Delft University of Technology

Quantitative Methods for Logistics

ME44206

Assignment Q2 2022

Group 9

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Contents

1 TSP Mathematical Formulation (Part A)

The notation used for this mathematical formulation is provided in Table 1

Table 1: Notation

Sets and indices		
N	Set of locations	$i, j \in N$
i, j	Indices for nodes/customers (1 = depot)	
Parameters		
RT_i	Earliest time for delivery at node i	[unit]
DT_i	Latest time for delivery at node i	[unit]
d_{ij}	Distance (shortest time required) from i to j, $d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$	[unit]
a_i	Demand of customer i	[product]
s_i	Service time at node i	[unit]
Variables		
x_{ij}	1 if arc (i,j) in the solution, 0 otherwise	
l_i	load of the vehicle at node i	[product]
t_i	Time at which the vehicle arrives at node i	[unit]

The mathematical formulation then follows as:

$$\min \quad \sum_{i \in N} \sum_{j \in N} d_{ij} x_{ij} \tag{1}$$

Subject to:

$$\sum_{i \in N} x_{ij} = 1 \qquad \forall j \in N, i \neq j \tag{2}$$

$$\sum_{j \in N} x_{ij} = 1 \qquad \forall i \in N, i \neq j \tag{3}$$

$$t_i + d_{ij} + s_i \leq t_j + M(1 - x_{ij}) \qquad \forall i, j \in \{2, ..., n\}, i \neq j \tag{4}$$

$$d_{1j} + s_1 \leq t_j \qquad \forall j \in \{2, ..., n\}, i \neq j \tag{5}$$

$$RT_j \leq t_j \leq DT_j \qquad \forall j \in N \tag{6}$$

$$a_i \leq l_i \qquad \forall i \in N \tag{7}$$

$$l_i - a_i - M(1 - x_{ij}) \leq l_j \leq l_i - a_i + M(1 - x_{ij}) \qquad i, j \in N, i \neq j \tag{8}$$

$$l_1 = 0 \tag{9}$$

$$x_{ij} \in \{0, 1\} \tag{10}$$

$$l_i \in Z \qquad \forall i \in N \tag{11}$$

$$t_i \in Z \qquad \forall i \in N \tag{12}$$

The objective function (1) is written as the minimization of the total distance. Constraint (2) ensures that the vehicle only leaves each node once, and constraint (3) ensures that the vehicle only enters each node once. Constraint (4) and (5) ensures that there are no sub-tours whilst guaranteeing that all needed time is considered (service time, travel time, and arrival time at the previous location). Constraint (6) makes sure that the vehicle arrives at each node within the opening and closing times of the node. Constraint (7) ensures that the vehicle capacity can hold the demand whilst constraint (8) ensures that the vehicle load at node j is less than the load of the vehicle at the previous node. Constraint(9) ensures that upon arrival of the vehicle at the depot, it is completely emptied, therefore defining the load it carries when it starts on its route. Constraints (10), (11), and (12) are that the variable x_{ij} is a binary variable and that l_i & t_i are natural numbers respectively.

2 TSP Results (Part B)

Using Gurobi with the data_small.txt file in Python, the following results for the total distance, route travelled, and additional information are reported in Table 2.

Table 2: TSP Results

Total distance travelled	78.34 units		
Sequence of the vehicle (nodes)	$0, 2, 1, 4, 6, 5, 3, 0$		
Location	Time window	Time of arrival	Load of vehicle at each location
Depot [0]	$0 - 1236$	0 [departure]	120
2	$15 - 67$	15	120
1	$65 - 146$	146	100
4	$255 - 324$	255	80
6	$448 - 505$	448	60
5	$534 - 605$	605	30
3	$621 - 702$	702	20
0	$0 - 1236$	1236	0

Figure 1 shows the route of the vehicle as it delivers products to each customer. An interesting note is that the vehicle waits until the depot (node 0) is about to close before reaching it. The vehicle could have arrived at nodes $5, 3, 0$ earlier but choose not to. As a side experiment, the team analyzed what would happen if we added the minimization of arrival time t_i. The results of that test are shown in Table 3. However we did not carry forward the minimization of t_i in the objective function in the next sections as adding it would mean that the optimization model would be trying to optimize both t_i and $d_{ij}x_{ij}$ which could lead to incorrect results.

