Spatial models for public health and economic strategies for COVID-19

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1 Our model including economic demographic

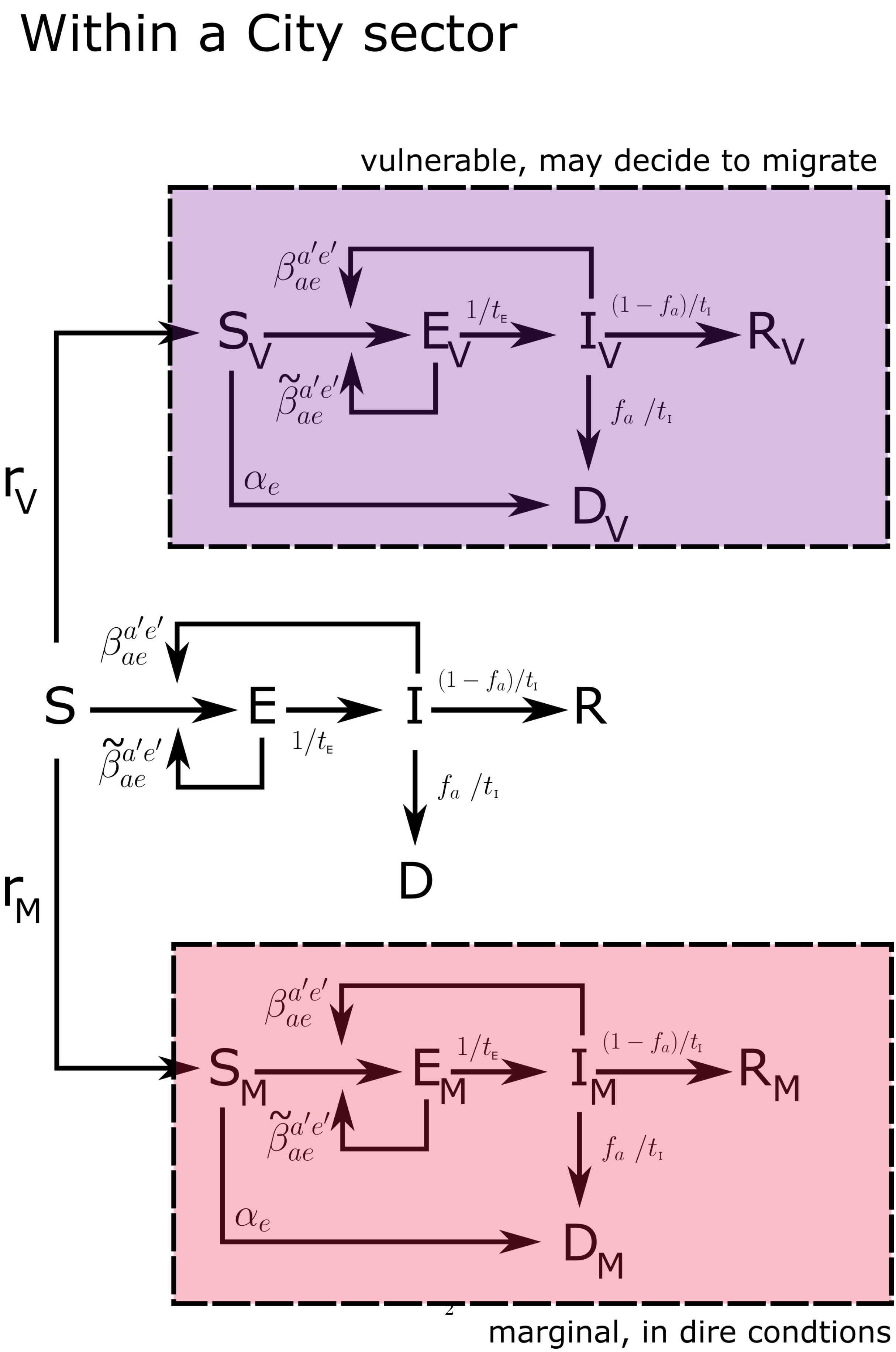
In our model we consider a city and several, n_v, villages. In each of them the population is divided into:

