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INDR 372

PRODUCTION PLANNING AND CONTROL

Homework 1

Question 1

Part A.

Here is the graph of total monthly domestic sales of Renault brand cars in Turkey from the beginning of 2013 to the end of 2022.

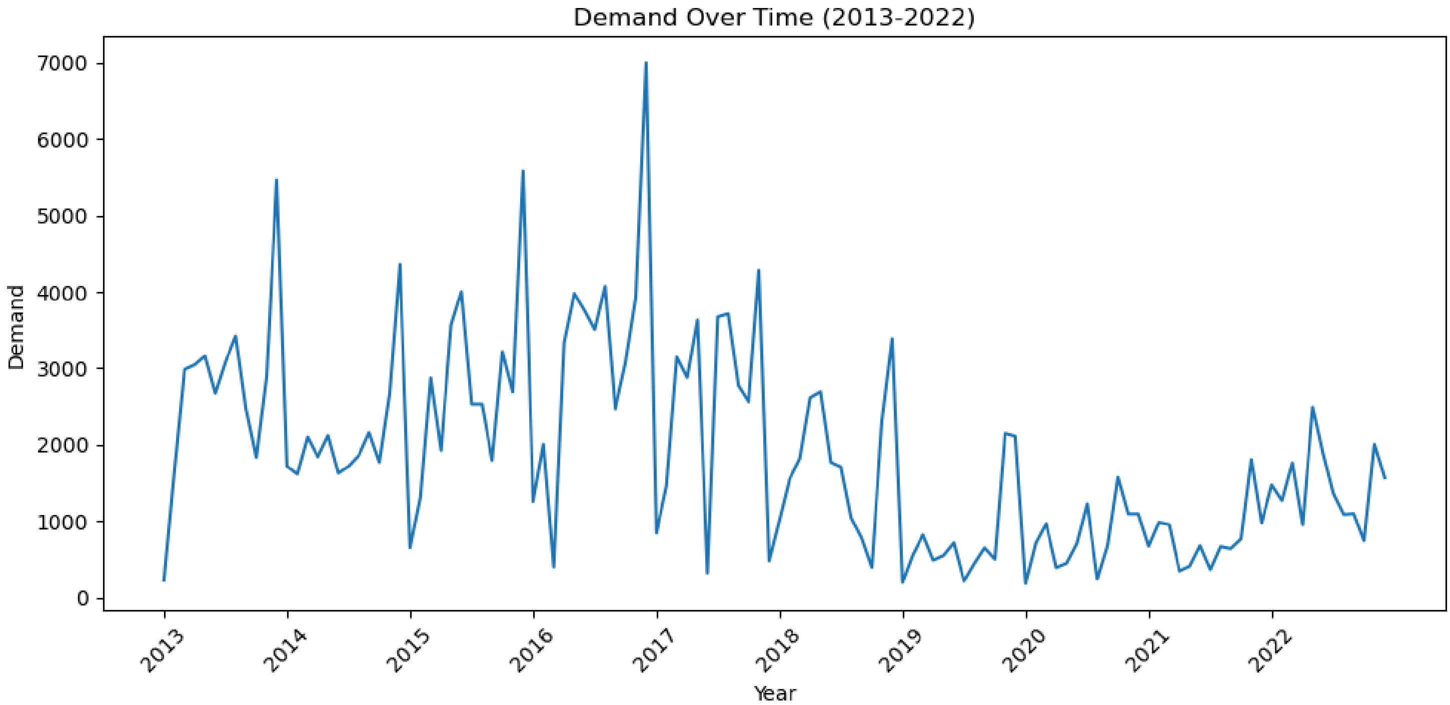


Figure 1: Demand of Renault Vehicles

Trend: Overall, there are no signs of a consistent long-term decrease or increase in sales, rather, demand fluctuates. Even though there are several peaks, they may be related to specific events or promotions that could have driven sales up at times. Thus, there is not enough evidence that indicate a trend going upward or downward between 2013 and 2022. There might be other factors influencing sales besides a simple trend. However, if we were to only comment on the interval of 2017-2022, then an overall visible downward trend is observable.

Seasonality: We can observe a pattern that repeats annually. There is a significant spike in demand at the beginning of each year, which indicates seasonality.

Part B.

Here are the graphs for the naive forecasts with tau = 1 and tau = 12 over the actual demand:

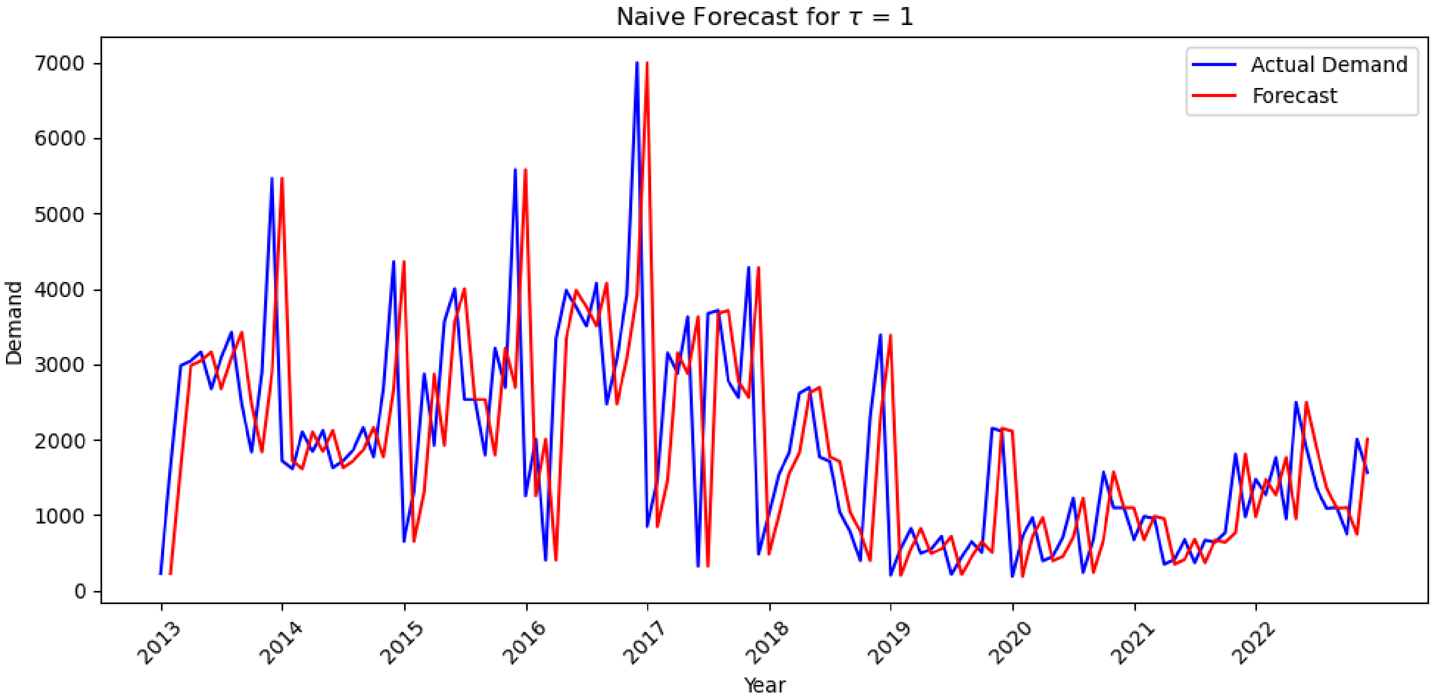


Figure 2: Naive Forecast (Tau = 1)

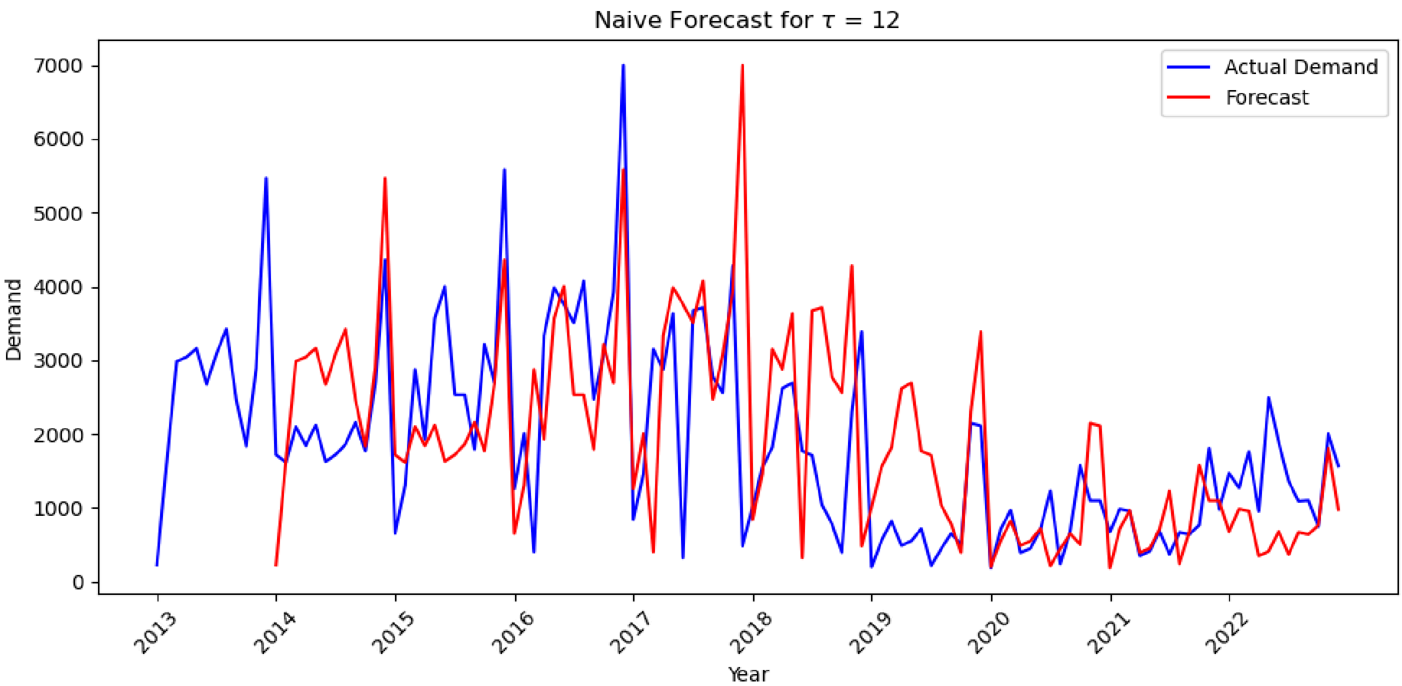


Figure 3: Naive Forecast (Tau = 12)

Here is the output of the error metric calculations for these forecasts for years 2019 until the end of 2022:

```
For tau = 1:
Mean Absolute Error (MAE): 529.4166666666666
Mean Absolute Percentage Error (MAPE): 109.72726757103048
Root Mean Squared Error (RMSE): 782.5992216113345
For tau = 12:
Mean Absolute Error (MAE): 606.7708333333334
Mean Absolute Percentage Error (MAPE): 88.86049187264734
Root Mean Squared Error (RMSE): 832.7095457000598
```

Figure 4: Error Metrics of Naive Forecasts

Part C.