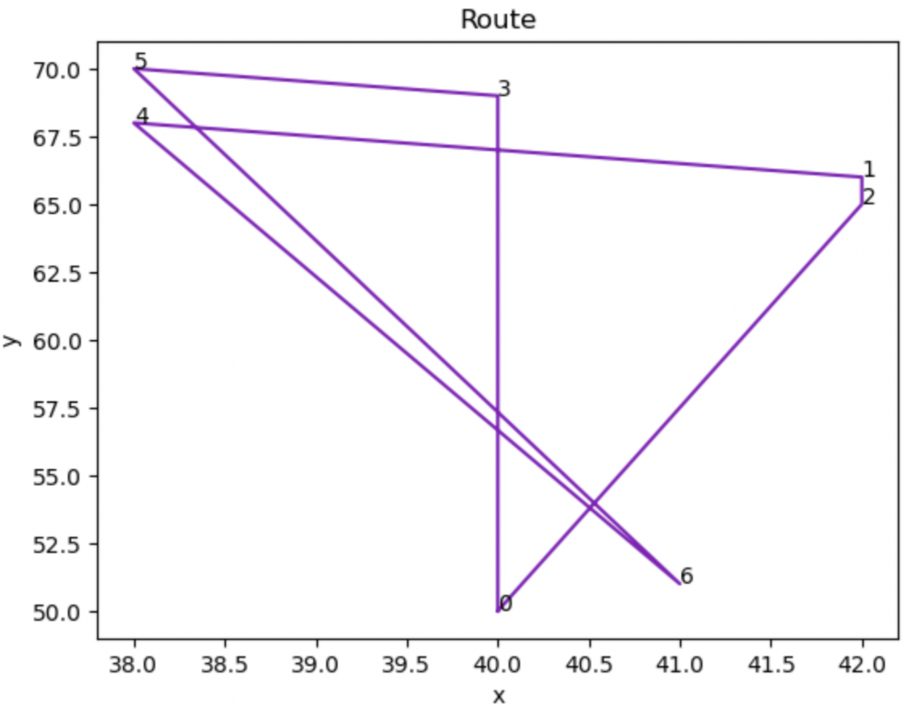


Figure 1: 2D plot of the route of the vehicle

Table 3: TSP Results with t minimization

Total distance travelled	78.34 units		
Sequence of the vehicle (nodes)	$0, 2, 1, 4, 6, 5, 3, 0$		
Location	Time window	Time of arrival	Load of vehicle at each location
Depot [0]	$0 - 1236$	0 [departure]	120
2	$15 - 67$	15	120
1	$65 - 146$	106	100
4	$255 - 324$	255	80
6	$448 - 505$	448	60
5	$534 - 605$	557	30
3	$621 - 702$	649	20
Depot [0]	$0 - 1236$	758	0

3 VRP Mathematical Formulation (Part C)

The new notation or modified notations are provided in Table 4.

Table 4: Notation

Sets and indices		
....		
v	Index for vehicle	
V	Set of vehicles	$v \in V$
Parameters		
....		
b_v	Capacity of vehicle v	[product]
K	Number of vehicles to be used	
Variables		
x_{ijv}	1 if v is travelling from customer i to j, 0 otherwise	
z_{iv}	1 if customer i is visited by vehicle v, 0 otherwise	
l_{iv}	load of vehicle v at node i	[product]
t_{iv}	Time at which the vehicle v arrives at node i	[unit]

The mathematical formulation then follows as:

$$\min \quad \sum_{v\in V}\sum_{i\in N}\sum_{j\in N} d_{ij}x_{ijv} \tag{13}$$

Subject to:

$$\sum_{v\in V} z_{iv} = 1 \qquad \forall i \in N \setminus \{1\} \tag{14}$$

$$\sum_{v\in V} z_{1v} \leq K \tag{15}$$

$$t_{iv} + d_{ij} + s_i \leq t_{jv} + M(1 - x_{ijv}) \qquad i, j \in \{2, ..., n\}, i \neq j, \forall v \in V \tag{16}$$

$$d_{1j} + s_1 \leq t_{jv} \qquad \forall j \in \{2, ..., n\}, i \neq j \tag{17}$$

$$RT_j \leq t_{jv} \leq DT_j \qquad \forall j \in N, \forall v \in V \tag{18}$$

$$\sum_{i\in N} a_i z_{iv} \leq b_v \qquad \forall v \in V \tag{19}$$

$$l_{iv} - a_i - M(1 - x_{ijv}) \leq l_{jv} \qquad i, j \in N, i \neq j, \forall v \in V \tag{20}$$

$$l_{jv} \leq l_{iv} - a_i + M(1 - x_{ijv}) \qquad i, j \in N, i \neq j, \forall v \in V \tag{21}$$

$$\sum_{v\in V} l_{1v} = 0 \tag{22}$$

$$\sum_{j\in N} x_{jiv} = \sum_{j\in N} x_{ijv} = z_{iv} \qquad \forall v \in V, \forall i \in N, i \neq j \tag{23}$$

$$x_{ijv}, z_{iv} \in \{0, 1\} \qquad \forall i, j \in N, \forall v \in V \tag{24}$$

$$l_{iv} \in Z \qquad \forall i \in N, \forall v \in V \tag{25}$$

In comparison to the mathematical model in part A, The objective function (13) is written as the minimization of total distance travelled by all vehicles, using an index for each vehicle v. Constraint (14) ensures that each node is visited only once by any vehicle (except the depot). Constraint (15) ensures that a minimum number of vehicles are used and that they all start at the depot. Constraint (19) ensures that the vehicle has at least the capacity to meet all the demands of the nodes it visits in its route. Constraint (23) is the flow conservation constraint meaning, if a vehicle visits a customer, it needs to exit the node after satisfying the demand. The rest of the constraints, (16), (17),(18), (20), (21), (22), (24), & (25) act in a similar manner to part A mathematical model with an additional index v to apply the constraints for an individual vehicle.

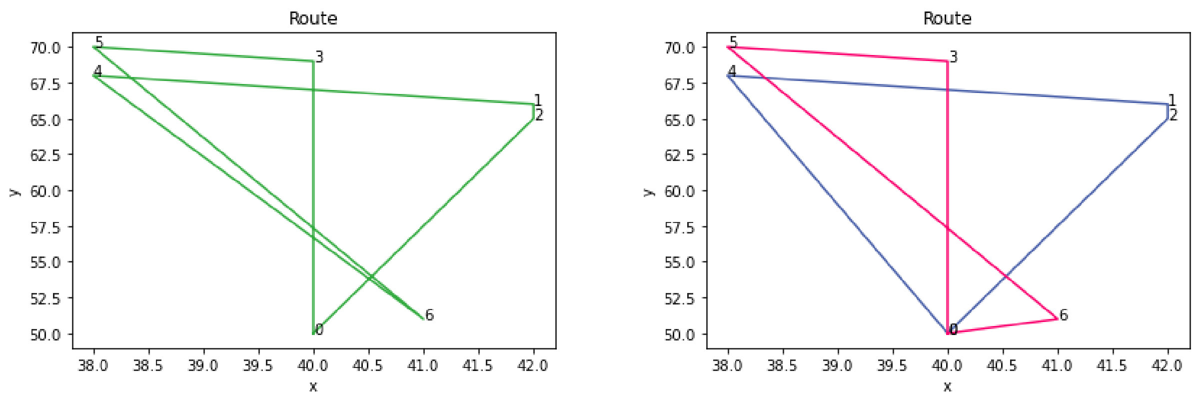
4 VRP Results (Part D)

Using Gurobi with the data_small.txt file in Python, the following results for the total distance, route travelled, and additional information are reported in Figures 2 thru 5. The section also does a short comparative evaluation between the 3 VRP cases and the result of the TSP problem (Part B).

