EE 583 Pattern Recognition HW5

Eda Özkaynar 2375582

Question 1

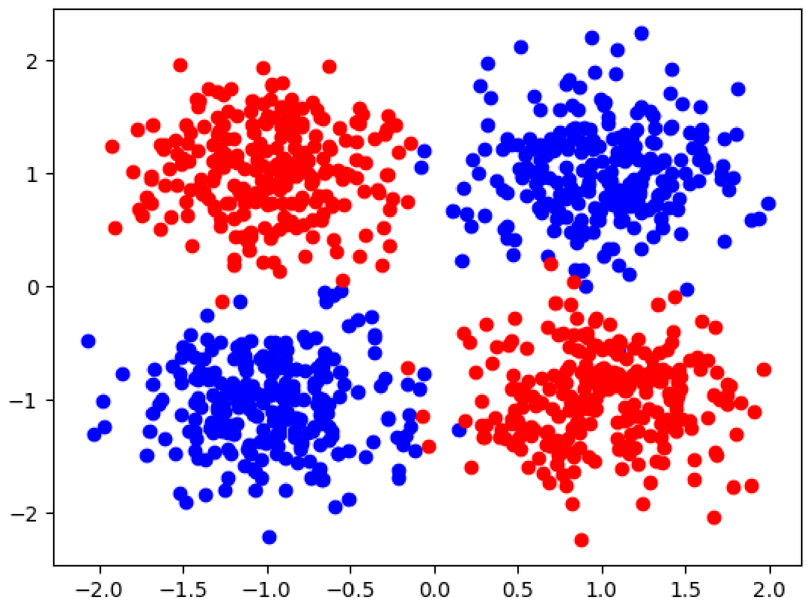


Fig. 1: Dataset

As can be seen in 2, this dataset consists of two distinct classes. However, its structure resembles an XOR problem, making it impossible to classify the data accurately using linear classifiers due to the non-linear decision boundaries required to separate the classes.

The network structure is a fully connected neural network, with one input layer, one hidden layer, and one output layer. The learning rate determines the step size at which the optimizer updates the model's weights. A small learning rate ensures gradual convergence but can be slow, while a large learning rate may result in instability. The optimizer determines how the model's parameters (weights and biases) are updated based on the gradients computed during backpropagation. In this case, Stochastic Gradient Descent (SGD) is used as the optimizer. Instead of utilizing the entire dataset in each iteration, SGD selects a single random training example or a small batch to calculate the gradient and update the model parameters, making it computationally efficient for large datasets. The loss function, which in this case is Mean Squared Error (MSE), quantifies the error between the model's predictions and the ground truth. During training, the optimizer aims to minimize this loss function by iteratively updating the model's parameters.

Metric	**Value**
Validation Loss (After Training)	0.217
Validation Accuracy	61.00%
Epoch 100 Train Loss	0.208