Here is the 3-Period Moving Averages Forecast over the actual demand:

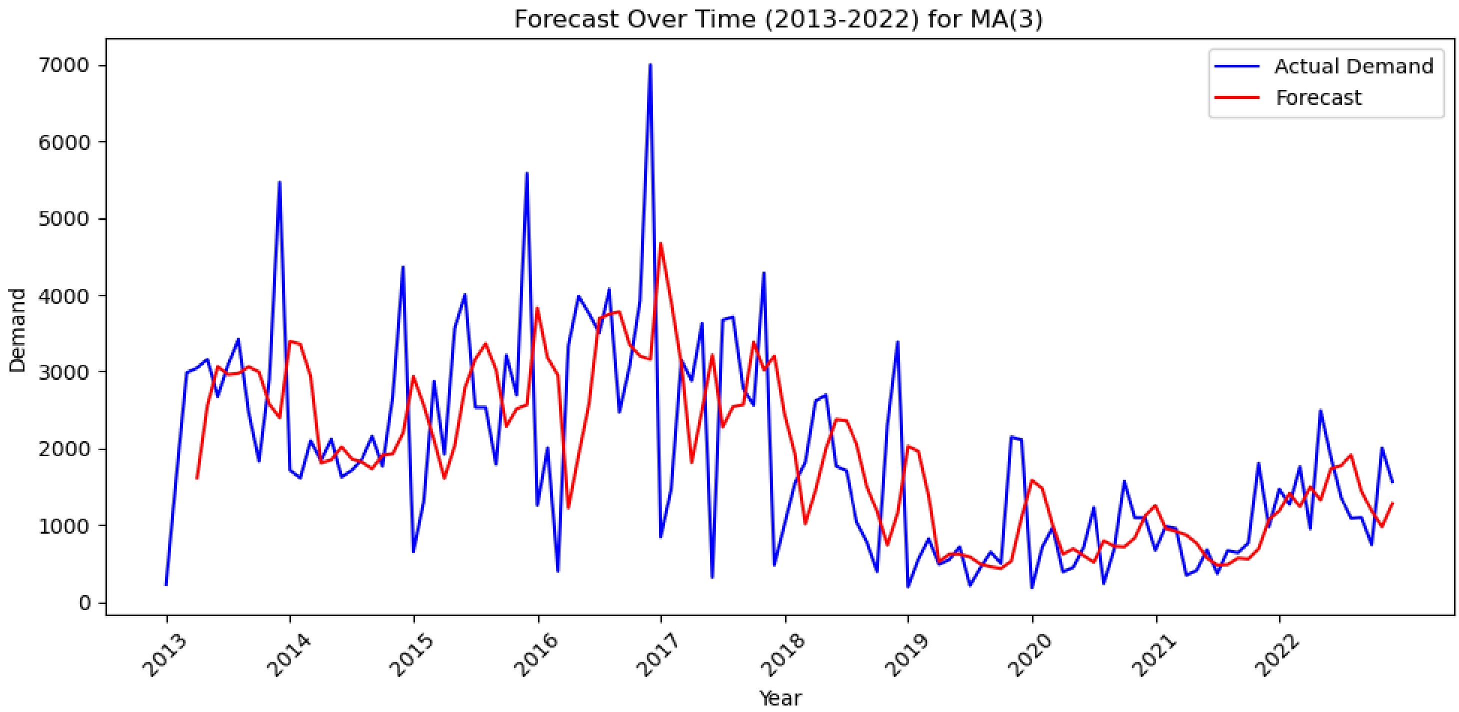


Figure 5: MA(3) Forecast Compared to Actual Demand

Here is the output of the error metric calculations for the MA(3) forecast for years 2019 until the end of 2022:

```
Three-Period Moving Average Forecast:
Mean Absolute Error (MAE): 460.8680555555555
Mean Absolute Percentage Error (MAPE): 82.30538388339788
Root Mean Squared Error (RMSE): 649.3309038427058
```

Figure 6: Error Metrics of MA(3) Forecast

By observing the decrease in error metrics compared to the Naive Forecasts, we can confidently state that the 3-period moving averages method is noticeably better at predicting the observed demand than the naive models.

Here is the output of the prediction interval calculations:

```
90 percent prediction intervals for the one-month ahead
forecasts using 3-month moving averages for year 2022:
[(-1305.452062412552, 3673.452062412552),
(-1070.1187290792188, 3908.7853957458856),
(-1249.1187290792188, 3729.7853957458856),
(-987.4520624125521, 3991.452062412552),
(-1160.7853957458854, 3818.1187290792186),
(-753.1187290792188, 4225.785395745886),
(-710.7853957458854, 4268.118729079219),
(-574.1187290792188, 4404.785395745886),
(-1042.1187290792188, 3936.7853957458856),
(-1305.7853957458854, 3673.1187290792186),
(-1511.7853957458856, 3467.1187290792186),
(-1205.7853957458854, 3773.1187290792186)]
```

Here is the graph of the %90 prediction intervals over the actual demand:

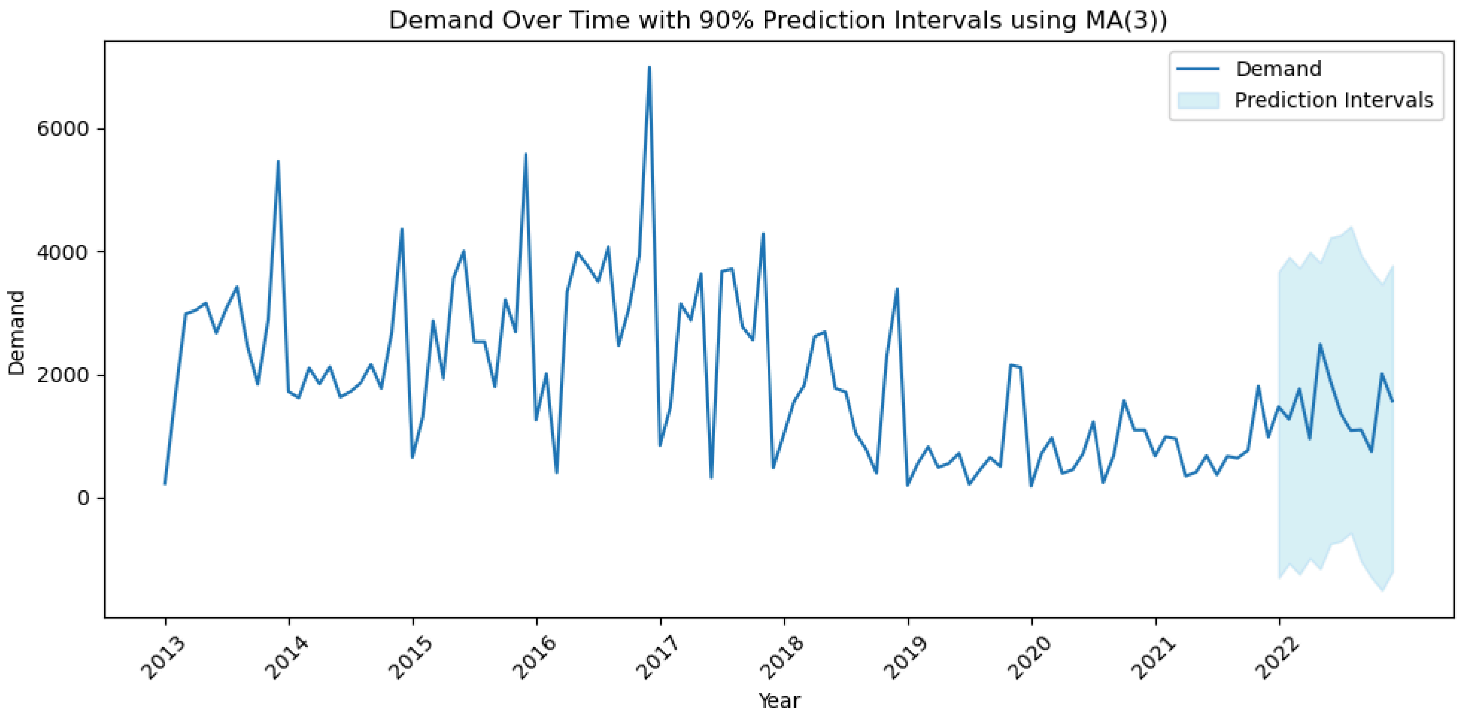


Figure 7: %90 Prediction Interval

The interval is quite large, however this can easily be attributed to the fact that especially until the beginning of 2019, there were also many large jumps and dips in demand which increased the RMSE, which is linearly related to the interval length, so this is understandable. It is also seen that MA(3) has been able to capture all the demand in 2022 within its prediction intervals.

Part D.

Here is the output that we generated for the MA(3) forecast. We fail to reject the null hypothesis that the mean of residuals is zero with significance level of 0.05.

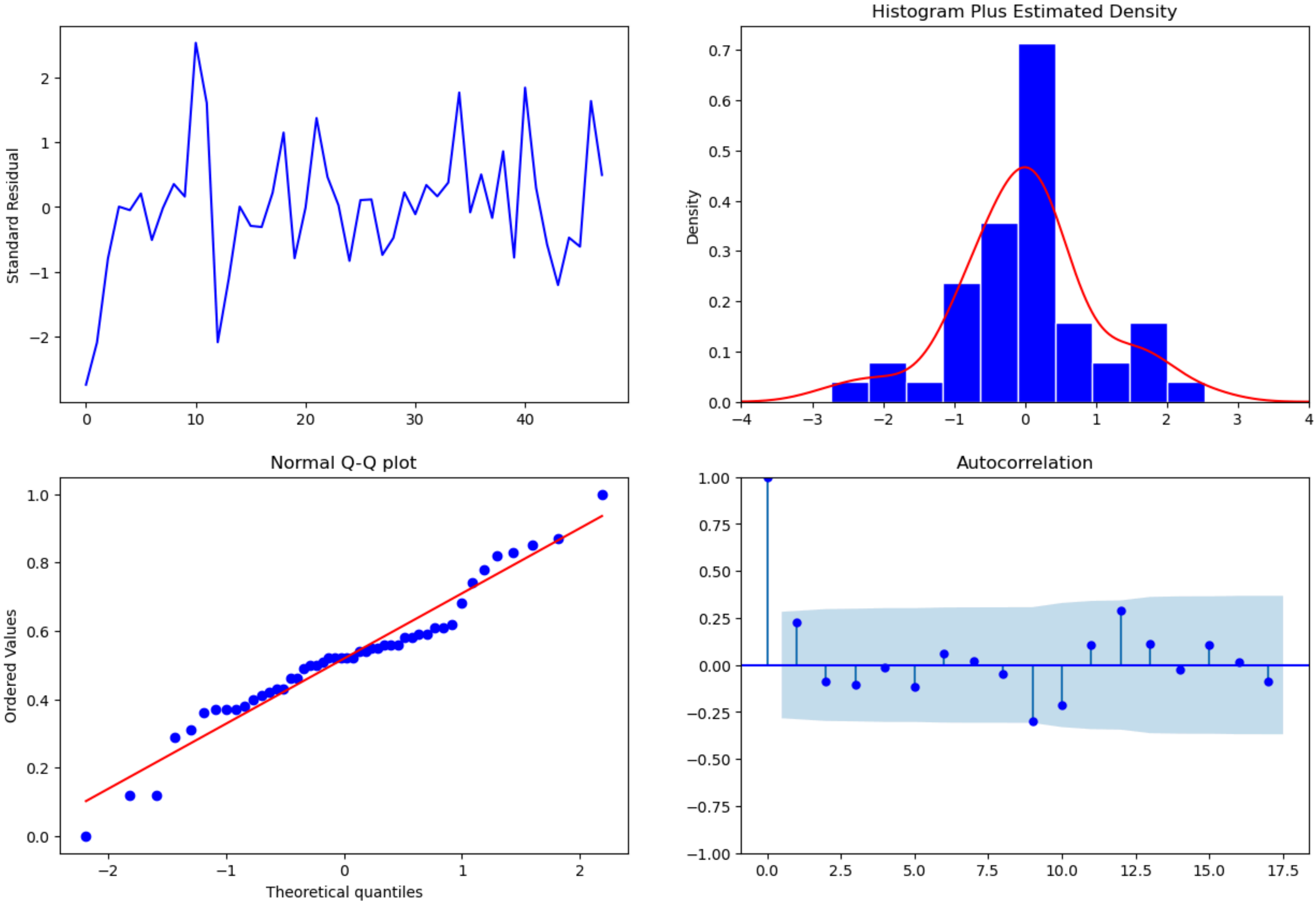


Figure 8: Residuals for MA(3) Forecast

When looking at the Normal Q-Q plot, we see that it does not fit well into the normal line, so we could conclude that the residuals are not normally distributed.

The autocorrelation plot shows us that there is no statistically significant correlation between any lag for the residuals, so we can say that the residuals are independently distributed.

Part E.

