*Please fill the blanks in every page. Thanks!

*Please use BOTH sides of paper.

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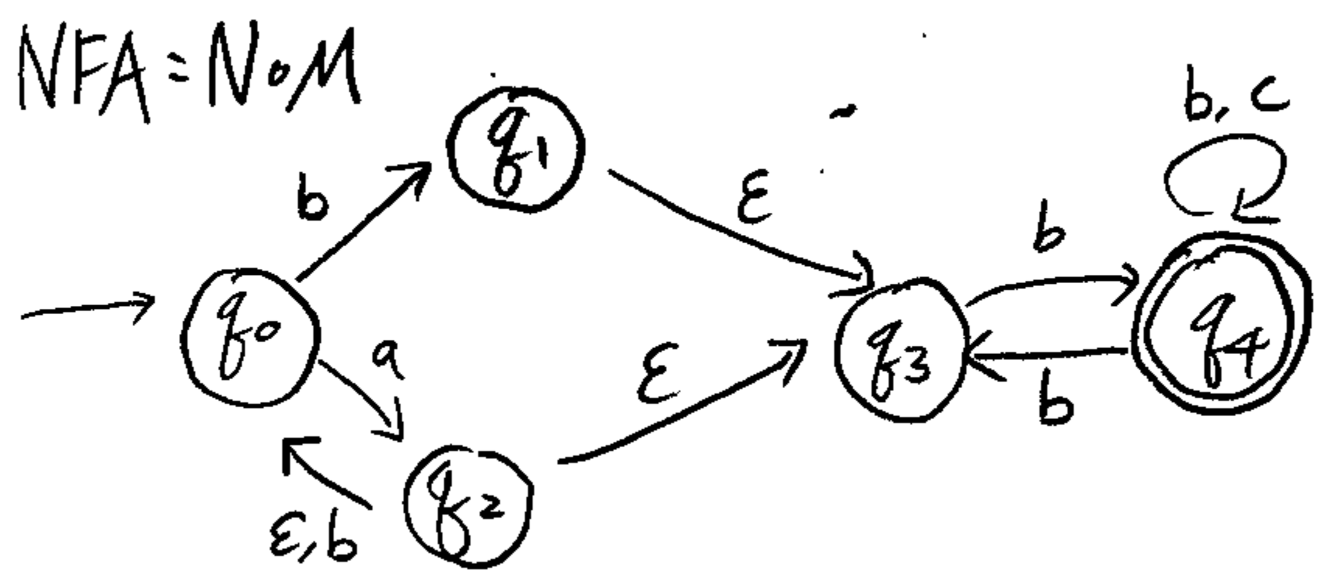
## Problem 1

1. NFA: NoM

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2. Formal definition of NoM.

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b, c\}$

$q_0 = q_0$

$F = \{q_4\}$

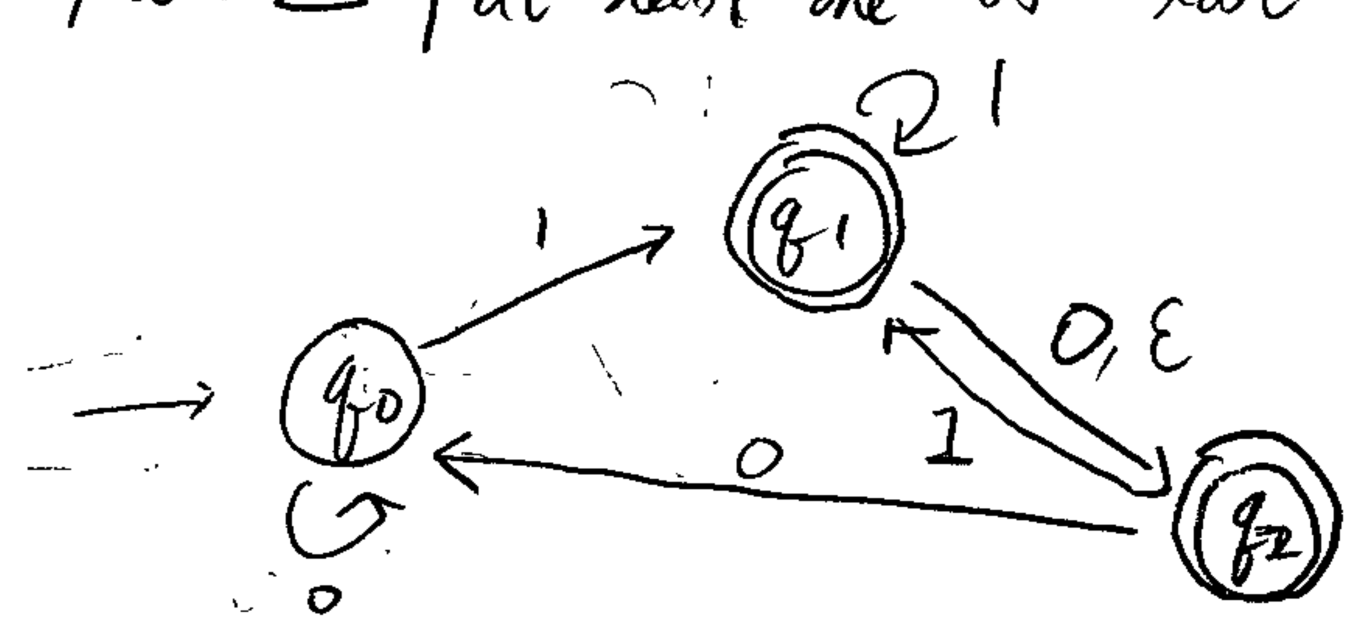
| $\delta$ | a | b | c | ε |
|---|---|---|---|---|
| $q_0$ | $\{q_2\}$ | $\{q_1\}$ | $\phi$ | $\phi$ |
| $q_1$ | $\phi$ | $\phi$ | $\phi$ | $\{q_3\}$ |
| $q_2$ | $\phi$ | $\{q_0\}$ | $\phi$ | $\{q_0, q_3\}$ |
| $q_3$ | $\phi$ | $\{q_4\}$ | $\phi$ | $\phi$ |
| $q_4$ | $\phi$ | $\{q_3, q_4\}$ | $\{q_4\}$ | $\phi$ |

