$$\text{minimize} \quad \sum_{i=1}^{N} c_i x_i \qquad i = 1, 2, \dots, N$$

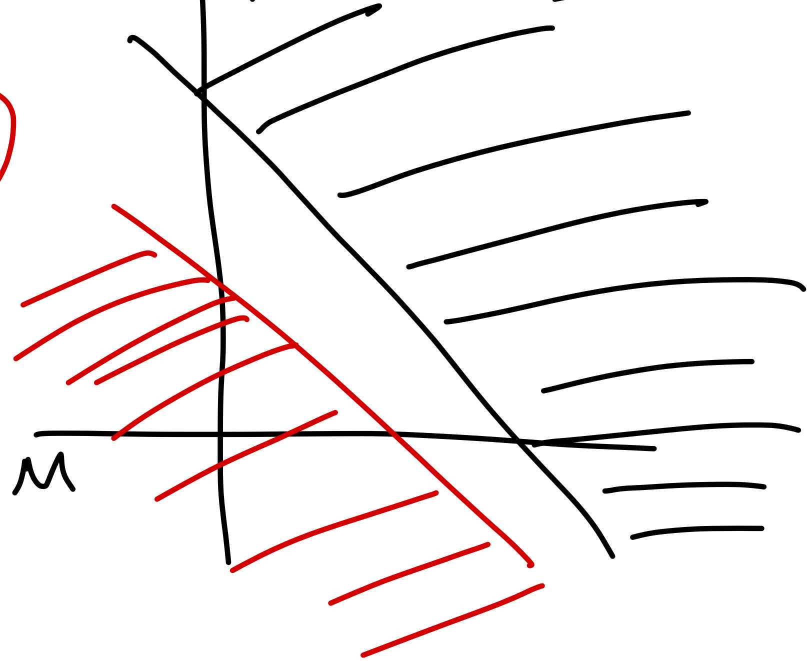
$$\text{subject to:} \quad \sum_{i=1}^{N} a_{ji} \cdot x_i \leq b_j \quad \forall j \qquad j = 1, 2, \dots, M$$

$$l_i \leq x_i \leq u_i \quad \forall i$$

$$x_i \in \mathbb{R}$$

min / minimize

$\{4, -3, 5, 7, 12, -20\}$



Model Peremeters

$J = \{1, 2, \dots, M\}$

$I = \{1, 2, \dots, N\}$

$j \in J$

$i \in I$

minimize $-5x$

subject to: $x \geq 0$

$c_1, c_2, \dots, c_N$

$b_1, b_2, \dots, b_M$

$a_{11}, a_{12}, \dots, a_{1N}$

$\vdots$

$a_{M1}, a_{M2}, \dots, a_{MN}$

$l_1, l_2, \dots, l_N$

$u_1, u_2, \dots, u_N$

binary decision variables $\Rightarrow$ $x_i \in \{0, 1\}$

integer decision variables $\Rightarrow$ $x_i \in \{0, 1, 2, \dots\}$

Integer Linear Programming (ILP) Problems

$f \rightarrow$ linear $\quad$ $g \rightarrow$ linear $\quad$ x_i's $\rightarrow$ binary, integer

f: $2.5x_1 + 3.7x_2$

g: $3.5x_1 + 0.8x_2 \le 10.8$

$$\overset{l_i}{4.7} \le x_i \le \overset{u_i}{5.8}$$

Mixed Integer Linear Programming (MILP) Problems

$f \rightarrow$ linear $\quad$ $g \rightarrow$ linear $\quad$ x_i's $\rightarrow$ real-valued, binary, integer

Nonlinear Programming (NLP) Problems

f might be nonlinear or g might be nonlinear

minimize $x_1^2 - 3x_1 . x_2 + \log(x_2)$

subject to: $x_1 + x_2 \leq 5$

$x_1 \geq 0$

$x_2 \geq 0$

$$\boxed{\text{minimize} \quad x^2 - 6x + 16}$$

$$\frac{\partial f(x)}{\partial x} = 2x - 6 = 0 \Rightarrow \underline{x^* = 3}$$

$$3^2 - \cancel{6}.3 + 16 = \underline{\underline{7}}$$

$$\frac{\partial^2 f(x)}{\partial x^2} = 2$$

minimize $x^2 - 6x + 16$

subject to: $x \geq 5$

Quadratic Programming (QP) Problems

$$\text{minimize} \sum_{i=1}^{N}\sum_{j=1}^{N} q_{ij} x_i x_j + \sum_{i=1}^{N} c_i x_i$$

