Quantitative Methods for Logistics—Assignment #1

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Manufacturing facility problem ---part A

The notation for this mathematical formulation is provided in Table 1.

Table 1: Notation

Sets and indices		
P	Set for the product types	$p \in P$
M	Set for the months	$m \in M$
Parameters		
s_{mp}	storage costs per month m of product p	[euro]
k_{mpc}	quantity of products p producible per month m per employee c	[product]
c_m	personnel costs per month m	[euro]
d_{pm}	demand of product p per month m	[product]
Variables		
h_{pm}	quantity of products *p* in holding per month *m*	[product]
z_{pm}	quantity of personnel *z* working on product *p* per month m	[employee/month]

The mathematical formulation then follows as:

$$\min\left(\sum_{m \in M}\sum_{p \in P} s_{mp}h_{pm} + \sum_{m \in M}\sum_{p \in P} z_{pm}c_m\right) \qquad (1)$$

Subject to:

$$h_{p1} = \left((z_{p1}k_{1pc}) - d_{p1}\right) \qquad \forall\ p \in P \qquad (2)$$

$$h_{pm} = \left((z_{pm}k_{mpc} + h_{p(m-1)}) - d_{pm}\right) \qquad \forall\ p \in P, m \in M\backslash\{1\} \qquad (3)$$

$$z_{pm},\ h_{pm} \geq 0 \qquad \forall\ p \in P, m \in M \qquad (4)$$

Objective function (1) is written as the minimization of the manufacturing costs. Constraint (2) ensures that the demand is met or exceeded in the 1st month, with any extra demand getting placed in holding. Constraint (3) ensures that the demand is met in all the months of the year by using products stored from the previous month and by producing products. Constraint (4) is a positivity constraint for the two variables.

Part B:

Using the matrix form of Gurobi with python, the following optimized values for the objective function (total manufacturing costs) are obtained, along with the total holding and personnel costs for the whole year.

- total manufacturing costs = 2.639.935,07 euros or 2,63M euros
- total personnel costs for the entire year = 2.424.921,3 euros or 2,42M euros
- total holding costs for the entire year = 215.013,4 euros

It is interesting to note that the optimization model decided that for the months of March thru August personnel costs were 0, most likely driven by the fact that it was too expensive to pay personnel for those periods. Thus, the manufacturing facility stockpiled enough products in order to meet the demand until September. After August, the model took advantage of the lower personnel costs for the rest of the calendar year and only produced what the demand was for each month (I.E. there was no more holding costs for the rest of the year).

Part C:

Due to new constraints, the following two items would change in the mathematical formulation (which is shown on the next page):

--z_{pm} would be an integer variable since personnel can only work on 1 product per month and they can only work full time (I.E. $z_{pm} \in N \quad \forall\, p \in P, m \in M$).

-- The total number of personnel each quarter is the same (to avoid additional costs):

$$\sum_{p \in P} z_{pm}$$

would be the same for 3 months at a time (Jan-Mar, April-June, July-Sept, Oct-Dec).

The new optimal solutions are the following:

- total manufacturing costs = 2.794.462,20 or 2,79M euros
- total personnel costs for the entire year = 2.578.000,0 euros or 2,57M euros
- total holding costs for the entire year = 216.462,2 euros

Due to the new personnel constraints, it logically made sense that the annual manufacturing costs were higher than in part B. The costs also went up since we still need to satisfy demand ever month, but we're limited on how many workers we can have for 3 months at a time. Hence there are workers in the months of July and August (which didn't exist in the part B model) which have high personnel costs. In total there is a 6% increase in total personnel costs and a 1% increase in total holding costs. Note that there are holding costs throughout the calendar year, caused most likely by the model trying to find a good optimization between keeping personnel for 3 months at time and meeting demand for all months.

Manufacturing facility problem ---part C.

The notation for this mathematical formulation is provided in Table 2.

Table 2: Notation

Sets and indices		
P	Set for the product types	$p \in P$
M	Set for the months	$m \in M$
Parameters		
s_{mp}	holding costs per month m of product p	[euro]
k_{mpc}	quantity of products p producible per month m per employee c	[product]
c_m	personnel costs per month m	[euro]
d_{pm}	demand of product p per month m	[product]
Variables		
h_{pm}	quantity of products *p* stored per month *m*	[product]
z_{pm}	quantity of personnel *z* working on product *p* per month m	[employee/month]

The mathematical formulation then follows as:

$$\min\left(\sum_{m \in M}\sum_{p \in P} s_{mp}h_{pm} + \sum_{m \in M}\sum_{p \in P} z_{pm}c_m\right) \tag{1}$$

