Spatial models for public health and economic strategies for COVID-19

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1 A short excurison into SIR models

Richard Neher's model [1]

$$\frac{d}{dt}S_a = -\beta(t)S_a(t)\sum_b I_b(t) \tag{1}$$

$$\frac{d}{dt}E_a = \beta(t)S_a(t)\sum_b I_b(t) - E_a(t)/t_l \tag{2}$$

$$\frac{d}{dt}I_a = E_a(t)/t_l - I_a(t)/t_i \tag{3}$$

$$\frac{d}{dt}R_a = m_a I_a(t)/t_i + (1-c_a)H_a(t)/t_h \tag{4}$$

$$\frac{d}{dt}H_a = (1-m_a)I_a(t)/t_i + (1-f_a)C_a(t)/t_C \tag{5}$$

$$\frac{d}{dt}C_a = c_a H_a(t)/t_h - C_a(t)/t_c \tag{6}$$

$$\frac{d}{dt}D_a = f_a C_a(t)/t_c \tag{7}$$

where age index, $a = 1, 2, ..., 9$ standing for age categories: $0-9, 10-19, ..., 80+$.

$$\beta_a(t) = R_0\zeta_a M_(t)(1+\varepsilon\, cos\left(\frac{2\pi(t-t_{max})}{t_i}\right)$$

Parameter	Symbol	Value	Units
avg interactions per day	R_0	2-3	per day
degree of isolation	ζ_a	0-1	
mitigation	$M(t)$	0-1	
seasonal driving	ε	0	
peak of seasonal effects	t_{max}	Jan 2020	time
average latency period	t_l	5	days
average infectious period	t_i	3	days
average hospitalisation time	t_h	4	days
average time in ICU	t_c	14	days
proportion of mild symptoms	m_a	0-1	
proportion requiring critical care	c_a	0-1	
proportion for which the disease is fatal	f_a	0-1	

Table 1: Parameters in the model

Age groups:	0-9	10-19	20-29	30 -39	40-49	50-59	60-69	70-79	80+
m_a	0.9995	0.9985	0.997	0.9955	0.988	0.975	0.925	0.86	0.75
c_a	0.05	0.1	0.1	0.15	0.2	0.25	0.35	0.45	0.55
f_a	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.5	0.5

Table 2: Age-specific parameters in the model

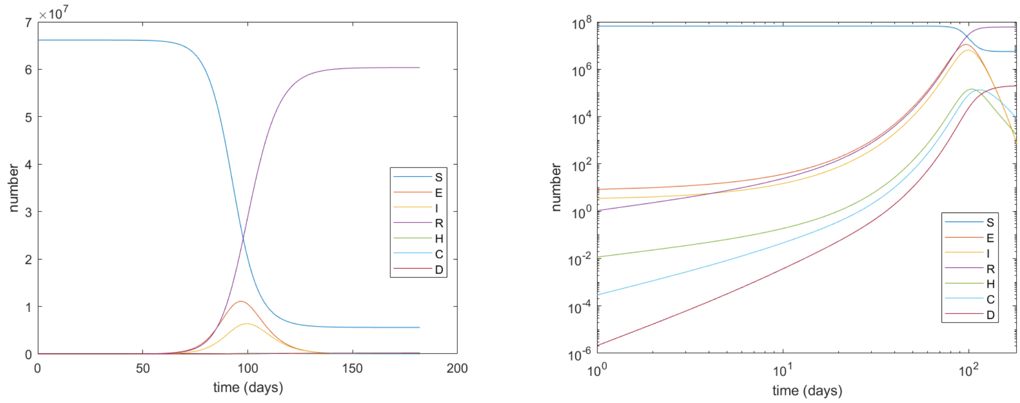


Figure 1: Test run for Karnataka with a population $N = 66165886$ and initial exposed/infected population of 10, i.e. $E_4 = 7$, $I_4 = 3$ with no mitigation, no imports.

