Delft University of Technology

Airport and Cargo Operations
AE4446

2D Bin Packing Assignment

Group 23

Authors	Student ID	Contribution statement
Lars Koetsier	(5372739)	Mathematical formulation, Python and Gurobi implementation
Ian Trout	(5851483)	Mathematical formulation, results, and discussion
Allard Krikke	(4724216)	Mathematical formulation, results
Kelvin Arbman	(4943589)	Introduction, results and managerial consideration

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1 Introduction

Transporting goods through air is often considered to be the most expensive mode of transport. Passenger airplanes have very limited space to store cargo, so it is essential that the space is optimized as much as possible. The belly space of airplanes are loaded with Unit Load Devices (ULD's), which in turn are loaded with various types of goods.

This research aims to optimize the costs related to packing the ULD's that are eventually stored in the cargo belly of an aircraft. The case study assumes that a person from the Revenue Management department has given the team a set of items I that need to be placed on a plane tomorrow. The team has to account for item constraints that restrict which packages can go together, as described below. In addition, all items need to be placed in ULD's such that the total cost is minimized (each ULD has a certain cost C_b). The space is assumed to be two-dimensional (X and Y). It is therefore a typical 2D Bin Packing Problem (2DBPP), where only the horizontal and vertical dimensions of the packages and ULD's are considered.

Solving models using optimization software can lead to various insights on packing ULD's, such as allowing ground handlers to test various configurations without having to waste time physically packing ULD's and to also test different package scenarios (E.G. more or less fragile and radioactive packages). As the aviation industry becomes more data driven, there is more data available to use for optimization models and thus, this data can be used to determine which package needs to be stored where inside each ULD in order to maximize profit (if one would consider package value) whilst minimizing costs (the amount of ULD's needed). In addition, using models to solve bin packing problems allows for better management of limited ULD space whilst being able to better predict how to correctly fill each ULD (which along with weight and balance, aircraft configuration, and build-up scheduling problems comprise the majority of air cargo loading problems).

Setting up this model requires defining numerous constraints to make sure the solution is valid in real life. Both dimensions X and Y of a package needs to fit within the remaining space left in the ULD. Certain packages can be rotated one quarter turn ($\frac{\pi}{2}$) around the y-axis. Additionally, a package can not overlap with another package. Items also need to have a stable position inside the ULD. Lastly, packages can be fragile, perishable and/or radioactive. Fragile packages can not have any packages stacked on top of them. Radioactive and perishable items can not be stored within the same ULD. The mathematical formulation of the 2DBPP model is fully described in the following section.

2 Mathematical Model (Part A)

The notation used for this mathematical formulation is provided in Table 1.

Table 1: Notation

Sets and indices		
B	Set of Bins	$b \in B$
I	Set of items	$i, j \in I$
Parameters		
$\overline{C}_b$	Cost of bin b	euros
$\overline{H}_b$	Height of bin b	cm
$\overline{L}_b$	Length of bin b	cm
H_i	Height of item i	cm
L_i	Length of item i	cm
$i_{rotatable}$	rotatable item i	[-]
i_{care}	fragile item i	[-]
i_{radio}	radioactive item i	[-]
i_{fresh}	perishable item i	[-]
Variables		
$p_{i,b}$	1 if item i is assigned to bin b, 0 otherwise	
z_b	1 if bin b is used , 0 otherwise	
$l_{i,j}$	1 if item i is to the left of item j, 0 otherwise	
$u_{i,j}$	1 if item i is under item j, 0 otherwise	
r_i	1 1 if it has the original orientation, 0 otherwise	
x_i	x coordinate of lower-left corner of item i w.r.t. the origin of bin b	
x'_i	x coordinate of upper-right corner of item i w.r.t. the origin of bin b	
y_i	y coordinate of lower-left corner of item i w.r.t. the origin of bin b	
y'_i	y coordinate of upper-right corner of item i w.r.t. the origin of bin b	
g_i	1 if item i lies on the ground of the bin it is assigned to, 0 otherwise	
per_b	Mark bin b as suitable for perishable items 1, or 0 otherwise	
rad_b	Mark bin b as suitable for radioactive items 1, or 0 otherwise	
$h_{i,j}$	1 if item j has a suitable height to support item i, 0 otherwise	
$o_{i,j}$	1 if there is a nonempty space between items i and j, 0 otherwise	
$s_{i,j}$	1 if item i is supported by item j, 0 otherwise	
$\eta^1_{i,j}$	0 if x_j is smaller than or equal to x_i, 1 otherwise	
$\eta^2_{i,j}$	0 if x'_j is bigger than or equal to x'_i, 1 otherwise	
$\beta^1_{i,j}$	1 if vertex 1 of item i is supported by item j, 0 otherwise	
$\beta^2_{i,j}$	1 if vertex 2 of item i is supported by item j, 0 otherwise	
$v_{i,j}$	y coordinate separation between items i and j	
$m_{i,j}$	1 if y'_j is greater than or equal to y_i, 0 otherwise	

