Dynamics Package Mathematical Nomenclature

David Hammen

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Contents

Introduction

This note describes the preferred nomenclature for the dynamics package documentation and describes a set of LaTeX macros that implement the nomenclature.

1 Nomenclature

This section describes the preferred nomenclature for the dynamics package documentation. The preferred nomenclature

- Uses International System (SI) units with standard SI abbreviations.
- Represents scalars in plain math font, vectors and matrices in bold math font, and quaternions in caligraphy font.
- Represents vectors, matrices, and quaternions in a standard, intuitive style with an obvious translation to variable names. For example, $\boldsymbol{x}_{A:a \to b}$ is the vector from point a to point b as expressed in reference frame A. This is denoted as the "arrow-separated" format. Alternative respresentations such as $\boldsymbol{x}_{a,b}$ ("comma-separated") and $\boldsymbol{x}_b^a$ ("stacked") were discussed but discarded for various reasons.
- Quaternions that represent a transformation from one frame to another are represented as unit left transformation quaternions, consistent with the quaternion functions defined in the Trick core.

The following tables depict the preferred nomenclature. In cases where alternative representation schemes exist, the preferred form is listed first followed by alternatives

1.1 Basics

Display style for vectors, matrices, and quaternions

Description	Remark	Example
Scalar	in plain math font	s
Vector	in bold math font	$\boldsymbol{x}$
Matrix	in bold math font	$\boldsymbol{T}$
Quaternion	in caligraphy font, uppercase	$\mathcal{Q}$

Scalar Symbols

Symbol	Represents	Units (MKS)
α	Angular acceleration	r/s^2
$\alpha, \beta, \theta, \phi, \psi$	Angle	r
ω	Angular rate	r/s
d, l, s	Distance or length	m
m	Mass	kg
v, s	Speed	m/s

Vector Symbols

Symbol	Represents	Units (MKS)
$\boldsymbol{\alpha}$	Angular acceleration	r/s^2
$\boldsymbol{\tau}$	Torque	Nm
$\boldsymbol{\omega}$	Angular velocity	r/s
$\boldsymbol{a}$	Acceleration	m/s^2
$\boldsymbol{L}$	Angular momentum	$Nms\ (kg\,m^2/s)$
$\boldsymbol{v}$	Velocity	m/s
$\boldsymbol{x}$	Position	m

Matrix Symbols

Symbol	Represents	Units (MKS)
$\boldsymbol{I}$	Inertia tensor	$kg\,m^2$
$\boldsymbol{T}$	Transformation matrix	$--$

Quaternion Symbols

Symbol	Represents	Units (MKS)
$\mathcal{Q}$	Left transformation quaternion	$--$

1.2 Adornments

Vectors

Description	**Remark**	**Example**
Vector from origin to b	Subscripted b	$\boldsymbol{x}_b$
Vector from a to b	Arrow, comma, stacked formats	$\boldsymbol{x}_{a\to b}$ $\quad \boldsymbol{x}_{a,b}$ $\quad \boldsymbol{x}_b^a$
Vector expressed in frame A		$\boldsymbol{x}_A$ $\quad \boldsymbol{x}_A$ $\quad {}_A\boldsymbol{x}$
Vector from a to b expressed in frame A		$\boldsymbol{x}_{A:a\to b}$ $\quad \boldsymbol{x}_{A:a,b}$ $\quad {}_A\boldsymbol{x}_b^a$
Time derivative of above, observer in frame A		$\dot{\boldsymbol{x}}_{A:a\to b}$ $\quad \dot{\boldsymbol{x}}_{A:a,b}$ $\quad {}_A\dot{\boldsymbol{x}}_b^a$
Vector time derivative, observer in frame A	Dot format	$\overset{\cdot A}{\boldsymbol{x}}$
	or d/dt format	$\frac{d}{dt_A}\boldsymbol{x}$

