DURHAM UNIVERSITY

Logic Coursework 2024/25: Written Work

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Module: *Computational Thinking* – Professor: *Prof. Barnaby Martin*
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Question 1

Answer the following questions about complete sets of logical connectives, in each case justifying your answer.
(i) Show $\{\neg, \rightarrow\}$ is a complete set of connectives.
(ii) Show $\{\rightarrow, 0\}$ is a complete set of connectives (where 0 is the constant false).
(iii) Is $\{\text{NAND}, \wedge\}$ a complete set of connectives?
(iv) Is $\{\wedge, \vee\}$ a complete set of connectives?

Question 2

Convert $(((p \rightarrow q) \rightarrow r) \rightarrow (s \rightarrow t))$ to
(i) Conjunctive Normal Form (CNF)
(ii) Disjunctive Normal Form (DNF)

Question 3

What is the purpose of Tseitin's Algorithm? Apply Tseitin's Algorithm to turn the propositional formula $(((x_1 \wedge x_2 \wedge x_3) \rightarrow (y_1 \wedge y_2 \wedge y_3)) \vee z)$ to CNF.

Question 4

State with justification if each of the following sentences of predicate logic is logically valid.
(i) $(\forall x \exists y \forall z\ E(x,y) \wedge E(y,z)) \rightarrow (\forall x \forall z \exists y\ E(x,y) \wedge E(y,z))$
(ii) $(\forall x \exists y \exists u \forall v\ E(x,y) \wedge E(u,v)) \rightarrow (\exists u \forall v \forall x \exists y\ E(x,y) \wedge E(u,v))$
(iii) $(\forall x \exists y \forall z\ R(x,y,z)) \rightarrow (\exists x \forall y \exists z\ R(x,y,z))$
(iv) $((\forall x \forall y \exists z (E(x,y) \wedge E(y,z))) \rightarrow (\forall x \forall y \forall z (E(x,y) \vee E(y,z))))$

Question 5

Evaluate the given sentence on the respective relation E over domain $\{0, 1, 2\}$
(i) $\forall x \forall y \forall z \exists w (E(x, w) \wedge E(y, w) \wedge E(z, w))$
(ii) $\exists x \forall y \forall z \exists w (E(x, w) \wedge E(y, w) \wedge E(z, w))$
(iii) $\forall y \exists x \forall z \exists w (E(x, w) \wedge E(y, w) \wedge E(z, w))$
(iv) $\exists x \exists y \exists z \forall w (E(x, w) \wedge E(y, w) \wedge E(z, w))$
(v) $\forall x_1 \exists x_2 \forall y_1 \exists y_2 \forall z_1 \exists z_2 \forall z \exists y \ E(x_1, x_2) \wedge E(x_2, w) \wedge E(y_1, y_2) \wedge E(y_2, w) \wedge E(z_1, z_2) \wedge E(z_2, w) \wedge E(z, w)$
(vi) $\forall x_1 \exists x_2 \forall y_1 \exists y_2 \forall z_1 \forall z \exists z_2 \exists y \ E(x_1, x_2) \wedge E(x_2, w) \wedge E(y_1, y_2) \wedge E(y_2, w) \wedge E(z_1, z_2) \wedge E(z_2, w) \wedge E(z, w)$