The mathematical formulation then follows as:

$$\min \quad \sum_{b \in B} z_b \cdot \overline{C}_b \tag{1}$$

Subject to:

$$x'_i = x_i + L_i \cdot r_i + H_i \cdot (1 - r_i) \qquad \forall i \in I \tag{2}$$

$$y'_i = y_i + H_i \cdot r_i + L_i \cdot (1 - r_i) \qquad \forall i \in I \tag{3}$$

$$l_{i,j} + l_{j,i} + u_{i,j} + u_{j,i} \geq p_{i,b} + p_{j,b} - 1 \qquad \forall i, j \in I, b \in B, i \neq j \tag{4}$$

$$x_j \geq x_i + L_i \cdot r_i + H_i \cdot (1 - r_i) - M \cdot (1 - l_{i,j}) \qquad \forall i, j \in I, b \in B, i \neq j \tag{5}$$

$$x_j \leq x_i + L_i \cdot r_i + H_i \cdot (1 - r_i) + M \cdot l_{i,j} \qquad \forall i, j \in I, b \in B, i \neq j \tag{6}$$

$$y_j \geq y_i + L_i \cdot (1 - r_i) + H_i \cdot r_i - M \cdot (1 - u_{i,j}) \qquad \forall i, j \in I, b \in B, i \neq j \tag{7}$$

$$y_j \leq y_i + L_i \cdot (1 - r_i) + H_i \cdot r_i + M \cdot u_{i,j} \qquad \forall i, j \in I, b \in B, i \neq j \tag{8}$$

$$x_i + L_i \cdot r_i + H_i \cdot (1 - r_i) \leq \sum_{b \in B} \overline{L} \cdot p_{i,b} \qquad \forall i \in I, b \in B \tag{9}$$

$$y_i + L_i \cdot (1 - r_i) + H_i \cdot r_i \leq \sum_{b \in B} \overline{H} \cdot p_{i,b} \qquad \forall i \in I, b \in B \tag{10}$$

$$\sum_{b \in B} p_{i,b} = 1 \qquad \forall i \in I \tag{11}$$

$$z_b \geq p_{i,b} \qquad \forall i \in I, b \in B \tag{12}$$

$$\sum_{j \in I} \beta^1_{i,j} + \sum_{j \in I} \beta^2_{i,j} + 2g_i = 2 \qquad \forall i \in I \tag{13}$$

$$\sum_{j \in I} \beta^1_{i,j} = \sum_{j \in I} \beta^2_{i,j} \qquad \forall i \in I \tag{14}$$

$$y_i \leq M \cdot (1 - g_i) \qquad \forall i \in I \tag{15}$$

$$x_j \geq x_i - M \cdot (1 - \eta^1_{i,j}) \qquad \forall i, j \in I, i \neq j \tag{16}$$

$$x_j \leq x_i + M \cdot \eta^1_{i,j} \qquad \forall i, j \in I, i \neq j \tag{17}$$

$$x'_i \geq x'_j - M \cdot (1 - \eta^2_{i,j}) \qquad \forall i, j \in I, i \neq j \tag{18}$$

$$x'_i \leq x'_j + M \cdot \eta^2_{i,j} \qquad \forall i, j \in I, i \neq j \tag{19}$$

$$y'_j - y_i \leq v_{i,j} \qquad \forall i, j \in I \tag{20}$$

$$y_i - y'_j \leq v_{i,j} \qquad \forall i, j \in I \tag{21}$$

$$y'_j \geq y_i - M \cdot (1 - m_{i,j}) \qquad \forall i, j \in I, i \neq j \tag{22}$$