Subject to:

$$h_{p1} = \left((z_{p1}k_{1pc}) - d_{p1}\right) \qquad \forall\ p \in P \tag{2}$$

$$h_{pm} = \left((z_{pm}k_{mpc} + h_{p(m-1)}) - d_{pm}\right) \qquad \forall\, p \in P, m \in M\backslash\{1\} \tag{3}$$

$$\sum_{p \in P} z_{pm} - \sum_{p \in P} z_{p(m+1)} = 0 \qquad \forall\, m \in \{1,2\} \tag{4}$$

$$\sum_{p \in P} z_{pm} - \sum_{p \in P} z_{p(m+1)} = 0 \qquad \forall\, m \in \{4,5\} \tag{5}$$

$$\sum_{p \in P} z_{pm} - \sum_{p \in P} z_{p(m+1)} = 0 \qquad \forall\, m \in \{7,8\} \tag{6}$$

$$\sum_{p \in P} z_{pm} - \sum_{p \in P} z_{p(m+1)} = 0 \qquad \forall\, m \in \{10,11\} \tag{7}$$

$$z_{pm},\ h_{pm} \geq 0 \qquad \forall\, p \in P, m \in M \tag{8}$$

$$z_{pm} \in N \qquad \forall\, p \in P, m \in M \tag{9}$$

Objective function (1) is written as the minimization of the manufacturing costs. Constraint (2) ensures that the demand is met or exceeded in the 1st month, with any extra demand getting placed in holding. Constraint (3) ensures that the demand is met in all the months of the year by using products stored from the previous month and by producing products. Constraints (4), (5), (6), and (7) ensure that the total number of personnel does not change for each quarter. Constraint (8) is a positivity constraint and constraint (9) denotes the variable as an integer decision variable.

Part D:

We need to check by predicting a hypothesis, then changing our mathematical model accordingly, observing the results, and then repairing the model/implementation. One should be able to estimate which direction the objective value and decision variables should change (going up or down) and by what amount (approximate value). Table 3 below depicts an overview of the verification tests performed and the desired outcome vs the actual result.

Table 3: Overview of verification tests for optimization model C

Verification test #	Description	Expected	Result	OK
1	Decrease holding cost to 1 euro for product 1	$\sum_{m \in M} \sum_{p \in P} s_{mp} h_{pm}$ < 216.462 & Z < 2.794.462,20	$\sum_{m \in M} \sum_{p \in P} s_{mp} h_{pm}$ = 113.347,1 & Z = 2.691.349,20	Pass
2	Increase demand for product 1 by 10x for all months	Z > 2.794.462,20	Z = 12.694.952,20	Pass
3	Increase k_{mpc} by 10x for all products	$\sum_{m \in M} \sum_{p \in P} z_{pm} c_m$ < 2.578.000 & 0 < Z < 2.794.462,20	$\sum_{m \in M} \sum_{p \in P} z_{pm} c_m$ = 286.500 & Z = 330.172,20	Pass
4	Decrease c_m for the months of June, July, and August to 1000 euros	$\sum_{m \in M} \sum_{p \in P} z_{pm} c_m$ < 2.578.000 & 0 < Z < 2.794.462,20	$\sum_{m \in M} \sum_{p \in P} z_{pm} c_m$ = 1.659.500 & Z = 2.190.152	Pass

Discussion:

The result of verification test #1 is to see the relationship between holding cost and production cost and how they are intertwined. From part C, the relationship between production costs/holding costs is 11.9 which means that production costs are the driving factor for the total manufacturing costs over the year. With verification test #1, the ratio goes up to 22.7 which makes sense since the cost of holding went down but personnel costs stayed the same, thus it makes up a larger portion of the total manufacturing costs.

The result of verification test #2 is to see the relationship between demand and the decision variables. I expect to see monthly production costs and holding costs go up accordingly with no substantial change in the ratio between the two costs. With the total holding cost for the entire year being 1.107.952,20 euros, the new ratio of production costs/holding costs is 10.4. This is slightly lower than the original part C which means that production costs start to have a smaller significance once demand for product 1 increases (still very high compared to holding costs). An interesting note is that the objective function Z is roughly 10 times more expensive with product 1 demand going up by 10 times. This makes sense since product 1 became the governing cost for the facility and therefore resources needed to be dedicated to it in order to meet demand.

Verification test #3 was to see how the productivity of the employees would affect personnel costs and total costs. Both monthly production and holding costs are greatly reduced and so is the ratio of total production costs/holding costs: 6.56. This means that as the productivity increases, the choice between holding and producing becomes closer to equal. Given that there still is fluctuation in how personnel cost differ each month, personnel costs are still going to be higher than holding costs.