## Problem 2

$\Sigma = \{0, 1\}$

$A = \{ w \in \Sigma^* \mid$ at least one of last 2 character of $w$ is $1\}$

(a)

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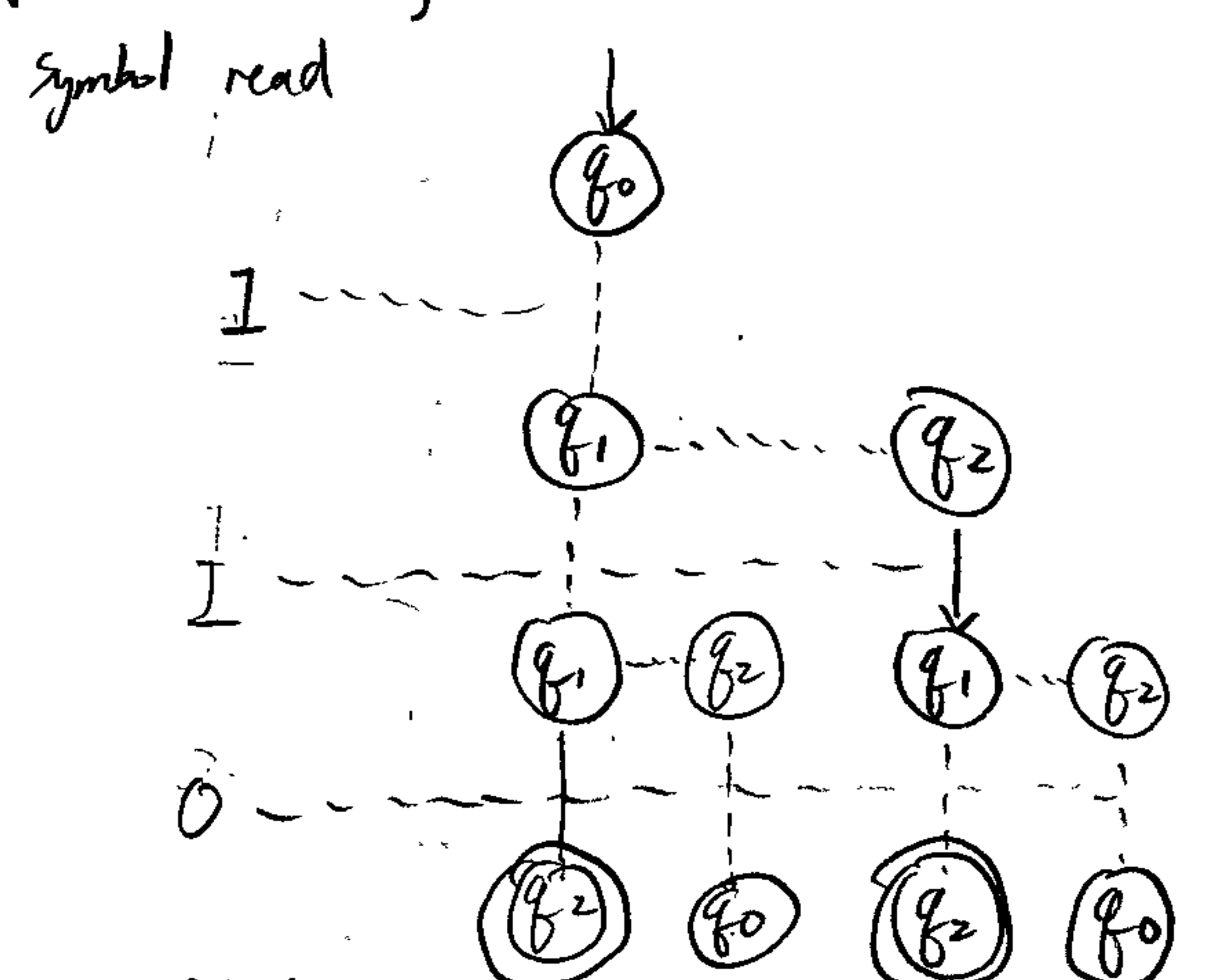


(b) 1. There is no unreachable state in the above NFA.

2. We examine for any nondistinguishable states in the automaton.

So for all state pairs, we examine if they are distinguishable.

For state $q_0$ and $q_1$, input = 0 makes $q_1 \to q_2$ accept but not $q_0 \to q_0$

For state $q_1$ and $q_2$, input: 0 makes $q_1 \to q_2$ accept but not $q_2 \to q_0$

For state $q_0$ and $q_2$, input: ε makes $q_2$ accept but not $q_0$

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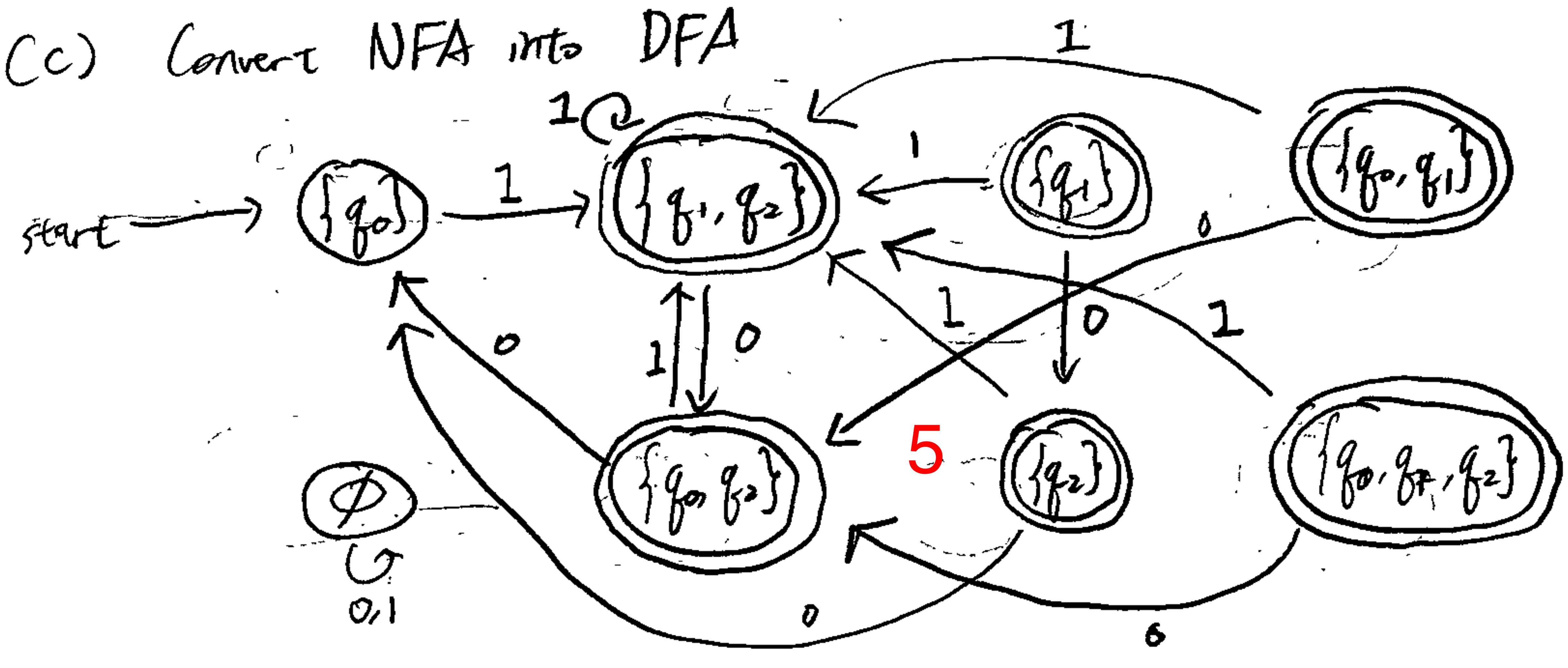
So all state pairs are distinguishable, therefore this NFA has smallest number of states.