Description	Part B: TSP problem	Part D: Capacitated VRP		
	1 Vehicle with no capacity limitation.	1 vehicle with a capacity of 120	2 vehicles with a capacity of 60 each	6 vehicles with a capacity of 20 each
Total distance travelled	78.34 units	78.34 units	80.60 units	Solution is infeasible
Sequence/ Path of each vehicles	0-2-1-4-6-5-3-0	0-2-1-4-6-5-3-0	Vehicle 1(V_1): 0-2-1-4-0 Vehicle 2(V_2): 0-6-5-3-0	

Figure 2: Distance travelled comparison of the TSP problem and 3 cases of capacitated VRP

In Figures 2 and 3 it can be observed the TSP and the case 1 of the capacitated VRP (1 vehicle with a capacity of 120) yields the same result in all aspects. This is due to the fact that the sum of all the demands at all nodes adds up to 120, i.e. the capacity of the vehicle.



(a) case 1: 1 vehicle with capacity of 120

(b) case 2 (2 vehicles with capacity of 60 each

Figure 3: Routes of vehicles for case 2 solved by VRP and SDVRP

In case 2, with 2 vehicles of homogeneous capacity of 60, it results in 2 vehicles travelling in different paths, with V_1 traversing $0-2-1-4-0$ and V_2 travelling the route $0-6-5-3-0$. The routes are shown in Figure 3(b). Upon comparison, it is observed, due to the capacity constraint of 60, the original tour is split in 2 tours, but with the sequence of the nodes visited remaining the same. The presence of this capacity constraint, increases the distance travelled by 2.88%, from 78.34 units to 80.60 units.

In Figure 4, it is observed that due to the presence of time windows, vehicles leave sequentially rather than simultaneously. Therefore, since the sequence of visit is identical to the TSP and case 1 of the capacitated VRP, except the final arrival time of V_1 at the depot, the arrival time at all the nodes is identical to the TSP and case 1 of the capacitated VRP.

Finally, in Figure 5, the load carried by each vehicle depends upon it completely meeting the demand of nodes it visits on it's route (ensured by constraint 19) and by the time it finally returns to the depot, the whole vehicle is emptied(defined by constraint 22).

Nodes	Time window		Time of arrival at each node				
			Part B: TSP problem	Part D: Capacitated VRP			
	Ready time	Due time	1 vehicle with no capacity limitation.	1 vehicle with a capacity of 120	2 vehicles with a capacity of 60		6 Vehicles with capacity of 20
					V_1	V_2	
1	65	146	146	146	146	--	Solution is infeasible
2	15	67	15	15	55	--	
3	621	702	702	702	--	702	
4	255	324	255	255	255	--	
5	534	605	605	605	--	605	
6	448	505	448	448	--	448	
0	0	1236	1236	1236	1236	1236	

Figure 4: Arrival time comparison of the TSP problem(B) and 3 cases of capacitated VRP

Node No.	Load at each node				
	Part B: TSP problem	Part D: Capacitated VRP			
	1 vehicle with no capacity limitation.	1 vehicle with a capacity of 120	2 vehicles with a capacity of 60		6 Vehicles with capacity of 20
			V_1	V_2	
1	120	120	40	--	Solution is infeasible
2	100	100	60	--	
3	80	80	--	20	
4	60	60	20	--	
5	30	30	--	30	
6	20	20	--	60	
0	0	0	0	0	

Figure 5: Vehicle load comparison of the TSP problem(B) and 3 cases of capacitated VRP

In case 3, with 6 vehicles of homogeneous capacity of 20, the solution is infeasible. This is due the presence of the constraint (14), which ensures that each node has to be visited only once. The demand of node 6 is higher than the vehicle maximum capacity of 20, thus violating constraint(19), which means the model can't satisfy the demand of the customer at node 6 in one visit, resulting in infeasibility.

5 SDVRP Mathematical Formulation (Part E)

For the split delivery vehicle route problem, notations used in Part C are still used in this formulation in addition to a new variable, p_{iv} which is defined to represent the portion of the demand of customer i delivered by vehicle v, as shown in Table 5.