$$y'_j \leq y_i + M \cdot m_{i,j} \qquad \forall i, j \in I, i \neq j \tag{23}$$

$$v_{i,j} \leq y'_j - y_i + M \cdot (1 - m_{i,j}) \qquad \forall i, j \in I, i \neq j \tag{24}$$

$$v_{i,j} \leq y_i - y'_j + M \cdot m_{i,j} \qquad \forall i, j \in I, i \neq j \tag{25}$$

$$h_{i,j} \leq v_{i,j} \qquad \forall i, j \in I, i \neq j \tag{26}$$

$$v_{i,j} \leq h_{i,j} \cdot M \qquad \forall i, j \in I, i \neq j \tag{27}$$

$$o_{i,j} = l_{i,j} + l_{j,i} \qquad \forall i, j \in I, i \neq j \tag{28}$$

$$(1 - s_{i,j}) \leq h_{i,j} + o_{i,j} \qquad \forall i, j \in I, i \neq j \tag{29}$$

$$2 \cdot (1 - s_{i,j}) \geq h_{i,j} + o_{i,j} \qquad \forall i, j \in I, i \neq j \tag{30}$$

$$p_{i,b} - p_{j,b} \leq 1 - s_{i,j} \qquad \forall i, j \in I, b \in B, i \neq j \tag{31}$$

$$p_{j,b} - p_{i,b} \leq 1 - s_{i,j} \qquad \forall i, j \in I, b \in B, i \neq j \tag{32}$$

$$\beta^1_{i,j} \leq s_{i,j} \qquad \forall i,j \in I, i \neq j \quad (33)$$
$$\beta^2_{i,j} \leq s_{i,j} \qquad \forall i,j \in I, i \neq j \quad (34)$$
$$\eta^1_{i,j} + \eta^2_{i,j} \leq 2 \cdot (1 - \beta^1_{i,j}) \qquad \forall i,j \in I, i \neq j \quad (35)$$
$$\eta^1_{i,j} + \eta^2_{i,j} \leq 2 \cdot (1 - \beta^2_{i,j}) \qquad \forall i,j \in I, i \neq j \quad (36)$$
$$s_{i,j} \leq (1 - i_{care}) \qquad \forall i,j \in I, i \neq j \quad (37)$$
$$r_i \geq (1 - i_{rotatable}) \qquad \forall i \in I \quad (38)$$
$$i_{fresh} - M \cdot (1 - p_{i,b}) \leq per_b \qquad \forall i \in I, b \in B \quad (39)$$
$$i_{radio} - M \cdot (1 - p_{i,b}) \leq rad_b \qquad \forall i \in I, b \in B \quad (40)$$
$$per_b + rad_b \leq 1 \qquad \forall b \in B \quad (41)$$
$$p_{i,b}, z_b, l_{i,j}, u_{i,j}, r_i, g_i, per_b, rad_b, h_{i,j}, o_{i,j}, s_{i,j}, \eta^1_{i,j}, \eta^2_{i,j}, \beta^1_{i,j}, \beta^2_{i,j}, m_{i,j} \in \{0,1\} \quad (42)$$
$$x_i, x'_i, y_i, y'_i, v_{ij} \in Z \qquad \forall i,j \in I \quad (43)$$

The objective function (1) is written as the minimization of bin costs for all the bins. Constraints (2) and (3) define the upper right x and y coordinates of each item based on the rotation, length, and height of the object. Constraint (4) ensures that if two items are in the same bin (right-hand side of the constraint is 1), at least one of the four decision variables on the left-hand side should be 1.

Constraints (5) and (6) force mutual positioning along x-axis ensuring that items cannot overlap each other in the ULD's. Constraints (7) and (8) force mutual positioning on the y-axis.

Constraints (9) and (10) ensure that the lower-right vertex and the upper-right vertex of item i are contained within bin b.

Constraint (11) ensures that each item is assigned to a bin. Constraint (12) ensures that a bin is flagged 'used' as soon as an item is assigned to that bin.

Constraint (13) ensures that every item must be supported. This already happens if it is on the ground ($g_i = 1$). Otherwise, it must be supported either by one item (if it is wide enough), two distinct items, or one item and the cut of the bin. Constraint (15) ensures the proper y coordinate if the item is on the ground. Constraints (16), (17), (18), and (19) ensure that there is no overlap, that is, two items cannot occupy a same portion of the bin space.

