Spatial models for public health and economic strategies for COVID-19

KI, SK, MR

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1 An short excurison into SIR models

Richard Neher's model [1]

$$\frac{d}{dt}S_a = -\beta(t)S_a(t)\sum_b I_b(t) \tag{1}$$

$$\frac{d}{dt}E_a = \beta(t)S_a(t)\sum_b I_b(t) - E_a(t)/t_l \tag{2}$$

$$\frac{d}{dt}I_a = E_a(t)/t_l - I_a(t)/t_i \tag{3}$$

$$\frac{d}{dt}R_a = m_a I_a(t)/t_i + (1-c_a)H_a(t)/t_h \tag{4}$$

$$\frac{d}{dt}H_a = (1-m_a)I_a(t)/t_i + (1-f_a)C_a(t)/t_C \tag{5}$$

$$\frac{d}{dt}C_a = c_a H_a(t)/t_h - C_a(t)/t_c \tag{6}$$

$$\frac{d}{dt}D_a = f_a C_a(t)/t_c \tag{7}$$

where age index, $a = 1, 2, ..., 9$ standing for age categories: $0-9, 10-19, ..., 80+$.

$$\beta_a(t) = R_0 \zeta_a M_(t)(1 + \varepsilon\, cos\left(\frac{2\pi(t - t_{max})}{t_i}\right)$$

Parameter	Symbol	Value	Units
avg interactions per day	R_0	2-3	per day
degree of isolation	ζ_a	0-1	
mitigation	$M(t)$	0-1	
seasonal driving	ε	0	
peak of seasonal effects	t_{max}	Jan 2020	time
average latency period	t_l	5	days
average infectious period	t_i	3	days
average hospitalisation time	t_h	4	days
average time in ICU	t_c	14	days
proportion of mild symptoms	m_a	0-1	
proportion requiring critical care	c_a	0-1	
proportion for which the disease is fatal	f_a	0-1	

Table 1: Parameters in the model

Age groups:	0-9	10-19	20-29	30 -39	40-49	50-59	60-69	70-79	80+
m_a	0.9995	0.9985	0.997	0.9955	0.988	0.975	0.925	0.86	0.75
c_a	0.05	0.1	0.1	0.15	0.2	0.25	0.35	0.45	0.55
f_a	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.5	0.5

Table 2: Age-specific parameters in the model

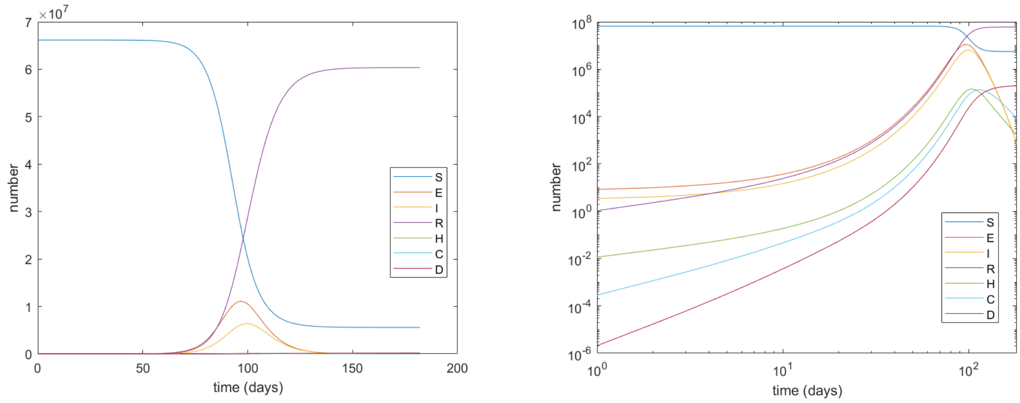


Figure 1: Test run for Karnataka with a population $N = 66165886$ and initial exposed/infected population of 10, i.e. $E_4 = 7$, $I_4 = 3$ with no mitigation, no imports.