Verification test #4 is to see how decreasing personnel costs during the summer months would change the way holding costs and the overall objective changed. This is similar to verification tests #3 and #1 where we see how a change in personnel or holding costs changes the other. An observation with the result of this test was that because the summer months are cheaper than the months of October to December, the result is that there are no personnel working for the last quarter of the year. The new ratio of total production costs/holding costs (3.12) is also the lowest of all the experiments compared to the original part C ratio of 11.9.

Part E:

From part C, we obtain Figure 1 which is a graph depicting the amount of personnel by product along with the amount of products held for each month.

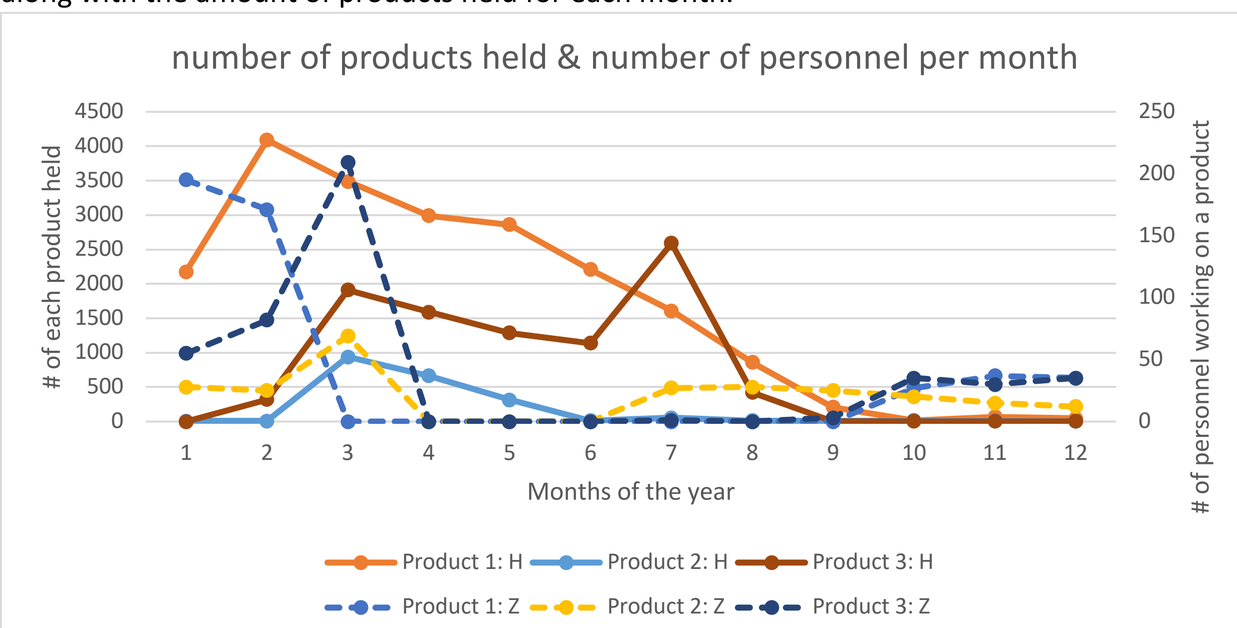
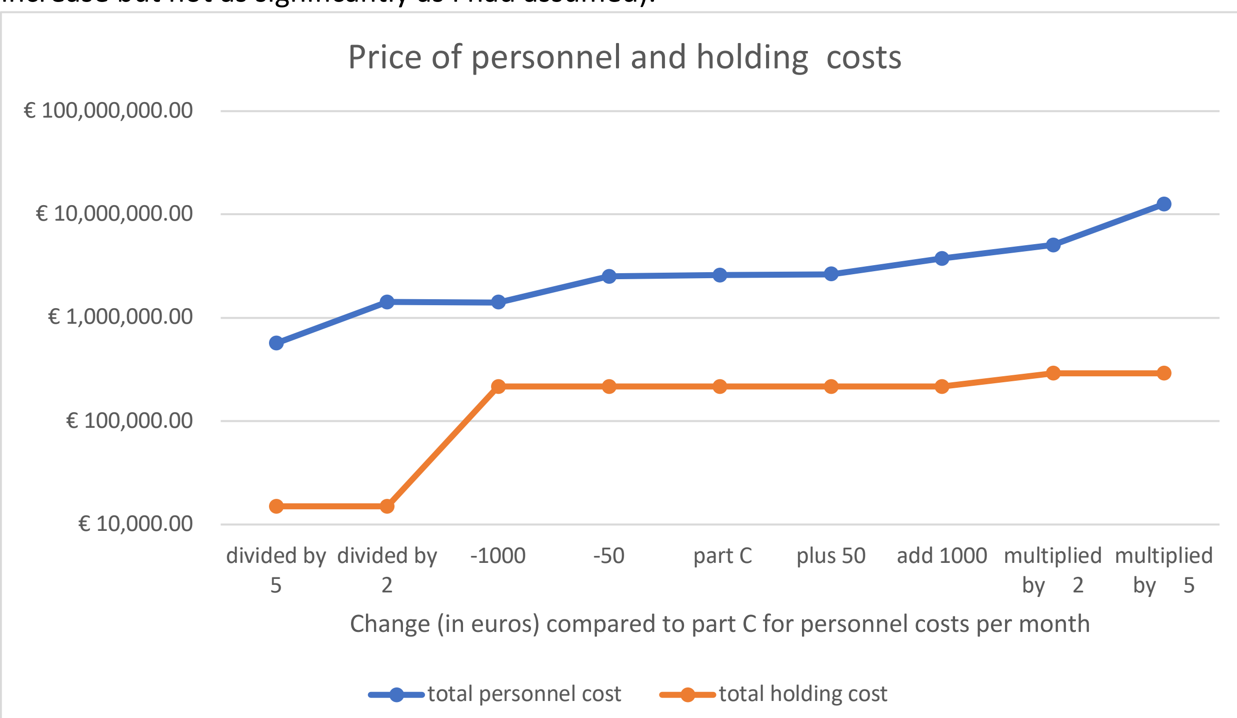