The vertical position of the top of each item is ensured by constraints (20) and (21) and constraints (22) and (23) are part of the big M constraint which is used to check for suitability (the next constraints mentioned).

Constraint (24), (25), (26) and (27) are used to determine whether item j has a suitable height for item i. Boxes i and j share a part of their orthogonal projection, which is made sure by constraint (28). Following the orthogonal projection, constraints (29) and (30) are to ensure that the bottom side of item i is supported by the top side of item j.

When two items are stacked on top of each other, they need to be within the same bin, which is done by constraints (31) and (32). Constraints (33) and (34) certify that item j supports one vertex of the basis of item i, when item j supports item i. Constraints (35) and (36) are to make sure that both vertices of the base of item j are fully supported by item i and visa-versa if those items support one another.

No other item can be stacked on top of a fragile item (37), an item cannot be rotated when that is not allowed (38) and perishable and radioactive items cannot be packed in the same bin (constraint (41)). Constraints (39) and (40) mark a bin as perishable or radioactive when a perishable or radioactive item is put in that bin. Finally, constraints (42) and (43) are that variables $p_{i,b}, z_b, l_{i,j}, u_{i,j}, r_i, g_i, per_b, rad_b, h_{i,j}, o_{i,j}, s_{i,j}, \eta^1_{i,j}, \eta^2_{i,j}, \beta^1_{i,j}, \beta^2_{i,j}, m_{i,j}$ are binary variables and that $x_i, x'_i, y_i, y'_i, v_{ij}$ are natural numbers respectively.

3 Packing Results (Part B)

Packing Results

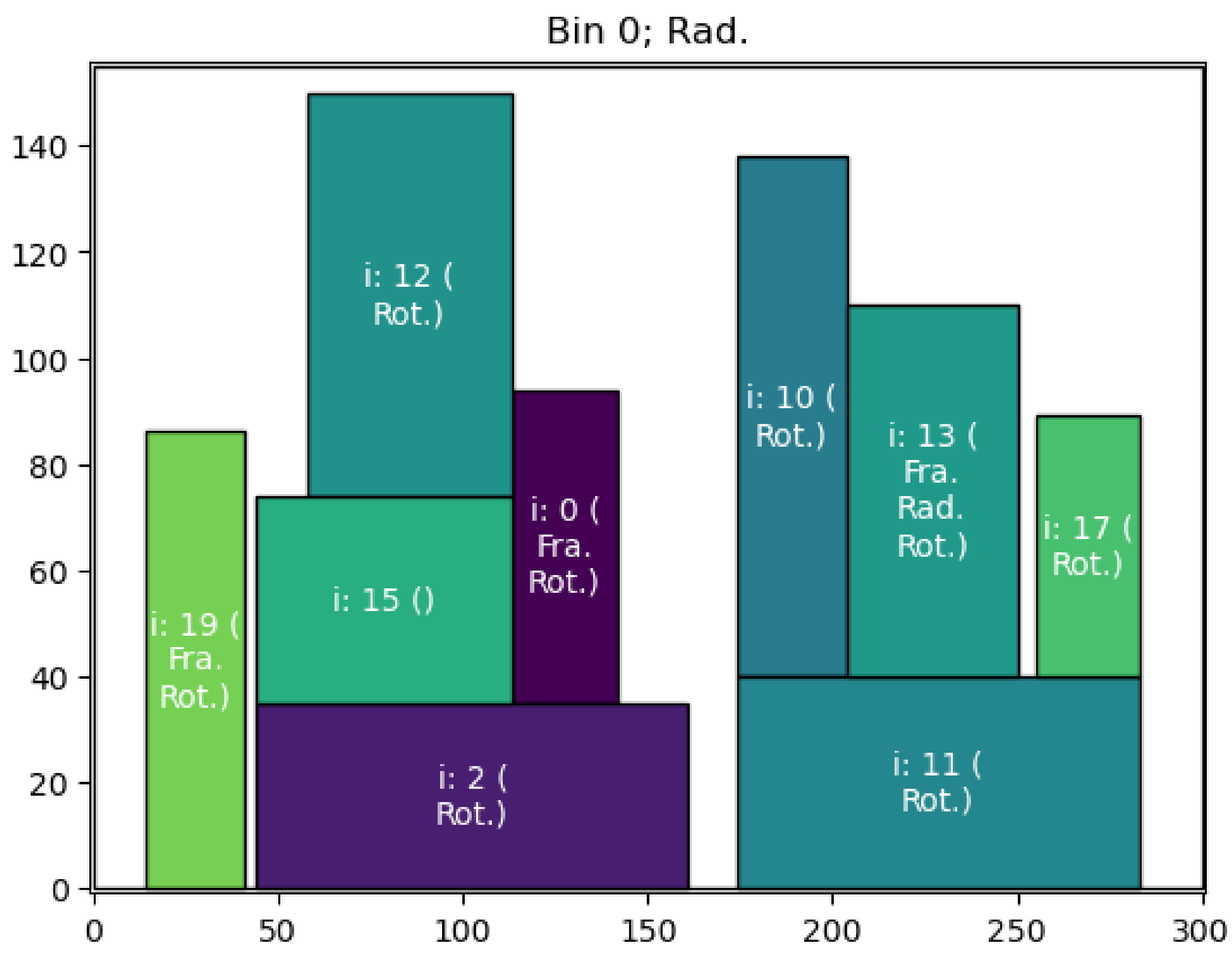
The data received from the Revenue Management team consisted of two different bin sizes, with the smaller sized bins each costing 200 euros and the larger ones costing 140 euros per bin. A total of 25 packages were given, of which three of them can not be rotated; six of them were fragile items, one of them was perishable, and two of them were labelled radioactive.