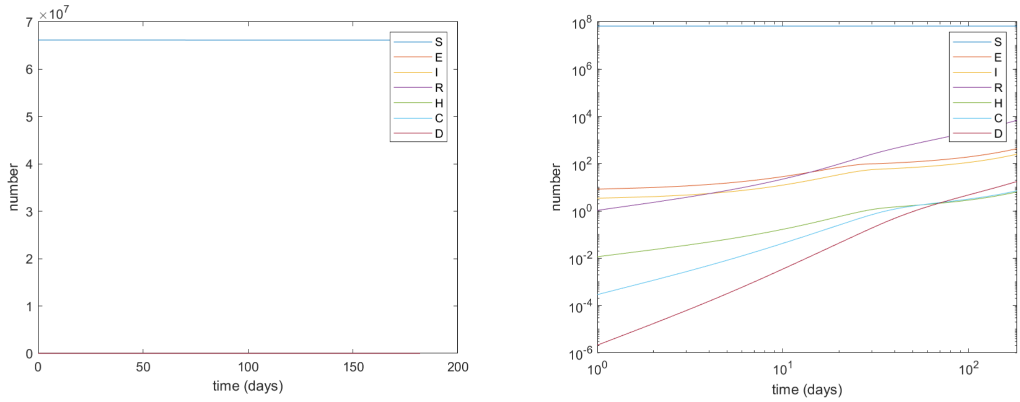


Figure 2: Test run for Karnataka with a population $N = 66165886$ and initial exposed/infected population of 10, i.e. $E_4 = 7$, $I_4 = 3$ with strong mitigation, no imports

