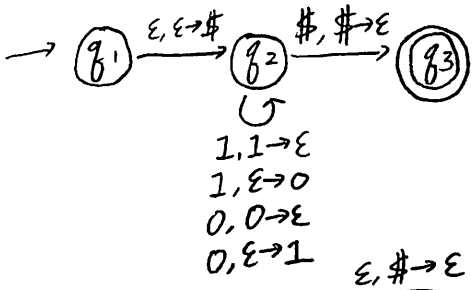
3.(c) Now we have the 8 rules of $A_{ik} \to A_{ij}A_{jk}$ removed. We now have $A_{11} \to \varepsilon$, $A_{22} \to \varepsilon$, $A_{12} \to 0A_{12}1$, $A_{12} \to \varepsilon$.
We have starting variable $A_{12}$, and we find that $A_{11}$, $A_{12}$ is not inside the right-hand side of variables on left-hand with $A_{12}$. So $A_{11}$ and $A_{22}$ are also redundant because they cannot be reached by $A_{12}$.
Now we have $A_{12} \to 0A_{12}1$, $A_{12} \to \varepsilon$.
We have successfully simplified (b) into (a).

4. $\{w\$ \mid w \in \{0,1\}^*, \#_0(w) = \#_1(w)\}$

(a) $T = \{0, 1, \$\}$




It is not a DPDA because in this PDA.
For $q = q_3$, $a \in \Sigma$, $x \in T$,
$\delta(q,a,x)$, $\delta(q,a,\varepsilon)$, $\delta(q,\varepsilon,x)$, $\delta(q,\varepsilon,\varepsilon)$
are all $\emptyset$, which violates the formal definition of DPDA.

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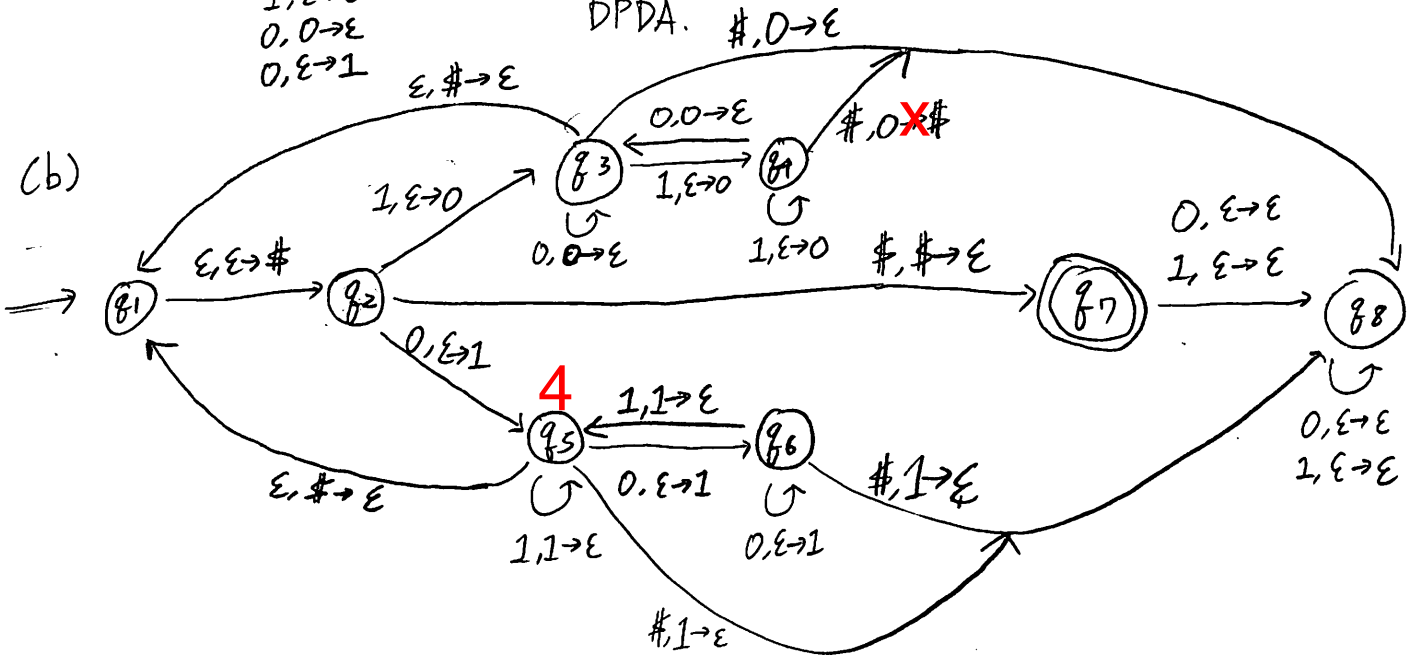
(b)

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(b)

Input: 0011$

start

$q_1, \phi$

$q_2, \#$

0 ----↓

$q_5, 1\#$

0 ----↓

$q_6, 11\#$

1 ----↓

$q_5, 1\#$

1 ---↓

$q_5, \#$

↓

$q_1, \phi$

$ ---↓

$q_2, \#$

↓

$q_7, \phi$

Accepted

3

Input: 1#

start

$q_1, \phi$

↓

$q_2, \#$

1 ----↓

$q_3, 0\#$

$ ----↓

$q_8, \#$

Not Accepted.