Optimization Problem

minimize $f(x_1, x_2, \ldots, x_N)$ → objective function

$x_1, x_2, \ldots, x_N$ → decision variables

$f: \mathbb{R}^N \to \mathbb{R}$

$g_1: \mathbb{R}^N \to \mathbb{R}$

$g_2: \mathbb{R}^N \to \mathbb{R}$

$\vdots$

$g_m: \mathbb{R}^N \to \mathbb{R}$

(s.t.) subject to:

constraints:

$$g_1(x_1, x_2, \ldots, x_N) \le b_1$$
$$g_2(x_1, x_2, \ldots, x_N) \le b_2$$
$$\vdots$$
$$g_M(x_1, x_2, \ldots, x_N) \le b_m$$

N = # of decision variables

$3x_1 + 5x_2 - 7x_3 \le 20$

M = # of constraints

bounds:

$$l_1 \le x_1 \le u_1 \quad \rightsquigarrow \quad -5 \le x_1 \le +4$$
$$l_2 \le x_2 \le u_2$$
$$\vdots$$
$$l_N \le x_N \le u_N$$

	Product#1	Product#2	Availability
Machine#1	1	0	4 hours
Machine#2	0	2	12 hours
Machine#3	3	2	18 hours
Profit	$3	$5	maximize profit

maximize $3x_1 + 5x_2$ $\Rightarrow$ minimize $-3x_1 - 5x_2$

subject to:

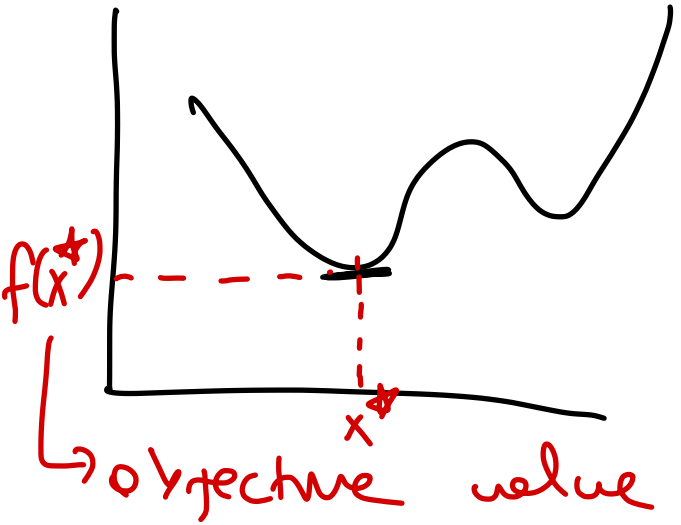
$1 \cdot x_1 + 0 \cdot x_2 \leq 4$ $\quad g_1(x_1, x_2) \leq b_1$