TABLE I: Training and Validation Metrics

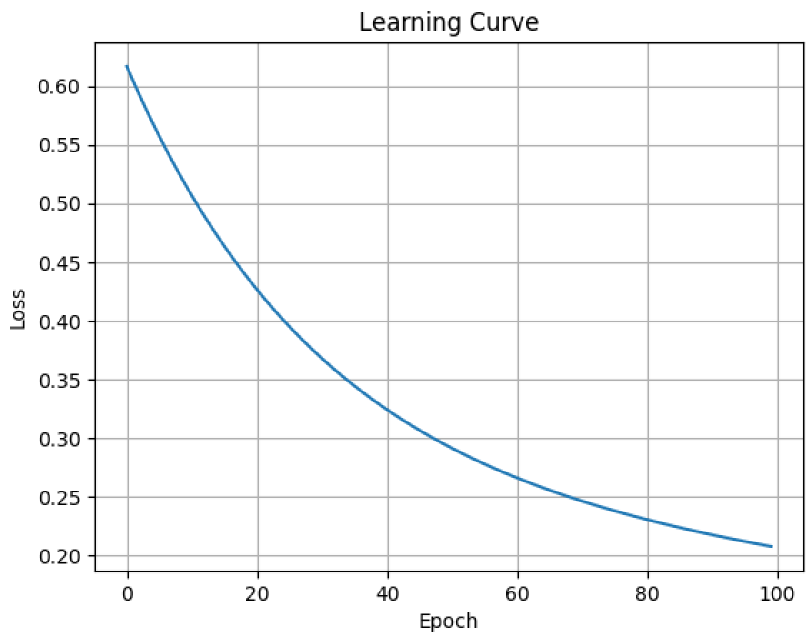


Fig. 2: Learning Curve for training data

QUESTION 2

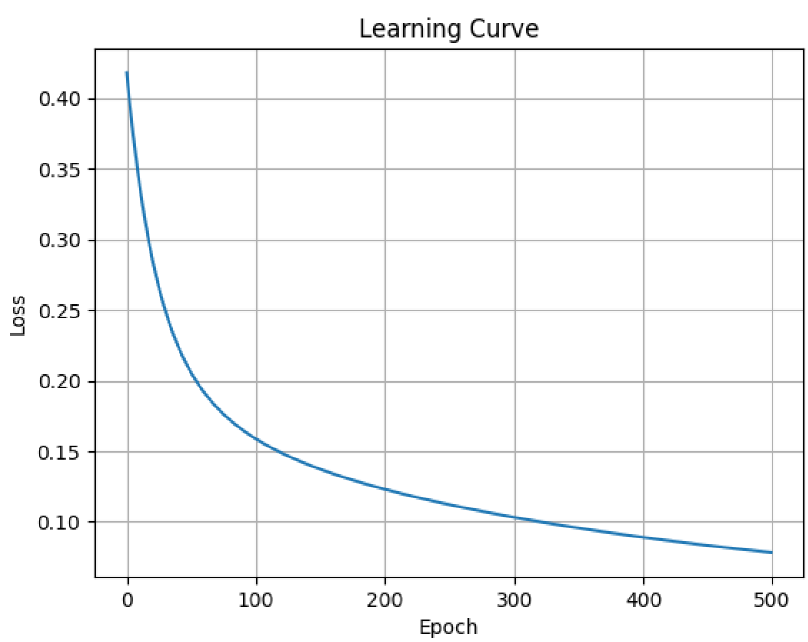


Fig. 3: Learning curve for 500 epoch

Metric	Value
Validation Loss (After Training)	0.093
Validation Accuracy	96.00%
Epoch 500 Train Loss	0.092

TABLE II: Training and Validation Metrics (Epoch 500)

When we compare I and II, we can easily observe that increasing the number of epochs improves the accuracy and reduces the loss. This occurs because the model is allowed more iterations to learn the underlying patterns in the data, effectively minimizing the error between the predicted outputs and the ground truth.

Question 3

A. *Modified Fully Connected Network*

- **Input Layer:**
  - Accepts input of size input_size.
- **First Hidden Layer:**
  - Fully connected layer with input size input_size and output size hidden_size.
  - Applies ReLU activation function.
- **Second Hidden Layer:**
  - Fully connected layer with input size hidden_size and output size $2 \times$ hidden_size.
  - Applies ReLU activation function.
- **Third Hidden Layer:**
  - Fully connected layer with input size $2 \times$ hidden_size and output size $4 \times$ hidden_size.
  - Applies ReLU activation function.
- **Output Layer:**
  - Fully connected layer with input size $4 \times$ hidden_size and output size num_classes.
  - No activation function is applied.