Table 5: Additional Notation

Variables	
....	
p_{iv}	The proportion of the ith customer demand delivered by vehicle v

The updates of the mathematical formulation for this part follows as:

Subject to new or modified constraints:

$$\sum_{v \in V} p_{iv} = 1 \qquad \forall i \in N \setminus \{1\} \tag{26}$$

$$\sum_{i \in N \setminus \{1\}} a_i p_{iv} \leq b_v \qquad \forall v \in V \tag{27}$$

$$l_{iv} - a_i p_{iv} - M(1 - x_{ijv}) \leq l_{jv} \qquad i, j \in N, i \neq j, \forall v \in V \tag{28}$$

$$l_{jv} \leq l_{iv} - a_i p_{iv} + M(1 - x_{ijv}) \qquad i, j \in N, i \neq j, \forall v \in V \tag{29}$$

$$z_{iv} \geq p_{iv} \qquad \forall v \in V, i \in N \setminus \{1\} \tag{30}$$

$$x_{ijv} \in \{0, 1\} \qquad \forall i, j \in N \, \forall v \in V \tag{31}$$

$$z_{iv} \in \{0, 1\} \qquad \forall i \in N \setminus \{1\}, \forall v \in V \tag{32}$$

$$0 \leq p_{iv} \leq 1 \qquad \forall i \in N \setminus \{1\}, \forall v \in V \tag{33}$$

The objective function is still written as the minimization of total distance, which is the same as expression (13) in part (C). Constraints (16)(17)(18) and (23) in part (C) still work here to ensure that every customer is visited by a vehicle within the time window provided and that each vehicle does not make sub-tours. Constraint (26) ensures that the sum of the delivery proportions at each customer equals 1. Constraint (27) makes sure that the proportion of the demand delivered at i does not exceed the vehicle capacity. Constraints (28), (29) define the current load of vehicle v at each node j. Constraints (30) and (33) ensure that the proportion delivered to each node by a vehicle v is between 0 and 1 and is dependant upon whether or not a vehicle visits that node. Constraints (31) and (32) are that the variables x_{ijv} and z_{iv} are binary variables respectively.

6 SDVRP Results (Part F)

Using Gurobi with the data_small.txt file in Python, the following results for the total distance, route traveled, and additional information are reported in Figures 6, 7 and 8.

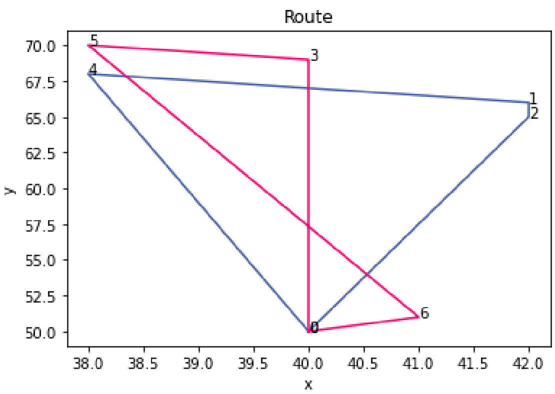
Figure 6 compares the differences between the VRP and the SDVRP considering the same cases mentioned in Part C (the total distance traveled & the path of each vehicle) while including the proportion of demand for each customer delivered by each vehicle. In case 1, there is only one vehicle with a capacity of 120, thus the SDVRP is degraded to the VRP in Part C and the results of the two solving methods are the same.

In case 2, however, the total distance traveled after considering split delivery is reduced by about 5.7%, from 80.60 units to 75.95 units. The demand for customer 2 is split, as shown in figure 7, with two vehicles delivering half of the demand each. The rest of the customers (1, 3, 4, 5, and 6) are still served by only one vehicle. There is a gain in efficiency by allowing split deliveries to a customer that has large demands. Also, it is noticeable that the routes of the vehicles change a lot after considering the split delivery. When split delivery is not allowed in Part C, distribution tasks are almost evenly distributed; the areas that are covered and the number of customers visited by each vehicle are close to each other. While in SDVRP, it is obvious that one vehicle takes on more distribution tasks. It is because SDVRP makes it possible to fully utilize the capacity of each vehicle so that the overall efficiency is improved despite the unequal distribution of distribution tasks between the two vehicles.

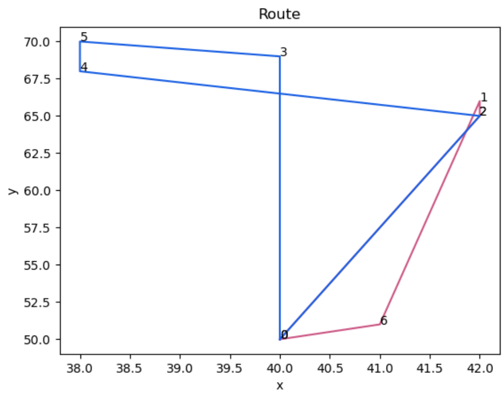
In case 3, the SDVRP does provide a solution (as opposed to VRP which was infeasible). In the solution, as shown in Figure 8, all 6 vehicles are used, but only the demand of customer 6 is split and met by 2 vehicles, with one vehicle serving 1/3 and the other one serving 2/3. Other customers are still served by only one vehicle. This is because each vehicle only has a carrying capacity of 20 units in this case, which is close to the needs of each customer, making it almost impossible to serve several customers per vehicle. In order to successfully meet the needs of all customers, all vehicle have to be utilized.

	VRP			**SDVRP**		
	Total distance	**Path of each vehicle**	**Proportion at each node for each vehicle**	**Total distance**	**Path of each vehicle**	**Proportion at each node for each vehicle**
Case 1: Veh. number: 1 Capa.: 120	78.34 units	**Vehicle 1 (V1)** 0-2-1-4-6-5-3-0	0-1-1-1-1-1-1-0	78.34 units	**Vehicle 1 (V1)** 0-2-1-4-6-5-3-0	0-1-1-1-1-1-1-0
Case 2: Veh. number: 2 Capa.: 60	80.60 units	**Vehicle 1 (V1)** 0-2-1-4-0 **Vehicle 2 (V2)** 0-6-5-3-0	0-1-1-1-0 0-1-1-1-0	75.95 units	**Vehicle 1 (V1)** 0-2-1-6-0 **Vehicle 2 (V2)** 0-2-4-5-3-0	0 - 0.5 - 1 - 1 - 0 0 - 0.5 - 1 - 1 - 1 - 0
Case 3: Veh. number: 6 Capa.: 20	No solution	No solution	No solution	180.31 units	**Vehicle 1 (V1)** 0-6-0 **Vehicle 2 (V2)** 0-3-0 **Vehicle 3 (V3)** 0-4-0 **Vehicle 4 (V4)** 0-6-5-0 **Vehicle 5 (V5)** 0-1-0 **Vehicle 6 (V6)** 0-2-0	0 - 0.67 - 0 0 - 1 - 0 0 - 1 - 0 0 - 0.33 - 1 - 0 0 - 1 - 0 0 - 1 - 0