The best smoothing constant for the exponential smoothing that we found was $\alpha = 0.5$. It yielded the smallest MAE, MAPE, and RMSE out of all the candidates $\{0.1, 0.2, ..., 1\}$. The exponentially smoothed forecasts of all the candidates are available in the .ipynb file. Here is the graph for the best smoothing constant of $\alpha = 0.5$:

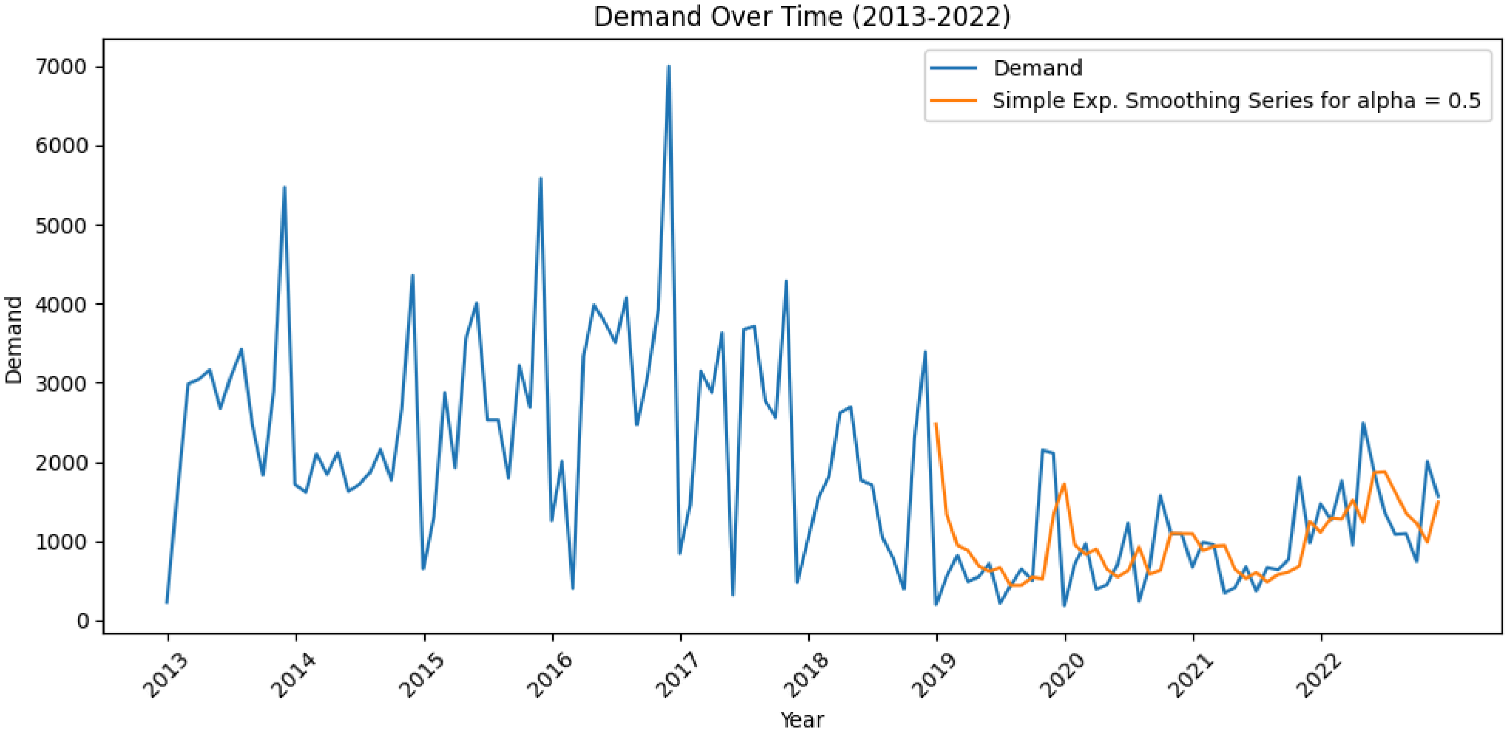
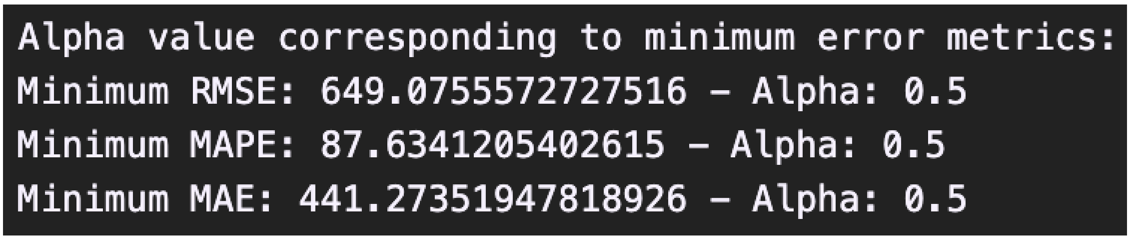


Figure 9: Exponential Smoothing (alpha = 0.5)

Here are the MAE, MAPE, and RMSE values for $\alpha = 0.5$:



```
Alpha value corresponding to minimum error metrics:
Minimum RMSE: 649.0755572727516 - Alpha: 0.5
Minimum MAPE: 87.6341205402615 - Alpha: 0.5
Minimum MAE: 441.27351947818926 - Alpha: 0.5
```

Figure 10: Error Metrics of Exponential Smoothing

Even for the best smoothing constant, the error metrics were very similar to the MA(3) method. This might mean that the demand is more influenced by the last 3 observations, and putting any sort of weight to later observations does not help at least positively in this case.

Here are the 90% prediction intervals for the simple exponential smoothing with a smoothing constant of $\alpha = 0.5$:

```
90 percent prediction intervals for the one-month ahead
forecasts using simple exp. smoothing for year 2022:
[(-1247.2652184939614, 3472.8828946325975),
(-1067.1696375286206, 3652.9784755979385),
(-1078.62184704595, 3641.5262660806093),
(-837.8479518046147, 3882.3001613219444),
(-1122.4610041839471, 3597.687108942612),
(-494.7675303736132, 4225.380582752946),
(-482.4207934684464, 4237.727319658113),
(-739.747425015863, 3980.400688110696),
(-1005.4107407895713, 3714.737372336988),
(-1133.2423986764254, 3586.905714450134),
(-1374.1582276198524, 3345.9898855067067),
(-863.616142091566, 3856.531971034993)]
```

Here is the graph of 90% intervals for the simple exponential smoothing with a smoothing constant of $\alpha = 0.5$ for a better visualization:

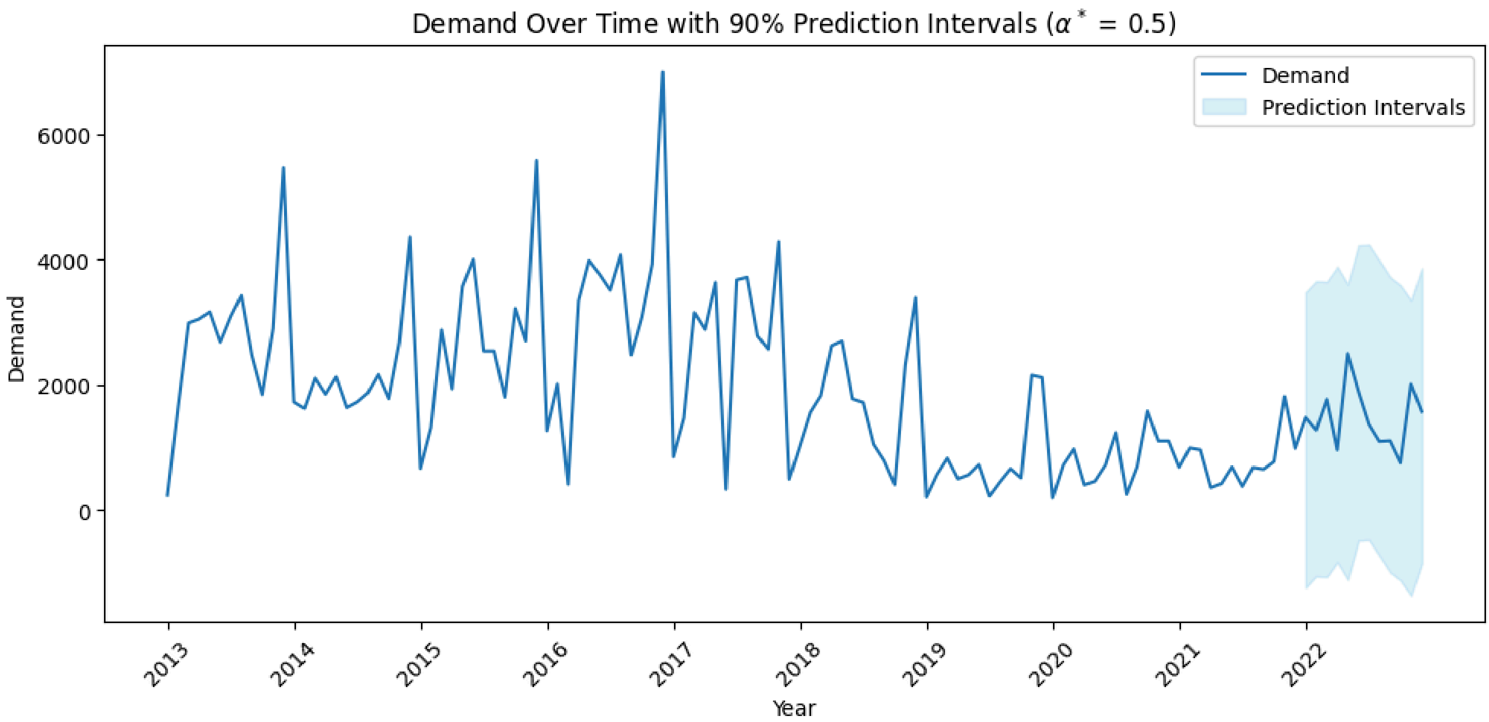


Figure 11: %90 Prediction Intervals for ES

The interval lengths are remarkably similar to the MA(3) case, but this is expected because their RMSE values turned out to be almost exactly the same. Other than that, the visualization of the intervals is simply the displaying of the original $\alpha = 0.5$ ES graph with the half of the interval length as positive and negative offsets, so there is not much to say other than the fact that it was able to predict all the values observed in 2022 within the suggested intervals.

Here is a graph that compares MA(3) forecast to $\alpha = 0.5$ ES forecast for more visuality:

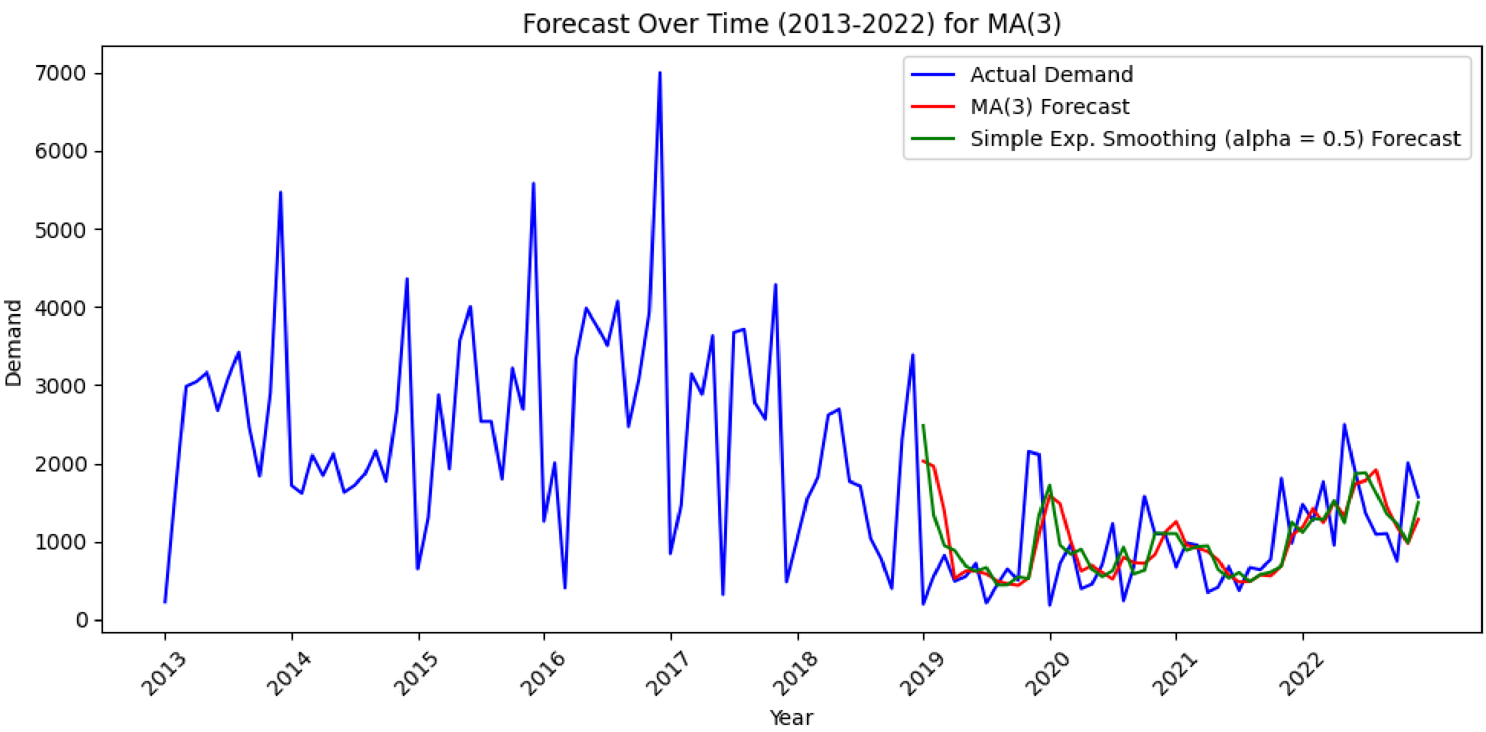


Figure 12: Exponential Smoothing vs. MA(3) Forecast

Part F.