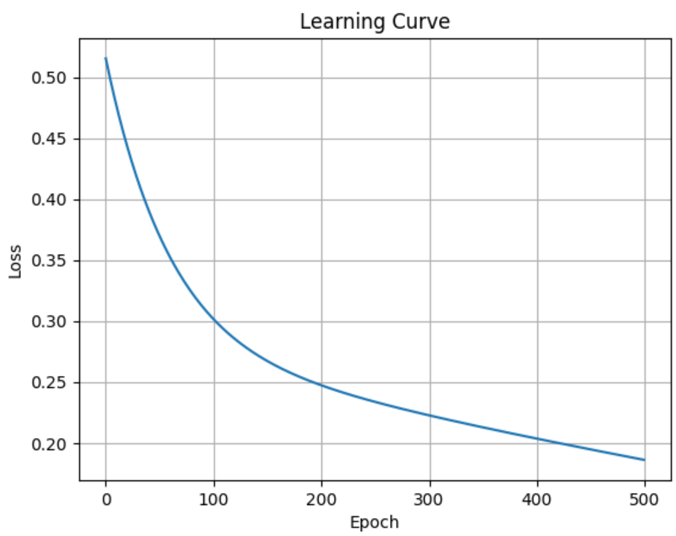


Fig. 4: Learning Curve of Modified FCN

Metric	Value
Validation Loss (After Training)	0.193
Validation Accuracy	85.00%
Epoch 500 Train Loss	0.182

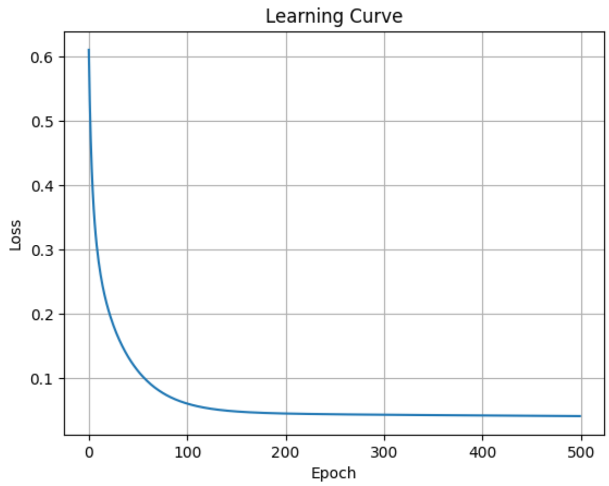
TABLE III: Training and Validation Metrics for Modified Fully Connected Network

Comparing II and III, we see that increasing the complexity of the model results in higher training loss and lower accuracy. This happens for a few reasons.

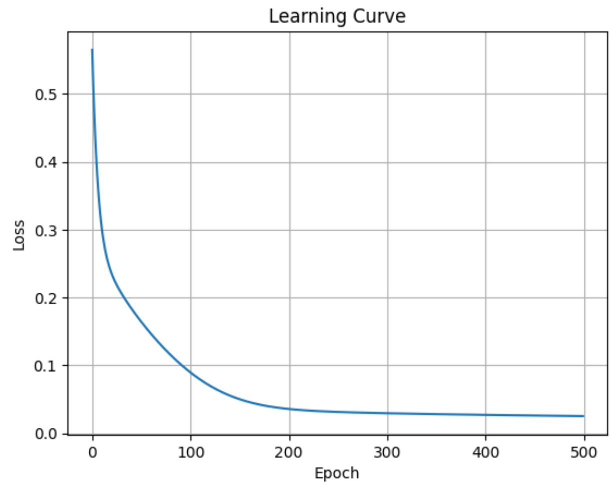
First, more complex models, with additional layers or more parameters, need larger datasets to learn effectively. If the training data isn't enough, the model struggles to fit the data properly, which increases the training loss. More parameters also make optimization harder, as the model might get stuck in local minima or saddle points.

Second, the drop in accuracy is likely due to overfitting. Complex models can capture very detailed patterns, but this often includes noise or irrelevant details in the training data. As a result, the model doesn't generalize well to unseen data, leading to lower accuracy on validation or test sets.

Question 4



(a) Learning curve for the First Fully Connected Network (Learning Rate = 0.01)



(b) Learning curve for the Modified Fully Connected Network (Learning Rate = 0.01)

Fig. 5: Learning Curves (Learning Rate = 0.01)

The learning rate is a critical hyperparameter that determines the speed and stability of model convergence during training. A higher learning rate allows faster convergence but may cause instability or poor performance if the model diverges or overshoots the optimal point. Conversely, a lower learning rate results in more stable convergence but requires more time to reach an accurate solution and may get stuck in saddle points or local minima.