Figure 6: Comparison between VRP and SDVRP for 3 cases

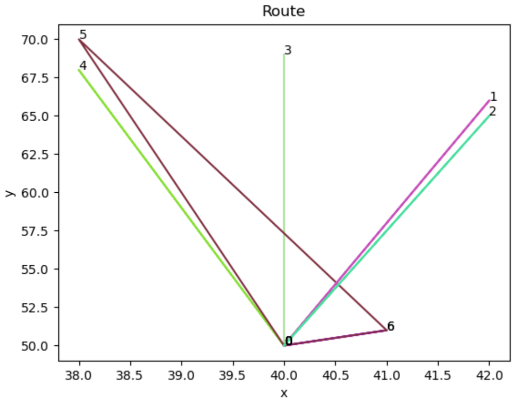


(a) Solved by VRP



(b) Solved by SDVRP

Figure 7: Routes of vehicles for case 2 solved by VRP and SDVRP



(a) case 3 route

Vehicle	Sequence of nodes (route)	load at each node	arrival time at each node
V1	0-2-1-4-6-5-3-0	120-100-80-60-30-20-0	15-146-255-448-605-702-1236
Below 2 vehicle scenario with capacity of 60 each			
V1	0-2-1-6-0	60-50-30-0	55-146-448-1236
V2	0-2-4-5-3-0	60-50-30-20-0	67-255-605-702-1236
Below 6 vehicle scenario with capacity of 20 each			
V1	0-4-0	20-0	255-1236
V2	0-3-0	20-0	702-1236
V3	0-6-0	20-0	448-1236
V4	0-1-0	20-0	146-1236
V5	0-2-0	20-0	67-1236
V6	0-6-5-0	20-10-0	448-605-1236

(b) overview

Figure 8: Route for case 3 & arrival time and load values at each node for each case

Figure 8(b) shows the 3 cases with each vehicle's arrival time and load at each node.

7 SDVRP With Vehicle Capacity: 200 (Part G)

- **Total travel distance:** 749.04
- **Vehicle numbers:** 11
- **Whether split deliveries are used:** No

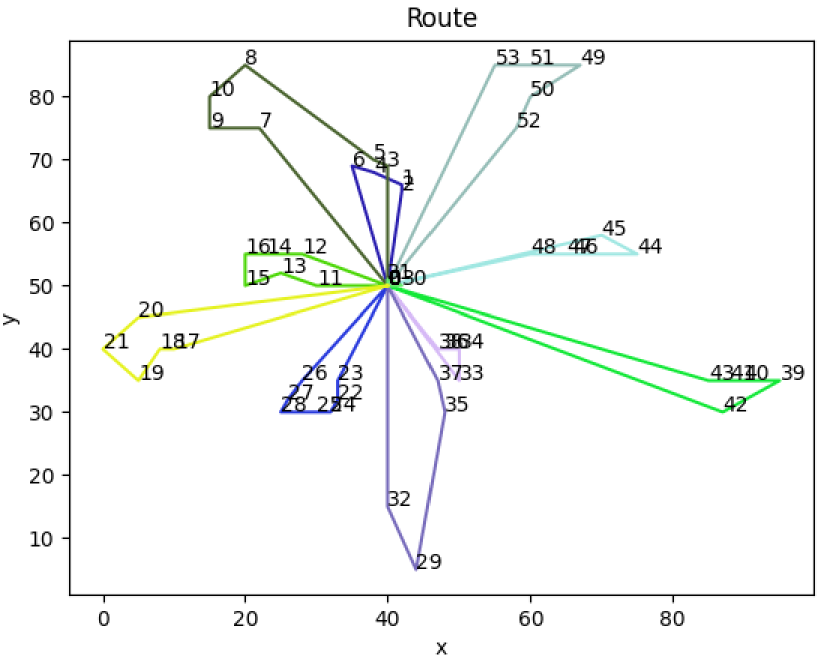


Figure 9: Routes for large data with a vehicle capacity of 200

This section used the large data set, hence why the results are more complex than in prior sections. Figure 10 depicts the routes of the 11 vehicles and their respective loads at each node and arrival times whilst Figure 9 depicts the routes of each vehicle in order to serve all 53 nodes/customers.