The best α and β values that we were able to find for double exponential smoothing are reported as follows:

```
Best MAPE: 69.80290423154662
Corresponding Alpha (MAPE): 0.3
Corresponding Beta (MAPE): 0.1
Best MAE: 541.8243398590141
Corresponding Alpha (MAE): 0.5
Corresponding Beta (MAE): 0.1
Best RMSE: 740.3038392206994
Corresponding Alpha (RMSE): 0.4
Corresponding Beta (RMSE): 0.1
```

Figure 13: Error Metrics of DES

Here is the visualization of the forecasts for all of the combinations:

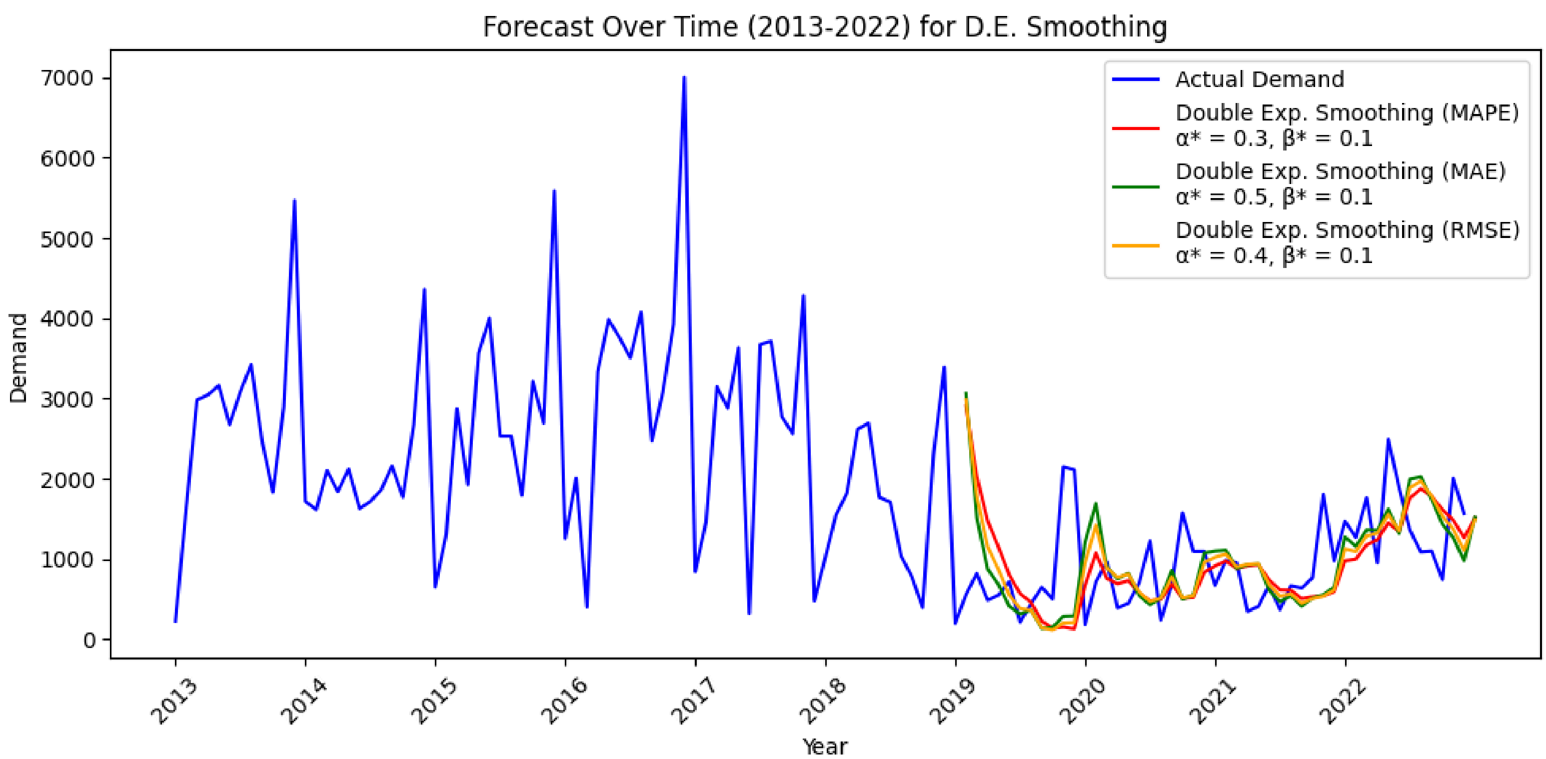


Figure 14: DES Forecasts for Best Alpha and Beta Combinations

To report the 90% prediction intervals, we decided to use the α, β combination that minimizes the RMSE. We did this to get the smallest (most precise) intervals for the predictions. Here are the values themselves and the visualization of the intervals:

```
90 percent prediction intervals for the one-month ahead
forecasts using double exponential smoothing (alpha = 0.4,
beta = 0.1):
(-1359.3815388805535, 3612.605627328696)
(-1390.9495415524175, 3581.0376246568317)
(-1196.7721048176243, 3775.2150613916247)
(-1162.2345019082284, 3809.7526643010206)
(-926.7423034104468, 4045.2448627988024)
(-1133.6970354995449, 3838.2901307097045)
(-596.2417366572067, 4375.745429552042)
(-514.9586312097006, 4457.028534999548)
(-701.3101660169941, 4270.677000192255)
(-950.5484235848751, 4021.438742624374)
(-1113.5491845063939, 3858.4379817028553)
(-1378.0474170032342, 3593.939749206015)
```

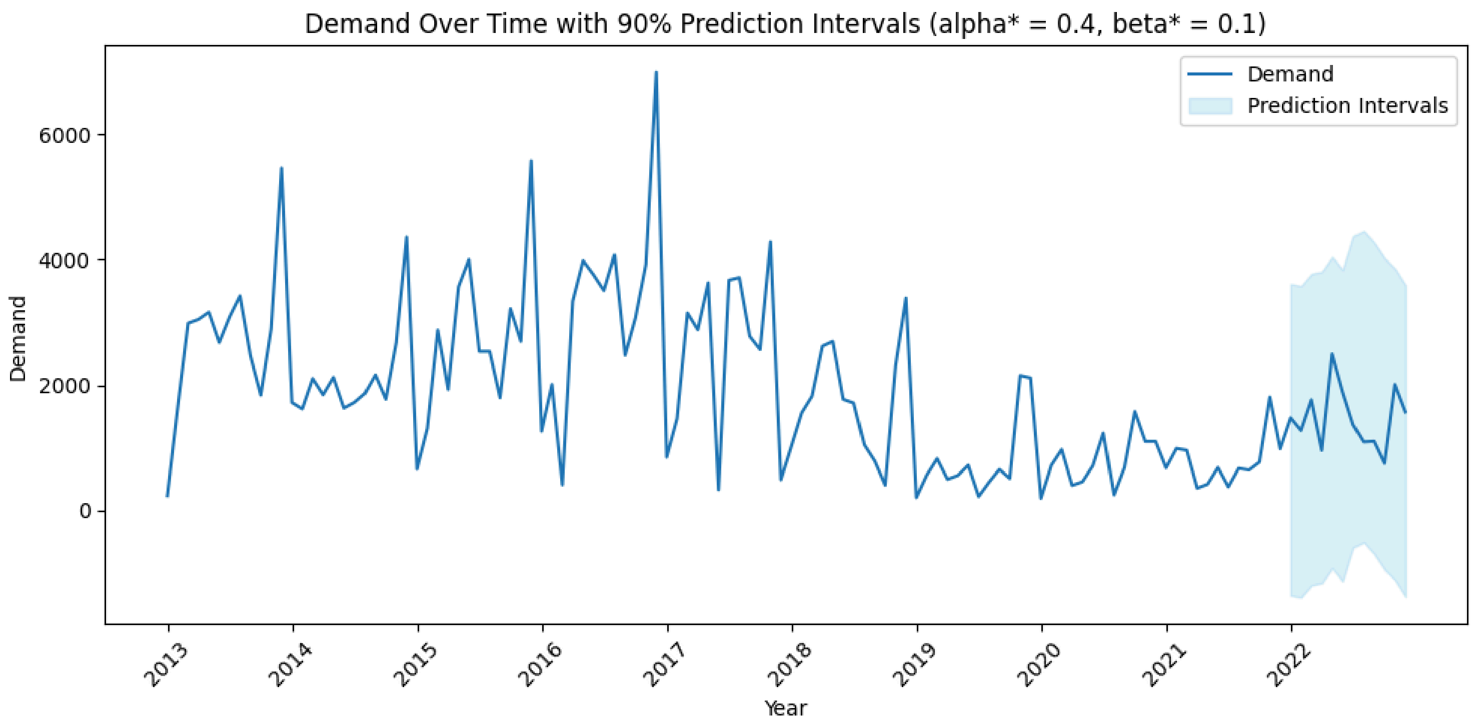


Figure 15: %90 Prediction Intervals for DES

There is not much to say that was not already said in MA(3) and ES reports. The interval is large because RMSE is large to the relatively high error. All values have been accurately predicted in the interval.

Part G.

Here is the output that we generated for the DES forecast with ($\alpha = 0.5, \beta = 0.1$). Once again, we fail to reject the null hypothesis that the mean of residuals is zero at significance level of 0.05.

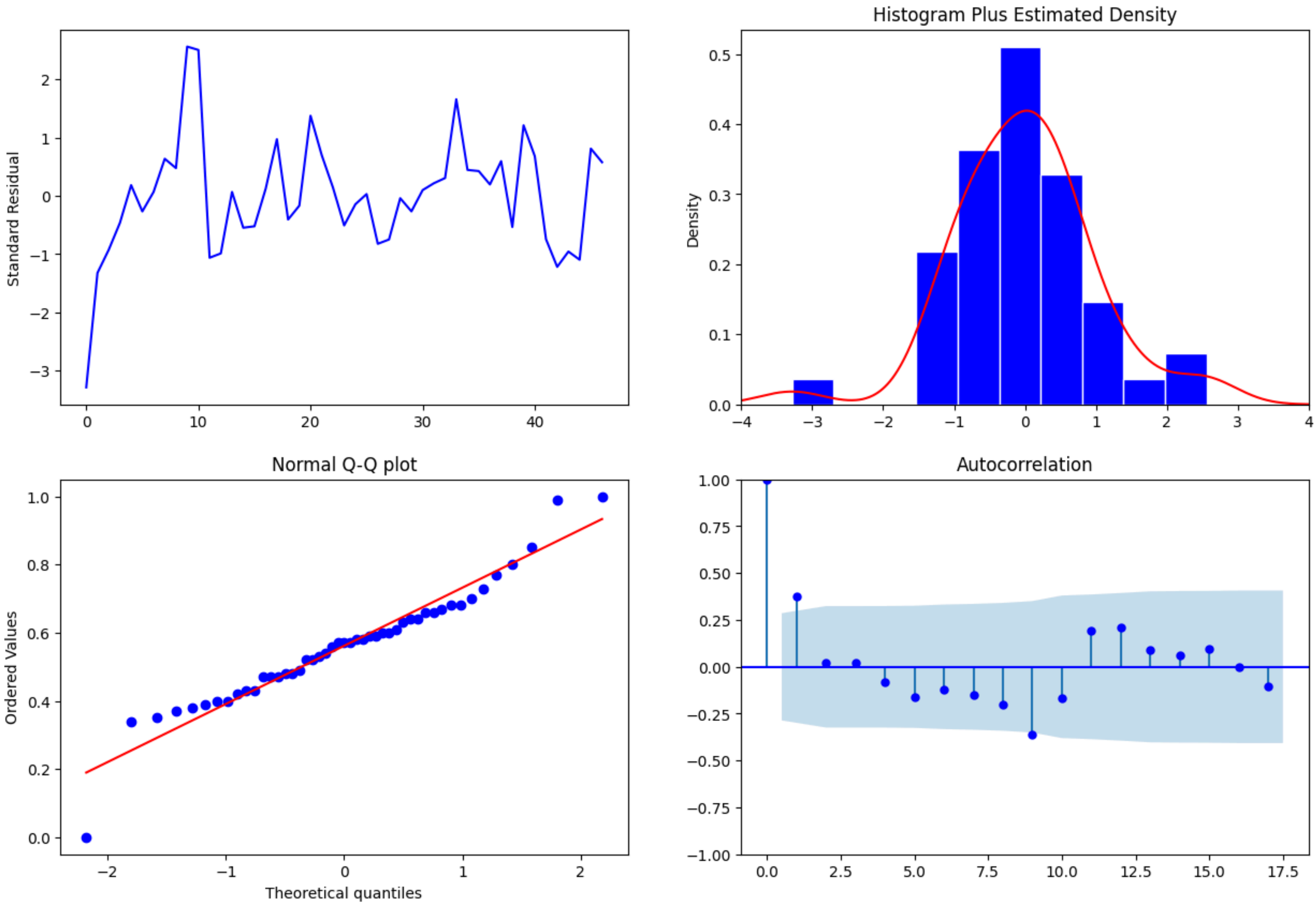


Figure 16: Residuals for DES Forecast

Looking at the Normal Q-Q plot, the residual values are still not a good fit to be considered to be coming from a normal distribution, with three clear outliers.

The autocorrelation plot tells us that there is a statistically significant correlation between lag-1 residuals. So, we cannot claim that the residuals are independent.

The error metrics were better than all of the models beforehand. However the drawback of this forecast with respect to this data seems to be that DES cannot catch up with large changes as fast as it should. Because of this, the last residual observed gives us a slight clue about the next residual that is going to be observed, which is unideal.