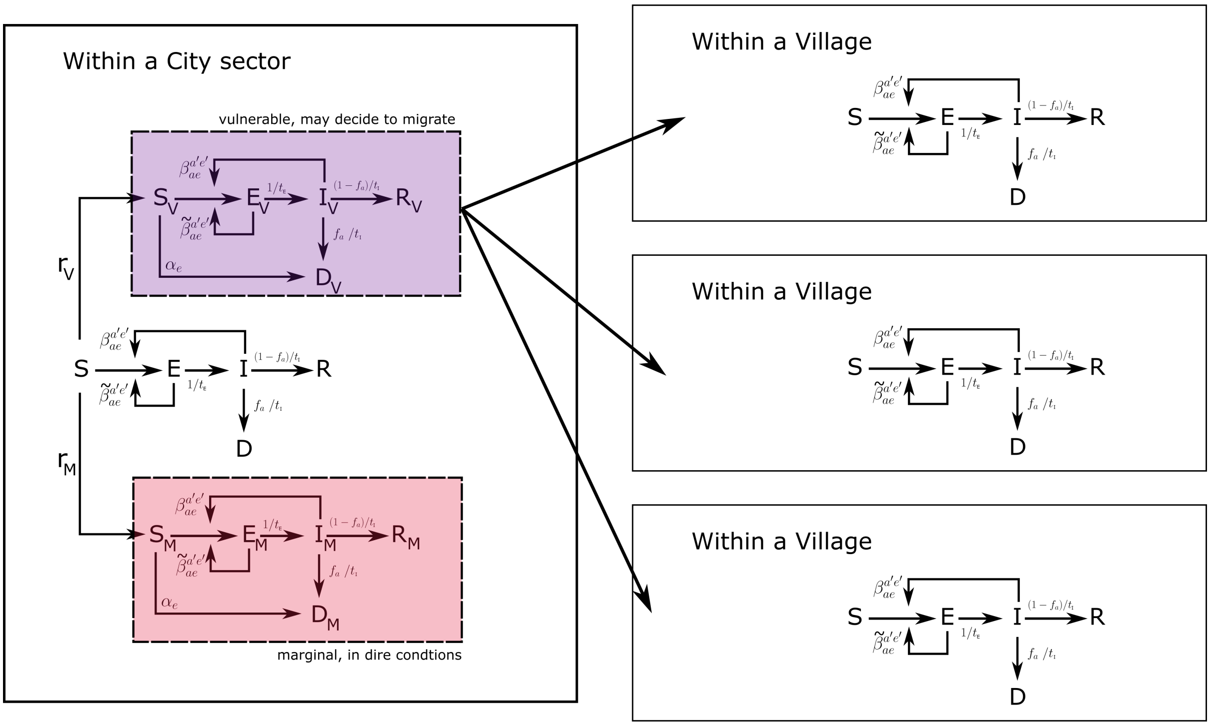
1. 3 economic categories: immobile poor, mobile poor and rich
2. 3 age categories: children (0-14), young (15-59) and old (60+)
3. 5 states: Susceptible (S), Exposed (E), Infectious (I), Recovered (R), Dead (D)

While we will label the states with a capital letter as indicated next to the states in the list above, we will use a subscript to denote age (a) and economic (e) categories. Therefore, $S_{ae}, I_{ae}, ...$ such that

$$N(t) = \sum_{a,e} N_{ae}, \qquad N_{ae} = S_{ae}(t) + E_{ae}(t) + I_{ae}(t) + R_{ae}(t) + D_{ae}(t)$$

for a city and villages individually. The dynamics is described by the following reaction graph:


Within a City sector
vulnerable, may decide to migrate
S_V
E_V
I_V
R_V
D_V
$\beta_{ae}^{a'e'}$
$\tilde{\beta}_{ae}^{a'e'}$
$1/t_E$
$(1-f_a)/t_I$
$f_a\ /t_I$
α_e
r_V
S
E
I
R
D
$\beta_{ae}^{a'e'}$
$\tilde{\beta}_{ae}^{a'e'}$
$1/t_E$
$(1-f_a)/t_I$
$f_a\ /t_I$
r_M
S_M
E_M
I_M
R_M
D_M
$\beta_{ae}^{a'e'}$
$\tilde{\beta}_{ae}^{a'e'}$
$1/t_E$
$(1-f_a)/t_I$
$f_a\ /t_I$
α_e
marginal, in dire condtions