Matrices

Description	**Remark**	**Example**
Transformation from A to B	Arrow, comma, stacked formats	$\boldsymbol{T}_{A\to B}$ $\quad \boldsymbol{T}_{A,B}$ $\quad {}^A_B\boldsymbol{T}$
Matrix product	No operator	$\boldsymbol{T}_{B\to C}\,\boldsymbol{T}_{A\to B}$
Single matrix transpose	Superscript $\top$	$\boldsymbol{T}^\top$
Matrix product transpose	Superscript $\top$	$(\boldsymbol{T}_{B\to C}\,\boldsymbol{T}_{A\to B})^\top$

Quaternions

Description	**Remark**	**Example**
Quaternion from A to B	Arrow, comma, stacked formats	$\mathcal{Q}_{A\to B}$ $\quad \mathcal{Q}_{A,B}$ $\quad {}^A_B\mathcal{Q}$
Quaternion product	No operator	$\mathcal{Q}_{B\to C}\,\mathcal{Q}_{A\to B}$
Single quaternion conjugate	Superscript $\star$	$\mathcal{Q}^\star$
Quaternion product conjugate	Superscript $\star$	$(\mathcal{Q}_{B\to C}\,\mathcal{Q}_{A\to B})^\star$
Quaternion components	Scalar + vector *or* four-vector	$\begin{bmatrix} q_s \\ \boldsymbol{q_v} \end{bmatrix}$ *or* $\begin{bmatrix} q_s \\ q_x \\ q_y \\ q_z \end{bmatrix}$

2 `dynmath` Macros

This section provides a brief description of the macros defined in `dynmath.sty` in the form of tables that describe the macros, their arguments, sample usage of the macros, and the displayed math that results.

Note: All of the macros defined in `dynmath.sty` assume LaTeX is in math mode.

2.1 Vector Macros

Command	Purpose	Arguments	Example	Display
`\vect`	Display a symbol that represents a vector (typically a lowercase letter)	Symbol	`\vect{x}`	$\boldsymbol{x}$
`\vhat`	Display a symbol that represents a unit vector	Symbol	`\vhat{x}`	$\hat{\boldsymbol{x}}$
`\vdot`	Time derivative of a vector	Symbol	`\vdot{x}`	$\dot{\boldsymbol{x}}$
`\framevect`	Vector represented in a specific frame	1. Frame 2. Vector	`\framevect B x`	$\boldsymbol{x}_B$
`\framevdot`	Time derivative of a vector represented in a specific frame	1. Frame 2. Vector	`\framevdot B x`	$\dot{\boldsymbol{x}}_B$
`\relvect`	Vector between two items	1. Vector 2. Source 3. Destination	`\relvect x a b`	$\boldsymbol{x}_{a \to b}$
`\relvdot`	Time derivative of a vector between two items	1. Vector 2. Source 3. Destination	`\relvdot x a b`	$\dot{\boldsymbol{x}}_{a \to b}$
`\framerelvect`	Vector between two items	1. Frame 2. Vector 3. Source 4. Destination	`\framerelvect B x a b`	$\boldsymbol{x}_{B:a \to b}$
`\framerelvdot`	Time derivative of a vector between two items	1. Frame 2. Vector 3. Source 4. Destination	`\framerelvdot B x a b`	$\dot{\boldsymbol{x}}_{B:a \to b}$
`\vectxyz`	Construct a vector from its components	Three components	`\vectxyz x y z`	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

2.2 Quaternion Macros

Command	Purpose	Arguments	Example	Display
`\quat`	Display a symbol that represents a quaternion (typically Q)	Symbol	`\quat{Q}`	$\mathcal{Q}$
`\qdot`	Time derivative of a quaternion	Symbol	`\qdot{Q}`	$\dot{\mathcal{Q}}$
`\quatconj`	Conjugate of a quaternion	Symbol	`\quatconj{Q}`	$\mathcal{Q}^\star$
`\quatconjdot`	Time derivative of the conjugate of a quaternion	Symbol	`\quatconjdot{Q}`	$\dot{\mathcal{Q}}^\star$

`\quatsv`	Construct a quaternion from a scalar and 3-vector	1. Scalar
		2. Vector
			`\quatsv`
			`{q_s}`
			`{\vect{q_v}}`
				$\begin{bmatrix} q_s \\ \boldsymbol{q_v} \end{bmatrix}$