Part H.

Here is how 3-month and 6-month ahead forecasts are visualized:

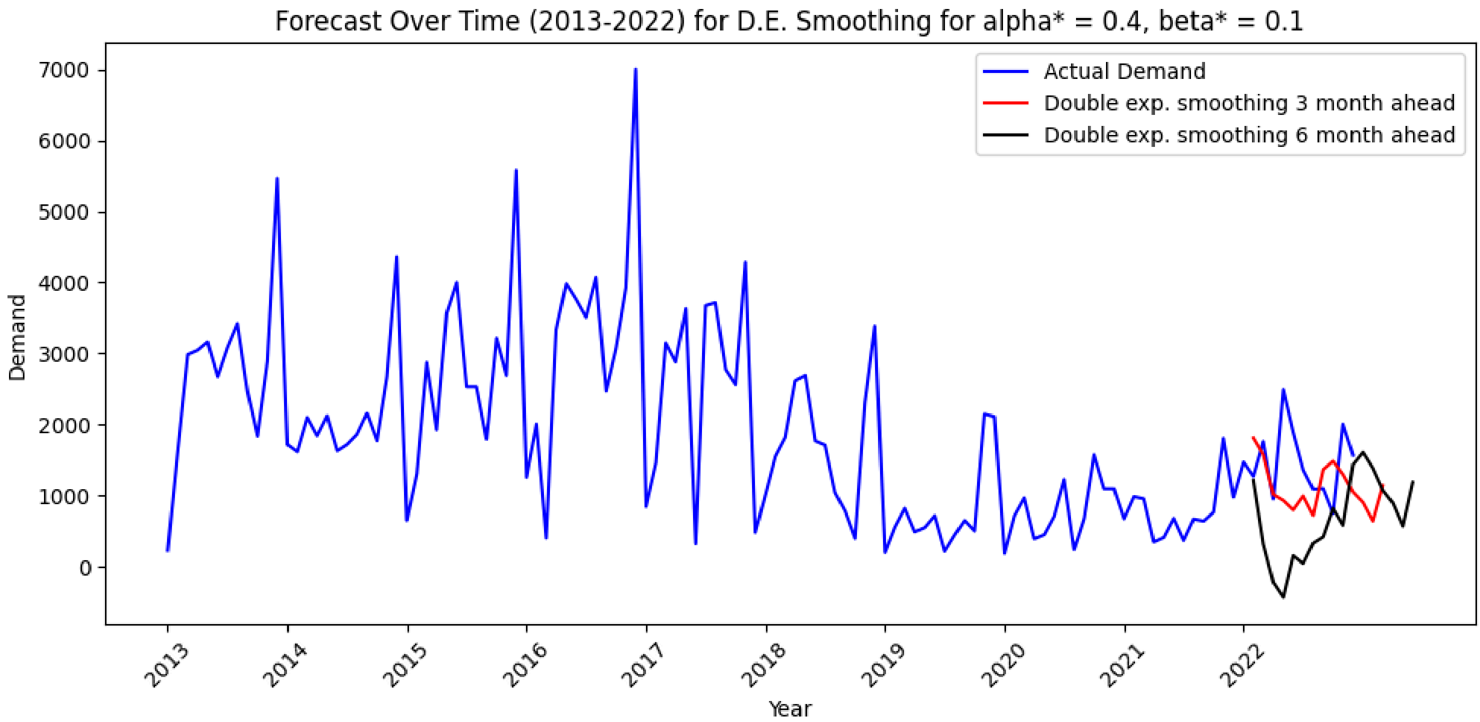


Figure 17: 3-Month and 6-Month Ahead DES Forecasts

Here are the error metrics for the 3-month and 6-month ahead forecasts:

```
For 3 lookahead
MAPE: 78.63348054642816
MAE: 614.6487217994933
RMSE: 827.7481341972621
For 6 lookahead
MAPE: 93.56796998976698
MAE: 710.7496706175447
RMSE: 909.5499629820614
```

Figure 18: Error Metrics for 3-Month and 6-Month ahead Forecasts

The 3-month lookahead performs better than the 6-month lookahead. We can claim that it gets harder to predict the future using DS as we increase our lookahead window.

Part I.

We performed the requested data transformation to take into account the effect of seasonality

$$U_t = D_t - D_{t-12}$$

Here is how the transformed series is visualised:

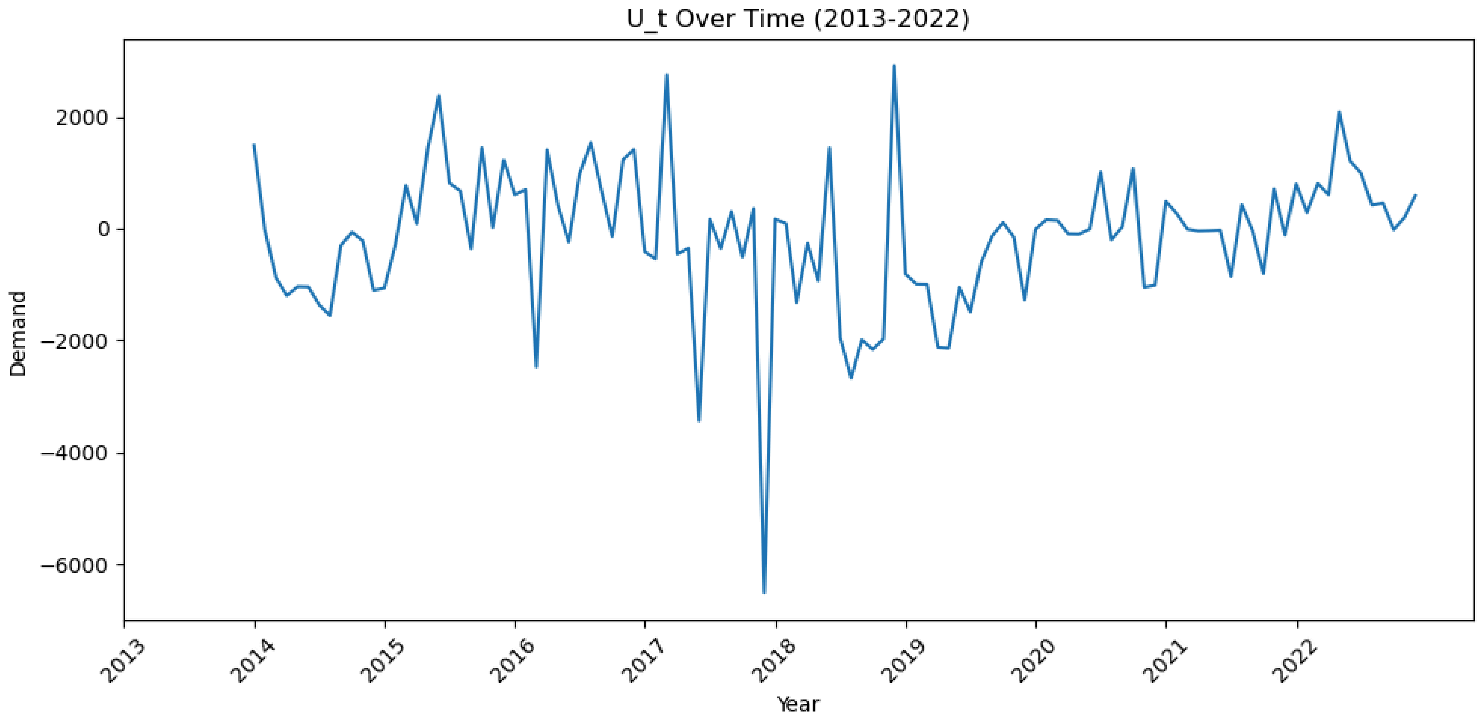


Figure 19: U_t Series

Trend: There seems to be no long-term upward or downward trend in the data after the transformation.

Seasonality: Since the data has been seasonally adjusted, we would expect the seasonality to be removed. However, we can still identify some fluctuations in the data that may indicate residual seasonality. Nevertheless, there does not seem to be a consistent repeating pattern at regular intervals that would indicate strong seasonality.

Outliers: There are several extreme values that deviate significantly from the rest of the data. For example, there are sharp peaks around early 2017, mid-2018, and late 2018. Similarly, there are notable drops in early 2014, early 2016, mid-2017, and late 2017.

The best smoothing constant for the exponential smoothing series that we found was $\alpha = 0.4$. It yielded the smallest MAE, MAPE, and RMSE out of all the candidates {0.1, 0.2, ..., 1}. The exponentially smoothed forecasts of all the candidates are available in the .ipynb file. Here is the graph of G_t for the best smoothing constant of $\alpha = 0.4$:

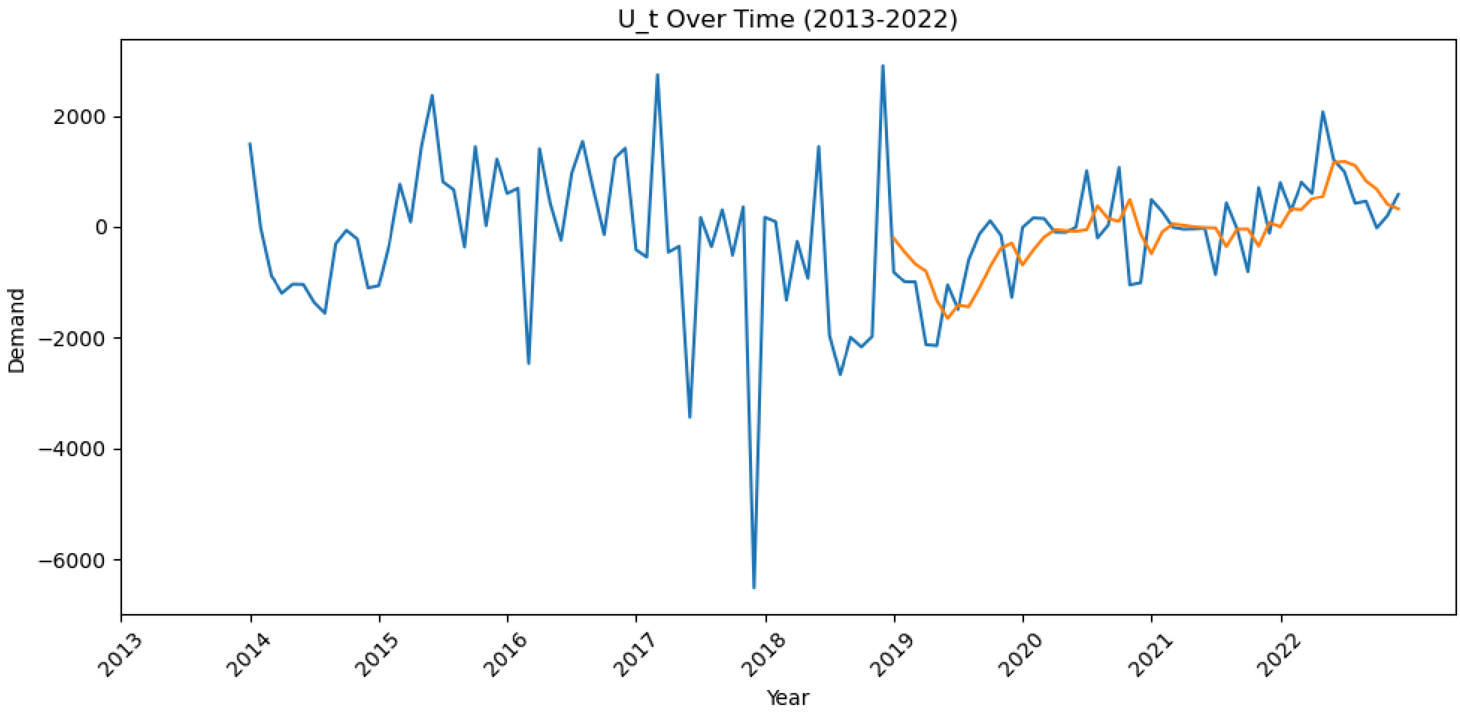


Figure 20: G_t Series over U_t

Here are the MAE, MAPE, and RMSE values for $\alpha = 0.4$:

```
Alpha value corresponding to minimum error metrics:
Minimum RMSE: 678.6200392461592 - Alpha: 0.4
Minimum MAPE: 326.50998844042635 - Alpha: 0.4
Minimum MAE: 534.6402760463778 - Alpha: 0.4
```

Figure 21: Error Metrics of G_t Series

To obtain a forecast F_t for D_t we were given two options:

1. $F_t = G_t + D_{t-12}$
2. $F_t = G_t + \gamma D_{t-12} + (1 - \gamma) F_{t-12}$

We decided to go with the second option since the second forecast also accounts for the first forecast when the smoothing constant is equal to one ($\gamma = 1$). We decided to try different smoothing constant candidates {0.1, 0.2, ..., 1} in order to minimize error metrics which would indicate that we have found the optimal gamma value.