Figure 1, 2 and 3 show the results of a single run of the model. Some information has been provided for each item, namely the item number (i) and whether or not it is fragile (Fra.), perishable (Per.), radioactive (Rad.) or can be rotated (Rot.). Note that these are the results of one single run, and as the model does not generate one specific solution, the coordinates might differ in the final output file.

As one can see in the figures, all 25 items can be placed inside one of the three bins provided. No items are floating inside the bin and are either supported by the bin itself or two vertices of other items. There are no items placed on top of the fragile items in all bins. Replicating different runs proved that rotatable items could in fact be rotated ninety degrees, and that items who do not own this characteristic are always placed in the same manner.

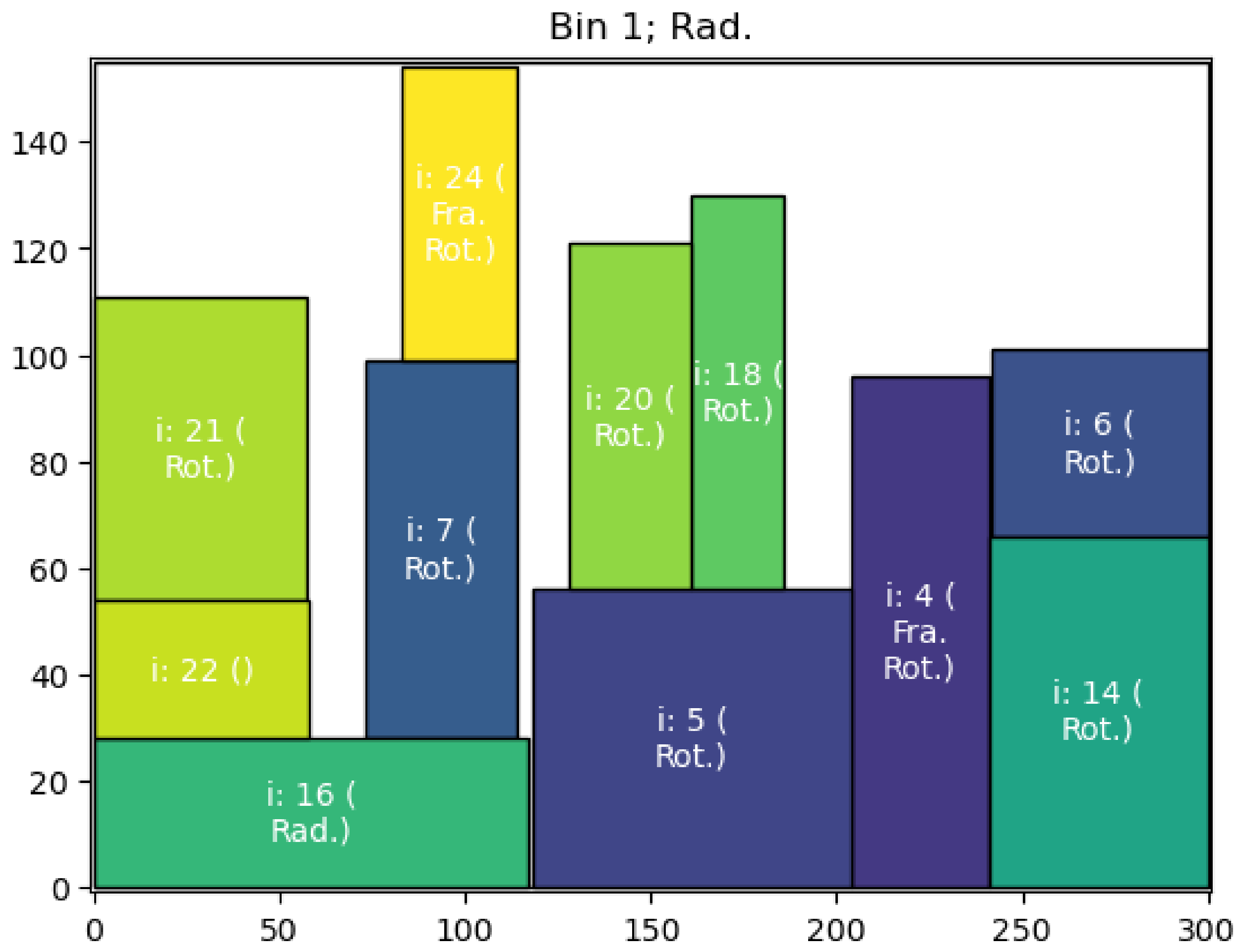
Bin 0 contains the radioactive items and therefore, does not contain any perishable items. There are three fragile items which do not have any items placed on top of them. As the figure suggests, there is space left to potentially put additional packages as well. This is due to set of limited number of packages that have been provided by the Revenue Management department.

