Logic, Interaction and Collective Agency

Lecture 5

ESSLLI'10, Copenhagen

Eric Pacuit Olivier Roy

TiLPS, Tilburg University
ai.stanford.edu/~epacuit

University of Groningen
philos.rug.nl/~olivier

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Correlations in UTI

Correlations arise from the Team Authority requirement:

$$\text{If } t_i = k \text{ then } \alpha_i = \alpha_i^k(s)$$

$\lambda_E(I_E)(I_O)$	M_O	H_O
$\bullet, M, MM$		
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$\vdots$		
$\bullet, H, MM$		
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Plan

1. Prior Beliefs and the Common Prior Assumption.
2. Correlations: strategies, beliefs, quantitatively, qualitatively.
3. General Discussion on Commitments and Intentions.
4. Wrap up.

Prior Beliefs and the Common Prior Assumption

Prior and posterior beliefs

- Different stages of information disclosure:

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Prior and posterior beliefs

- Different stages of information disclosure:

  1. *Ex ante*: *A priori*. No information. Possibly background beliefs about who you are playing against (background beliefs about the "context of interaction").
     Prior beliefs.

  2. *Ex interim*: the players receive a piece of (private) information. They know more.
     Posterior beliefs.

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  - In type spaces, in UTIs, in Bayesian games...

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$$\lambda_i(t_i)(s,t) = \frac{\Omega((s,t)\ \&\ t_i)}{\Omega(t_i)}$$

- We encountered priors many time already:
  - In type spaces, in UTIs, in Bayesian games...
  - We mostly used common priors:
    - $\Omega_i = \Omega_j$ for all $i, j \in \mathcal{A}$.
    - $\preceq_i = \preceq_j$ for all $i, j \in \mathcal{A}$ (in epistemic-plausibility models)

Common priors

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Common priors

- $\not\Rightarrow$ same posteriors! For the simple reason that agents can receive different private information.
- Olivier and I play card together. Before the cards are dealt, our common prior belief that the other end up with a Joker is $0.37 = 2/54$.
- We get 5 card each (and don't show them to each other). I end up with the 2 Jokers.
  - My posterior belief that Olivier has a Joker is 0.
  - Olivier's posterior belief that I have a Joker is $0.4 = 2/49$.

Common Prior Assumption

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"Harsanyi doctrine" [Aumann, 1976].

R. Aumann. *Agreeing to Disagree*. *Annals of Statistics*, Vol.4, No.6, 1976.

J.C. Harsanyi. *Games with incomplete informations played by bayesian players*. *Management Science* 14:159182, 320334, 486502, 1967-68.

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"Harsanyi doctrine" [Aumann, 1976].

- CPA not an innocuous assumption!

Explanation

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Why CPA?

- A controversial question!
    - See Morris (1995) for a thorough discussion. One methodological observation:
        - Better to explain differences in posterior on the basis of identifiable differences in information or plausible errors in information processing.
        - Resorting on differences in priors often appears ad hoc (the resulting theory is "too permissive").

S. Morris. *The Common Prior Assumption in Economic Theory*. *Economics and Philosophy*, 11(2): pgs. 227- 253, 1995.

Common Prior and Shared Perspective

- Common prior is already a form of shared background within a group, although a relatively weak one.
    - RCBR characterizes non-dominance; but RCBR + (non-correlated) CP characterizes Nash equilibrium.

Common Prior and Shared Perspective

- Common prior is already a form of shared background within a group, although a relatively weak one.
  - RCBR characterizes non-dominance; but RCBR + (non-correlated) CP characterizes Nash equilibrium.
- But the connection between group members can be strengthened if the priors are correlated.

Correlations

Correlated Strategies

	H	M
H	3, 3	0, 4
M	4, 0	1, 1

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We each can make our choice dependent on a flip of the coin:

Correlated Strategies

	H	M
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We each can make our choice dependent on a flip of the coin:

- A mixed strategy is a random variable assigning actions to the outcome of a coin toss.
- The coins are *independent*: (H, H) is never rational.

Correlated Strategies

	H	M
H	3, 3	0, 4
M	4, 0	1, 1

Suppose we base our choices on the *same coin*:

Correlated Strategies

	H	M
H	3, 3	0, 4
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Suppose we base our choices on the *same coin*:

- $\rho_E = (Head \Rightarrow H, Tail \Rightarrow M)$
- $\rho_O = (Head \Rightarrow H, Tail \Rightarrow M)$

Our choice are correlated, i.e. not independent

(In general, the players need not observe the *same* random event.)

Correlated Strategies

	H	M
H	3, 3	0, 4
M	4, 0	1, 1

	H	M
H	1/2	0
M	0	1/2

Convenient representation of correlated strategies in terms of the distribution of the external signal.