Here is the output for the result of this attempt:

```
Gamma value corresponding to minimum MAPE: 0.7
Gamma value corresponding to minimum RMSE: 0.5
Gamma value corresponding to minimum MAE: 0.6
```

Figure 22: Gamma Values

Here is how this forecast is visualized over the actual demand:

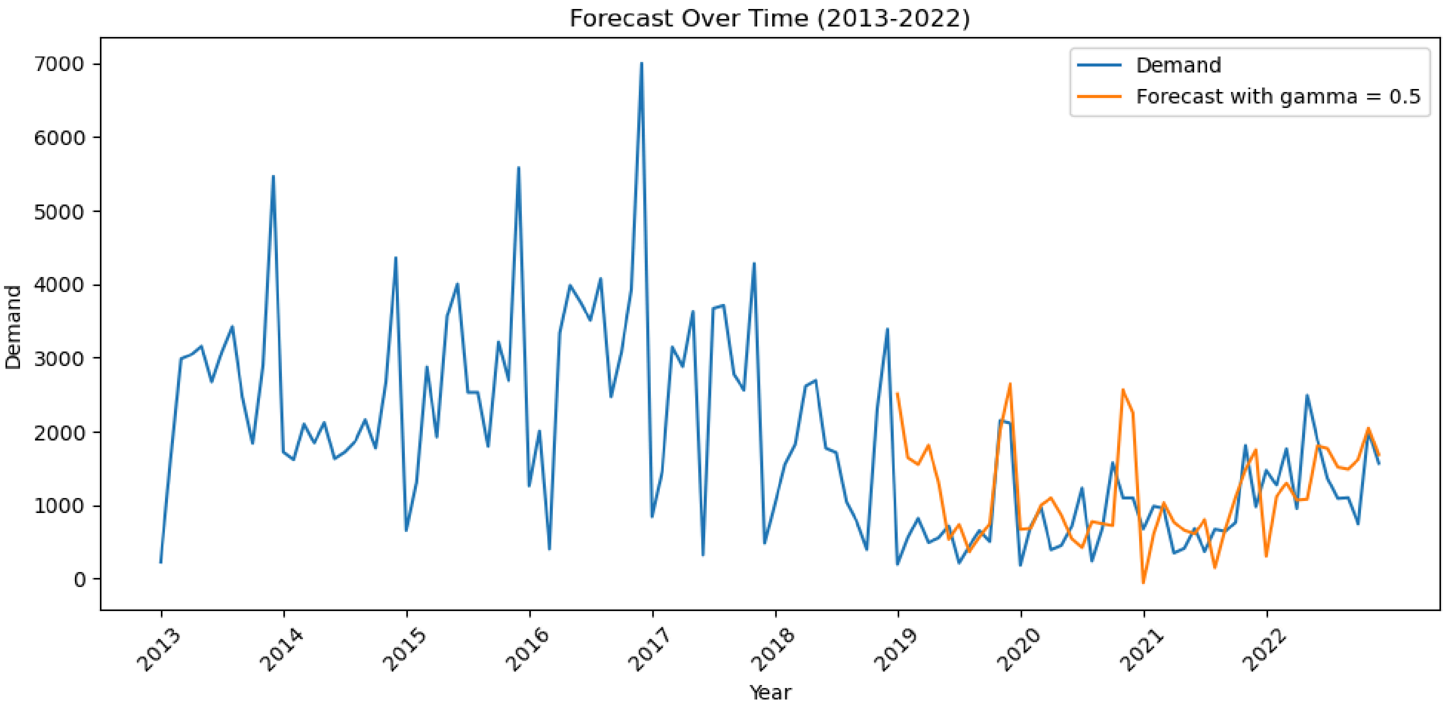


Figure 23: Forecast F_t $(\gamma = 0.5)$

Here are the error metrics for the optimal gamma value:

```
Error Metrics for Gamma = 0.5
MAPE:  93.85294299106762
RMSE:  696.423053182303
MAE:  515.0221715328368
```

Figure 24: Error Metrics of F_t

Part J.

Method	Specification	RMSE	MAPE
Naive Forecast	$\tau = 1$	782.60	109.73%
Naive Forecast	$\tau = 12$	832.71	88.86%
MA-3	-	649.33	82.31%
ES	$\alpha^* = 0.5$	649.08	87.63%
DES	$\alpha^* = 0.5, \beta^* = 0.1$	725.30	95.17%
Seasonal	$\alpha^* = 0.4, \gamma^* = 0.5$	696.42	93.85%

Table 1: Forecast Comparison

We can observe by looking at Table 1 that the best method for forecasting the monthly sales of Renault vehicles is 3-Month Moving Averages. It has the lowest MAPE and second-lowest RMSE metrics. The lowest MAPE we could come up with using these methods is 82.31%, so even the MA(3) model shows pretty high error metrics. The reason behind this may be the irregularity of the patterns in demand, the lack of a dominant trend and also little signs of seasonality resulting in forecasts with high error metrics. The worst performing forecast, as we were expecting, was the Naive Forecast that was looking at one month before.

Question 2

Part A.

Here is the data plotted:

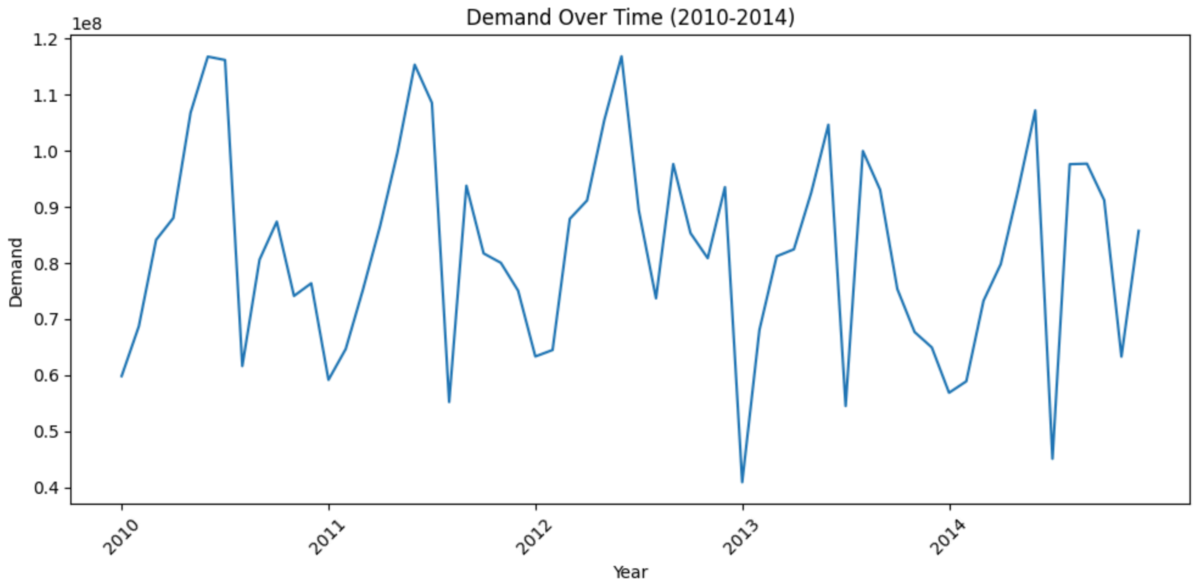


Figure 25: Demand of Domestic Beer

The seasonality is clear. The seasonality seems to be yearly, so the patterns repeat every 12 months. There is no significant trend to be observed.

Part B.

Here is the naive forecast that looks at the demand right before it to forecast the next period:

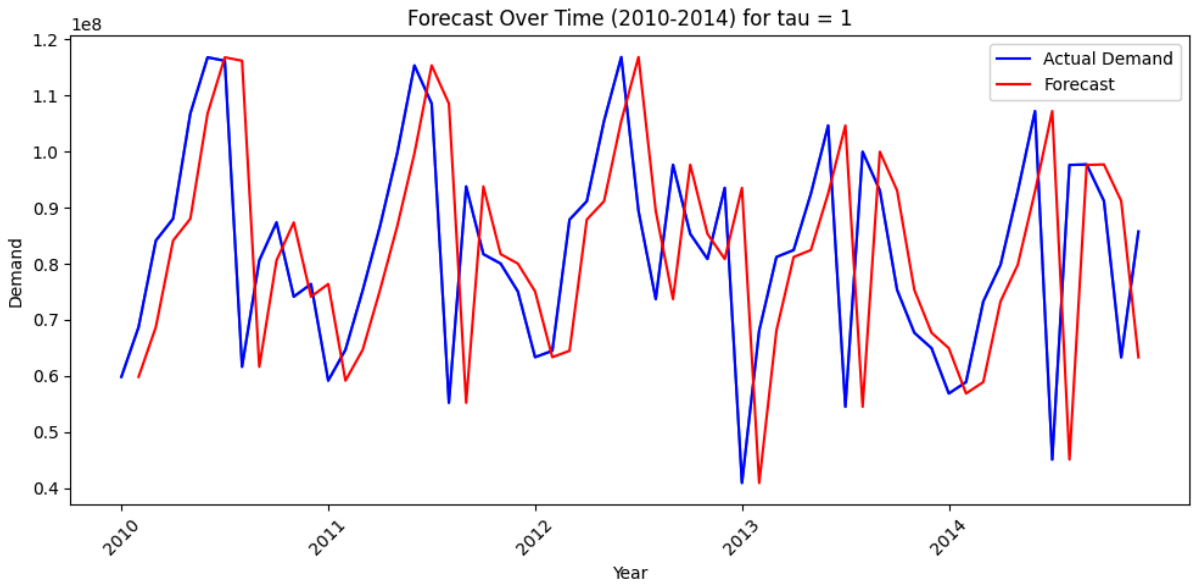


Figure 26: Naive Forecast (Tau = 1)

Here is the naive forecast that looks at the demand 12 months before it to forecast the next period:

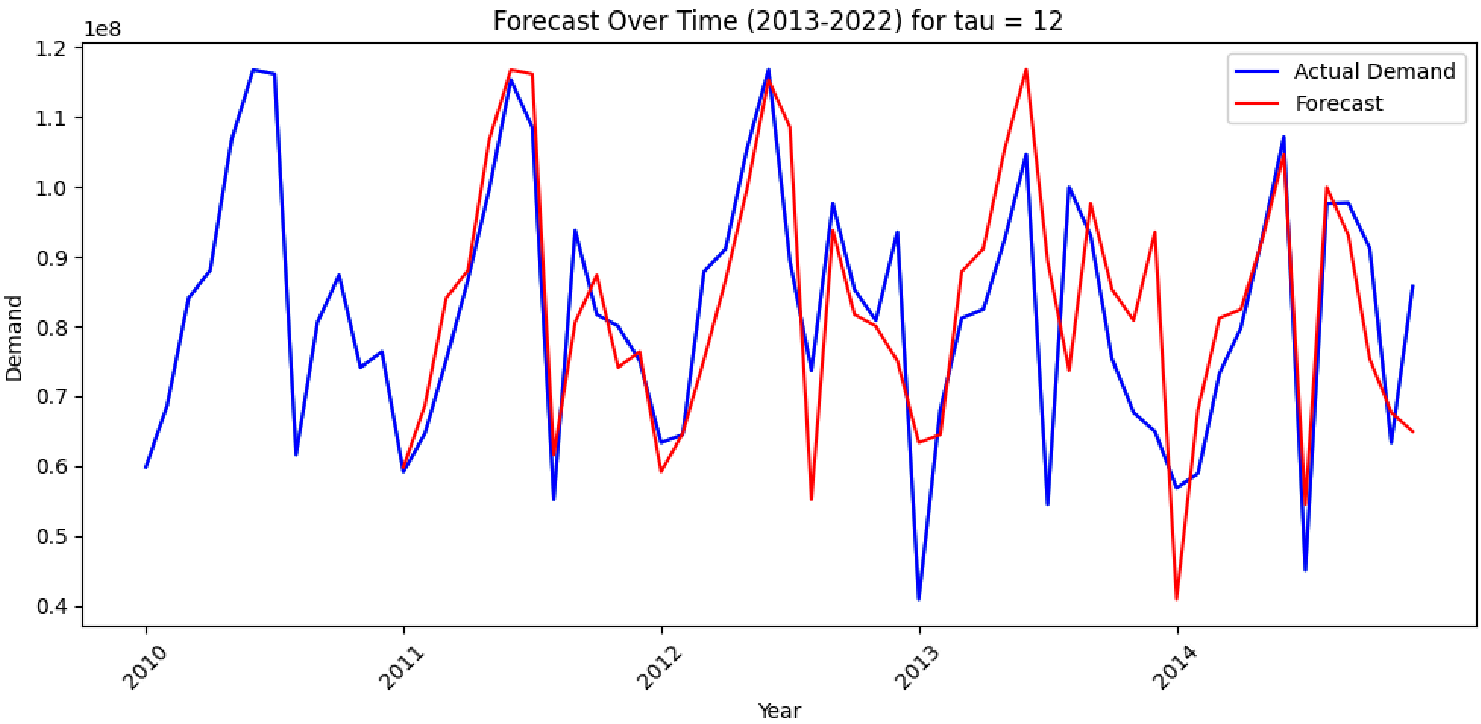


Figure 27: Naive Forecast (Tau = 12)

Here are the error metrics for both of the forecasts:

```
For tau = 1:
Mean Absolute Error (MAE): 17286475.833333332
Mean Absolute Percentage Error (MAPE): 24.533132381898707
Root Mean Squared Error (RMSE): 23299270.417937733
For tau = 12:
Mean Absolute Error (MAE): 9104424.75
Mean Absolute Percentage Error (MAPE): 12.501022374181906
Root Mean Squared Error (RMSE): 12085064.922952536
```

Figure 28: Error Metrics (for tau = 1 and tau = 12)

Part C.