Figure 1: Bin packing results: Bin 0



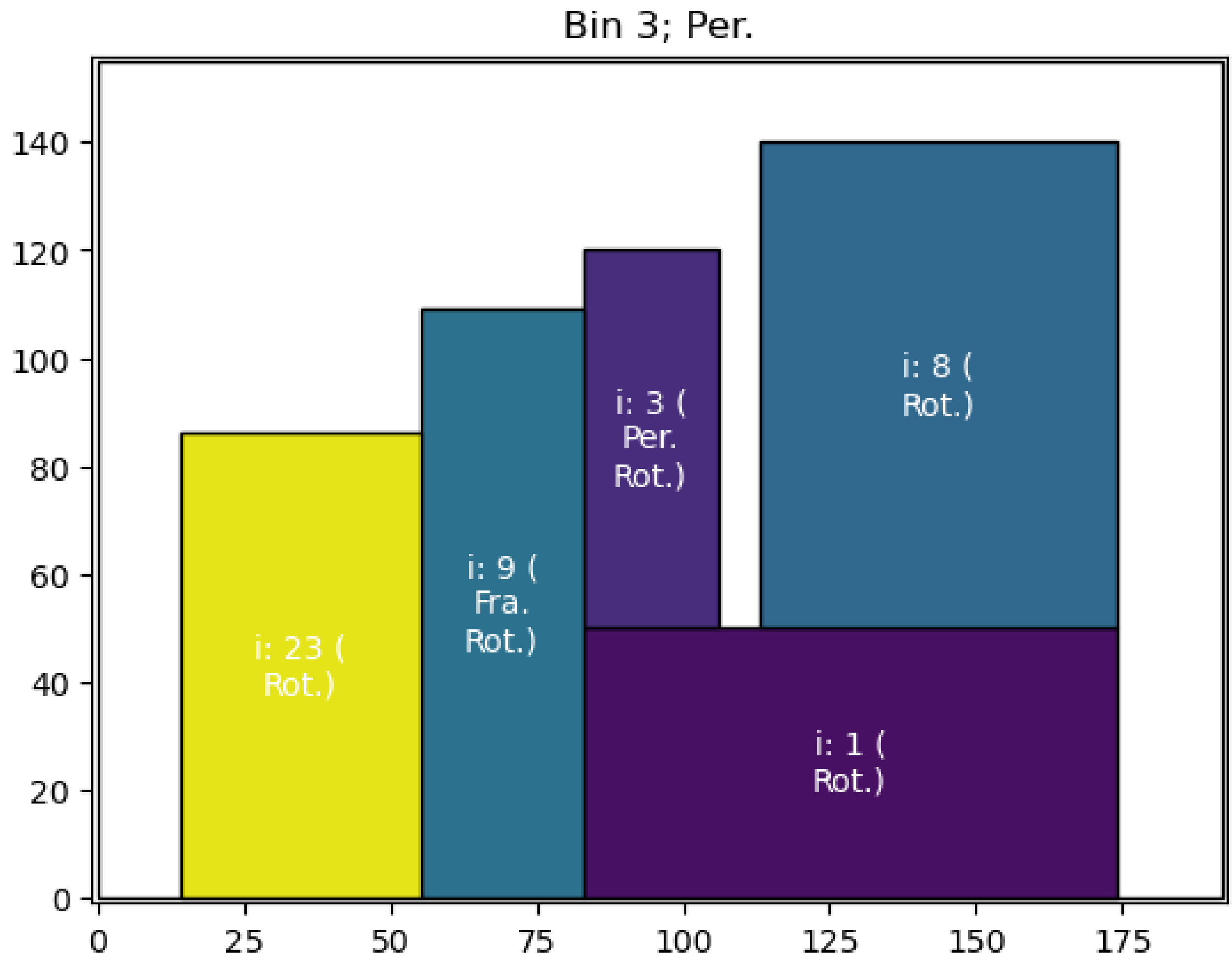
Some other radioactive items are placed inside the second ULD (bin 2), because the optimal solution could not find a more efficient way which would result into using only two bins. It therefore chose arbitrary to also put some radioactive items inside the second bin. Again, all fragile packages do not have any other items placed on top of them.