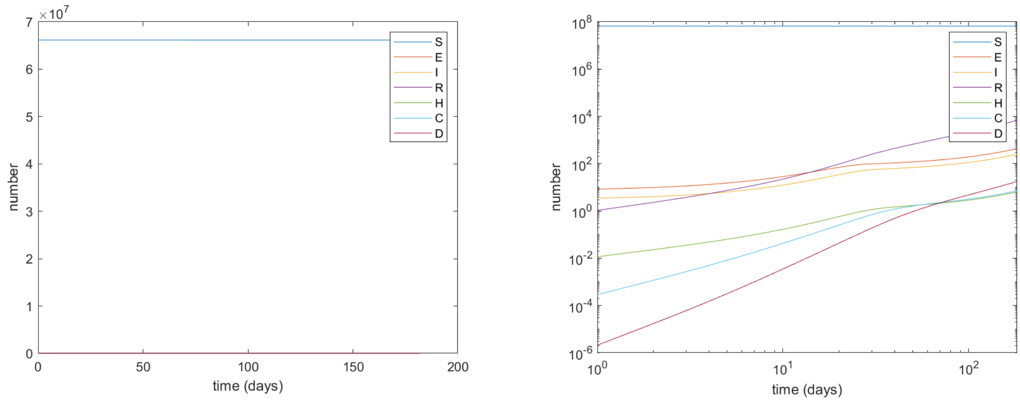


Figure 2: Test run for Karnataka with a population $N = 66165886$ and initial exposed/infected population of 10, i.e. $E_4 = 7$, $I_4 = 3$ with strong mitigation, no imports

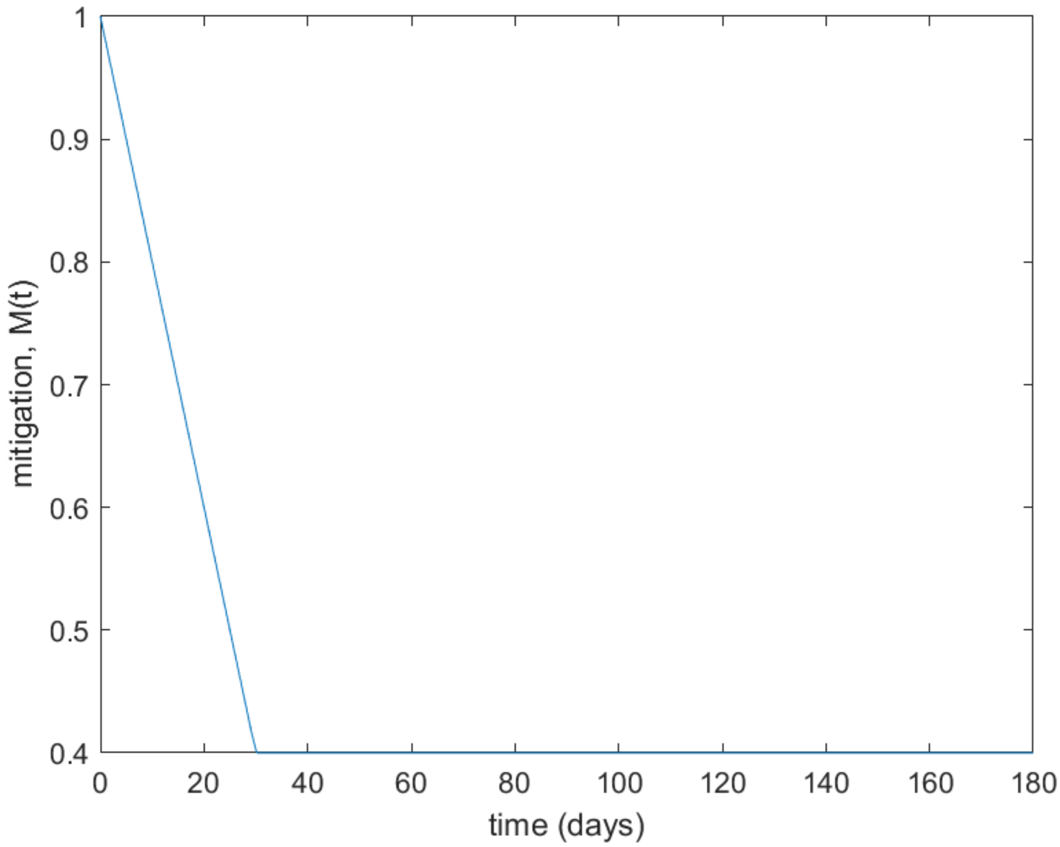


Figure 3: Mitigation curve for Fig. 2

2 Our model including economic demographic

the symbols had to be changed. If you think it's confusing, I suggest we remove the previous section or change the symbols in it?