and the corresponding equations within each module are

$$\frac{d}{dt}S_{ae} = -S_{ae}(t)\sum_{a',e'}\left(\beta^{ae}_{a'e'}(t)\frac{I_{a'e'}(t)}{N_{a'e'}(t)} + \tilde{\beta}^{ae}_{a'e'}(t)\frac{E_{a'e'}(t)}{N_{a'e'}(t)}\right) + \mathcal{M}^S_{ae}(t) \tag{1}$$

$$\frac{d}{dt}E_{ae} = S_{ae}(t)\sum_{a',e'}\left(\beta^{ae}_{a'e'}(t)\frac{I_{a'e'}(t)}{N_{a'e'}} + \tilde{\beta}^{ae}_{a'e'}(t)\frac{E_{a'e'}(t)}{N_{a'e'}}\right) - E_{ae}(t)/t_E + \mathcal{M}^E_{ae}(t) \tag{2}$$

$$\frac{d}{dt}I_{ae} = E_{ae}(t)/t_E - I_{ae}(t)/t_I + \mathcal{M}^I_{ae}(t) \tag{3}$$

$$\frac{d}{dt}R_{ae} = (1-f_a)I_{ae}(t)/t_I + \mathcal{M}^R_{ae}(t) \tag{4}$$

$$\frac{d}{dt}D_{ae} = f_a I_{ae}(t)/t_I + \alpha_e S_{ae}(t) \tag{5}$$

and the intermodule transitions are absorbed in to $\mathcal{M}$.

Notice that the parameters are dependent on age (a) and economic (e) category too.

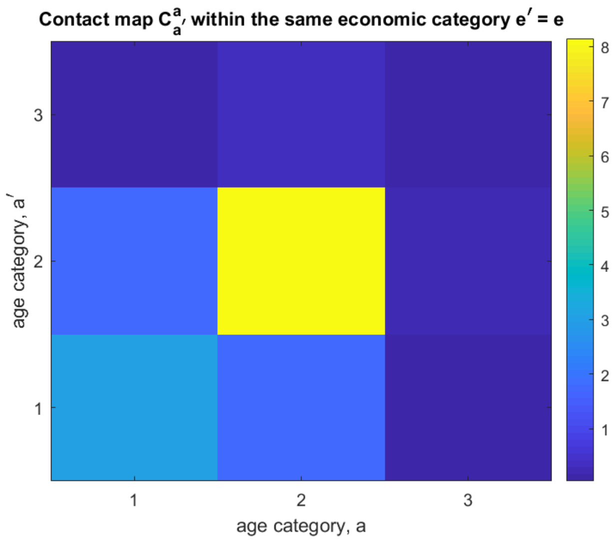
Parameter	**Symbol**
interactions rate between S_{ae} and $E_{a'e'}$	$\beta^{ae}_{a'e'}$
interactions rate between S_{ae} and $I_{a'e'}$	$\tilde{\beta}^{ae}_{a'e'}$
rate of deaths due to economic reasons (depends on economic category)	α_e
time spent in E (latency period)	t_E
time spent in I (infectious period)	t_I
proportion for which the disease is fatal (depends on age category)	f_a
transitions between modules	$\mathcal{M}$

Table 1: Parameters in our model

The epidemiological parameters, f_a, t_E, t_I have been taken from [1]. The rate of infection and economic death rate had to be deciphered from social contact maps [2] and reports of poverty-related deaths in India. The social contact map of India presented by Keisha et al is for 16 age categories and is described by a 16x16 matrix. For, our purposes, this had to be convolved with the age distribution of India to obtain a 3x3 matrix, $C_{a'}^{a}$. Economic categories were not considered in the study. We therefore defined β in the following manner:

1. within an economic group, i.e. $e' = e$, social contact map is as given in the study by Keisha et al, $\beta_{a'e}^{ae} = \lambda C_{a'}^{a}$. λ stands for the probability of infection on contact.
2. rich-poor interactions are limited to the young, $\beta_{a',2}^{a,2} = 0.5\lambda C_{a'}^{a}$
3. in all other cases, the degree of contact is zero.

With the above layout in place, we perform a few quick checks on the parameters.