Figure 2: Bin packing results: Bin 1



Lastly, the third bin (bin 3) has been used to store the single perishable item. Two fragile items have been placed correctly inside the bin and all items are supported.

Figure 3: Bin packing results: Bin 2



All major constraints are followed correctly, notably the different constraints on the packages, no overlapping packages, and the correct use of gravity supporting at least 2 edges of each package. Looking at Figures 1, 2 and 3 above, one does note that even though there's only 2 radioactive items, they are not in the same bin. Bin 2 is not that full but most likely it had to be used since certain packages had a large length or height. The results also prioritize Bins 0 and 1 as they are larger in length and cheaper before using bins 2 and 3. Another important notion might be that although all items are considered to be positioned stably within this research, some items could still fall over when a aircraft would tilt during the flight. For example, item i: 10 is an elongated oriented object, which has a relatively high centre of gravity which might lead it to fall over during flight.

4 Algorithmic results (Part C)

With the major constraints being adhered to, the final best solution came out to be **480 euros**, which represents the cost of the 3 bins used. This value is the best incumbent, whereas the best bound was found to be 280. Intuitively this makes sense, as the geometric properties of the packages makes it such that, at a minimum, the 2 larger sized bins will need to be used (assuming a relaxed model that doesn't restrict the item constraints). A simple geometric calculation of the dimensions of all the packages found that you would need a $76055m^2$ space in order to fit them all. The 2 larger bins are $93000m^2$, thus in theory all items should be able to fit in the larger/cheaper bins. Thus the solution is quasi-optimal given the additional constraints for each item.

The solution did not find a complete optimal solution within the two hour time limit set and therefore had an optimality gap of 41.7%. This optimality gap is quite high, reflecting the underlying nature of the bin packing problem (that it is NP-complete in nature, thus very difficult to solve once a certain number of variables/packages are introduced into the model). When the model was rerun with only 15 packages, an optimal solution was found and the computational time was only 15 seconds.

The model has multiple constraints that very much limit the possibility of using fewer bins than three, namely the radioactivity, perish-ability or the fragility of the items. The first two determine that there are at least two bins needed if there are at least one item of each, which is the case in this data set. The combination of defined constraints and the limited number of bins and items results into a situation where the model does not necessarily has to be very economical or parsimonious with the total space available, as one can see in figures 1, 2 and 3.