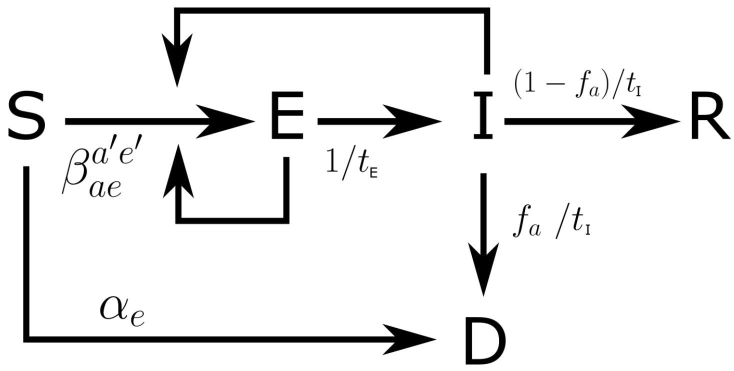
In our model we consider a city and several, n_v, villages. In each of them the population is divided into:

1. 3 economic categories: immobile poor, mobile poor and rich
2. 3 age categories: children (0-14), young (15-59) and old (60+)
3. 5 states: Susceptible (S), Exposed (E), Infectious (I), Recovered (R), Dead (D)

While we will label the states with a capital letter as indicated next to the states in the list above, we will use a subscript to denote age (a) and economic (e) categories. Therefore, $S_{ae}, I_{ae}, ...$ such that

$$N(t) = \sum_{a,e} N_{ae}, \qquad N_{ae} = S_{ae}(t) + E_{ae}(t) + I_{ae}(t) + R_{ae}(t) + D_{ae}(t)$$

for a city and villages individually. The dynamics is described by the following reaction graph:



and the corresponding equations are

$$\frac{d}{dt}S_{ae} = -S_{ae}(t)\sum_{a',e'}\left(\beta^{ae}_{a'e'}(t)\frac{I_{a'e'}(t)}{N_{a'e'}(t)} + \tilde{\beta}^{ae}_{a'e'}(t)\frac{E_{a'e'}(t)}{N_{a'e'}(t)}\right) + \mathcal{M}^S_{ae}(t) \tag{8}$$

$$\frac{d}{dt}E_{ae} = S_{ae}(t)\sum_{a',e'}\left(\beta^{ae}_{a'e'}(t)\frac{I_{a'e'}(t)}{N_{a'e'}} + \tilde{\beta}^{ae}_{a'e'}(t)\frac{E_{a'e'}(t)}{N_{a'e'}}\right) - E_{ae}(t)/t_E + \mathcal{M}^E_{ae}(t) \tag{9}$$

$$\frac{d}{dt}I_{ae} = E_{ae}(t)/t_E - I_{ae}(t)/t_I + \mathcal{M}^I_{ae}(t) \tag{10}$$

$$\frac{d}{dt}R_{ae} = (1-f_a)I_{ae}(t)/t_I + \mathcal{M}^R_{ae}(t) \tag{11}$$

$$\frac{d}{dt}D_{ae} = f_a I_{ae}(t)/t_I + \alpha_e S_{ae}(t) \tag{12}$$

Notice that the parameters are dependent on age (a) and economic (e) category too.

Parameter	**Symbol**
interactions rate between S_{ae} and $E_{a'e'}$	$\beta^{ae}_{a'e'}$
interactions rate between S_{ae} and $I_{a'e'}$	$\tilde{\beta}^{ae}_{a'e'}$
rate of deaths due to economic reasons (depends on economic category)	α_e
time spent in E (latency period)	t_E
time spent in I (infectious period)	t_I
proportion for which the disease is fatal (depends on age category)	f_a
import rate	$\mathcal{M}$