In our case, increasing the learning rate from 0.001 to 0.01 improved accuracy. This suggests that a learning rate of 0.001 was too small for the model to effectively converge within 500 epochs. The increase in learning rate also reduced the training loss for the modified FCN. However, the modified FCN exhibited a higher validation loss compared to the simpler FCN. This indicates that the modified FCN, being a more complex model, struggled to generalize and overfitted the training data slightly.

In summary, while increasing the learning rate improved both accuracy and training efficiency, it also highlighted the modified FCN's tendency to overfit due to its increased complexity.

Metric	**Value**
Validation Loss (After Training)	0.028
Validation Accuracy	100.00%
Epoch 500 Train Loss	0.040

TABLE IV: Training and Validation Metrics for First Fully Connected Model

Metric	**Value**
Validation Loss (After Training)	0.043
Validation Accuracy	97.00%
Epoch 500 Train Loss	0.030

TABLE V: Training and Validation Metrics for Modified Fully Connected Model

QUESTION 5

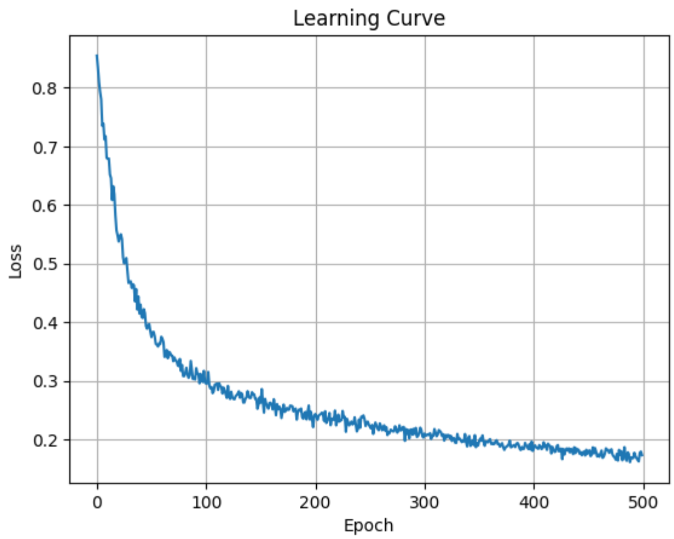


Fig. 6: Learning Curve (Dropout = 0.5)

Metric	Value
Epoch 500 Train Loss	0.173
Validation Loss (After Training)	0.101
Validation Accuracy	95.00%

TABLE VI: Training and Validation Metrics for FCN with dropout

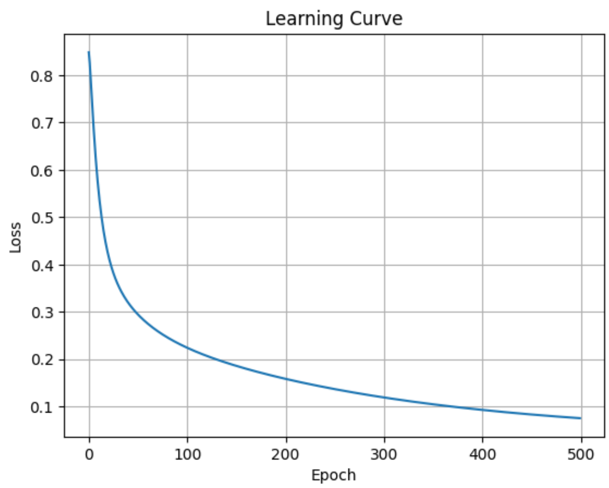
Dropout is a regularization technique that improves the model's ability to generalize by preventing overfitting. It works by randomly ”dropping out” (i.e., setting to zero) a fraction of the neurons during each forward and backward pass in training. This forces the network to rely on multiple independent representations of the data, rather than overfitting to specific patterns in the training set.

In our case, adding a dropout layer has effectively reduced overfitting, as evidenced by the validation error being lower than the training error. This indicates that the model is generalizing better to unseen data. However, dropout introduces some level of instability in the model's learning process, as the randomly dropped neurons can cause fluctuations in the loss during training. This is expected behavior and is a trade-off for improved generalization.

