DURHAM UNIVERSITY

Logic Coursework 2024/25: Written Work

Student name: *Huseyin Emre Ozden*

Module: *Computational Thinking* – Professor: *Prof. Barnaby Martin*
Due date: *March 25th, 2025*

Question 1

Answer the following questions about complete sets of logical connectives, in each case justifying your answer.
(i) Show $\{\neg, \to\}$ is a complete set of connectives.
(ii) Show $\{\to, 0\}$ is a complete set of connectives (where 0 is the constant false).
(iii) Is $\{\text{NAND}, \wedge\}$ a complete set of connectives?
(iv) Is $\{\wedge, \vee\}$ a complete set of connectives?

Answer. In order to determine whether the sets are complete, I will be showing whether $\wedge, \vee$ and $\neg$ can be expressed using the connectives in the set, in which case any logical expression can be written in CNF or DNF, meaning that it's a complete set.

Proof for part (i).
Can $\vee$ be expressed using $\{\neg, \to\}$?
Yes: $\neg p \to q \equiv \neg(\neg p) \vee q \equiv p \vee q$
Can $\wedge$ be expressed using $\{\neg, to\}$?
Yes: $\neg(p \to \neg q) \equiv \neg(\neg p \vee \neg q) \equiv p \wedge q$
Since $\neg$ is already in our set of logical connectives, we can then conclude that $\{\neg, \to\}$ is a complete set of logical connectives, as any logical expression can be expressed in CNF/DNF using the connectives within the set. □

Proof for part (ii).
Can $\neg$ be expressed using $\{\to, 0\}$?
Yes: $p \to 0 \equiv \neg p \vee 0 \equiv \neg p$
Can $\vee$ be expressed using $\{\to, 0\}$?
Yes: $(p \to 0) \to q \equiv \neg(p \to 0) \vee q \equiv \neg(\neg p) \vee q \equiv p \vee q$
Can $\wedge$ be expressed using $\{\to, 0\}$?
Yes: $(p \to (q \to 0)) \to 0 \equiv (p \to (\neg q \vee 0)) \to 0 \equiv (p \to \neg q) \to 0 \equiv (\neg p \vee \neg q) \to 0 \equiv \neg(\neg p \vee \neg q) \vee 0 \equiv p \wedge q$
Therefore $\{\to, 0\}$ is a complete set of logical connectives □

Question 2

Convert $(((p \to q) \to r) \to (s \to t))$ to
(i) Conjunctive Normal Form (CNF)
(ii) Disjunctive Normal Form (DNF)

Question 3

What is the purpose of Tseitin's Algorithm? Apply Tseitin's Algorithm to turn the propositional formula $(((x_1 \wedge x_2 \wedge x_3) \to (y_1 \wedge y_2 \wedge y_3)) \vee z)$ to CNF.

Question 4

State with justification if each of the following sentences of predicate logic is logically valid.
(i) $(\forall x \exists y \forall z\ E(x,y) \wedge E(y,z)) \to (\forall x \forall z \exists y\ E(x,y) \wedge E(y,z))$
(ii) $(\forall x \exists y \exists u \forall v\ E(x,y) \wedge E(u,v)) \to (\exists u \forall v \forall x \exists y\ E(x,y) \wedge E(u,v))$
(iii) $(\forall x \exists y \forall z\ R(x,y,z)) \to (\exists x \forall y \exists z\ R(x,y,z))$
(iv) $((\forall x \forall y \exists z (E(x,y) \wedge E(y,z))) \to (\forall x \forall y \forall z (E(x,y) \vee E(y,z))))$

Question 5

Evaluate the given sentence on the respective relation E over domain $\{0,1,2\}$
(i) $\forall x \forall y \forall z \exists w (E(x,w) \wedge E(y,w) \wedge E(z,w))$
(ii) $\exists x \forall y \forall z \exists w (E(x,w) \wedge E(y,w) \wedge E(z,w))$
(iii) $\forall y \exists x \forall z \exists w (E(x,w) \wedge E(y,w) \wedge E(z,w))$
(iv) $\exists x \exists y \exists z \forall w (E(x,w) \wedge E(y,w) \wedge E(z,w))$
(v) $\forall x_1 \exists x_2 \forall y_1 \exists y_2 \forall z_1 \exists z_2 \forall z \exists y\ E(x_1,x_2) \wedge E(x_2,w) \wedge E(y_1,y_2) \wedge E(y_2,w) \wedge E(z_1,z_2) \wedge E(z_2,w) \wedge E(z,w)$
(vi) $\forall x_1 \exists x_2 \forall y_1 \exists y_2 \forall z_1 \forall z \exists z_2 \exists y\ E(x_1,x_2) \wedge E(x_2,w) \wedge E(y_1,y_2) \wedge E(y_2,w) \wedge E(z_1,z_2) \wedge E(z_2,w) \wedge E(z,w)$