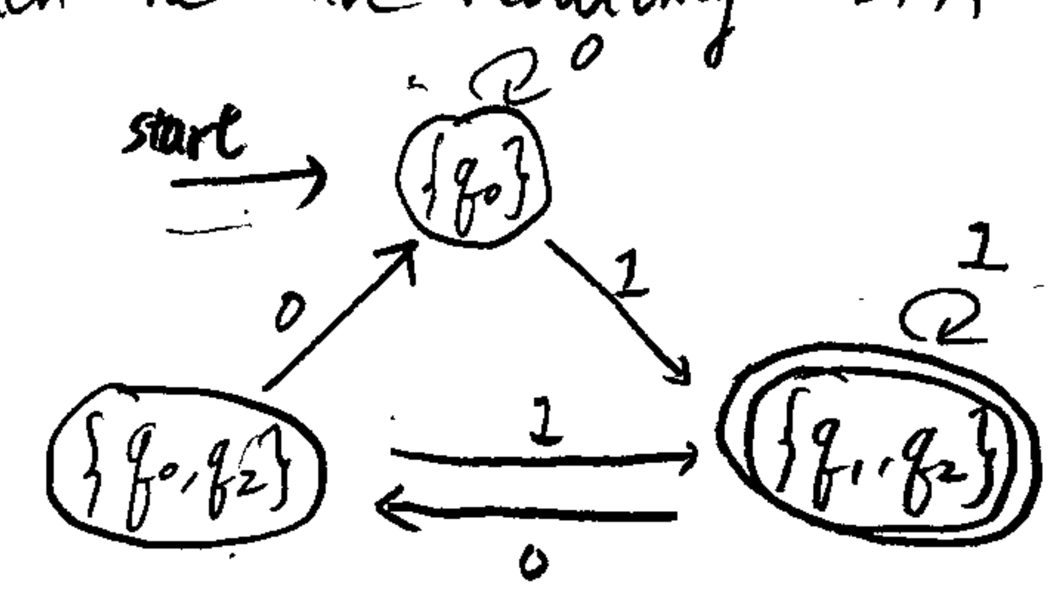
## Problem 2

(c) Convert NFA into DFA




(d) First we eliminate unreachable states, which are states with no in degree. We eliminate $\{q_1\}$, $\{q_0, q_1\}$, $\{q_0, q_1, q_2\}$, $\emptyset$. Then we find out that $\{q_2\}$ now has no in-degree, so also remove it.

Then we have resulting DFA.




As there is no nondistinguishable states mentioned in (a), which we distinguish $\{q_0\}$, $\{q_1, q_2\}$ by $z = 0$

$\{q_1, q_2\}$, $\{q_0, q_2\}$ by $z = 0$

$\{q_0\}$, $\{q_0, q_2\}$ by $z = \varepsilon$

This is the minimal DFA.

It should be unique because if it is not unique, then we would find nondistinguishable states on the DFA and we can reduce it even smaller. However since we examined and know that all states are distinguishable, the DFA should be minimal and unique.