In principle the number of S-E contacts would be much larger than S-I contacts and therefore we will take $\tilde{\beta} = \gamma\beta$, where $0 \leq \gamma \leq 1$. Mitigation strategies bring down the amount of interactions, i.e. $\beta \to \beta\zeta$, where $0 \leq \zeta \leq 1$. According to some reports, India has 2.5 million poverty-related deaths in a year []. This means that without mitigation strategies in place,

$$\sum_{e=1}^{2} \alpha_e^o \sim \frac{2.5 \times 10^6}{365} \text{per day}, \quad \alpha_3 = 0 \text{ i.e. rich are not affected}$$

However, mitigation strategies like lock-down are going to increase this rate as more poor people are likely to fall off the grid without being able to commute to work daily. For the time being, we factor this in as,

$$\alpha_e = \frac{\alpha_e^o}{\zeta^n}$$

where $\alpha_e^o = 2.2 \times 10^{-3}$ per day. A more sophisticated model is to be discussed.

With the above layout in place, we perform a few quick checks on the parameters.

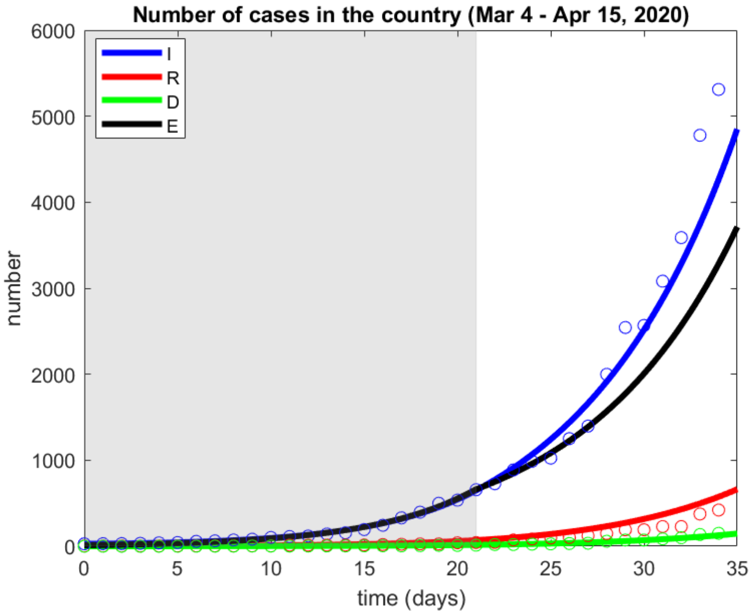


Figure 1: A comparison of simulations to the real data obtained from JHU CSSE [3]. Initial number of infectious (I) cases were 28 on Mar 4, 2020 and we assumed that the number of exposed (E) people were 12 (about 0.4 times the number of infectious cases). We also assumed that all the inital cases belonged to the young , rich category. Lock-down is implemented on Mar 25, 2020. Parameters used for simulation (NOT best fit): $t_E = 5$, $t_I = 42$, $f_a = [0.0910, 0.1867, 0.6194]$, $\lambda = 0.019$, $\gamma = 0$, $\zeta = 0.85$.