5 Managerial considerations (Part D)

The modeled 2DBPP problem correctly displays how packages can be stacked on top of each other whilst adhering to gravity constraints. In addition to that the model allows for additional constraints to be added such as fragile items or items that can't be in the same bin (E.G. radioactive versus fresh produce). It however does not get at any odd, irregular shapes as the model was simplified in order to provide straightforward insights into bin packing in general.

In reality however, loading cargo inside the belly space of a aircraft is very much a three dimensional problem. Adding a third dimension is therefore needed in order to generate more realistic results.

Secondly, weight is a rather important factor within air transport. Weight related constraints do not only limit the total weight that can be put inside one of the bins, it is also important that all bins are well balanced. In other words, both the balance of weight between different ULD's as well as the weight within one ULD should be taken into consideration. If this is not considered, this may lead to crushed packages.

As mentioned before, all items are considered to be stably placed inside the ULD's, however some might still fall over during flight because of the relatively high centre of gravity of that item, having no stability provided from a horizontal perspective. Updating the model with this consideration would lead to less damaged goods.

Lastly, the time aspect of the different items has been ignored within this research. Time is a very critical and costly aspect within the aviation industry in general, but could also affect the allocation of items in bins. If the arriving pattern of incoming items has a rather large variation, it might change the distribution of items across the different bins quite significantly.

It is therefore suggested this model should be used as a good base to learn about compatibility of different types of goods and the optimal allocation of items inside a bin. The aforementioned additions would help improve the validity of the model even further.

6 Reflection and Conclusion (Part E)

Overall, this research ran a MILP model in order to optimize the packing order for 25 packages that had to go into ULD's. The model ran for 2 hours and gave a solution with a 41.7% optimality gap. The best incumbent result was 480 euros, which was the use of 3 bins. Section 6 above discussed various real world implications of this model and what other aspects may be important when considering applying this MILP model to real world applications. The next sections discuss the individual team member's thoughts about the assignment.

Kelvin Arbman

Generally, I think the assignment was quite a challenge. Especially setting up and understanding the mathematical model was a relatively tough part of the assignment, because many of the constraints had something to do with making the model valid compared to reality (i.e. the gravity constraints), rather than implementing capacity-related constraints. However, I enjoyed applying the knowledge I retrieved during this course as well as prior courses regarding MILP on an aviation industry related problem.

Personally, I wouldn't specifically Drop an element of the assignment. What I'd maybe like to see Added to next years assignment, are more (student)-assistants, so that we can discuss certain subjects more closely with someone. I'd definitely Keep the current explanation of the assignment, which was very clear and extensive during both the lecture as well as in the assignment file. I would Improve the preparation for the assignment a bit, so that the submission date doesn't need to be beyond the exam weeks. But taking into consideration that it was the first time, I think it was a very well executed assignment.

Lars Koetsier

It was quite challenging to amend the mathematical model that was provided in the lectures. However, I have learned yet again very much about the implementation of such a dimensional problem on paper and in Python and Gurobi.

I wouldn't drop any specific part of the assignment, as the base of it was already set up rather good. If there would be one thing that I would improve, then I would choose the preparation of the assignment, so that the original delivery date could have been met. I would keep the current form of instructions, giving a general lecture about the optimization method, after which you introduce the assignment. I would dedicate a little bit more time however to discussing several problems that different groups face during the process.

Ian Trout

This assignment was well thought out and was nice to have a real world application with a simulated package and bin set. What could have been improved was the dominance for our bin set (1 type of bin was cheaper and larger) which forced the MILP model to always choose that type of bin. A thing to keep would be the simplification of the bins (none had an angled cut) since that lowered the complexity of the required constraints. It was also nice that in the course work, half of the model was already specified, which allowed the team to focus only on the additional 'gravity' constraints.

Allard Krikke

Overall the assignment was well described and detailed and it was nice to have a different kind of programming challenge compared to other courses from the TIL master. I think we slightly underestimated the size of the final mathematical model and the Python/Gurobi challenges that arose from there, but that did not lead to any problems in the end. Apart from that it was nice to learn more about the 2D bin-packing problem and the python implementation of it.

I personally think the assignment has the right study load and reflects the course well. I wouldn't drop or add anything. Maybe weight and/or the stability of items could be added, but that does add quite some more complexity to the model. As already mentioned, the biggest improvement is the final deadline, but as the issue is known that can be easily solved for next year's students.