Warehouse Location

$i = 1, 2, \ldots, N$

N "possible" or "candidate" warehouse locations

$j = 1, 2, \ldots, M$

M customers

f_i = investment cost for warehouse "i"

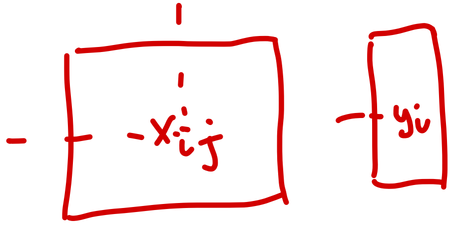
c_{ij} = cost of serving customer "j" from warehouse "i"

d_j = demand of customer "j"

x_{ij} = amount of product sent to customer "j" from warehouse "i"

$$y_i = \begin{cases} 1 & \text{if warehouse "}i\text{" is opened} \\ 0 & \text{otherwise} \end{cases}$$

minimize $\underbrace{\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}\cdot x_{ij}}_{\text{operational cost}} + \underbrace{\sum_{i=1}^{N} f_i\, y_i}_{\text{investment cost (fixed)}}$



subject to:

$$\overbrace{\sum_{i=1}^{N} x_{ij}}^{\text{total amount received by customer "j"}} = d_j \qquad j = 1, 2, \dots, M$$

$$\underbrace{\sum_{j=1}^{M} x_{ij}}_{\text{total amount sent by warehouse "i"}} - y_i \overbrace{\left[\sum_{j=1}^{M} d_j\right]}^{\text{TD: total demand}} \le 0 \qquad i = 1, 2, \dots, N$$

$$0 \le x_{ij} \le +\infty \qquad i = 1, 2, \dots, N, \; j = 1, 2, \dots, M$$

$$y_i \in \{0, 1\} \qquad i = 1, 2, \dots, N$$

# of decision variables = $N.M + N$

Assume $N=2$ and $M=3$

$$\sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}.x_{ij}$$

minimize $\boxed{c_{11}x_{11}+c_{12}x_{12}+c_{13}x_{13}+c_{21}x_{21}+c_{22}x_{22}+c_{23}x_{23}}$

$+\boxed{f_1.y_1+f_2.y_2}$ $\quad \sum_{i=1}^{N} f_i.y_i$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

minimize $c^T.x$

$$\overbrace{\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{21} & c_{22} & c_{23} & f_1 & f_2 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \end{bmatrix}}^{c^T}$$

$$x = \begin{bmatrix} x_{11} \rightarrow x_1 \\ x_{12} \rightarrow x_2 \\ x_{13} \rightarrow x_3 \\ x_{21} \rightarrow x_4 \\ x_{22} \rightarrow x_5 \\ x_{23} \rightarrow x_6 \\ y_1 \rightarrow x_7 \\ y_2 \rightarrow x_8 \end{bmatrix}$$

$$Ax = b$$

$$\sum_{i=1}^{N} x_{ij} = d_j \qquad j = 1, 2, \dots, M$$

$$A_1 \begin{bmatrix} 1x_{11} + 0x_{12} + 0x_{13} + 1x_{21} + 0x_{22} + 0x_{23} + 0y_1 + 0y_2 \\ 0x_{11} + 1x_{12} + 0x_{13} + 0x_{21} + 1x_{22} + 0x_{23} + 0y_1 + 0y_2 \\ 0x_{11} + 0x_{12} + 1x_{13} + 0x_{21} + 0x_{22} + 1x_{23} + 0y_1 + 0y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{b_1}$$

$$x_{11} + x_{21} = d_1$$
$$x_{12} + x_{22} = d_2$$
$$x_{13} + x_{23} = d_3$$

demend satisfaction constraints

$M \times (NM + N)$

$$\begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{M \times M} & \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{M \times M} & \underbrace{\begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix}}_{M \times N} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\sum_{j=1}^{M} x_{ij} - y_i \underbrace{\left[\sum_{j=1}^{M} d_j\right]}_{TD} \leq 0 \qquad i = 1, 2, \dots, N$$