1.1 Case: absence of economic module in the model, $\alpha_e = 0$

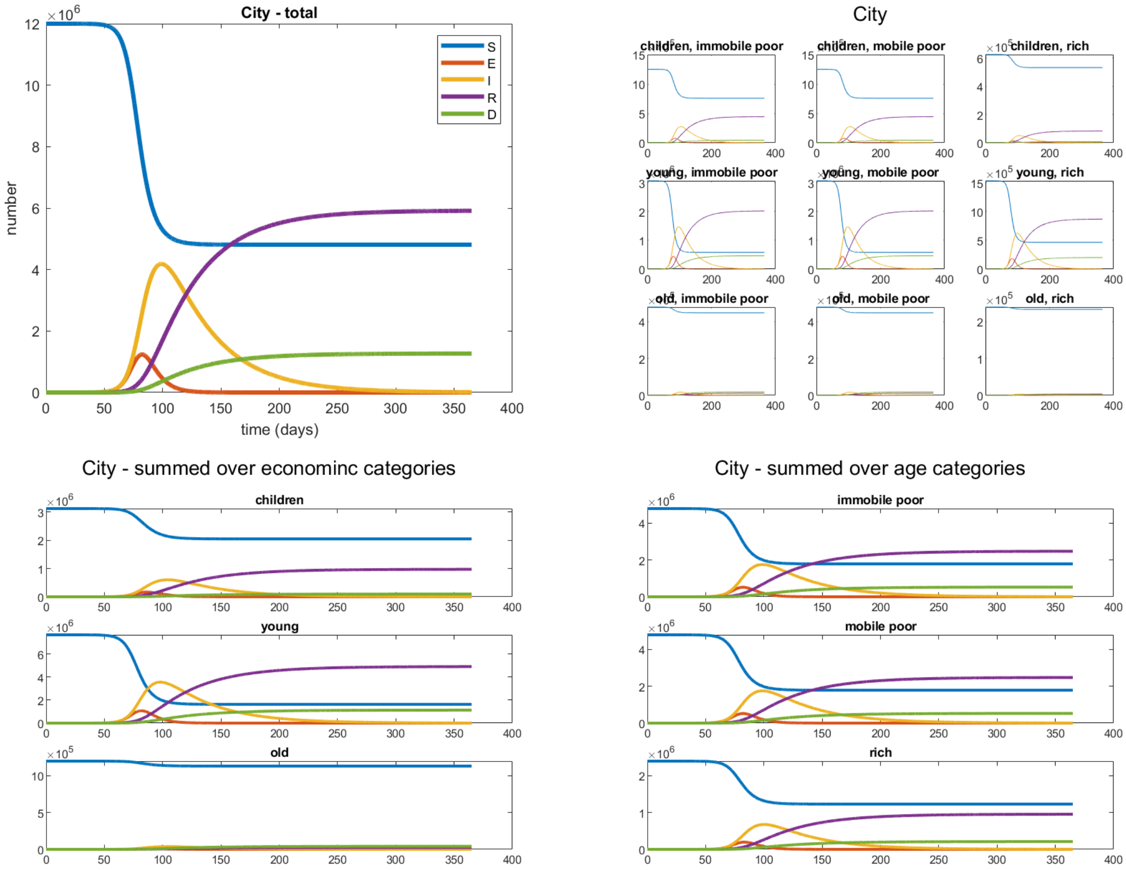


Figure 2: Without mitigation strategies, i.e. $\zeta = 1$ throughout. For full-size images here: `here`.