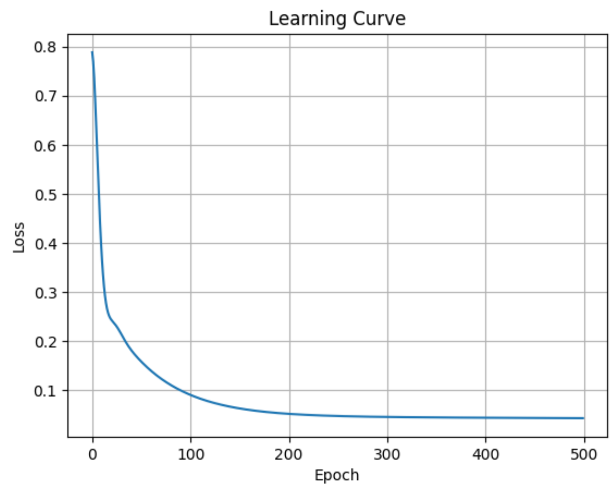
QUESTION 6

Momentum	Epoch 500 Train Loss	Validation Loss (After Training)	Validation Accuracy
0.50	0.075	0.074	98.00%
0.85	0.043	0.039	99.00%
0.90	0.041	0.038	99.00%
0.99	0.025	0.024	98.00%

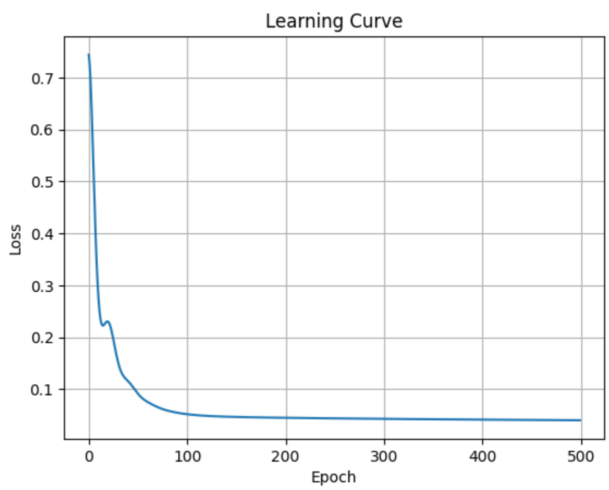
TABLE VII: Training and Validation Metrics for Different Momentum Values



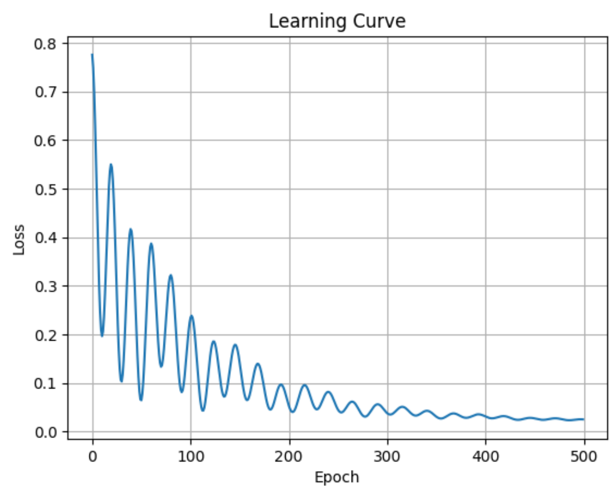
(a) Learning Curve (momentum = 0.50)



(b) Learning Curve (momentum = 0.85)



(c) Learning Curve (momentum = 0.90)



(d) Learning Curve (momentum = 0.99)