Figure 1: personnel and product holding values by month for Pformula .

Even though Figure 1 does not directly show the shadow prices involved behind the model choices, we can see how personnel is 0 in the middle of the year (when personnel costs are high) because the high amount of stored products allows for that. The shadow price for the holding cost is lower than the personnel costs during the middle of the year, which is why the model favors reducing the number of personnel working as much as possible.

My theory was that at a certain point with increased personnel costs, holding costs will become more advantageous (I.E the shadow prices for holding costs will be higher than those of personnel costs). In Figure 2, you can see the experimentation results from changing the personnel costs per month with a range of 1/5 the cost of the part C answer to x5 the cost of the part C answer. I did this because as is shown on the graph, holding costs remain relatively stable (either 15.000 euros on the left-hand side or 216.462 or 290.462 euros on the right-hand side) and I was trying to identify at what point do the personnel costs cause a change in the total holding cost. You can see that between a personnel cost reduction of 1000 euros per month and 1/2 of part C, there is distinct reduction in total holding cost while the change in total personnel cost remains constant. That region is where holding costs have a high shadow price and should be explored further. My theory was incorrect however that by increasing personnel costs, you would cause the total holding cost to increase significantly (note it does increase but not as significantly as I had assumed).



<u>Figure 2: changes in the total holding and personnel costs based on changes to the monthly personnel cost parameters. Note the y-axis is in logarithmic scale for better clarity of the 2 lines.</u>

This report experimented with a holding cost adjustment range from 1/5th the cost of the parameters in part C to 10x the cost of the holding parameters in part C. This again was driven by my knowledge that the personnel costs are far larger and thus more significant in the optimization model. As shown below in Figure 3, you can see how the personnel costs remain the same except for a small increase between the 2x and 5x increase in holding costs per month. That zone should be analyzed more closely as there is a significant drop in the total holding cost most likely due to the decision to hire more personnel (change in shadow price) since it becomes the significant parameter.