Table 3: Parameters in our model

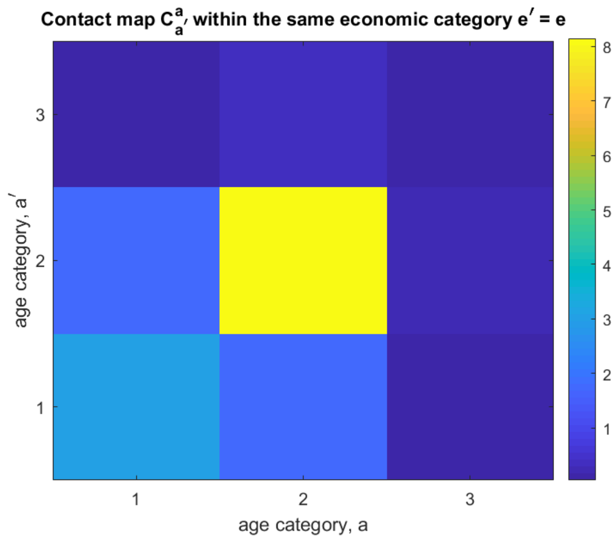
The epidemiological parameters, f_a, t_E, t_I have been taken from [1]. The rate of infection and economic death rate had to be deciphered from social contact maps [2] and reports of poverty-related deaths in India. The social contact map of India presented by Keisha et al is for 16 age categories and is described by a 16x16 matrix. For, our purposes, this had to be convolved with the age distribution of India to obtain a 3x3 matrix, $C^a_{a'}$. Economic categories were not considered in the study. We therefore defined β in the following manner:

1. within an economic group, i.e. $e' = e$, social contact map is as given in the study by Keisha et al, $\beta^{ae}_{a'e} = \lambda C^a_{a'}$. λ stands for the probability of infection on contact.

2. rich-poor interactions are limited to the young, $\beta^{a,2}_{a',2} = 0.5\lambda C^a_{a'}$

3. in all other cases, the degree of contact is zero.

With the above layout in place, we perform a few quick checks on the parameters.



In principle the number of S-E contacts would be much larger than S-I contacts and therefore we will take $\tilde{\beta} = \gamma\beta$, where $0 \leq \gamma \leq 1$. Mitigation strategies bring down the amount of interactions, i.e. $\beta \to \beta\zeta$, where $0 \leq \zeta \leq 1$. According to some reports, India has 2.5 million poverty-related deaths in a year []. This means that without mitigation strategies in place,

$$\sum_{e=1}^{2} \alpha_e^o \sim \frac{2.5 \times 10^6}{365} \text{per day}, \quad \alpha_3 = 0 \text{ i.e. rich are not affected}$$

However, mitigation strategies like lock-down are going to increase this rate as more poor people are likely to fall off the grid without being able to commute to work daily. For the time being, we factor this in as,

$$\alpha_e = \frac{\alpha_e^o}{\zeta^n}$$

where $\alpha_e^o = 2.2 \times 10^{-3}$ per day. A more sophisticated model is to be discussed.

With the above layout in place, we perform a few quick checks on the parameters.

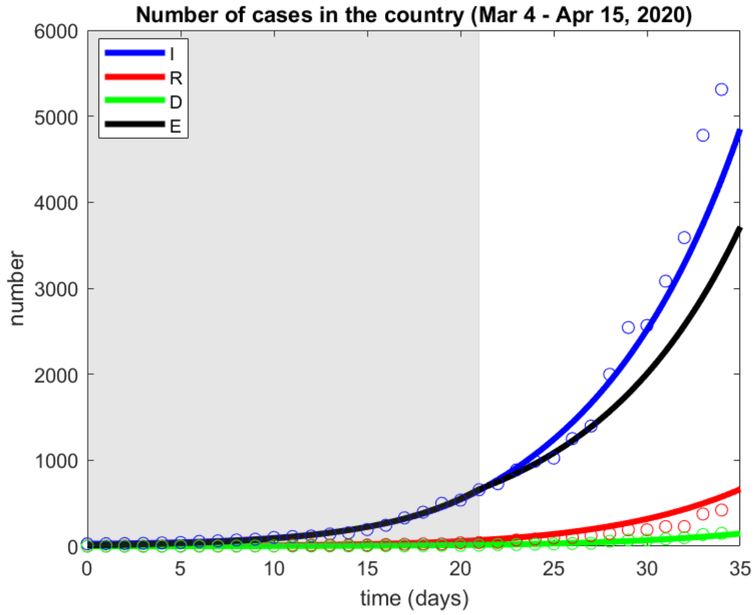


Figure 4: A comparison of simulations to the real data obtained from JHU CSSE [3]. Initial number of infectious (I) cases were 28 on Mar 4, 2020 and we assumed that the number of exposed (E) people were 12 (about 0.4 times the number of infectious cases). We also assumed that all the inital cases belonged to the young , rich category. Lock-down is implemented on Mar 25, 2020. Parameters used for simulation (NOT best fit): $t_E = 5$, $t_I = 42$, $f_a = [0.0910, 0.1867, 0.6194]$, $\lambda = 0.019$, $\gamma = 0$, $\zeta = 0.85$.