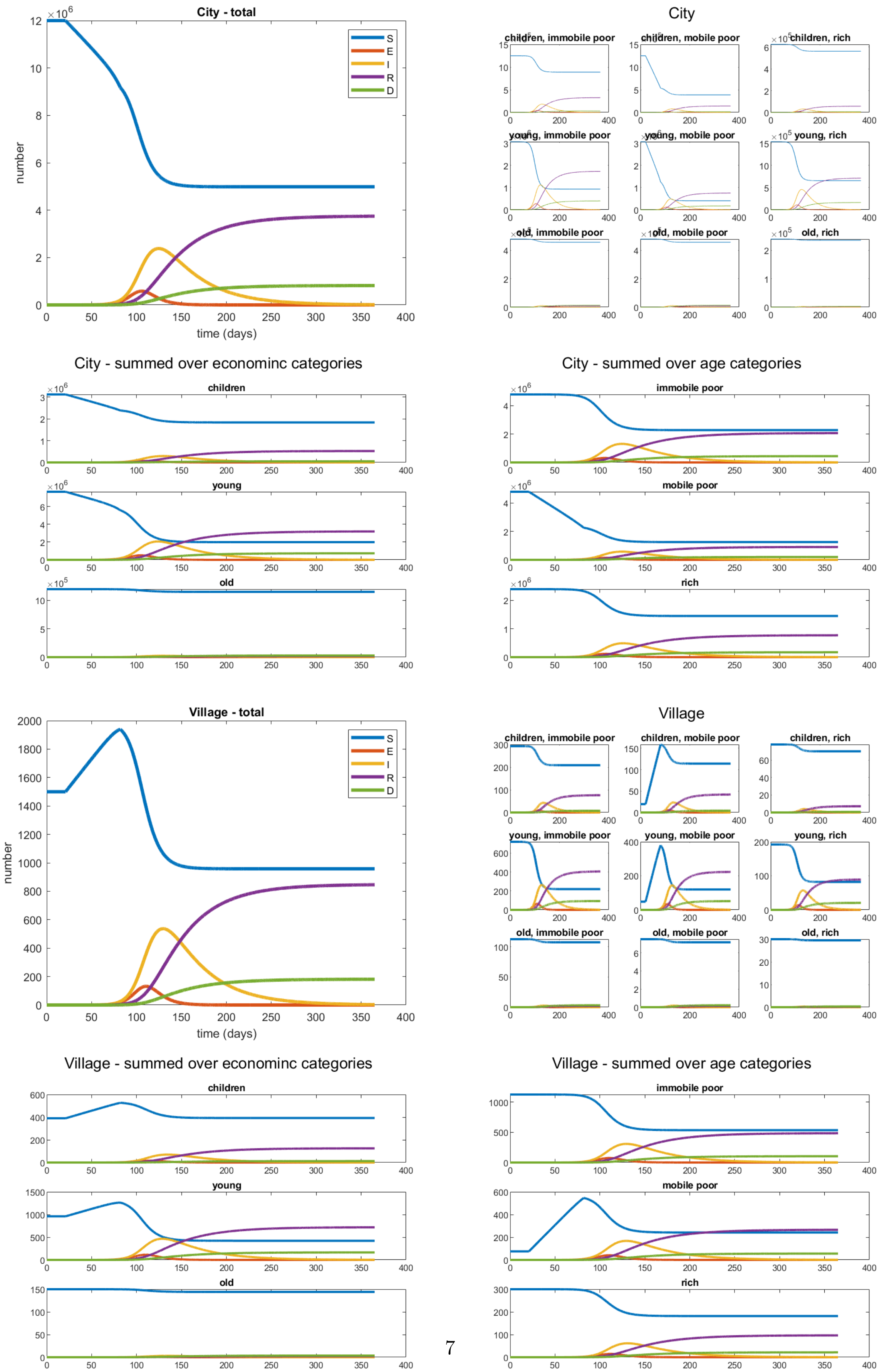
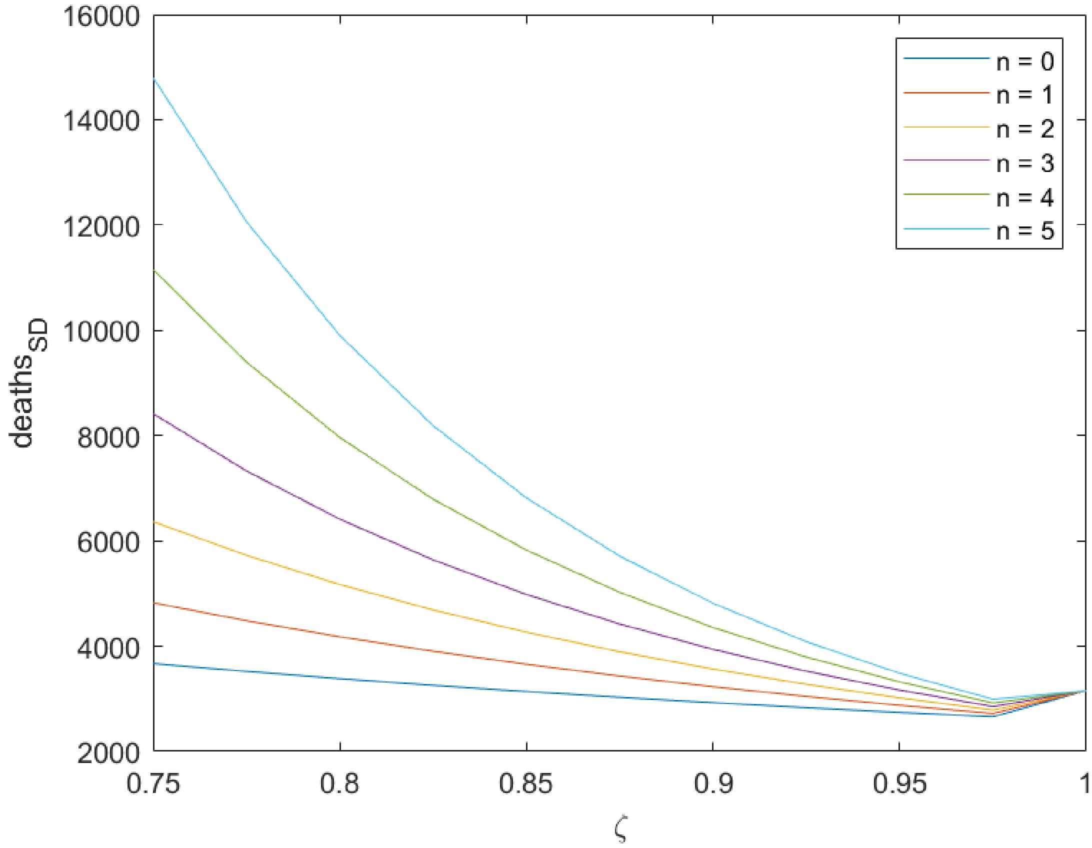