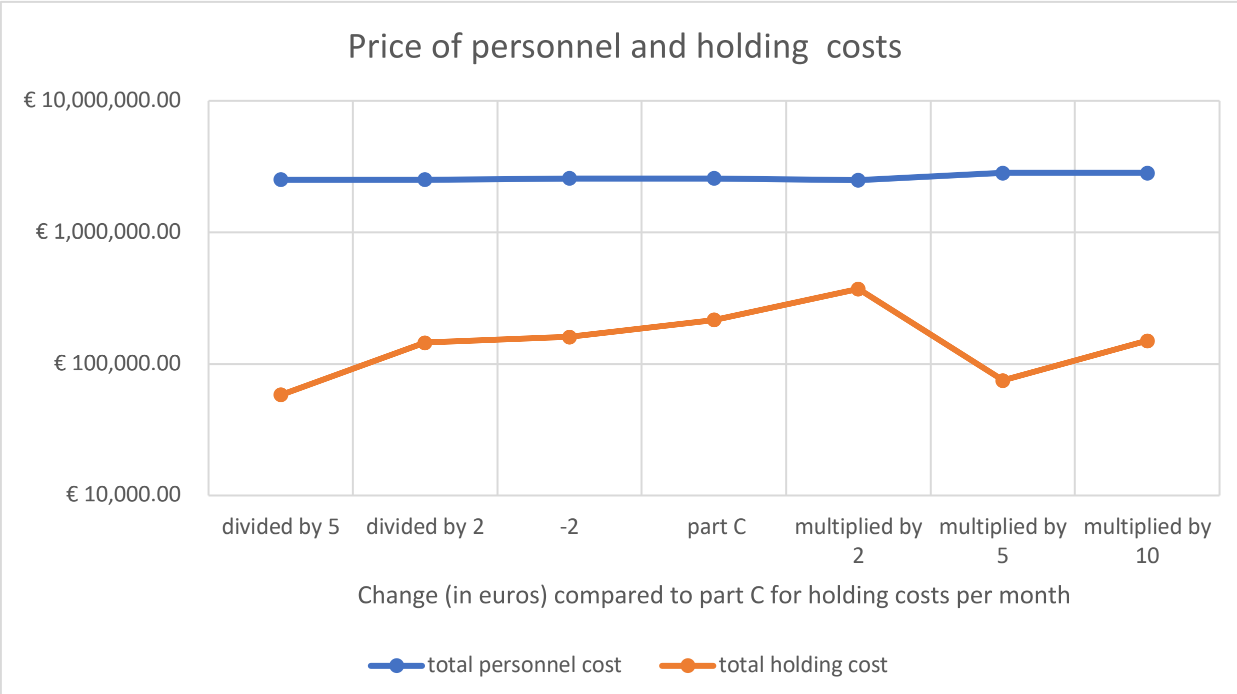


Figure 3: effect on total holding cost and total personnel cost by changes in holding cost monthly parameters. Note the y-axis is in logarithmic scale for better clarity of the 2 lines.

Part F:

Due to the new constraint, there would be a new parameter introduced and with that a new variable would be added in the mathematical formulation.
The new parameter is the following:

---f_z would be the new 2000 euro firing cost per worker that can only occur at the beginning of each quarter (January, April, July or October).

The new mathematical formulation consists of an updated objective function, a new functional constraint, a modified functional constraint, a new variable x_m, and the new parameter f_z as shown below:

Parameters

.....

f_z	firing cost per personnel	[euro]

Variables

.....

x_m	cost of firing employees	[euro]

New objective function:

$$\min\left(\sum_{m \in M}\sum_{p \in P} s_{mp}h_{pm} + \sum_{m \in M}\sum_{p \in P} z_{pm}c_m + \sum_{m \in M} x_m\right) \tag{1}$$

Subject to:

…..

New or modified functional constraints:

$$\left(\sum_{p \in P} z_{pm} - \sum_{p \in P} z_{p(m+1)}\right) f_z \leq x_m \qquad \forall\, m \in M \tag{8}$$

$$z_{pm},\ h_{pm}, x_m \geq 0 \qquad \forall\, p \in P, m \in M \tag{9}$$

$$z_{pm} \in N \qquad \forall\, p \in P, m \in M \tag{10}$$

Objective function (1) is written as the minimization of the manufacturing costs which now includes the total firing cost. Constraint (8) ensures that firing costs for employees is accounted for. Constraint (9) ensures positivity for all decision variables and (10) means that the decision variable for personnel is an integer variable.

The new optimal solutions are the following:

- total manufacturing costs = 2.964.182,20 euros
- total personnel costs for the entire year = 2.824.500 euros
- total holding costs for the entire year = 37.682,20 euros
- the total firing cost (new variable): 102.000,00 euros

Due to the new firing costs, it logically made sense that the annual manufacturing costs were higher than in part C. The costs also went up since we still need to satisfy demand every month, but we're limited on how many workers we can have for 3 months at a time and how many workers we should fire. The model output never hires any additional people after the 1st month, although it does fire people each quarter. The model must decide between hiring enough workers for the whole year in order to produce enough for holding as well as production. The model decided to not hold a lot of products (37k is much lower than 216K from part C) and invest in more personnel in the beginning of the year. The model then decided to start firing only the personnel it needed to as the months passed.

Part G:

In this section we're going to be experimenting with different values of the firing cost, as shown below in Figure 4. We choose the range of firing costs to go all the way down to 50 euros and then up to a 10.000 euro addition on top of the original 2000 euro value. This was chosen to see if the firing cost could become as expensive as the total personnel cost if increased and could become cheaper than the total holding costs if decreased.

