OPPENHEIM

DEVANANTH V - EP20BTECH11004

Abstract—This is the solution of the question 3.21 (b) and 2.26 (b) of Oppenheim

1 ASSIGNMENT-1 (3.21 (B))

1) Consider a linear time-invariant system with impulse response

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (1.1)$$

and input

$$x[n] = \begin{cases} 1, & 0 \leq n \leq (N-1) \\ 0 & \text{otherwise} \end{cases} \quad (1.2)$$

(b) Determine the output y[n] by computing the inverse z-transform of the product of z-transforms of x[n] and h[n].
Solution: :

$$H(z) = \sum_{n=-\infty}^{\infty} a^n z^{-n} = \frac{1}{1 - az^{-1}} \quad |z| > |a| \quad (1.3)$$

$$X(z) = \sum_{n=-\infty}^{N-1} z^{-n} = \frac{1 - z^{-n}}{1 - z^{-1}} \quad |x| > 0 \quad (1.4)$$

Therefore,

$$Y(z) = \frac{1 - z^{-N}}{(1 - az^{-1})(1 - z^{-1})} = \frac{1}{(1 - az^{-1})(1 - z^{-1})} - \frac{z^{-N}}{(1 - az^{-1})(1 - z^{-1})} \quad |z| > |a| \quad (1.5)$$

Now,

$$\frac{1}{(1 - az^{-1})(1 - z^{-1})} = \frac{\frac{1}{1 - a^{-1}}}{1 - az^{-1}} + \frac{\frac{1}{1 - a}}{1 - z^{-1}} = \left(\frac{1}{1 - a}\right)\left(\frac{1}{1 - z^{-1}} - \frac{a}{1 - az^{-1}}\right) \quad (1.6)$$

So,

$$y[n] = \left(\frac{1}{1 - a}\right)[u[n] - a^{n+1}u[n] - \quad (1.7)$$

$$u[n - N] - a^{n-N+1}u[n - N]] \quad (1.8)$$

$$= \frac{1 - a^{n+1}}{1 - a}u[n] - \frac{1 - a^{n-N+1}}{1 - a}u[n - N] \quad (1.9)$$

$$y[n] = \begin{cases} 0 & n < 0 \\ \frac{1 - a^{n+1}}{1 - a} & 0 \leq n \leq N - 1 \\ a^{n+1}\left(\frac{1 - a^{-N}}{a - 1}\right) & n \geq N \end{cases} \quad (1.10)$$

2 ASSIGNMENT-2 (2.26(B))

1) Which of the following discrete-time signals could be eigenfunctions of any stable LTI system?

(b) $e^{j2\pi n}$
Solution: :

$$x[n] = e^{j2\pi\omega n} \quad (2.1)$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]e^{j2\omega(n-k)} \quad (2.2)$$

$$= e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j2\omega k} \quad (2.3)$$

$$= e^{j2\omega n}.H\left(e^{j2\omega}\right) \quad (2.4)$$