Here is the residual analysis of naive forecast for tau = 1:

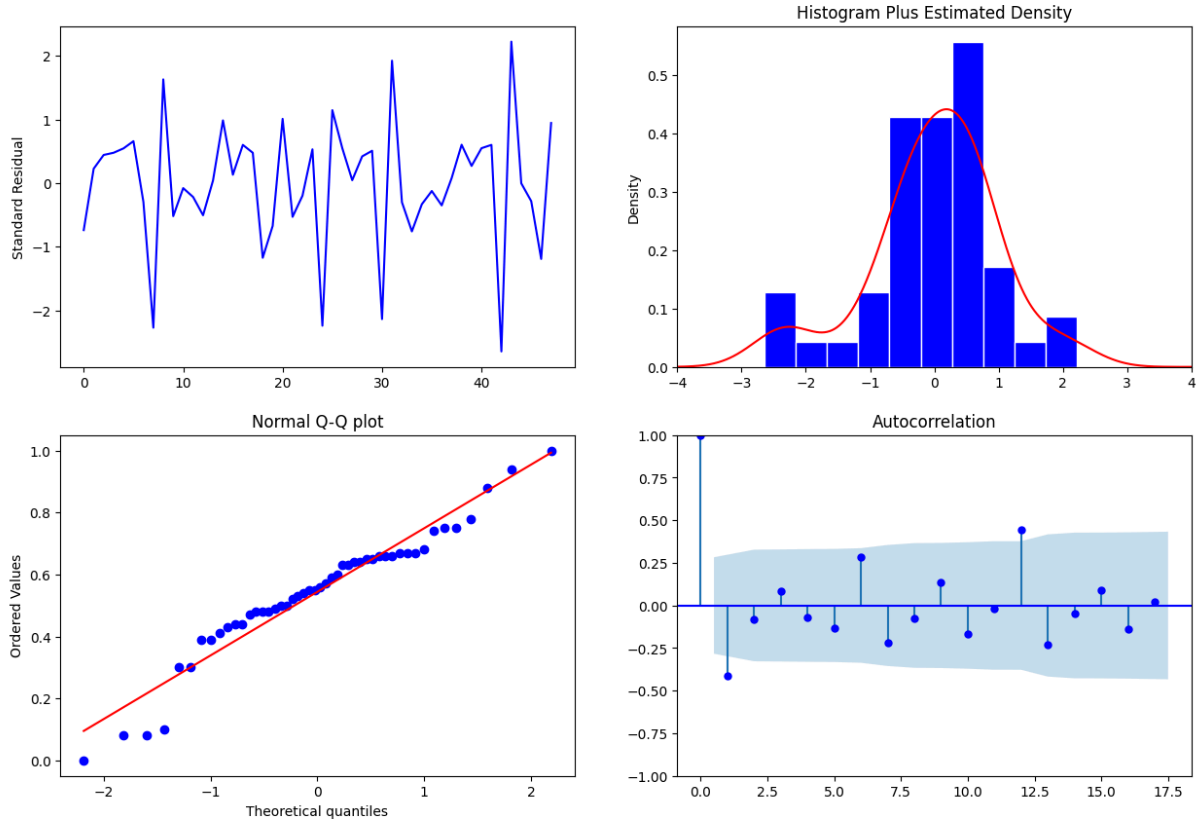


Figure 29: Residuals for Naive Forecast with Tau = 1

We cannot reject the null hypothesis that the mean of residuals is zero at significance level of 0.05.

The normal Q-Q plot shows that the residuals do not fit the normal distribution nicely, so the residuals likely does not come from a normal distribution.

The autocorrelation plot shows us that at lag-1 and lag-12, there is a statistically significant correlation between the residuals. This is expected, especially at lag-12, because there is a clear 12 month seasonality, so any residual observed when predicting with tau = 1 naive forecast is also reflected in the residual that was observed 12 months ago. This is a clear drawback of this naive forecast.

Here is the residual analysis of naive forecast for tau = 12:

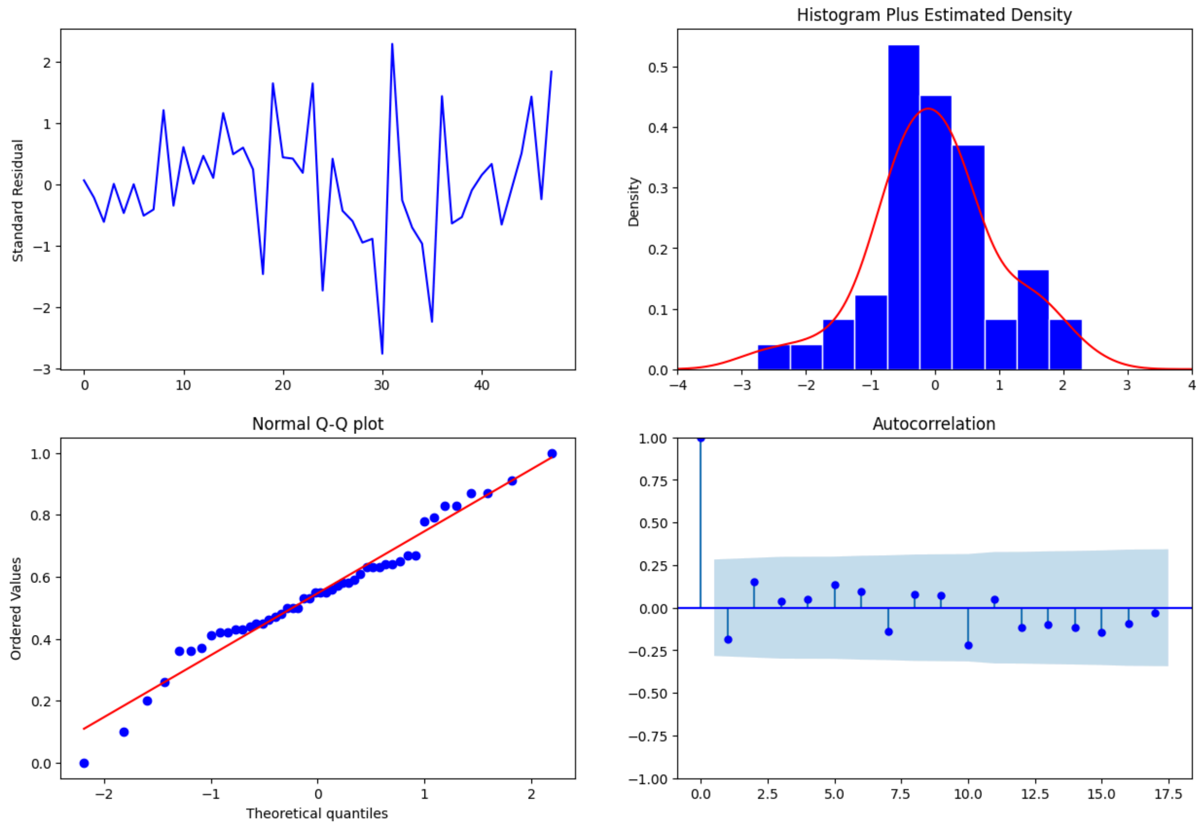


Figure 30: Residuals for Naive Forecast (tau = 12)

We cannot reject the null hypothesis that the mean of residuals is zero at significance level of 0.05.

The normal Q-Q plot shows that the residuals do not fit the normal distribution nicely, so the residuals likely do not come from a normal distribution. Although it can be noted that the fit is better than the tau = 1 case.

The autocorrelation plot shows us that no statistically significant correlation exists at any of the lags, so this time we can claim that the residuals are independent.

There is no clear drawback of this forecast other than the outliers in the residuals, and the non-normality of them.

Part D.

Here are the best α, β and γ values for each of the error metrics:

```
Best Smoothing Constants:

Best MAPE: 13.439903421307337
Corresponding Alpha (MAPE): 0.1
Corresponding Beta (MAPE): 0.4
Corresponding Gamma (MAPE): 0.9

Best MAE: 9619281.001392981
Corresponding Alpha (MAE): 0.1
Corresponding Beta (MAE): 0.4
Corresponding Gamma (MAE): 0.9

Best RMSE: 13397575.39660953
Corresponding Alpha (RMSE): 0.1
Corresponding Beta (RMSE): 0.1
Corresponding Gamma (RMSE): 0.9
```

Figure 31: Error Metrics for Triple Exponential Smoothing

Here are is the visualization of the forecasts on alongside the demand:

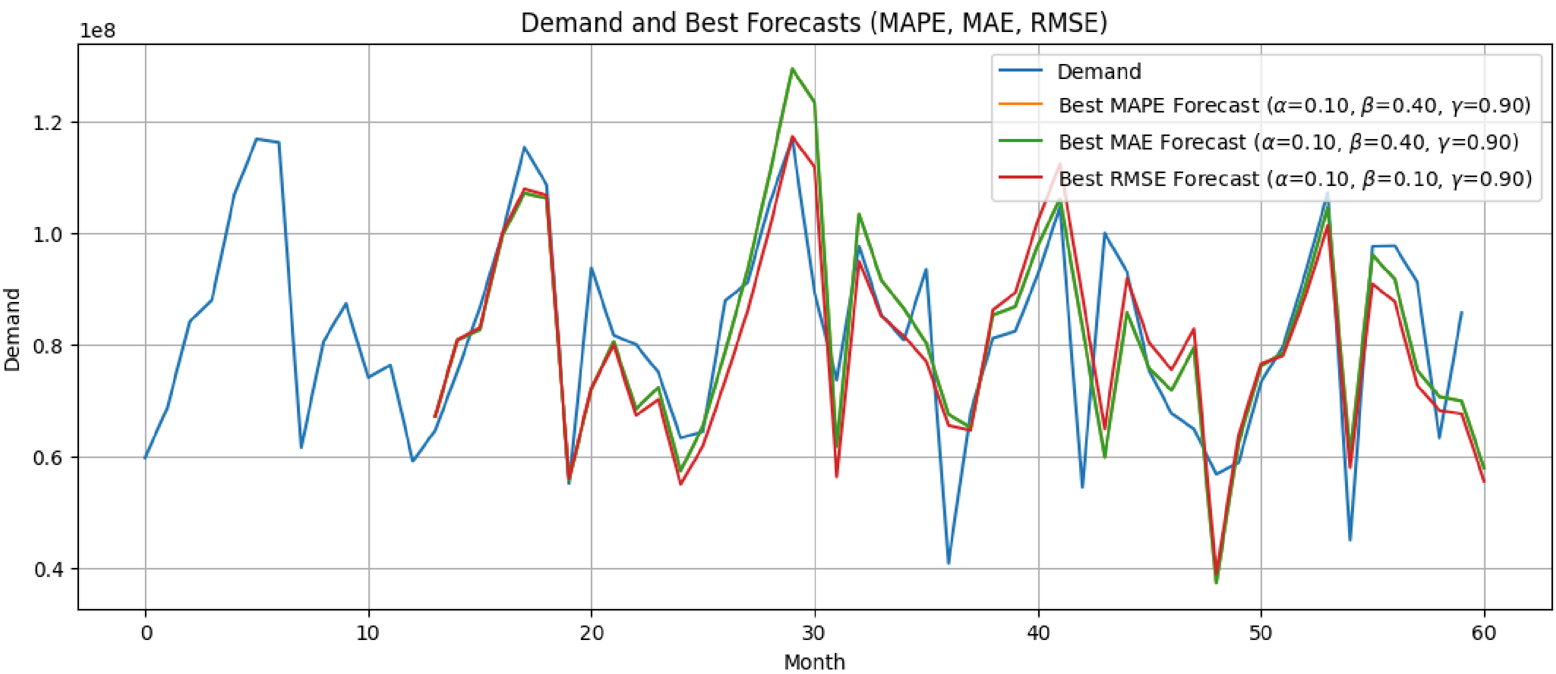


Figure 32: Graph of Triple Exponential Smooting Forecast

We decided to use the α, β and γ values that correspond to the best RMSE metric to get the prediction intervals with the smallest interval. Here are the prediction intervals for $\alpha = 0.1$, $\beta = 0.1$ and $\gamma = 0.9$:

```
90 percent prediction intervals for the one-month ahead
forecasts using triple exponential smoothing (alpha = 0.1,
beta = 0.1, gamma = 0.9):
(17913532.42868094, 60307417.56238669)
(42665451.67488194, 85059336.8085877)
(55441064.9439921, 97834950.07769784)
(56870659.73097909, 99264544.86468485)
(67515919.91953641, 109909805.05324215)
(80103845.09170252, 122497730.22540826)
(36832381.385113776, 79226266.51881953)
(69691250.69648412, 112085135.83018985)
(66507308.265390776, 108901193.39909652)
(51595428.10890535, 93989313.24261111)
(47078805.50377046, 89472690.6374762)
(46455954.95928306, 88849840.09298882)
```