`\quattrot`	Construct a transformation quaternion from an angle and unit vector	1. Angle
		2. Unit vector
			`\quattrot`
			`\theta`
			`{\vhat u}`
				$\begin{bmatrix} \cos\frac{\theta}{2} \\ -\sin\frac{\theta}{2}\hat{\boldsymbol{u}} \end{bmatrix}$

`\conjop`	Conjugate operator	None	`\conjop \quat Q`	$\operatorname{conj}\mathcal{Q}$
`\scalarpart`	Scalar part operator	None	`\scalarpart \quat Q`	$\operatorname{scalar}\mathcal{Q}$
`\vectorpart`	Vector part operator	None	`\vectorpart \quat Q`	$\operatorname{vect}\mathcal{Q}$
`\QBI`	Inertial-to-body quaternion	None	`\QBI`	$\mathcal{Q}_{I\to B}$

2.3 Matrix Macros

Command	**Purpose**	**Arguments**	**Example**	**Display**
`\mat`	Display a symbol that represents a matrix (typically an uppercase letter)	Symbol	`\mat{T}`	$\boldsymbol{T}$
`\framemat`	Matrix represented in some reference frame	1. Frame 2. Matrix	`\framemat B \inertia`	$\boldsymbol{I}_B$
`\diagmatrix`	3×3 diagonal matrix	Three diagonal elements	`\diagmatrix 1 2 3`	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
`\identmatrix`	3×3 identity matrix	None	`\identmatrix`	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
`\inertia`	Inertia matrix	None	`\inertia`	$\boldsymbol{I}$

2.4 Multiplication Macros

The preferred nomenclature for depicting the product of two (or more) composite objects is "implicit multiplication": The operands are written with no intervening operator. However, a small amount of white space between the operands helps to distinguish the operands. The multiplication macros provide a default amount of white space between operands.

Command	**Purpose**	**Arguments**	**Example**	**Display**

`\MxM`	Product of two matrices	Matrix 1
		Matrix 2
			`\MxM`
			`{\tmat B C}`
			`{\tmat A B}`
				$\boldsymbol{T}_{B\to C}\,\boldsymbol{T}_{A\to B}$

`\QxVxQ`	Product of a quaternion, a vector, and a quaternion	Quaternion 1
		Vector
		Quaternion 2
			`\QxVxQ`
			`{\quat Q}`
			`{\vect x}`
			`{\quatconj Q}`
				$\mathcal{Q}\begin{bmatrix}0\\ \boldsymbol{x}\end{bmatrix}\mathcal{Q}^{\star}$

Similar macros are defined for

- the product of three matrices (`MxMxM`)
- the product of a matrix and vector (`MxV`)
- the product of two or three quaternions (`QxQ`, `QxQxQ`)
- the product of a quaternion and a vector (`QxV`, `VxQ`)

The multiplication macros take an optional argument via which an explicit multiplication operator can be specified. For example,

Command	**Purpose**	**Option**	**Example**	**Display**

`\QxQ`	Implicit product of two quaternions	None	`\QxQ`
			`{\quat{Q}_1}`
			`{\quat{Q}_2}`
				$\mathcal{Q}_1\,\mathcal{Q}_2$

`\QxQ`	Explicit product of two quaternions	`\circ`	`\QxQ[\circ]`
			`{{\quat Q}_1}`
			`{{\quat Q}_2}`
				$\mathcal{Q}_1{\circ}\mathcal{Q}_2$

2.5 Miscellaneous Macros

Command	Purpose	Arguments	Example	Display
`\abs`	Absolute value	Scalar expression	`\abs{x}`	$\lvert x\rvert$
`\norm`	Euclidian norm	Vector or quaternion expression	`\norm{\vect x}`	$\lVert \boldsymbol{x} \rVert$
`\framedot`	Frame-dependent time derivative	1. Frame 2. Expression	`\framedot B {\vect x}`	$\overset{\cdot B}{\boldsymbol{x}}$