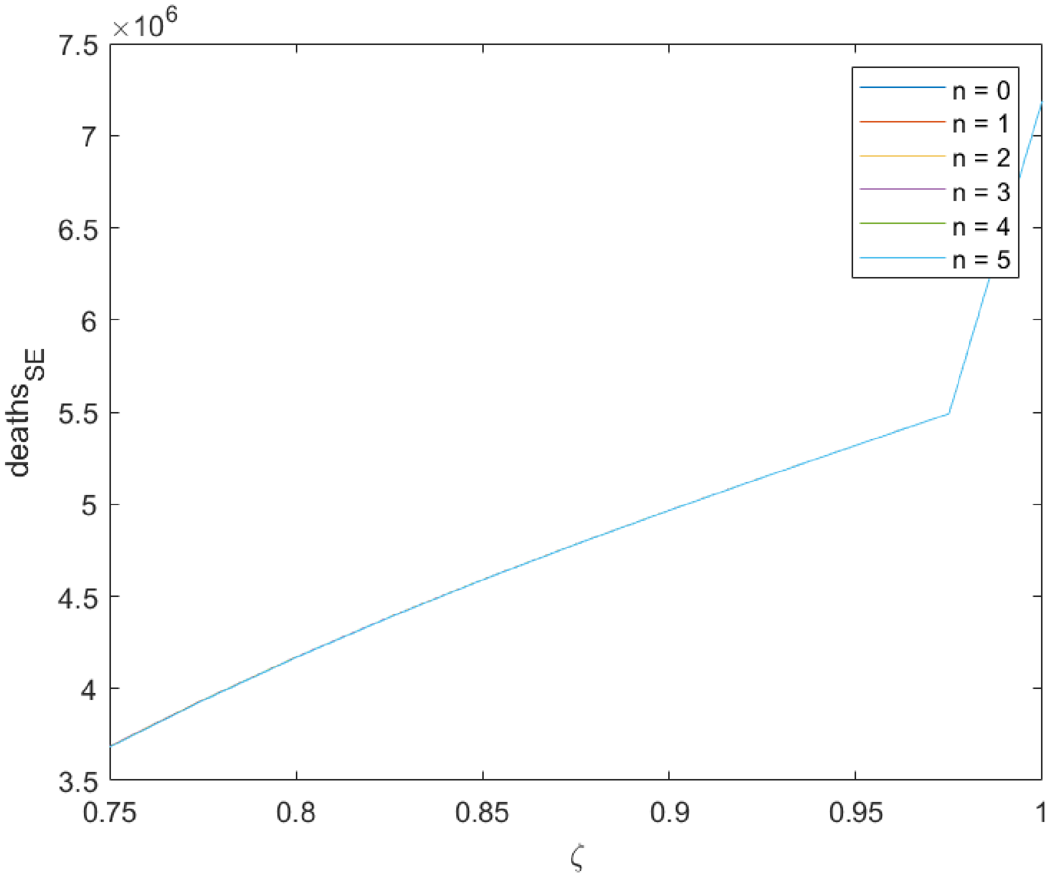
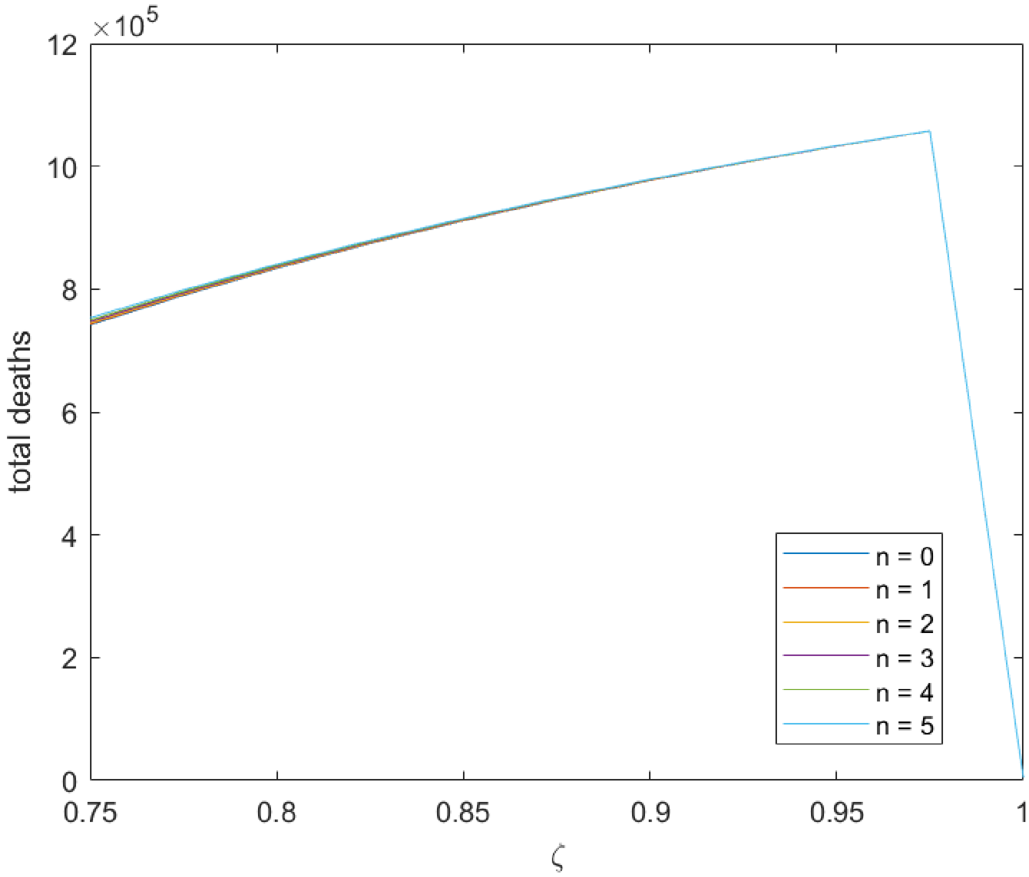


