OPPENHEIM

DEVANANTH V - EP20BTECH11004

Contents

***Abstract*—This is the solution of the question 3.21 (b) and 2.26 (b) of Oppenheim**

1 Assignment-1 (3.21 (b))

1) Consider a linear time-invarient system with impulse response

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

and input

$$x[n] = \begin{cases} 1, & 0 \leq n \leq (N-1) \\ 0 & \text{otherwise} \end{cases} \tag{1.2}$$

(b) Determine the output y[n] by computing the inverse z-transform of the product of z-transforms of x[n] and h[n].
Solution: :

$$H(z) = \sum_{n=-\infty}^{\infty} a^n z^{-n} = \frac{1}{1-az^{-1}} \quad |z| > |a| \tag{1.3}$$

$$X(z) = \sum_{n=-\infty}^{N-1} z^{-n} = \frac{1-z^{-n}}{1-z^{-1}} \quad |x| > 0 \tag{1.4}$$

Therefore,

$$Y(z) = \frac{1-z^{-N}}{\left(1-az^{-1}\right)\left(1-z^{-1}\right)} = \frac{1}{\left(1-az^{-1}\right)\left(1-z^{-1}\right)} - \frac{z^{-N}}{\left(1-az^{-1}\right)\left(1-z^{-1}\right)} \quad |z| > |a| \tag{1.5}$$

Now,

$$\frac{1}{\left(1-az^{-1}\right)\left(1-z^{-1}\right)} = \frac{\frac{1}{1-a^{-1}}}{1-az^{-1}} + \frac{\frac{1}{1-a}}{1-z^{-1}} = \left(\frac{1}{1-a}\right)\left(\frac{1}{1-z^{-1}} - \frac{a}{1-az^{-1}}\right) \tag{1.6}$$

So,

$$y[n] = \left(\frac{1}{1-a}\right)[u[n] - a^{n+1}u[n] - \tag{1.7}$$

$$u[n-N] - a^{n-N+1}u[n-N]] \tag{1.8}$$

$$= \frac{1-a^{n+1}}{1-a}u[n] - \frac{1-a^{n-N+1}}{1-a}u[n-N] \tag{1.9}$$

$$y[n] = \begin{cases} 0 & n < 0 \\ \frac{1-a^{n+1}}{1-a} & 0 \leq n \leq N-1 \\ a^{n+1}\left(\frac{1-a^{-N}}{a-1}\right) & n \geq N \end{cases} \tag{1.10}$$

2 Assignment-2 (2.26(b))

1) Which of the following discrete-time signals could be eigenfunctions of any stable LTI system?

(b) $e^{j2\pi n}$

Solution: :

$$x[n] = e^{j2\pi\omega n} \tag{2.1}$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] e^{j2\omega(n-k)} \tag{2.2}$$

$$= e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k] e^{-j2\omega k} \tag{2.3}$$

$$= e^{j2\omega n}.H\left(e^{j2\omega}\right) \tag{2.4}$$

Thus it is a Eigenfunction