Vehicle	Sequence of nodes (route)	load at each node	arrival time at each node
V1	0-52-50-49-51-53-0	100-80-70-50-20-0	31-126-441-622-726-1236
V2	0-2-1-4-6-0	120-70-30-10-0	15-106-278-448-1236
V3	0-30-31-0	120-80-0	257-436-1236
V4	0-43-41-40-39-42-0	90-60-40-30-10-0	77-170-260-360-731-1236
V5	0-17-18-19-21-20-0	120-90-50-40-10-0	32-124-344-522-700-1236
V6	0-38-36-34-33-0	160-120-70-20-0	12-103-218-317-1236
V7	0-11-13-15-16-14-12-0	100-90-50-40-30-10-0	10-224-358-500-593-777-1236
V8	0-32-29-35-37-0	80-40-20-10-0	35-294-693-875-1236
V9	0-23-22-24-25-28-27-26-0	170-130-110-60-30-20-10-0	17-110-412-600-786-880-1001-1236
V10	0-48-47-46-44-45-0	80-70-50-40-20-0	21-138-238-369-465-1236
V11	0-7-9-10-8-5-3-0	130-100-80-70-30-20-0	31-183-345-475-605-702-1236

Figure 10: Route, arrival time, and load information per vehicle

Reasons for not using split deliveries:
Since each vehicle has a capacity of 200, each departure can satisfy at least four nodes/demand points fully. The highest demand is 80 at node 31 which can be handled by any of the vehicles in the fleet. One can also note from Figure 9 that each vehicle has its own responsible area, the points in each area form a cluster, and the distance between the poles of each area is very far. If a vehicle goes to other areas for delivery, more time is wasted. Therefore the model does not use split delivery at all.

8 SDVRP With Vehicle Capacity: 100 (Part H)

After running the large data set with Gurobi and a fleet of vehicles with a capacity of 100 each, we get the following objective function result but only after timing out after 30 minutes (even with a fleet of 15 vehicles).

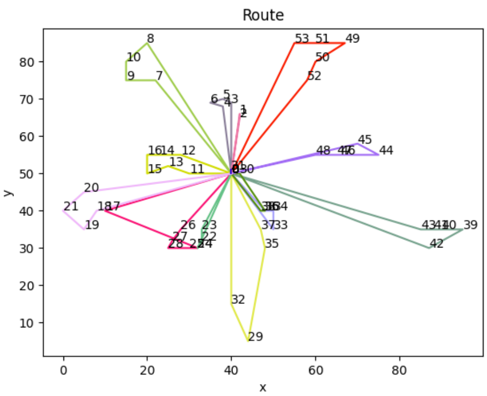
Capacity	Running time	Number of used vehicles	Total travel distance	# of split delivery nodes
100	time limit reached	14	879.67	1
150	time limit reached	11	769.99	0
200	10.5s	11	749.04	0
250	12.73s	11	749.04	0

Table 6: Experiment with different capacity

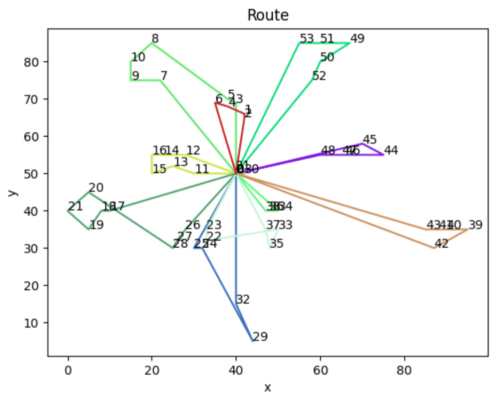
Table 6 and Figure 11 show the route sequence and the total travel distance for the scenario with vehicles that have a capacity of 100. Note that only node 36 is visited twice. The difficulty with this case is that depending upon the amount of vehicles in our fleet, there are many different possible routing options given that each vehicle can fully satisfy one or two demand nodes(customers). In addition, with the possibility of split deliveries, vehicles can drop off only a proportion of the demand requested by a certain node(customer). This is shown in Figure 12(a). It's for these reasons that at the end of the 30 minutes, there is still an optimality gap of 11.5%.

Vehicle	Sequence of nodes (route)	load at each node	arrival time at each node
V1	0-43-41-40-39-42-0	90-60-40-30-10-0	47-140-260-360-731-1236
V2	0-2-1-0	90-40-0	15-106-1236
V3	0-31-0	80-0	385-1236
V4	0-7-9-10-8-0	100-70-50-40-0	31-250-345-475-1236
V5	0-38-36-30-0	100-60-40-0	12-103-257-1236
V6	0-52-50-49-51-53-0	100-80-70-50-20-0	31-126-372-561-726-1236
V7	0-36-34-33-0	100-70-20-0	76-218-313-1236
V8	0-48-47-46-44-45-0	80-70-50-40-20-0	21-144-238-373-469-1236
V9	0-4-6-5-3-0	60-40-30-20-0	324-504-598-702-1236
V10	0-32-29-35-37-0	80-40-20-10-0	35-347-632-875-1236
V11	0-23-19-21-20-0	90-50-40-10-0	17-344-482-700-1236
V12	0-17-18-28-27-26-0	100-70-30-20-10-0	32-124-725-880-1001-1236
V13	0-22-24-25-0	100-80-30-0	68-405-541-1236
V14	0-11-13-15-16-14-12-0	100-90-50-40-30-10-0	10-224-358-453-546-732-1236