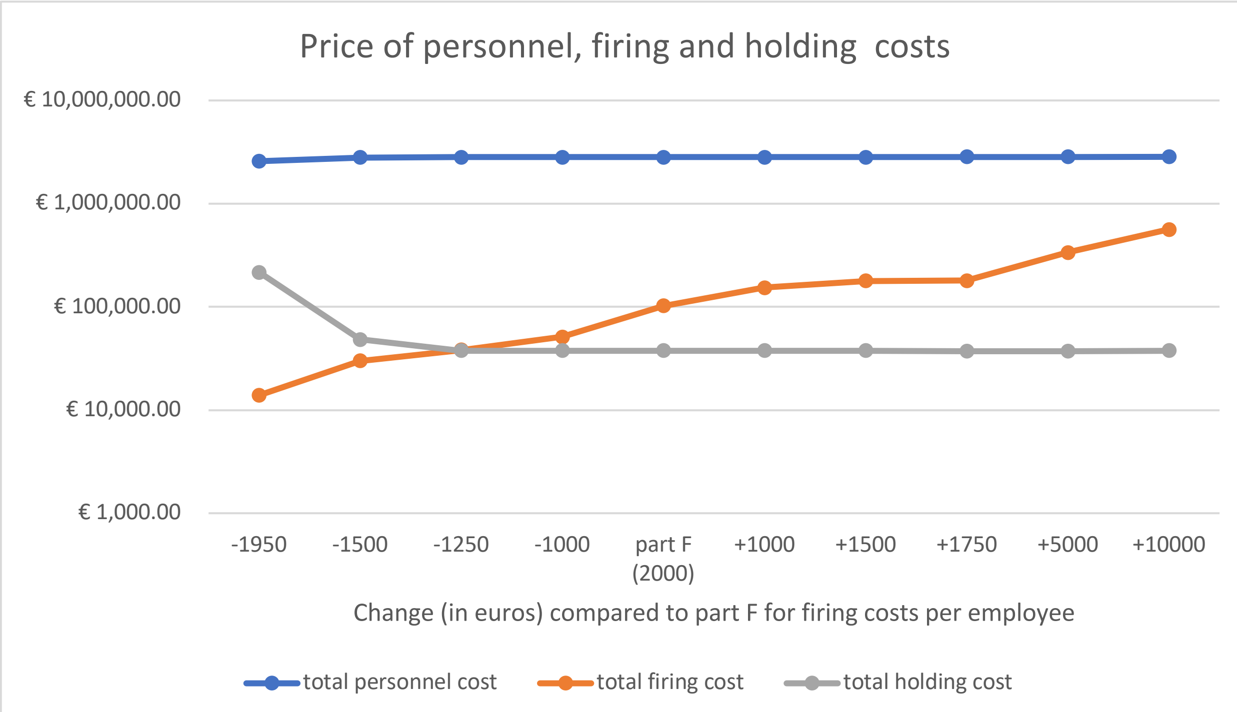


Figure 4: graph depicting the change in firing cost per employee and the resulting total personnel and total holding costs. Note the y-axis is in logarithmic scale for better clarity.

We can see from Figure 4 that the firing costs directly effect the total manufacturing costs as they pass the 750 euro (below the 2000 euro per personnel parameter set in part F) mark. It is hard to tell from the graph, but the total personnel costs only increase at a firing cost of 750 euros and again at 3600 euros. Total holding costs remain stable except when the firing cost becomes lower than 750 euros. At that stage, then holding costs increase greatly to become the answer in part C (216.462,2 euros) when the firing cost gets close to zero. This makes sense as the more expensive it is to fire employees, the more the optimization model will choose to hold less products and try and keep all personnel that has been hired.

Part H:

This problem removes some constraints (the quarters when firing can occur) but adds in a training cost and a contract duration where personnel can't be fired. The assumption is that everyone is a new employee in January so there will be a training cost in that month regardless. Please see below for the new parameters, variables, modified objective function, and modified constraints:

Parameters		
….		
b	contract duration	[month]
t_m	training cost per training per month	[euro/month]
bigM	large integer for LP conversion	

Variables

……

x_m	number of fired employees per month	[employee]
y_m	training occurrence	[euro/month]

The mathematical formulation then follows as:

$$\min\left(\sum_{m\in M}\sum_{p\in P} s_{mp}h_{pm} + \sum_{m\in M}\sum_{p\in P} z_{pm}c_m + f_z\sum_{m\in M} x_m + \sum_{m\in M} t_m\, y_m\right) \qquad (1)$$

Subject to:

….