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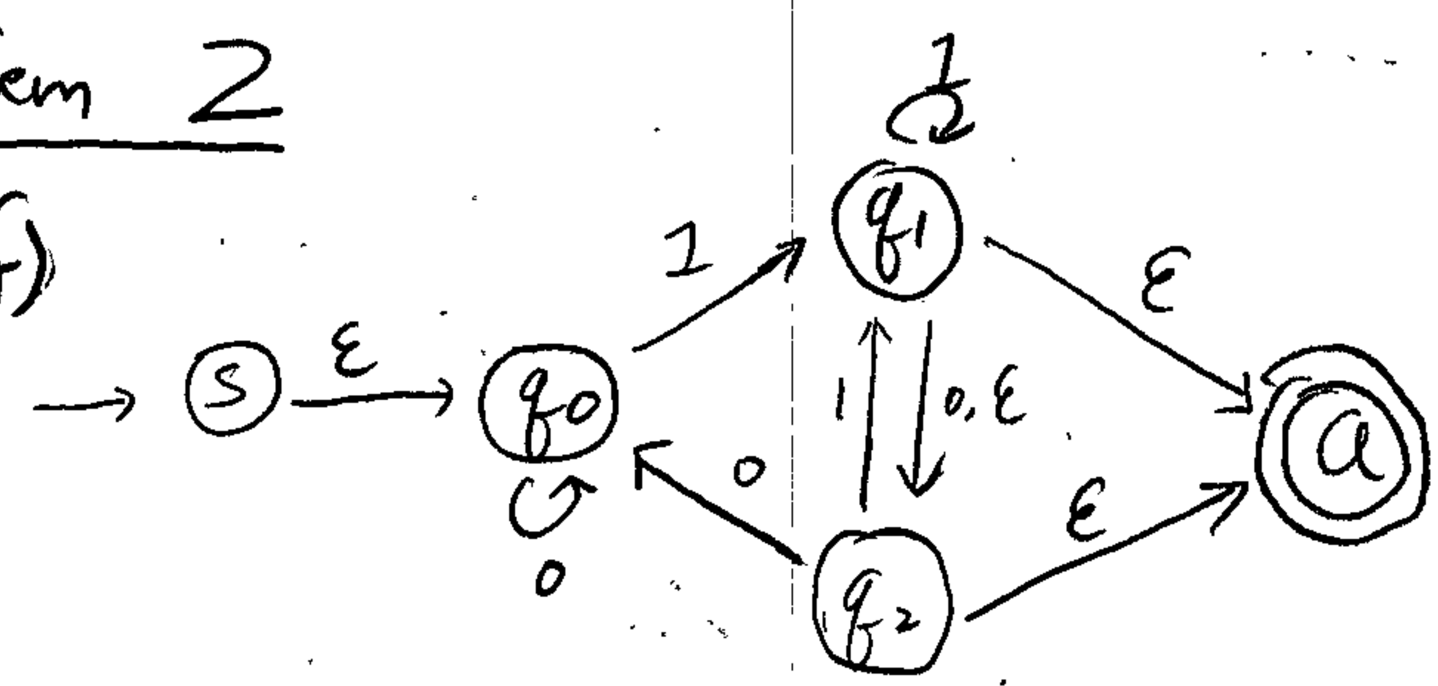
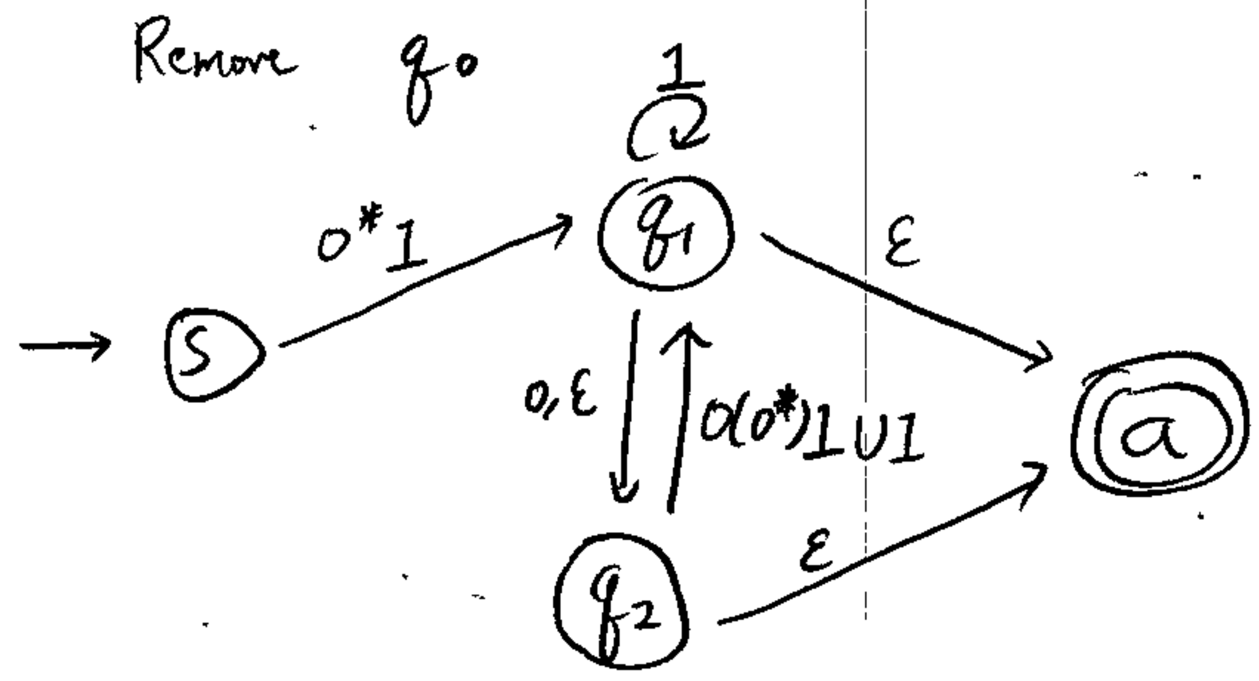
## Problem 2

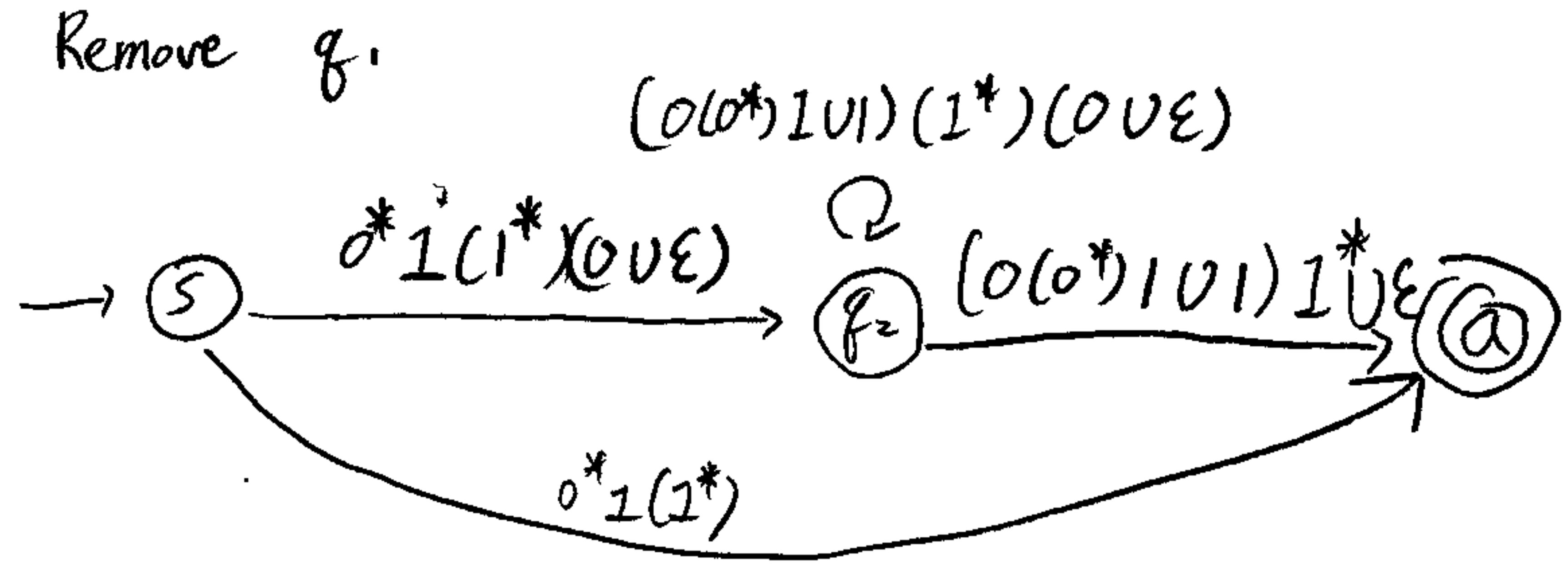
(f)