$$A_2 \left[\begin{array}{l} 1x_{11} + 1x_{12} + 1x_{13} + 0x_{21} + 0x_{22} + 0x_{23} - TD\,y_1 + 0y_2 \\ 0x_{11} + 0x_{12} + 0x_{13} + 1x_{21} + 1x_{22} + 1x_{23} + 0y_1 - TD\,y_2 \end{array}\right] \begin{array}{l} \leq \\ \leq \end{array} \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{b_2}$$

$$x_{11} + x_{12} + x_{13} - (TD) \cdot y_1 \leq 0$$

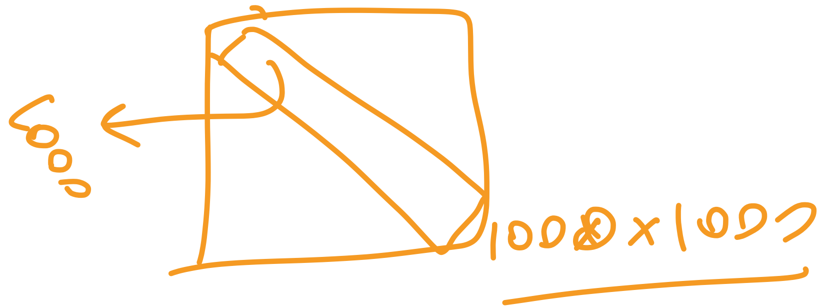
$$x_{21} + x_{22} + x_{23} - (TD) \cdot y_2 \leq 0$$

$$\left[\underbrace{\begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{matrix}}_{N \times M} \;\; \underbrace{\begin{matrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}}_{N \times M} \;\; \underbrace{\begin{matrix} -TD & 0 \\ 0 & -TD \end{matrix}}_{N \times N}\right] \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ y_1 \\ y_2 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$
\underbrace{\begin{bmatrix} A_1 \\ \hline A_2 \end{bmatrix}}_{5\times 8}
\underbrace{\begin{bmatrix} x \end{bmatrix}}_{8\times 1}
\begin{bmatrix} = \\ = \\ = \\ \le \\ \le \end{bmatrix}
\begin{bmatrix} b_1 \\ \hline b_2 \end{bmatrix}
$$

A_1: 3×8, A_2: 2×8, b_1: 3×1, b_2: 2×1

$$
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & -TD & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & -TD
\end{bmatrix}
\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ y_1 \\ y_2 \end{bmatrix}
\begin{bmatrix} = \\ = \\ = \\ \le \\ \le \end{bmatrix}
\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ 0 \\ 0 \end{bmatrix}
$$



$$l \leq x \leq u$$

$$0 \leq x_{ij} \leq +\infty \qquad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, M$$

$$y_i \in \{0, 1\} \qquad i = 1, 2, \ldots, N$$

$$\underset{(NM+N)\times 1}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}} \leq \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \\ y_1 \\ y_2 \end{bmatrix} \leq \begin{bmatrix} \left.\begin{matrix} +\infty \\ +\infty \\ +\infty \\ +\infty \\ +\infty \\ +\infty \end{matrix}\right\} NM \times 1 \\ \left.\begin{matrix} 1 \\ 1 \end{matrix}\right\} N \times 1 \end{bmatrix}$$

General form of A

$$
\begin{array}{c|cccc|cccc|c|cccc|ccc|}
 & x_{11} & x_{12} & \cdots & x_{1m} & x_{21} & x_{22} & \cdots & x_{2m} & \cdots\cdots & x_{N1} & x_{N2} & \cdots & x_{Nm} & y_1 & \cdots & y_N \\
\hline
1 & 1 & & & & 1 & & & & & 1 & & & & & & \\
2 & & 1 & & 0 & & 1 & & 0 & & & 1 & & 0 & & & \\
\vdots & & & \ddots & & & & \ddots & & \cdots & & & \ddots & & & 0 & \\
 & 0 & & & & 0 & & & & & 0 & & & & & & \\
m & & & & 1 & & & & 1 & & & & & 1 & & & \\
\hline
m+1 & 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 & -TD & & \\
m+2 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & & 0 & 0 & \cdots & 0 & & -TD & 0 \\
\vdots & & 0 & 0 & 0 & & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & & 0 & \ddots & \\
m+N & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & & 1 & 1 & \cdots & 1 & & & -TD \\
\end{array}
$$

$(m+N)\times(Nm+N)$