Fig. 7: Learning Curves for different momentum values

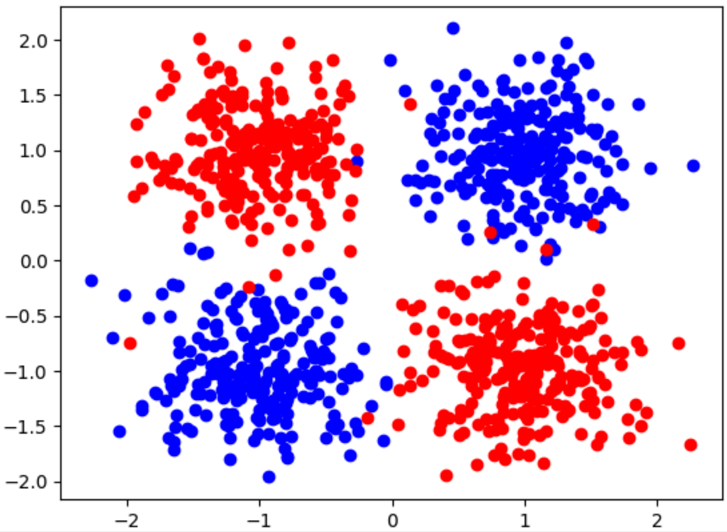
Momentum is a technique used in the Gradient Descent (GD) algorithm and is designed to speed up the optimization process and make it more stable. Momentum adds ”acceleration” to parameter updates by taking into account past gradients. This allows optimization to be performed more effectively, especially in steep slope and oscillating regions. Momentum is typically used with γ=0.9 as the default and provides better results than standard Gradient Descent in most cases. In Figure 7, one can see that the model in 7c converges faster than models in 7a and 7b. However, in Figure 7d, the effect of high momentum causes the model to take larger steps with each iteration. This causes the loss function to initially fluctuate chaotically. Excessively high momentum value causes instability in the optimization process. Although the training loss reaches the lowest level, the verification accuracy is 98.00%, which is lower than other high momentum values.

```
import torch
import numpy as np
from sklearn.datasets import make_blobs
import matplotlib.pyplot as plt


#DEFINE YOUR DEVICE
device = torch.device('cuda:0' if torch.cuda.is_available() else 'cpu')

print(device) #if cpu, go Runtime-> Change runtime type-> Hardware accelerator GPU -> Save -> Redo previous steps
```

```
cuda:0
```

```
#CREATE A RANDOM DATASET
centers = [[1, 1], [1, -1], [-1, -1], [-1, 1]] #center of each class
cluster_std=0.4 #standard deviation of random gaussian samples

x_train, y_train = make_blobs(n_samples=1000, centers=centers, n_features=2, cluster_std=cluster_std, shuffle=True)
y_train[y_train==2] = 0 #make this an xor problem
y_train[y_train==3] = 1 #make this an xor problem
x_train = torch.FloatTensor(x_train)
y_train = torch.FloatTensor(y_train)

x_val, y_val = make_blobs(n_samples=100, centers=centers, n_features=2, cluster_std=cluster_std, shuffle=True)
y_val[y_val==2] = 0 #make this an xor problem
y_val[y_val==3] = 1 #make this an xor problem
x_val = torch.FloatTensor(x_val)
y_val = torch.FloatTensor(y_val)


#CHECK THE BLOBS ON XY PLOT
plt.scatter(x_train[y_train==0,0],x_train[y_train==0,1],marker='o',color='blue')
plt.scatter(x_train[y_train==1,0],x_train[y_train==1,1],marker='o',color='red')
```

```
<matplotlib.collections.PathCollection at 0x791b0b2f4f70>
```




```
#DEFINE NEURAL NETWORK MODEL
class FullyConnected(torch.nn.Module):
  def __init__(self, input_size, hidden_size, num_classes):
    super(FullyConnected, self).__init__()
    self.input_size = input_size
    self.hidden_size  = hidden_size
    self.fc1 = torch.nn.Linear(self.input_size, self.hidden_size)
    self.fc2 = torch.nn.Linear(self.hidden_size, num_classes)
    self.relu = torch.nn.ReLU()
    self.sigmoid = torch.nn.Sigmoid()
  def forward(self, x):
    hidden = self.fc1(x)
    relu = self.relu(hidden)
    output = self.fc2(relu)
    return output


class FullyConnected2(torch.nn.Module):
    def __init__(self, input_size, hidden_size, num_classes):
      super(FullyConnected2, self).__init__()
      self.input_size = input_size
      self.hidden_size  = hidden_size
      self.fc1 = torch.nn.Linear(self.input_size, self.hidden_size)
```