Remove q0




Remove q1




Remove q2

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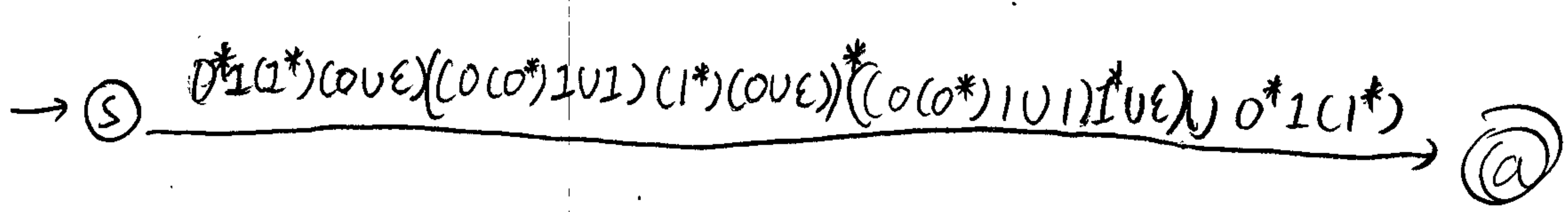
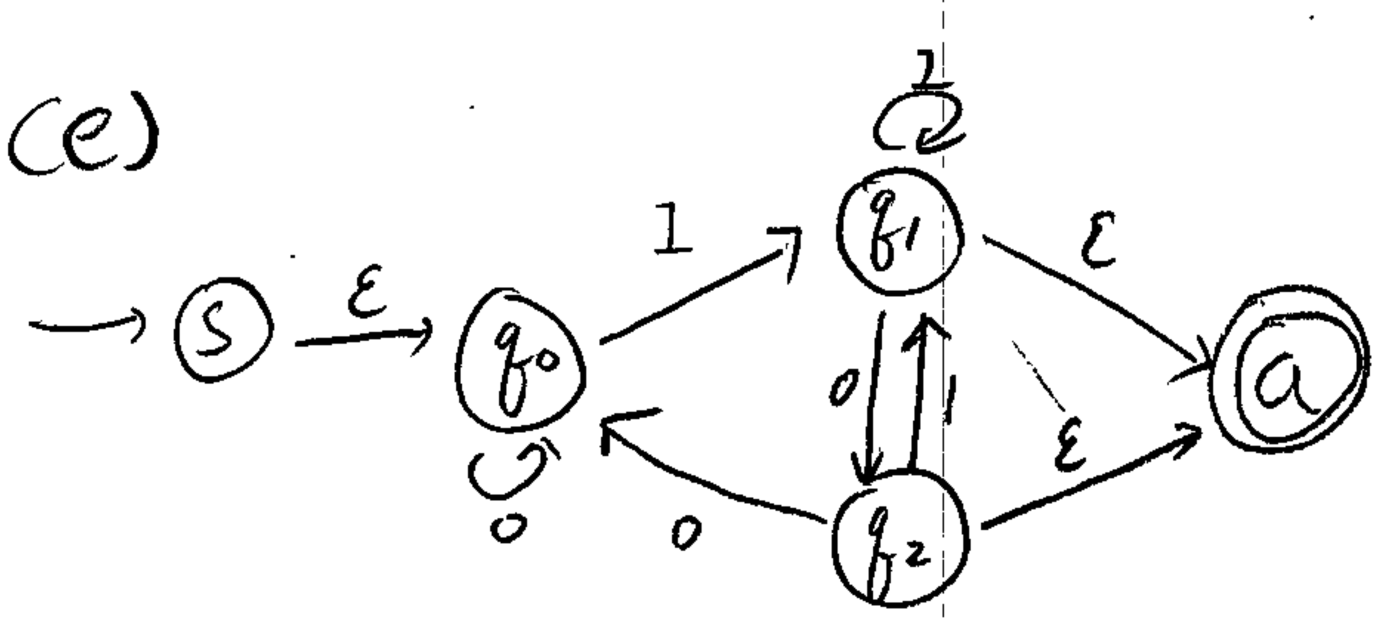
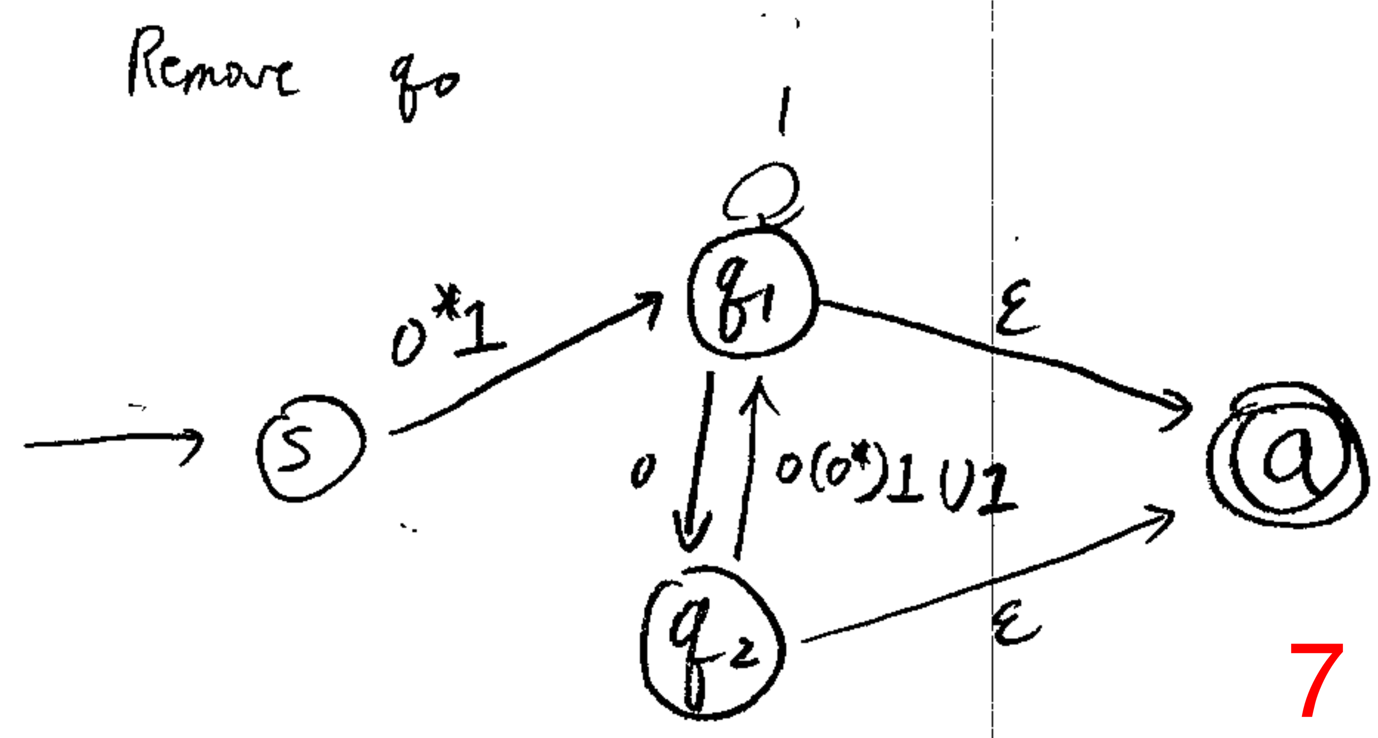