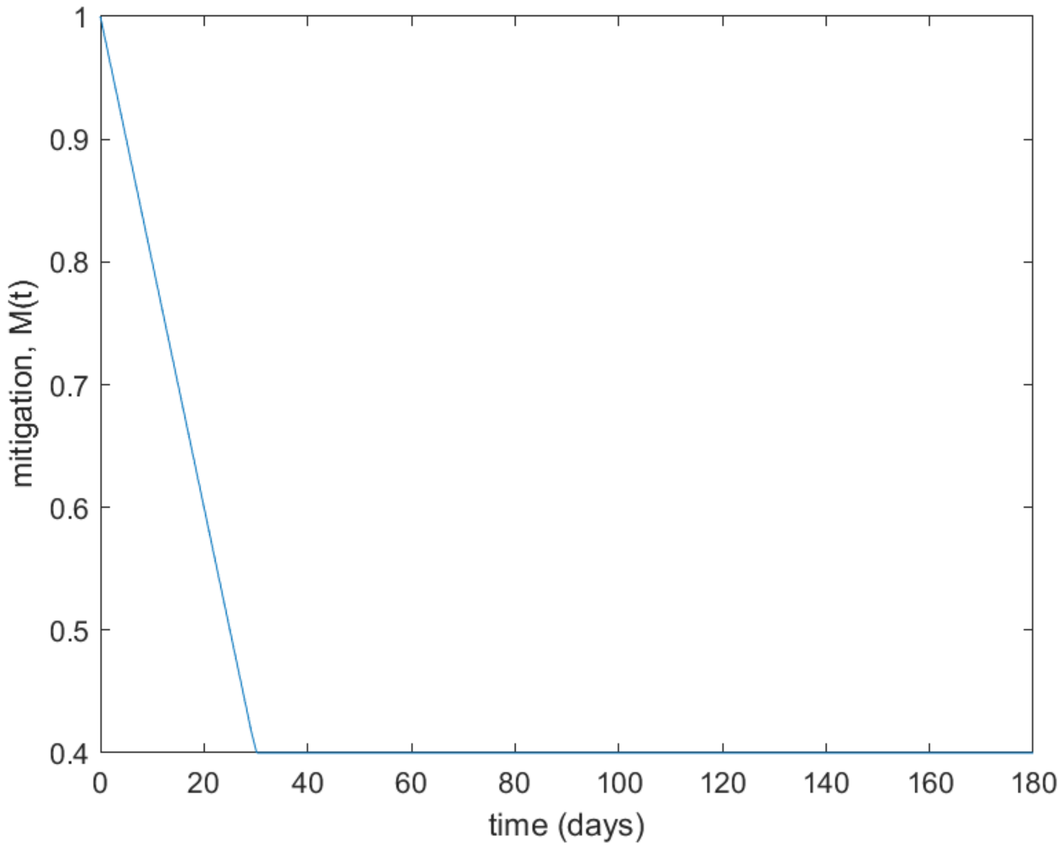


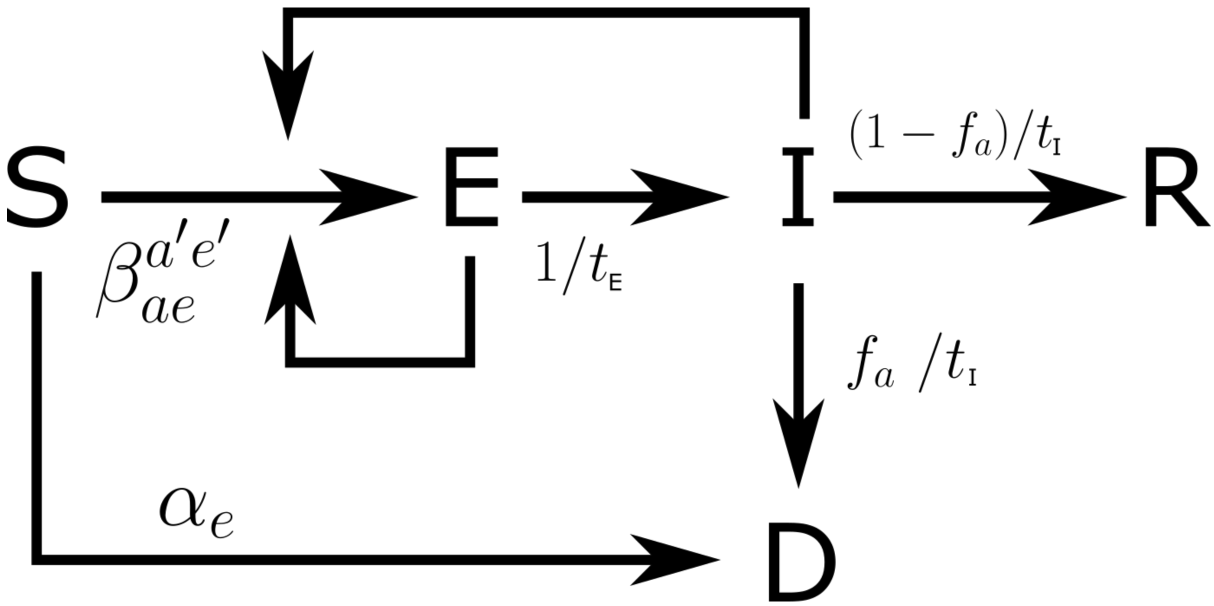
Figure 3: Mitigation curve for Fig. 2

2 Our model including economic demographic

In our model we consider a city and several, N_v, villages. In each of them the population is divided into:

1. 3 economic categories, into immobile poor, mobile poor and rich
2. 3 age categories, into children (0-14), young (15-59) and old (60+)
3. 5 state, Susceptible (S), Exposed (E), Infectious (I), Recovered (R), Dead (D)

While we will label the states with a capital letter as indicated next to the states in the list above, we will use a subscript to denote age (a) and economic (e) categories. Therefore, $S_{ae}, I_{ae}, \ldots$. The dynamics is given by the following reaction graph:



and the corresponding dynamics are given by the equations below

$$\frac{d}{dt}S_{ae} = -\sum_{a',e'} \beta^{ae}_{a'e'} S_{ae}(t)\Big(I_{a'e'}(t) + E_{a'e'}(t)\Big) + \mathcal{M}^S_{ae}(t) \tag{8}$$

$$\frac{d}{dt}E_{ae} = \sum_{a',e'} \beta^{ae}_{a'e'} S_{ae}(t)\Big(I_{a'e'}(t) + E_{a'e'}(t)\Big) - E_{ae}(t)/t_E + \mathcal{M}^E_{ae}(t) \tag{9}$$

$$\frac{d}{dt}I_{ae} = E_{ae}(t)/t_E - I_{ae}(t)/t_I + \mathcal{M}^I_{ae}(t) \tag{10}$$

$$\frac{d}{dt}R_{ae} = (1-f_a)I_{ae}(t)/t_I + \mathcal{M}^R_{ae}(t) \tag{11}$$

$$\frac{d}{dt}D_{ae} = f_a I_{ae}(t)/t_I + \alpha_e S_{ae}(t) \tag{12}$$

Notice that the parameters have subscripts too. This shows that they dependent on the age (a) and economic (e) category too. The full list of these parameters and their meaning is provided below

Parameter	**Symbol**
interactions rate between S_{ae} and $I_{a'e'}$ / $E_{a'e'}$	$\beta^{ae}_{a'e'}$
time spent in E (latency period)	t_E
time spent in I (infectious period)	t_I
proportion for which the disease is fatal	f_a
import rate	$\mathcal{M}$

Table 3: Parameters in the model

This to capture the effect of city's mobile poor population's migration from the city to these villages.

We add an economic demographic beside age to all the species in the model, e.g. $S_{a,e}$ where $a = 1, 2, 3$ standing for age categories 0-14, 15-59, 60+, $e = 1, 2, 3$ for poor, middle-class, rich.

References

[1] https://neherlab.org/covid19/about