New functional constraints:

$$\left(\sum_{p\in P} z_{pm}\right) \le \ (bigM)y_m \qquad \forall\ m \in \{1\} \qquad (4)$$

$$\left(\sum_{p\in P} z_{pm} - \sum_{p\in P} z_{p(m-1)}\right) \le \ (bigM)y_m \qquad \forall\ m \in M\backslash\{1\} \qquad (5)$$

$$\left(\sum_{p\in P} z_{pm} - \sum_{p\in P} z_{p(m-1)}\right) \ge \ 0 \qquad \forall\ m \in B\backslash\{1\} \qquad (6)$$

$$\left(\sum_{p\in P} z_{p(m-b)} - \sum_{11-b}^{m} x_m\right) \ge \ x_m \qquad \forall\ m \in (M-B)\backslash\{1\} \qquad (7)$$

$$\left(\sum_{p\in P} z_{pm} - \sum_{p\in P} z_{p(m+1)}\right) \le \ x_m \qquad \forall\ m \in M \qquad (8)$$

$$z_{pm},\ h_{pm},\ x_m\ \ge 0 \qquad \forall\, p \in P, m \in M \qquad (9)$$

$$z_{pm}\ \in N \qquad \forall\, p \in P, m \in M \qquad (10)$$

$$y_m\ \in \{0,1\} \qquad \forall\, p \in P, m \in M \qquad (11)$$

The new objective function (1) is written as the minimization of the manufacturing costs. New constraints (4) & (5) express that if there are new employees hired, then a training will occur. New constraint (6) forces the company to not fire workers before their contract duration. New constraint (7) forces the amount of fired workers in the current month to be smaller than the difference between the total number of workers working at the beginning of the contract and the sum of fired workers that occurred between the current month and the end of the contract period. New constraint (8) is that if there are less workers, then they have been fired. Constraint (9) is a positivity constraint, while constraint (10) is an integer variable for personnel and constraint (11) is a binary variable for the training cost.

The new optimal solutions are the following:

- total manufacturing costs = 3,066,462.20 euros
- total personnel costs for the entire year = 2.808.500 euros
- total holding costs for the entire year = 81.962,20 euros
- the total firing cost: 166.000,00 euros
- the total training cost (new constraint): 10.000,00 euros

Due to the new training costs, it logically made sense that the annual manufacturing costs were higher than in part C. The costs also went up since we still need to satisfy demand every month, but we're limited on the cost of firing workers and more importantly the contract duration. The model output only hires additional people in November, when personnel costs are low, in order to finish meeting all the demand for the year. The model optimized to hold products in order to meet the contract duration, training, and firing parameters which restrict personnel numbers.

Part I:

In this section we're going to be experimenting with different durations of the contract, as shown below in Figure 5 and different training costs as shown below in Figure 6. This will show the relationship between the variables and the decisions that the optimization model makes when parameters change.

For contract durations, we choose the range of contract durations to be between 1 and 10 months long (part H problem had it set at 6 month) due to my hypothesis that a shorter contract duration will have a benefit on the overall costs.