Now we have regex $= 0^*1(1^*)(0\cup\varepsilon)((0(0^*)1\cup1)(1^*)(0\cup\varepsilon))^*(0(0^*)1\cup1)1^*\cup 0^*1(1^*)$

(e)




Remove q0

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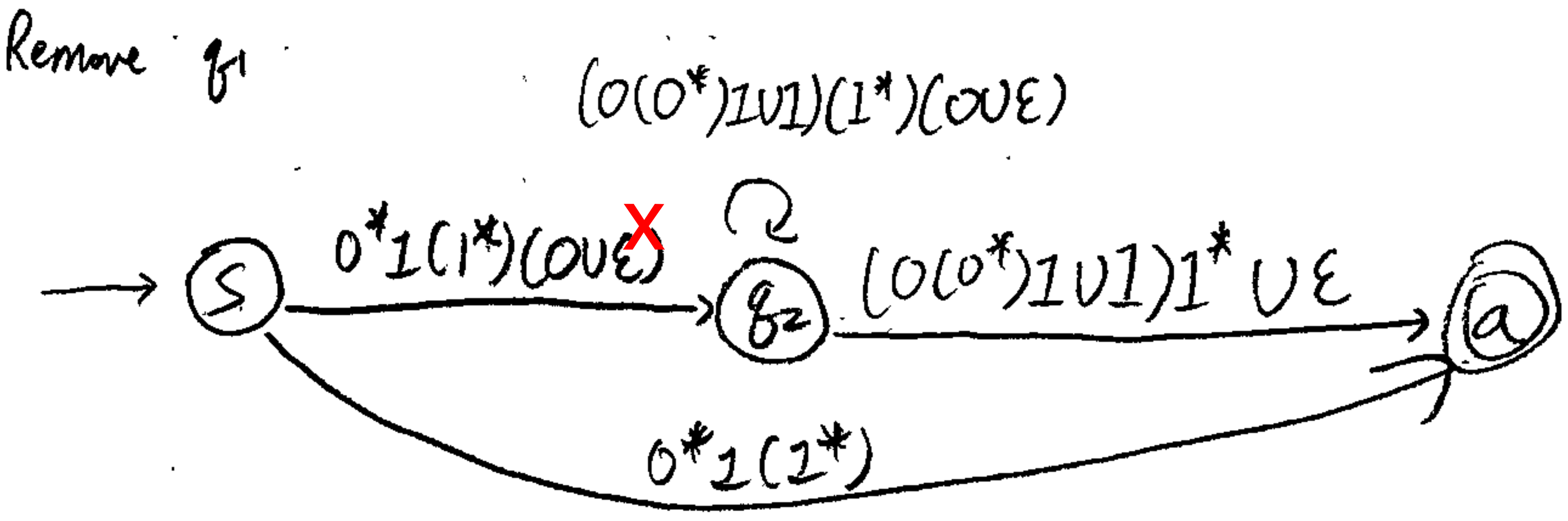
Remove q1



(e) Remove $q_2$

$$\rightarrow (s) \xrightarrow{0^*1(1^*)(0\cup\varepsilon)((0(0^*)1\cup 1)(1^*)(0\cup\varepsilon))^*((0(0)^*1\cup 1)1^*\cup\varepsilon)\cup 0^*1(1^*)} ((a))$$

Now we have regex: $0^*1(1^*)(0\cup\varepsilon)((0(0^*)1\cup 1)(1^*)(0\cup\varepsilon))^*((0(0^*)1\cup 1)1^*\cup\varepsilon)\cup 0^*1(1^*)$

## Problem 3.

$B = \{1^n \mid n = 2^k, k \geq 0\}$

Assume $B$ regular.

Let $p$ be the pumping length of $B$ given by the pumping lemma.

Let $s = 1^{2^p}$, by the pumping lemma we have $s = xyz$.

By condition 2 and 3 of the lemma, $y = 1^k$, $0 < k \leq p$.