2.1 Case: absence of economic module in the model, $\alpha_e = 0$

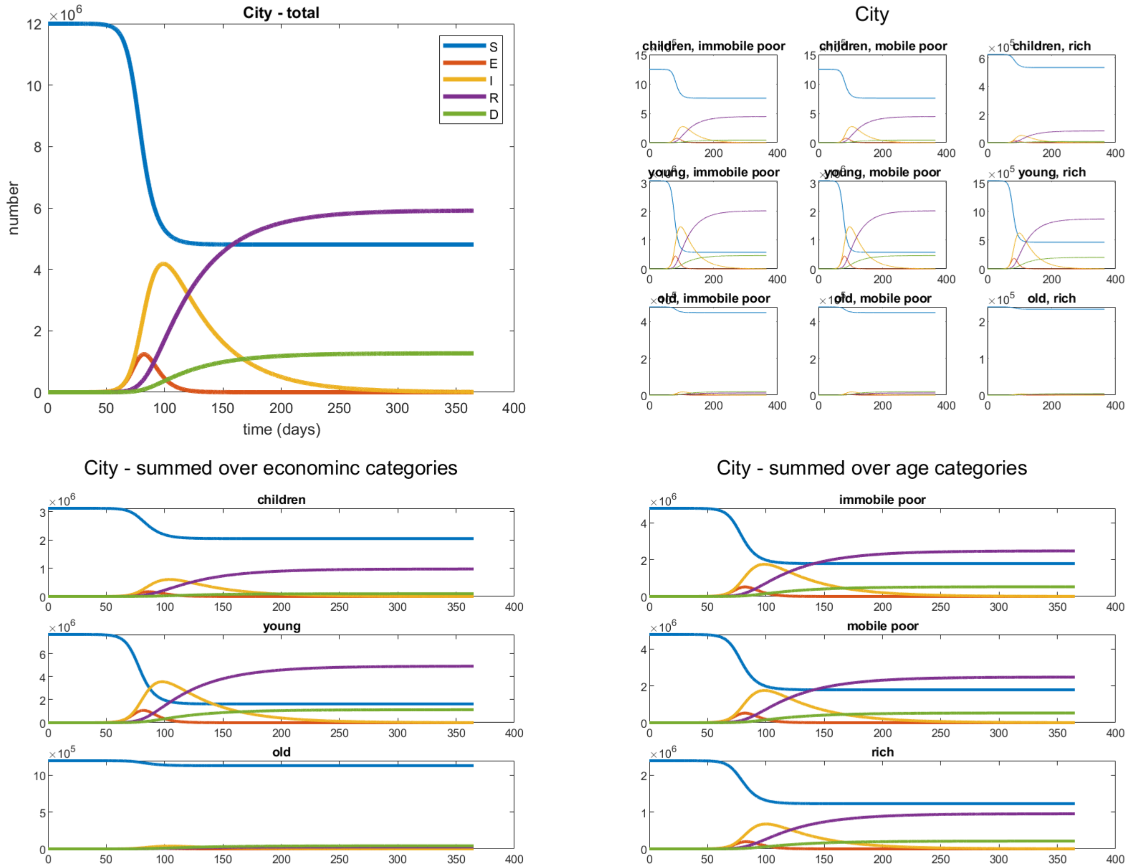


Figure 5: Without mitigation strategies, i.e. $\zeta = 1$ throughout. For full-size images here: `here`.