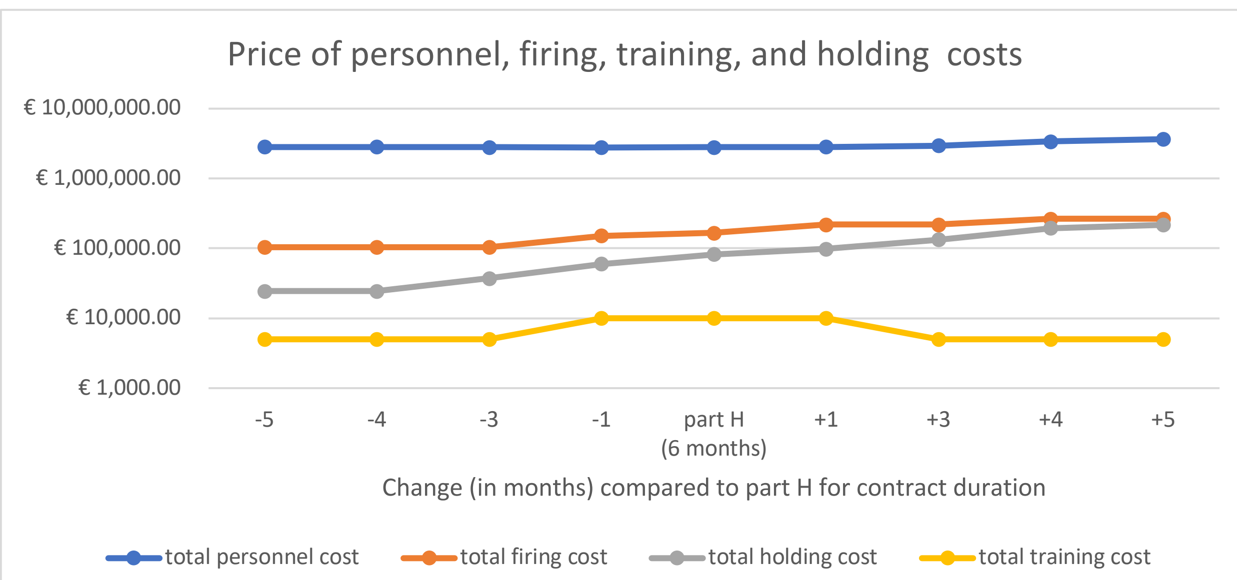
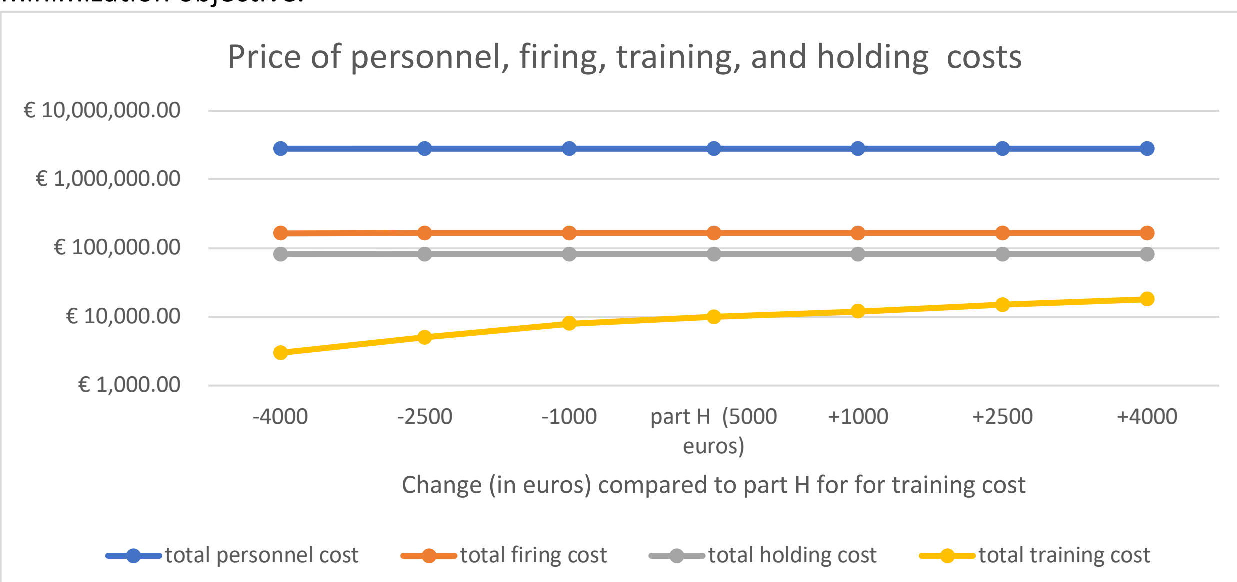


Figure 5: graph depicting the change in firing cost per employee and the resulting total personnel and total holding costs. Note the y-axis is in logarithmic scale for better clarity.

It is hard to tell but Figure 5 shows that holding costs become nearly as costly as firing costs if the contract duration is set at 11 months. The model is trying to optimize at that point between how many people it has to hire to meet demand vs also storing enough products since it can only fire workers in month 11. In addition, the training costs go from 5000 to 10000 euros when the contract durations are between 5 and 7 months long which shows the constraint that training costs have on the model. The model doesn't try to hire more workers besides the initial hiring in January.

Based on experimenting with the contract duration lengths, my theory was that as you increase or decrease the training cost, not a lot of changes would occur for the over costs given how little the model decided to incur training costs as shown in Figure 5. In Figure 6, you can see my experimentation with changing the training costs by a range of 1000 to 9000 euros. I did this because as is shown on the graph, all of the other costs (holding, firing, personnel) remain relatively stable and I was trying to identify if at a certain high value of training cost, would that cause a change in the other costs. There is a small change in the other costs between a training cost of 1000 euros and 2500 euros but otherwise all other costs remain the same. This means that the shadow price for training cost is low and does not have much effect on the overall minimization objective.



<u>Figure 6: graph representing changes in the different costs based on changes to the training cost amount. Note the y-axis is in logarithmic scale for better clarity of the lines.</u>