Now consider $s' = xy^5z = 1^{2^p+4k}$, if $p > 3$, $2^{p+1} > 2^p + 4k > 2^p$

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if $p \leq 2$, $2^p + 4k \neq 2^j$ for any $j \in \mathbb{N}$ p≥3?

So $s' \notin B$, thus $B$ is not regular.

## Problem 4

$\Sigma = \{0,1\}$, $\varepsilon$ is not a substring.

$d(w) = |(\# \text{ of } 1\text{'s}) - (\# \text{ of } 0\text{'s})|$

(a) $C = \{w \in \Sigma^* \mid d(w) \leq 2\}$

Assume $C$ regular.

Let $p$ be the pumping length of $C$ given by the pumping lemma.

Let $s = 0^p1^p$ where $d(s) = 0$, by the pumping lemma we have $s = xyz$.

And by condition 2 and 3 of the lemma, $y = 0^k$, $0 < k \leq p$.

Now consider $s' = xy^4z = 0^{p+3k}1^p$, $d(s') = 3k > 2$, so $s' \notin C$

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Thus $C$ is not regular.

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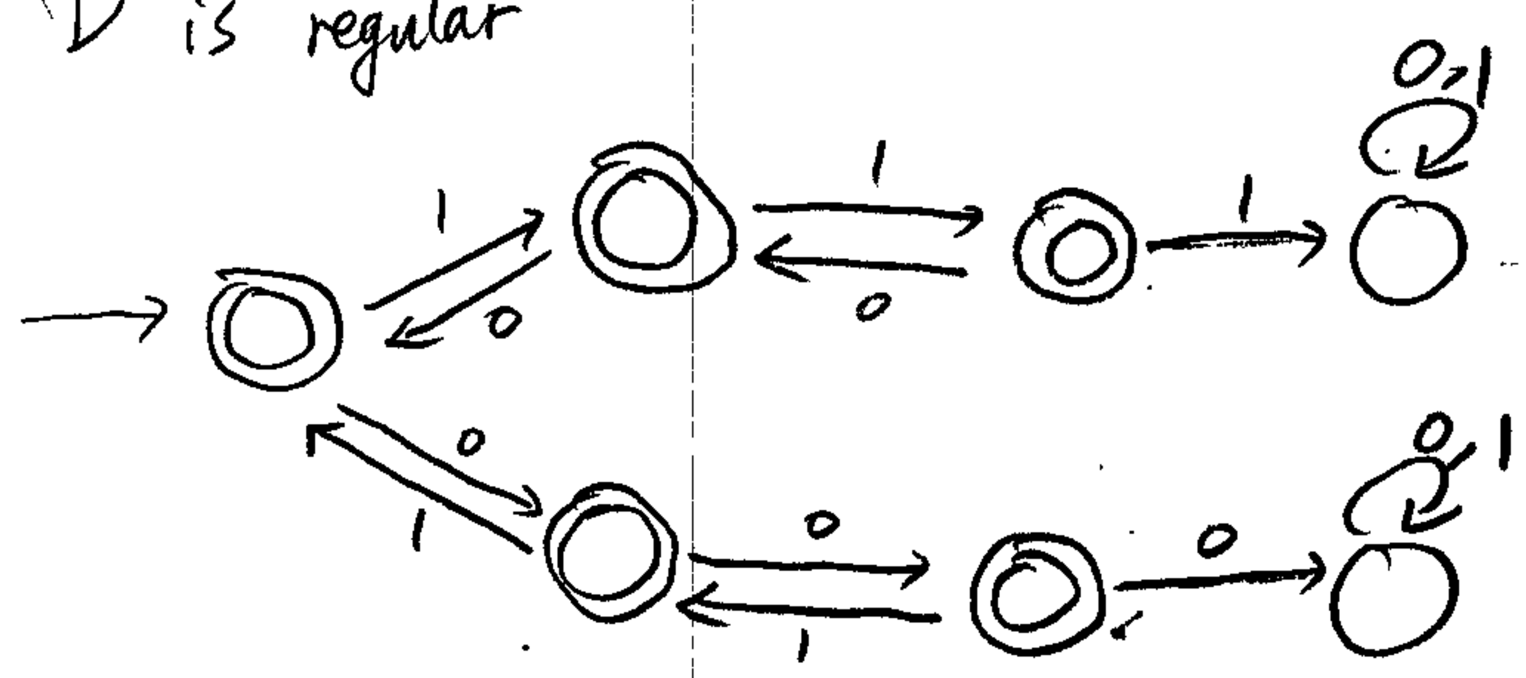
## Problem 4

(b) $D = \{ w \in \Sigma^n \mid n \geq 1 \text{ and } d(w_{1:j}) \leq 2 \text{ for } 1 \leq j \leq n \}$