Figure 11: Route, arrival time, and load for each vehicle



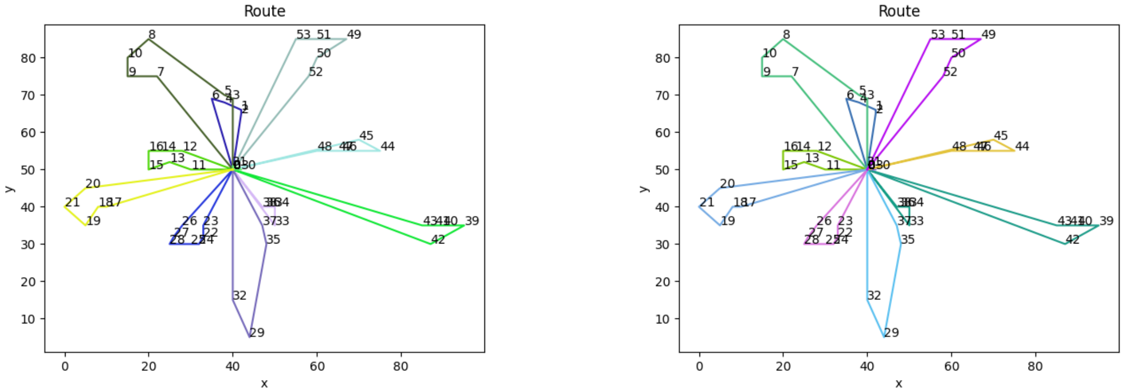
(a) Routes of capacity 100



(b) routes of capacity 150

Figure 12: Routes where the computational time limit was reached

This report did experiment by changing the capacity of the vehicles, as shown in Table 6 and Figures 12 and 13. Interesting things to note are that even with a vehicle capacity of 150, split deliveries are no longer used. In addition, once the vehicle capacity is 150 or greater, the number of vehicles used remains constant, meaning that there is stability in the results (and the total travel distance is close or is the same in the case of a capacity 200 or above).



(a) Routes of capacity 200

(b) routes of capacity 250

Figure 13: Routes where the computational time limit was not reached

9 HSDVRP Mathematical Formulation (Part I)

For a heterogeneous fleet that is doing split deliveries, notations used in Part E are still used with the addition of a new parameter and variable to consider the fixes cost for each vehicle that is being used, $c_v y_v$. This new addition to the objective function and 2 new constraints are provided in Table 7.

Table 7: Additional Notation

Parameters		
...		
c_v	Fixed cost of each vehicle v	[euro]
Variables		
....		
y_v	1 if vehicle v is used in the solution, 0 otherwise	

9.1 Weighted Sum Method

The updated objective function and mathematical formulation then follows as:

$$\min \quad (0.2 \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} d_{ij} x_{ijv} + \sum_{v \in V} c_v y_v) \tag{34}$$

Subject to:

$$\frac{\sum_{i \in N \setminus \{0\}} z_{iv}}{N-1} \leq y_v \qquad \forall v \in V \tag{35}$$

$$y_v \in \{0,1\} \qquad \forall v \in V \tag{36}$$

The objective function (34) is written as the minimization of the total distance and total fixed costs, whereas constraint (35) ascertains that a certain vehicle v is used ('y_v' which is constraint (36)).

The changed objective function (34) is written as the minimization of total distance and cost associated with using a vehicle. Here, instead of keeping the weights for the distance and the cost are equal, we need to find a weight for distance that can transform it into a cost. Increasing the weight for the distance traveled in the objective function would give more priority to the distance traveled, hence reducing the total distance traveled. According to L. Marujo, G. Goes, M. D'Agosto, A. Ferreira, M. Winkenbach, R. Bandeira (https://www.sciencedirect.com/science/article/pii/S1361920917308295), the average cost per km travelled in an urban setting for fleet of cargo bikes and vans was $0,20$.

10 HSDVRP Results (Part J)

10.1 set with 25 vehicles

Table 8 and Figure 14 show the route sequence and the total travel distance for the scenario with a heterogeneous fleet with different fixed costs. The model did not reach optimality, so after 30 minutes, the model stopped with a total fixed cost of 44900 euros and an optimality gap of 2.52%.

Total Distance	Total vehicles used	20 capacity used	100 capacity used	Nodes with split
973.41	20	9	11	7

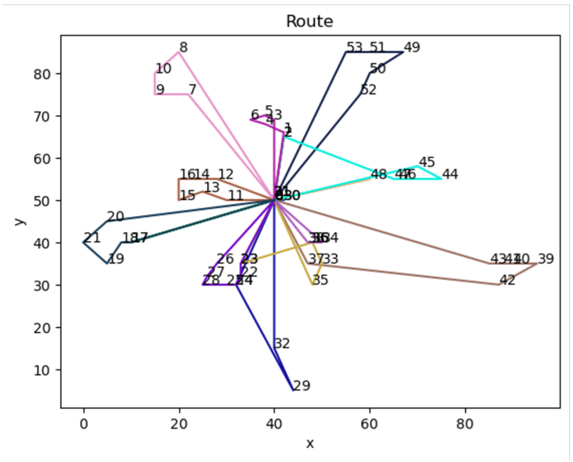
Table 8: Result of HSDVRP

Vehicle	Node sequence	Load sequence	Time sequence
v1	0-48-0	10-0	0-21-1236
v2	0-30-0	20-0	0-257-1236
v3	0-30-0	20-0	0-257-1236
v4	0-31-0	20-0	0-436-1236
v5	0-31-0	20-0	0-436-1236
v6	0-2-0	20-0	0-15-1236
v7	0-31-0	20-0	0-436-1236
v8	0-31-0	20-0	0-436-1236
v9	0-17-0	20-0	0-32-1236
v10	0-11-13-15-16-14-12-0	100-90-50-40-30-10-0	0-10-169-388-483-593-777-1236
v11	0-32-29-24-0	100-60-40-0	0-35-286-412-1236
v12	0-7-9-10-8-0	100-70-50-40-0	0-31-250-345-528-1236
v13	0-43-41-40-39-42-37-0	100-70-50-40-20-10-0	0-47-170-203-298-731-875-1236
v14	0-23-36-33-35-0	100-70-30-10-0	0-17-122-281-632-1236
v15	0-52-50-49-51-53-0	100-80-70-50-20-0	0-31-126-441-561-726-1236
v16	0-38-36-34-0	100-60-50-0	0-12-103-218-1236
v17	0-2-47-46-44-45-0	100-70-50-40-20-0	0-15-130-238-420-516-1236
v18	0-23-22-24-25-28-27-26-0	100-90-70-60-30-20-10-0	0-17-110-359-541-786-880-1001-1236
v19	0-1-4-6-5-3-0	100-60-40-30-20-0	0-65-288-505-605-702-1236
v20	0-17-18-19-21-20-0	100-90-50-40-10-0	0-32-124-283-482-700-1236

Figure 14: Route, arrival time, and load for each vehicle

In addition, Figure 15 shows the 2D plot of all vehicles as well as the results comparison between the case of a homogeneous fleet versus heterogeneous fleet.