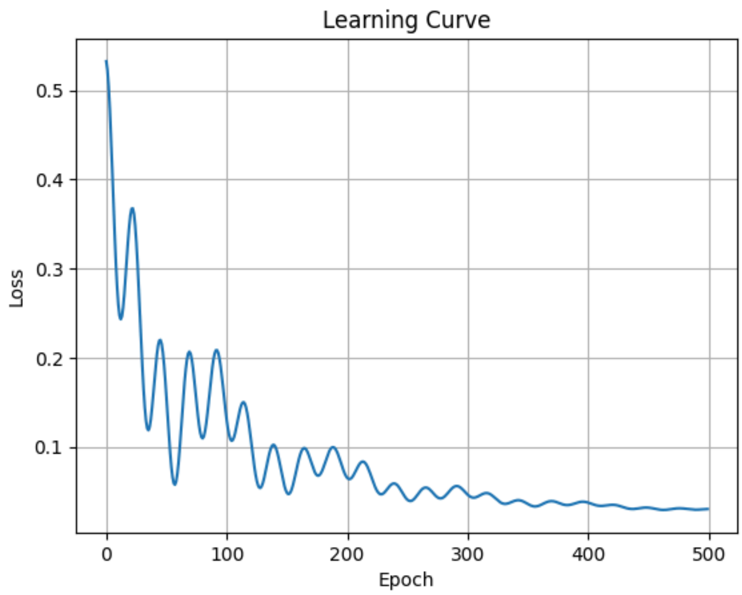
```
      self.fc2 = torch.nn.Linear(self.hidden_size, self.hidden_size*4,)
      self.fc3 = torch.nn.Linear(self.hidden_size*4, self.hidden_size*2,)
      self.fc4 = torch.nn.Linear(self.hidden_size*2, num_classes)
      self.relu = torch.nn.ReLU()
      self.sigmoid = torch.nn.Sigmoid()
    def forward(self, x):
      hidden = self.fc1(x)
      relu = self.relu(hidden)
      hidden2 = self.fc2(relu)
      relu2 = self.relu(hidden2)
      hidden3 = self.fc3(relu2)
      relu3 = self.relu(hidden3)
      output = self.fc4(relu3)
      return output


# Fully Connected Network for Question 5
class FullyConnected3(torch.nn.Module):
  def __init__(self, input_size, hidden_size, num_classes):
    super(FullyConnected3, self).__init__()
    self.input_size = input_size
    self.hidden_size  = hidden_size
    self.fc1 = torch.nn.Linear(self.input_size, self.hidden_size)
    self.fc2 = torch.nn.Linear(self.hidden_size, num_classes)
    self.relu = torch.nn.ReLU()
    self.sigmoid = torch.nn.Sigmoid()
    self.dropout = torch.nn.Dropout(0.5)
  def forward(self, x):
    hidden = self.fc1(x)
    relu = self.relu(hidden)
    dropped = self.dropout(relu)
    output = self.fc2(dropped)
    return output


#CREATE MODEL
input_size = 2
hidden_size = 64
num_classes = 1

model = FullyConnected(input_size, hidden_size, num_classes)
model.to(device)
```

```
FullyConnected(
  (fc1): Linear(in_features=2, out_features=64, bias=True)
  (fc2): Linear(in_features=64, out_features=1, bias=True)
  (relu): ReLU()
  (sigmoid): Sigmoid()
)
```

```
#DEFINE LOSS FUNCTION AND OPTIMIZER
learning_rate = 0.001
momentum = 0.99

loss_fun = torch.nn.MSELoss()
optimizer = torch.optim.SGD(model.parameters(), lr = learning_rate, momentum = momentum)


#TRAIN THE MODEL
model.train()
epoch = 500
x_train = x_train.to(device)
y_train = y_train.to(device)

loss_values = np.zeros(epoch)

for i in range(epoch):
    optimizer.zero_grad()
    y_pred = model(x_train)    # forward
    #reshape y_pred from (n_samples,1) to (n_samples), so y_pred and y_train have the same shape
    y_pred = y_pred.reshape(y_pred.shape[0])

    loss = loss_fun(y_pred, y_train)

    loss_values[i] = loss.item()
    print('Epoch {}: train loss: {}'.format(i, loss.item()))
    loss.backward() #backward
    optimizer.step()
```

```
Epoch 447: train loss: 0.03152424842119217
Epoch 448: train loss: 0.031557682901620865
Epoch 449: train loss: 0.031530290842056274
Epoch 450: train loss: 0.03144175931811333
Epoch 451: train loss: 0.031295470893383026
Epoch 452: train loss: 0.031097887083888054
Epoch 453: train loss: 0.030858393758535385
Epoch 454: train loss: 0.03058871626853943
Epoch 455: train loss: 0.03030208870768547