Figure 3: With mitigation strategy implemented 21 days after $t = 0$, i.e. $\zeta = 1$ for $t \leq 21$, and $\zeta = 0.85$ for $t > 21$. For full-size images here: `here`.


16000
14000
12000
10000
8000
6000
4000
2000
deaths$_{SD}$
0.75
0.8
0.85
0.9
0.95
1
ζ
n = 0
n = 1
n = 2
n = 3
n = 4
n = 5



×10^6
7.5
7
6.5
6
5.5
5
4.5
4
3.5
deaths_SE
0.75
0.8
0.85
0.9
0.95
1
ζ
n = 0
n = 1
n = 2
n = 3
n = 4
n = 5



$\times 10^5$
12
10
8
6
4
2
0
total deaths
0.75
0.8
0.85
0.9
0.95
1
ζ
n = 0
n = 1
n = 2
n = 3
n = 4
n = 5

Things to-do:

1. parameters and their toggles
2. the economic module is fairly simple and the model is giving obvious results. Discuss with Amit.
3. differences in the public health infrastructure: f_a different in villages and city? or infections in villages cause migration back to city.
4. having a social net might ensure α_e does not ramp up due to mitigation and migration also does not play a strong role in that case

1.2 Take home messages - summarize results

References

[1] https://neherlab.org/covid19/about

[2] Prem, Kiesha, Alex R. Cook, and Mark Jit. "Projecting social contact matrices in 152 countries using contact surveys and demographic data." PLoS computational biology 13.9 (2017): e1005697.

[3] https://github.com/CSSEGISandData/COVID-19