Here is the visualization of the prediction intervals:

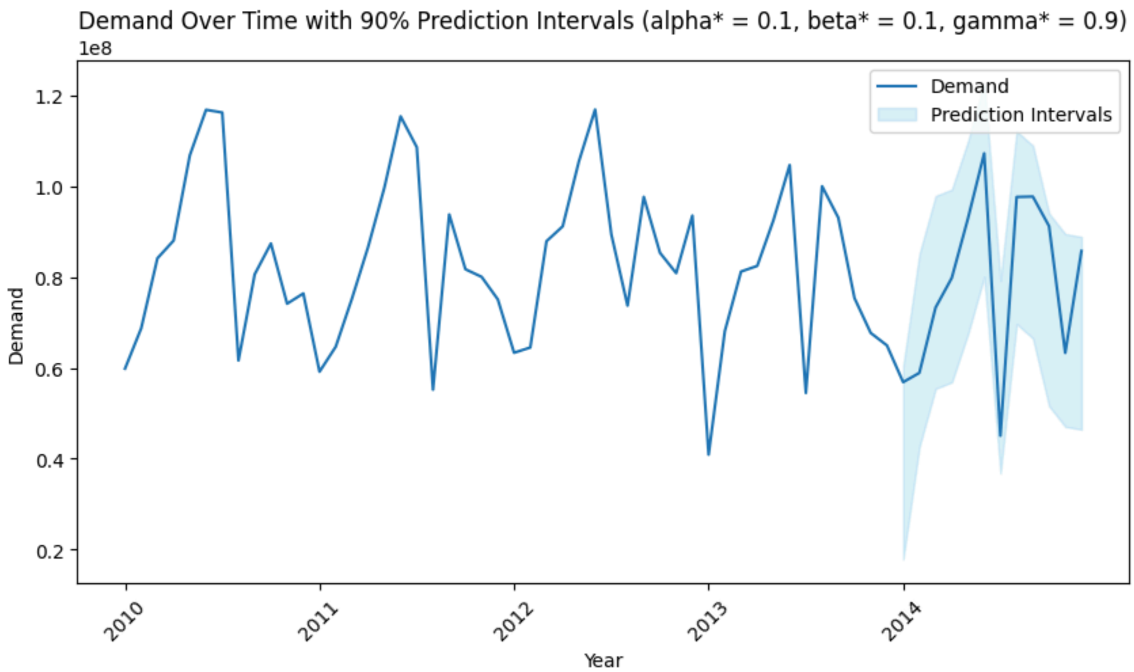


Figure 33: %90 Prediction Intervals for TES

The prediction intervals seem to have captured all of the observed demand within 2024 in its intervals.

Part E.

For the 3-month and 6-month ahead forecast with TES, we decided to use the α, β and γ values that correspond to the best MAE and MAPE metrics since these smoothing constants accord with two out of three of our error metrics.

Here is the visualization of 3-month ahead Triple Exponential Smoothing Forecast for α = 0.1, β = 0.4 and γ = 0.9:

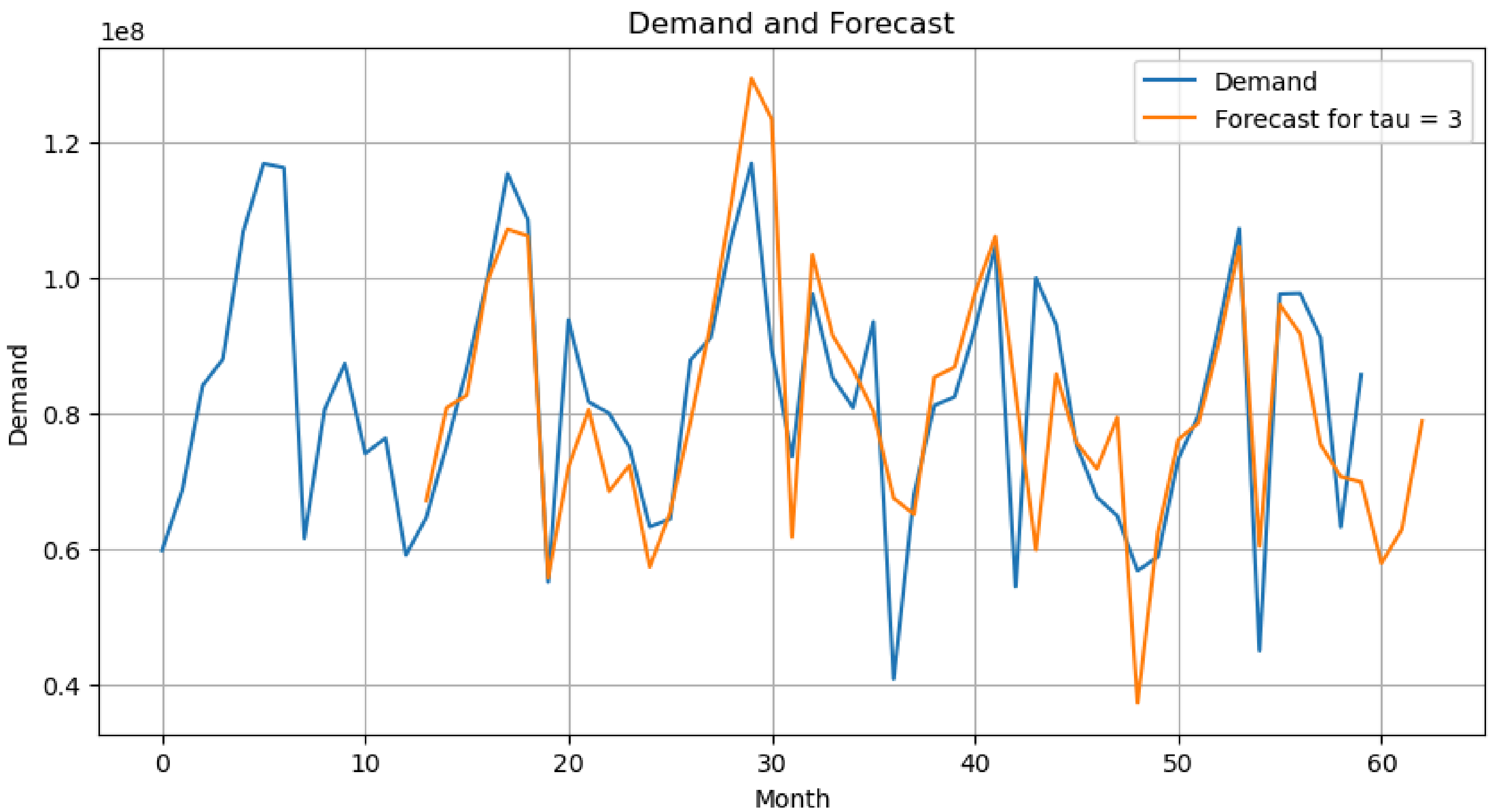


Figure 34: 3-Month Ahead TES Forecast

Here is the visualization of 6-month ahead Triple Exponential Smoothing Forecast for α = 0.1, β = 0.4 and γ = 0.9:

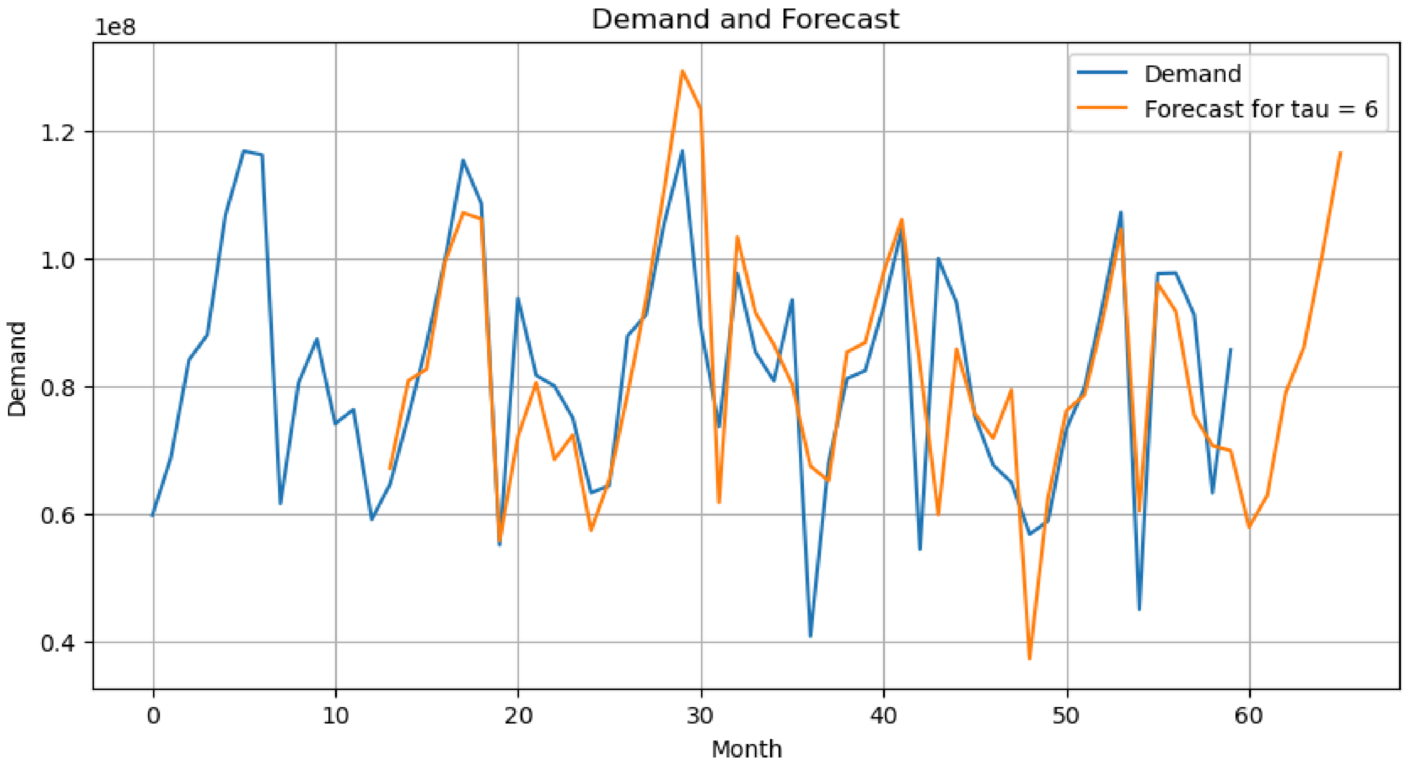


Figure 35: 6-Month Ahead TES Forecast

It can be seen that the last two graphs show the same Forecasts until the beginning of the 63rd month. The forecasts for before this month are identical. This is expected, because triple exponential smoothing uses the seasonality factors that are exactly 12 periods before to predict the next period. So, predicting at any lookahead time that is less than 12 periods should be expected to turn out the same.

Here are the error metrics for 3-Month and 6-Month ahead forecasts:

```
Error Metrics for 3-Month Ahead Forecast
MAPE:  13.439903421307337
RMSE:  13547209.416337851
MAE:  9619281.001392981
```

Figure 36: Error Metrics of 3-Month Ahead Forecast

```
Error Metrics for 6-Month Ahead Forecast
MAPE:  13.439903421307337
RMSE:  13547209.416337851
MAE:  9619281.001392981
```

Figure 37: Error Metrics of 6-Month Ahead Forecast

The error metrics are identical, because the forecasts for the observed periods are also identical.

Part F.

Method	Specification	RMSE	MAPE
Naive Forecast 1	tau = 1	23299270.42	24.53%
Naive Forecast 2	tau = 12	12085064.92	12.50%
TES	$\alpha = 0.1, \beta = 0.4, \gamma = 0.9$	13397575.39	13.44%

Table 2: Forecast Comparison

We see that the best forecast was the lag-12 naive forecast. It is surprising to see that it has outperformed the triple exponential model. We can attribute this to the fact that there was no strong trend to be seen, even when considering the seasonality. So, TES trying to account for the seasonality might have been a detriment to the forecasting model. The naive forecast for lag-1 was the worst performing, which is obvious to see why, as it did not consider the seasonality whatsoever.