D is regular

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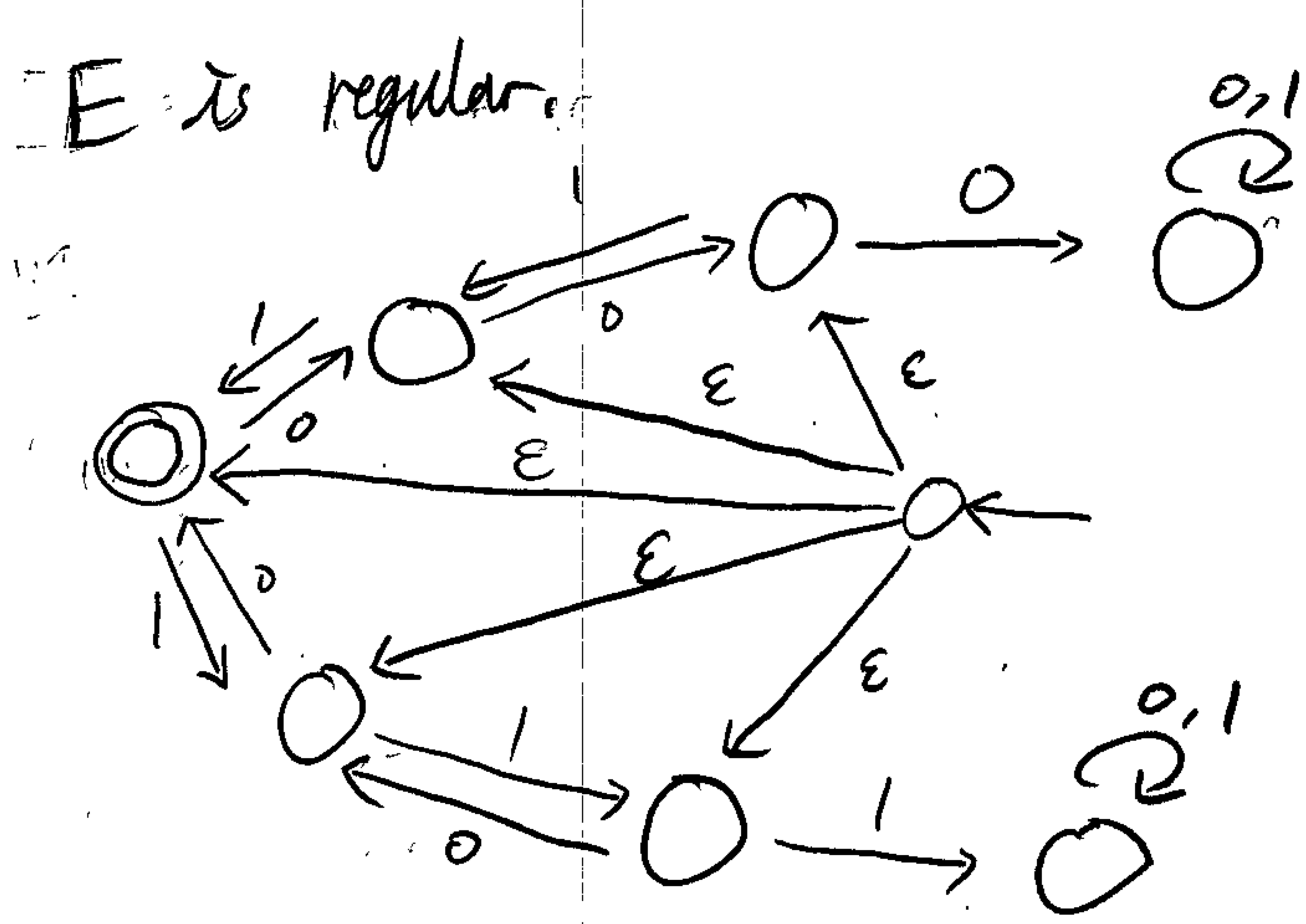
This DFA recognizes D because the state of the DFA don't need to increase as n (length of string) goes larger, and is able to classify $d(w) = 0, 1, 2$.

(c) $E = \{ w \in \Sigma^n \mid n \geq 1 \text{ and } d(w_{i:n}) \leq 2 \text{ for } 1 \leq i \leq n \}$

E is regular.

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This is a NFA constructed using the reverse operation. Since $D^R = E$. So if we claim and prove that regular language is closed under reverse operation, since D is regular, E will be regular too. (claim and prove at next page)

Claim: Regular language is closed under reverse

Proof: Given a regular language $A$. Since $A$ is regular, there is DFA

$M = \langle Q, \Sigma, \delta, q_0, F \rangle$ that recognizes $A$.

From this construct an NFA $M'$ that recognizes $A^R$.

Construction: $M' = \langle Q \cup \{q_0'\}, \Sigma, \delta', q_0', \{q_0\} \rangle$, where

$$\delta'(q_0', \epsilon) = F \qquad (\delta'.1)$$

$$\delta'(q_0', a) = \phi \quad \text{all } a \in \Sigma \qquad (\delta'.2)$$

$$\delta'(p, a) = \{q \mid \delta(q, a) = P \quad \text{all } p \in Q, a \in \Sigma \qquad (\delta'.3)$$

Also, claim that $L(M') = A^R$. We prove this by showing

(1) $w \in L(M) \rightarrow w^R \in L(M')$

(2) $w \in L(M') \rightarrow w^R \in L(M)$

(1) Since $w \in L(M)$, we know $w = w_1 w_2 \cdots w_n$, and exist states $r_0, r_1, \cdots, r_n$ s.t. $r_0 = q_0$, $r_n \in F$, and $\forall i, 0 < i \leq n$, $r_i = \delta(r_{i-1}, w_i)$

In this case, $M'$ will accept $w^R$, which will be rewritten as $\epsilon w_n w_{n-1} \cdots w_1$ with state sequence $q_0', r_n, r_{n-1}, \cdots, r_1$. Note that $q_0'$ and $r_1 = q_0$ are initial and final states of $M'$, so to complete the argument of $w^R$ accepted by $M'$ we only need to show each transaction is valid.

First transaction: $r_n \in \delta'(q_0', \epsilon)$, establishing $r_n \in F$.

The remainder: $r_{i-1} \in \delta'(r_i, w_i)$, by $(\delta'.3)$ this becomes

$r_{i-1} \in \{q \mid \delta(q, w_i) = r_i\}$. This follows immediately from $\delta(r_{i-1}, w_i) = r_i$

which was established by $w \in L(M)$.

(2) Since $w \in L(M')$, we know that $w = w_1 w_2 \cdots w_n$ and exist states $r_0, r_1 \cdots r_n$ s.t. $r_0 = q_0'$, $r_n \in \{q_0\}$ and $r_{i+1} \in \delta'(r_i, w_{i+1})$

Furthermore, since clauses $(\delta'.1)$ and $(\delta'.2)$ define all transition on $q_0'$, we know that $w_1 = \epsilon$ and $r_1 \in F$. Since all other transition are defined by clauses $(\delta'.3)$, we know that states $r_1, r_2 \cdots r_n$ are in $Q$, the state space of DFA $M$.

Show $w^R \in L(M)$. by showing that $M$ accepts $w_n, w_{n-1}, \cdots w_2$. with state sequence $r_n, r_{n-1} \cdots r_1$.

First note that $r_n$ is $q_0$ and $r_1 \in F$. It remains to show that $r_{i-1} = \delta(r_i, w_i)$

Since $w \in L(M')$, we know that $r_i \in \delta'(r_{i-1}, w_i)$, that is $r_i \in \{q \mid \delta(q, w_i) = r_{i-1}\}$

So $\delta(r_i, w_i) = r_{i-1}$ as required.