QCQP → Quadratically constrained quadratic programming

Capital Budgeting

- investment decisions (options)

c_i : contribution resulting from investment "i"

a_{ji} : resource "j" needed for investment "i"

b_j : upper limit for resource "j"

$$x_i = \begin{cases} 1 & \text{if we decide to make investment "}i\text{"} \\ 0 & \text{otherwise} \end{cases}$$

minimize $-c_1 x_1 - c_2 x_2 - \dots - c_N x_N$

maximize $c_1 \cdot x_1 + c_2 \cdot x_2 + \dots + c_N x_N$

subject to:

$$a_{11} \cdot x_1 + a_{12} x_2 + \dots + a_{1N} x_N \le b_1$$

$$\vdots$$

$$a_{M1} x_1 + a_{M2} x_2 + \dots + a_{MN} x_N \le b_M$$

$$x_i \in \{0, 1\} \quad \forall i$$

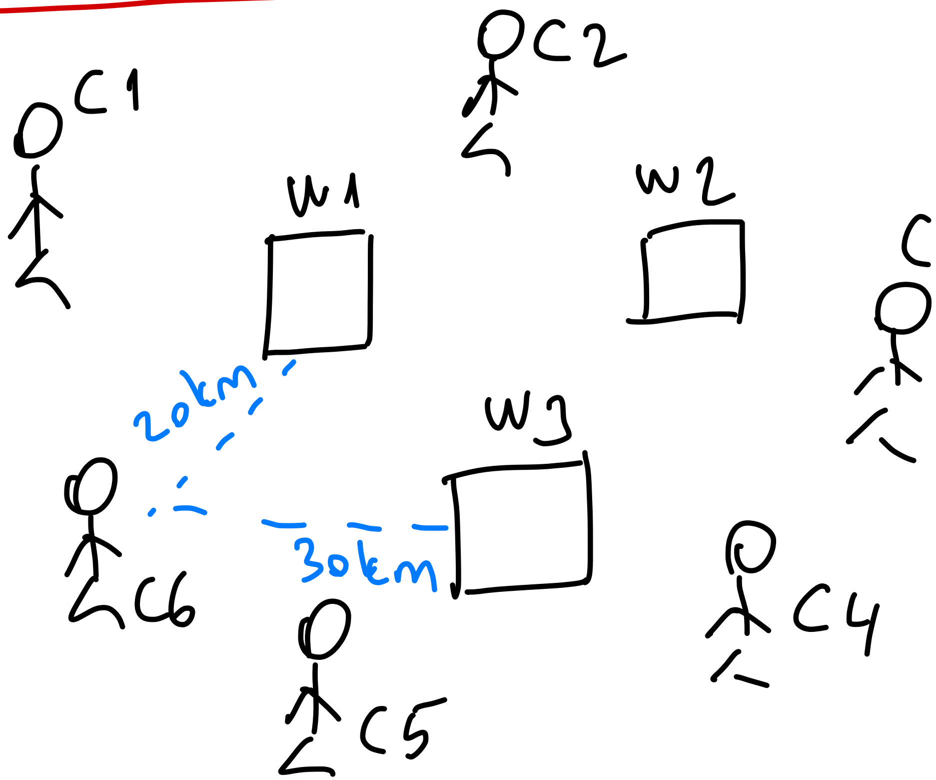
redundant $\Longleftarrow \boxed{\cancel{0 \le x_i \le 1 \ \forall i}}$

minimize $-\sum_{i=1}^{N} c_i x_i$

subject to: $\sum a_{ji} x_i \le b_j \ \forall j$

$x_i \in \{0,1\} \ \forall i$

Warehouse Location



$3 \Leftarrow N$ "possible" warehouse locations

$6 \Leftarrow M$ customers

d_j = demand of customer "j"

f_i = operational cost of warehouse "i"

c_{ij} = cost of serving customer "j" from warehouse "i"

$$y_i = \begin{cases} 1 & \text{if warehouse "i" is opened} \\ 0 & \text{otherwise} \end{cases}$$

x_{ij} = amount that I send from warehouse "i" to customer "j"

$$\text{minimize} \quad \underbrace{\sum_{i=1}^{N} f_i \cdot y_i}_{\text{investment cost}} + \underbrace{\sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} \cdot x_{ij}}_{\text{operational cost}}$$

$$\text{subject to:} \quad \sum_{i=1}^{N} x_{ij} = d_j \qquad j = 1, 2, \ldots, M$$

$$\underbrace{\sum_{j=1}^{M} x_{ij}}_{\text{amount we sent from warehouse } i} - y_i \left(\sum_{j=1}^{M} d_j \right) \leq 0 \qquad i = 1, 2, \ldots, N$$

$$y_i \in \{0, 1\} \qquad i = 1, 2, \ldots, N$$

$$x_{ij} \geq 0 \qquad i = 1, 2, \ldots, N$$
$$j = 1, 2, \ldots, M$$