Epoch 456: train loss: 0.030012544244527817
Epoch 457: train loss: 0.029734168201684952
Epoch 458: train loss: 0.02948031760752201
Epoch 459: train loss: 0.029262932017445564
Epoch 460: train loss: 0.029091693460941315
Epoch 461: train loss: 0.028973760083317757
Epoch 462: train loss: 0.02891341969370842
Epoch 463: train loss: 0.028911741450428963
Epoch 464: train loss: 0.028966829180717468
Epoch 465: train loss: 0.02907375991344452
Epoch 466: train loss: 0.029224848374724388
Epoch 467: train loss: 0.029410339891910553
Epoch 468: train loss: 0.029618840664625168
Epoch 469: train loss: 0.02983803302049637
Epoch 470: train loss: 0.030055468901991844
Epoch 471: train loss: 0.03025909699499607
Epoch 472: train loss: 0.030438033863902092
Epoch 473: train loss: 0.030582968145608902
Epoch 474: train loss: 0.03068666160106659
Epoch 475: train loss: 0.030744504183530807
Epoch 476: train loss: 0.030754433944821358
Epoch 477: train loss: 0.030717190355062485
Epoch 478: train loss: 0.030636079609394073
Epoch 479: train loss: 0.030516676604747772
Epoch 480: train loss: 0.030366623774170876
Epoch 481: train loss: 0.030194994062185287
Epoch 482: train loss: 0.030011780560016632
Epoch 483: train loss: 0.0298272967338562
Epoch 484: train loss: 0.029651544988155365
Epoch 485: train loss: 0.02949369139969349
Epoch 486: train loss: 0.029361506924033165
Epoch 487: train loss: 0.02926097996532917
Epoch 488: train loss: 0.029195992276072502
Epoch 489: train loss: 0.029168222099542618
Epoch 490: train loss: 0.029176976531744003
Epoch 491: train loss: 0.029219264164566994
Epoch 492: train loss: 0.02929018810391426
Epoch 493: train loss: 0.02938300371170044
Epoch 494: train loss: 0.029489651322364807
Epoch 495: train loss: 0.029601242393255234
Epoch 496: train loss: 0.029708653688430786
Epoch 497: train loss: 0.029802991077303886
Epoch 498: train loss: 0.0298761073499918
Epoch 499: train loss: 0.029921049252152443
```

```
#PLOT THE LEARNING CURVE
plt.plot(loss_values)
plt.title('Learning Curve')
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.grid('on')
```



```
#TEST THE MODEL
model.eval()

x_val = x_val.to(device)
y_val = y_val.to(device)

y_pred = model(x_val)
#reshape y_pred from (n_samples,1) to (n_samples), so y_pred and y_val have the same shape
y_pred = y_pred.reshape(y_pred.shape[0])
after_train = loss_fun(y_pred, y_val)
print('Validation loss after Training' , after_train.item())

correct=0
total=0
for i in range(y_pred.shape[0]):
  if y_val[i]==torch.round(y_pred[i]):
    correct += 1
  total +=1

print('Validation accuracy: %.2f%%' %((100*correct)//(total)))
```

```
Validation loss after Training 0.030277404934167862
Validation accuracy: 100.00%
```

Kodlamaya başlayın veya yapay zeka ile kod oluşturun.