csce350 — Data Structures and Algorithms
Fall 2020 — Test 0

Evan

first/given name

Owre

last/family name

time downloaded

- ✓ Read the questions carefully and make sure to give the answers asked for. Pay particular attention to **boldface** words. Don't give a beautiful answer to the wrong question.
- ✓ You have 75 minutes to complete this test.
- ✓ A single $8.5'' \times 11''$ sheet of notes in your own handwriting is required.
- ✓ No calculators nor other reference material are allowed.
- ✓ Show enough work to convince your instructor that you know what you're doing. Mark your answers clearly.
- ✓ Make sure you have all 7 pages, including the three reference sheets at the end.
- ✓ If you continue an answer onto the back of the page, note this clearly so the instructor can find it.
- ✓ Partial credit will be awarded for incorrect answers that demonstrate partial understanding of the relevant concepts. Therefore, it is to your advantage to explain your reasoning and show your work. However, meaningless or irrelevant writing will not earn partial credit.

Problem	Value	Your Score
1	2	
2	2	
3	6	
Total	10	

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code. I certify that I have neither given nor received unauthorized aid on this exam.

signature

Problem 1 *(2 points)*
Simplify this expression:

$$2+2$$

Show your work.

Problem 2 *(2 points)*
Recall that there are four Teenage Mutant Ninja Turtles, whose names are Leonardo, Michelangelo, Donatello, and Raphael.

Draw a picture of the best turtle and **explain** why it is Donatello.

Problem 3 *(6 points)*
The remaining 6 points on this assignment will be awarded for completing the steps on the following checklist:

- ☐ **Download** the exam within the allotted 72-hour window.
- ☐ **Write** the time you downloaded the exam on the cover page. *This is primarily for your own reference; we will verify the actual access time for your exam document.*
- ☐ **Sign** the cover page to confirm that you have completed the test using only the allowed resources. *For Test A, Test B, and the Final Exam, submissions for which this signature is missing will receive a 0 score.*
- ☐ **Include** the required sheet (8.5" by 11", single side) of notes as the final page of your submission. *You are required to use exactly one sheet of notes. Not zero. Not two. If you do not feel the need for notes, you may use a blank sheet.*
- ☐ **Label** your notes page with your name in the top left corner.
- ☐ **Scan** the exam back into a single PDF file. Include the cover sheet but omit the formula sheets. *This may be done, for example, using the Google Drive app on an Android Device, or the Notes app on an iOS device, or even an old school flatbed scanner.*
- ☐ **Upload** the completed exam to

  http://dropbox.cse.sc.edu/

  at most two hours after first accessing the download link. *For Test A, Test B, and the Final Exam, submissions after the expiration of this two-hour window will receive a 0 score.*

APPENDIX A

Useful Formulas for the Analysis of Algorithms

This appendix contains a list of useful formulas and rules that are helpful in the mathematical analysis of algorithms. More advanced material can be found in [Gra94], [Gre07], [Pur04], and [Sed96].

Properties of Logarithms

All logarithm bases are assumed to be greater than 1 in the formulas below; $\lg x$ denotes the logarithm base 2, $\ln x$ denotes the logarithm base $e = 2.71828\ldots$; x, y are arbitrary positive numbers.

1. $\log_a 1 = 0$

2. $\log_a a = 1$

3. $\log_a x^y = y \log_a x$

4. $\log_a xy = \log_a x + \log_a y$

5. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$

6. $a^{\log_b x} = x^{\log_b a}$

7. $\log_a x = \dfrac{\log_b x}{\log_b a} = \log_a b \log_b x$

Combinatorics

1. Number of permutations of an n-element set: $P(n) = n!$

2. Number of k-combinations of an n-element set: $C(n, k) = \dfrac{n!}{k!(n-k)!}$

3. Number of subsets of an n-element set: 2^n

Important Summation Formulas

1. $\sum_{i=l}^{u} 1 = \underbrace{1+1+\cdots+1}_{u-l+1 \text{ times}} = u-l+1$ (l, u are integer limits, $l \le u$); $\sum_{i=1}^{n} 1 = n$

2. $\sum_{i=1}^{n} i = 1+2+\cdots+n = \frac{n(n+1)}{2} \approx \frac{1}{2}n^2$

3. $\sum_{i=1}^{n} i^2 = 1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{1}{3}n^3$

4. $\sum_{i=1}^{n} i^k = 1^k+2^k+\cdots+n^k \approx \frac{1}{k+1}n^{k+1}$

5. $\sum_{i=0}^{n} a^i = 1+a+\cdots+a^n = \frac{a^{n+1}-1}{a-1}$ ($a \ne 1$); $\sum_{i=0}^{n} 2^i = 2^{n+1}-1$

6. $\sum_{i=1}^{n} i2^i = 1\cdot 2+2\cdot 2^2+\cdots+n2^n = (n-1)2^{n+1}+2$

7. $\sum_{i=1}^{n} \frac{1}{i} = 1+\frac{1}{2}+\cdots+\frac{1}{n} \approx \ln n+\gamma$, where $\gamma \approx 0.5772\ldots$ (Euler's constant)

8. $\sum_{i=1}^{n} \lg i \approx n \lg n$

Sum Manipulation Rules

1. $\sum_{i=l}^{u} ca_i = c\sum_{i=l}^{u} a_i$

2. $\sum_{i=l}^{u} (a_i \pm b_i) = \sum_{i=l}^{u} a_i \pm \sum_{i=l}^{u} b_i$

3. $\sum_{i=l}^{u} a_i = \sum_{i=l}^{m} a_i + \sum_{i=m+1}^{u} a_i$, where $l \le m < u$

4. $\sum_{i=l}^{u} (a_i - a_{i-1}) = a_u - a_{l-1}$

Approximation of a Sum by a Definite Integral

$$\int_{l-1}^{u} f(x)dx \leq \sum_{i=l}^{u} f(i) \leq \int_{l}^{u+1} f(x)dx \quad \text{for a nondecreasing } f(x)$$

$$\int_{l}^{u+1} f(x)dx \leq \sum_{i=l}^{u} f(i) \leq \int_{l-1}^{u} f(x)dx \quad \text{for a nonincreasing } f(x)$$

Floor and Ceiling Formulas

The *floor* of a real number x, denoted $\lfloor x \rfloor$, is defined as the greatest integer not larger than x (e.g., $\lfloor 3.8 \rfloor = 3$, $\lfloor -3.8 \rfloor = -4$, $\lfloor 3 \rfloor = 3$). The *ceiling* of a real number x, denoted $\lceil x \rceil$, is defined as the smallest integer not smaller than x (e.g., $\lceil 3.8 \rceil = 4$, $\lceil -3.8 \rceil = -3$, $\lceil 3 \rceil = 3$).

1. $x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$

2. $\lfloor x + n \rfloor = \lfloor x \rfloor + n$ and $\lceil x + n \rceil = \lceil x \rceil + n$ for real x and integer n

3. $\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$

4. $\lceil \lg(n + 1) \rceil = \lfloor \lg n \rfloor + 1$

Miscellaneous

1. $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ as $n \to \infty$ (Stirling's formula)

2. Modular arithmetic (n, m are integers, p is a positive integer)

$$(n + m) \bmod p = (n \bmod p + m \bmod p) \bmod p$$
$$(nm) \bmod p = ((n \bmod p)(m \bmod p)) \bmod p$$