$0 \cdot x_1 + 2 \cdot x_2 \leq 12$ $\quad g_2(x_1, x_2) \leq b_2$

$3 \cdot x_1 + 2 \cdot x_2 \leq 18$ $\quad g_3(x_1, x_2) \leq b_3$

$0 \leq x_1$

$0 \leq x_2$



↳ objective value at x^*

$3x_1 + 2x_2 \leq 18$

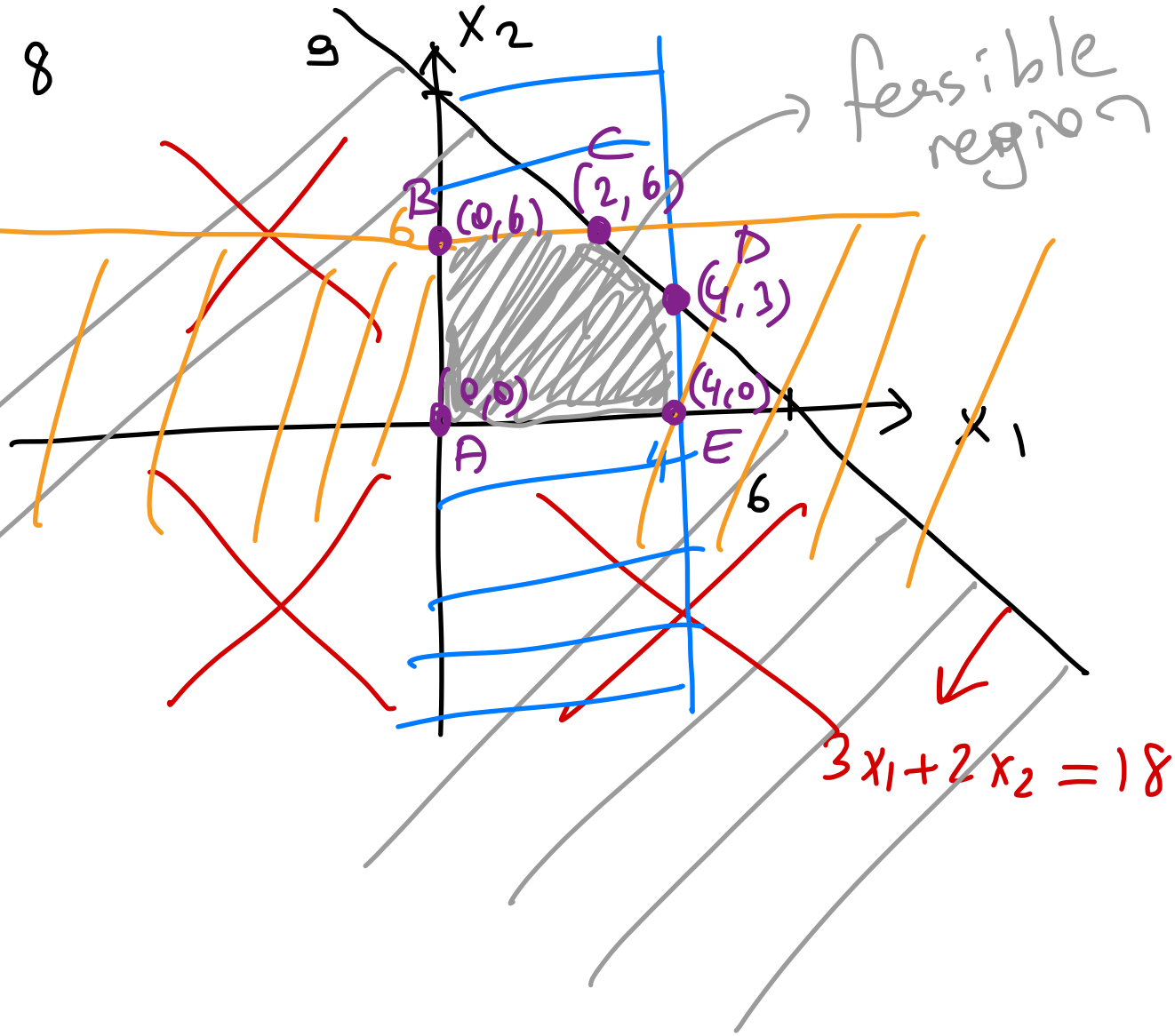
$f(A) \Rightarrow 3.0 + 5.0 = 0$

$f(B) \Rightarrow 3.0 + 5.6 = 30$

$f(C) \Rightarrow 3.2 + 5.6 = 36$

$f(D) \Rightarrow 3.4 + 5.3 = 27$

$f(E) \Rightarrow 3.4 + 5.0 = 12$



Linear Programming (LP) Problems

cost coefficients

minimize $c_1 x_1 + c_2 x_2 + \cdots + c_N x_N$

subject to:

$$\begin{aligned}
a_{11} x_1 + a_{12} x_2 + \cdots + a_{1N} x_N &\le b_1 \\
a_{21} x_1 + a_{22} x_2 + \cdots + a_{2N} x_N &\le b_2 \\
&\vdots \\
a_{M1} x_1 + a_{M2} x_2 + \cdots + a_{MN} x_N &\le b_M
\end{aligned}$$

$$\begin{aligned}
l_1 \le x_1 \le u_1 \\
l_2 \le x_2 \le u_2 \\
\vdots \\
l_N \le x_N \le u_N
\end{aligned}$$

N = # of decision variables

M = # of constraints