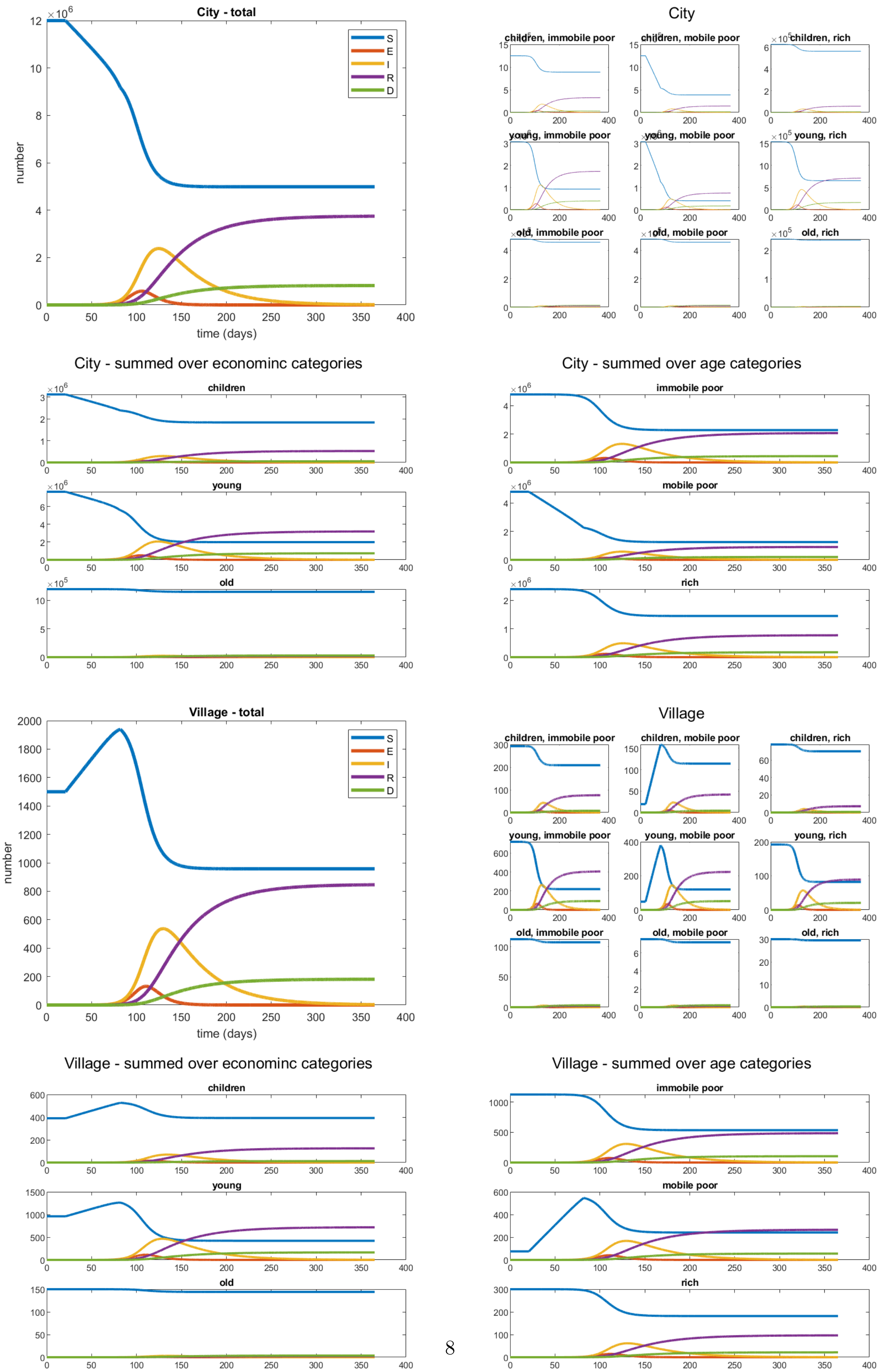


Figure 6: With mitigation strategy implemented 21 days after $t = 0$, i.e. $\zeta = 1$ for $t \leq 21$, and $\zeta = 0.85$ for $t > 21$. For full-size images here: `here`.

Things to-do:

1. parameters and their toggles
2. the economic module is fairly simple and the model is giving obvious results. Discuss with Amit.
3. differences in the public health infrastructure: f_a different in villages and city? or infections in villages cause migration back to city.
4. having a social net might ensure α_e does not ramp up due to mitigation and migration also does not play a strong role in that case

2.2 Take home messages - summarize results

References

[1] https://neherlab.org/covid19/about

[2] Prem, Kiesha, Alex R. Cook, and Mark Jit. "Projecting social contact matrices in 152 countries using contact surveys and demographic data." PLoS computational biology 13.9 (2017): e1005697.

[3] https://github.com/CSSEGISandData/COVID-19