In HSDVRP, the objective function is the total cost while considering unit cost for unit distance travelled. therefore, minimizing this function allows for reducing the use of larger vehicles as they contribute more to the total cost. However, setting the weight of the cost for unit distance travelled above the fixed costs would result in lesser distance travelled.



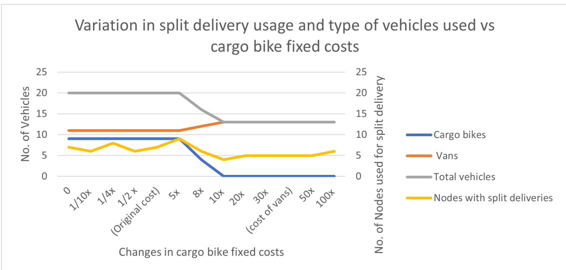
(a) HSDVRP vehicle plot

Description	Part H: SDVRP	Part J: HSDVRP
Total distance travelled	879.67	973.41
Number of vehicles used	14	20

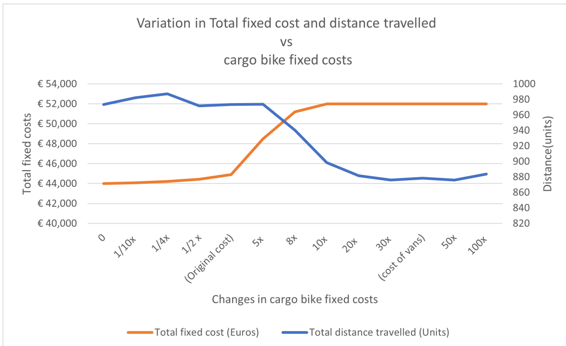
(b) Comparison between SDVRP and HSDVRP

Figure 15: Vehicle routes for HSDVRP and comparison table between SDVRP and HSDVRP

10.2 Experimentation with different small vehicle fixed cost



(a) Variation in usage of vehicles and nodes used for split delivery



(b) Variation in changes to total fixed costs and total distance travelled

Figure 16: Variation in parameters subject to the experimentation of the fixed costs of the cargo bikes

For this experiment, The small vehicles(capacity 20) are termed as cargo bikes, and large vehicle(capacity 100) as vans. The fixed cost of the cargo bikes was varied from 0 to 100x of the original cost of 100 euros. The fixed cost was made to 0 to check if it uses all the 10 vehicles when they are free to use. However as seen in Figure 16(a), it still doesn't result in usage of the all 10 vehicles. Some of the possible explanations could be, that it hasn't yet reached the optimal solution within the 1800-second time limit (optimality gap 2.8%) or that the vans would have to be used regardless in order to satisfy demand, so using one more cargo bike would mean an increase in total distance traveled. The other ranges were added to see the full variation in the usage of the cargo bikes such as to observe the tipping point when there was no benefit of using the cargo bikes.

The small variations in parameters observed in the number of nodes used for split deliveries (Figure 16(a)) and total distance traveled (Figure 16(b)) could be attributed to different optimality gaps attained for each of the solutions. The usage of cargo bikes remains constant until it is 5 times the original cost i.e 500 euros. However, once the cost rises above 500, the usage of cargo bikes gradually descends until, at 1000 euros for fixed cost, there are no cargo bikes being used. The low capacity of the cargo bike, coupled with rising fixed costs, make it eventually become unchosen by the optimization model. One of the possible reasons, could be attributed to the increase in the cost per unit of demand it can satisfy i.e. cost to unit ratio(CUR). At the 500 euro price, the cost to satisfy a demand is 25 $\frac{euro}{unit}$ capacity($\frac{500euro}{20unit}$ of capacity), as compared to much expensive 40 $\frac{euro}{unit}$ of capacity offered by vans.

At the 800 euro mark, the CUR is same at 40 $\frac{euro}{unit}$ for both the cargo bike and vans ($\frac{800euro}{20unit}$ for cargo bikes and $\frac{4000euro}{100unit}$ capacity for vans), this results in a lowering of the benefit to using cargo bikes to satisfy the demands of each customer, since every vehicle has to return back to the depot once emptied, meaning, having more cargo bikes would result in an increase in empty returns back to the depot, thus increasing the distance without satisfying much demand as compared to vans. Any further increase past this point results in higher (CUR) of the cargo bikes as compared to vans, making the cargo bike more expensive to satisfy one unit of demand without being able to travel much. As a result, the total number of vehicles starts to decline post the 500 euros price until it starts to converge at the 1000 euro price, where only vans are used.

The above-mentioned increase in higher-capacity vans has further implications. Since these vans have higher capacity and therefore can travel further before getting emptied, this results in 2 things. First, less nodal demand is satisfied using split delivery as observed in Figure 16(a). Secondly, the total distance traveled also goes on a decline as observed in Figure 16(b). Both the declines coincide with the 500 euro price of the cargo bikes. Also as observed in Figure16(b), the total fixed costs rise due to the gradual increase in cargo bike fixed costs till the 1000 euro price. After that, it increases and then eventually stabilizes, due to an increase in the usage of vans from 11 to the eventual stabilization at 13 vans.