Correlated Equilibrium

Let $\mathbb{G}$ be a game in strategic form and W a (measurable) set of outside signals and $\Omega(S)$ a probability distribution over S.

- A correlated strategy for player i is a function $\rho_i : W \longrightarrow S_i$ assigning to each signal an action in S_i.
- The expected value for player i of the profile ρ:

$$EV_i(\rho) = \sum_{w \in W} \Omega(w) v_i(\rho(w))$$

- A correlated equilibrium is a profile ρ_i of correlated strategies such that, for all i:

$$\rho_i \in argmax_{\rho_i'}(\rho_i', \rho_{-i})$$

R.J. Aumann. *Correlated equilibrium as an expression of bayesian rationality*. *Econometrica*, 55(1-18), 1987.

Nash equilibrium $\subsetneq$ Correlated Equilibrium

	H	M
H	3, 3	0, 0
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Nash equilibrium $\subsetneq$ Correlated Equilibrium

	H	M
H	6, 6	2, 7
M	7, 2	0, 0

Nash equilibrium $\subsetneq$ Correlated Equilibrium

	H	M
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- Three Nash equilibria: (M, H), (H, M) and ((2/3 M, 1/3 H), (2/3 M, 1/3 H)).

Nash equilibrium $\subsetneq$ Correlated Equilibrium

-

	H	M
H	6, 6	2, 7
M	7, 2	0, 0

	H	M
H	Blue Card	Red card
M	Green Card	

- Now suppose we make our choice dependent on drawing one of three cards.

Nash equilibrium $\subsetneq$ Correlated Equilibrium

	H	M
H	6, 6	2, 7
M	7, 2	0, 0

	H	M
H	1/3	1/3
M	1/3	

- Now suppose we make our choice dependent on drawing one of three cards.

Nash equilibrium $\subsetneq$ Correlated Equilibrium

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	H	M
H	1/3	1/3
M	1/3	

- Now suppose we make our choice dependent on drawing one of three cards.
- This is a correlated equilibrium, with expected value of 5 for each of us.

Nash equilibrium $\subsetneq$ Correlated Equilibrium

▶

	H	M
H	6, 6	2, 7
M	7, 2	0, 0

	H	M
H	$1/3$	$1/3$
M	$1/3$	

- Now suppose we make our choice dependent on drawing one of three cards.
- This is a correlated equilibrium, with expected value of 5 for each of us.
- No Nash equilibrium can give you that.

Nash equilibrium $\subsetneq$ Correlated Equilibrium

	H	M
H	6, 6	2, 7
M	7, 2	0, 0

	H	M
H	$1/3$	$1/3$
M	$1/3$	

- Now suppose we make our choice dependent on drawing one of three cards.
- This is a correlated equilibrium, with expected value of 5 for each of us.
- No mixing can give you the probability distribution ρ on $\Pi_i S_i$.

Nash equilibrium $\subsetneq$ Correlated Equilibrium

	H	M
H	6, 6	2, 7
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	H	M
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M	1/3	

- Now suppose we make our choice dependent on drawing one of three cards.
- This is a correlated equilibrium, with expected value of 5 for each of us.
- The probability distribution ρ on $\Pi_i S_i$ cannot be factorized in independent probability distributions on S_i's.

Correlations in Belief

- Small step from correlated strategies to correlated priors.

	H	M
H	3, 3	0, 0
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	H	M	Ω	M	H
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M	0, 0	1, 1	M	0	1/2

  - $\mathcal{T} = \{t_O, t_E\}$.

Correlations in Belief

- Small step from correlated strategies to correlated priors.

	H	M	Ω	M	H
H	3, 3	0, 0	H	1/2	0
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- $\mathcal{T} = \{t_O, t_E\}$.
- This example hides some conceptual complications...

Explanation

Beliefs about correlations

X	L	R
U	1, 1, 3	1, 0, 3
D	0, 1, 0	0, 0, 0

Y	L	R
U	1, 1, 2	0, 0, 0
D	0, 0, 0	1, 1, 2

Z	L	R
U	1, 1, 0	1, 0, 0
D	0, 1, 3	0, 0, 3

- Three players: Ann, Bob and Charlie.

A. Brandenburger and A. Friedenberg. *Intrinsic Correlations in Games*. J.E.T., vol. 141, 2008.

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- Ann chooses the column.

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Z	L	R
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D	0, 1, 3	0, 0, 3

- Bob chooses the row.

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U	1, 1, 2	0, 0, 0
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- Bob chooses the row.

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Z	L	R
U	1, 1, 0	1, 0, 0
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- Charlie chooses the matrix.

Beliefs about correlations

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D	0, 1, 0	0, 0, 0

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- Strategy **Y** is rationalizible for Charlie.

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- Strategy **Y** is rationalizible for Charlie.
- Take: $\lambda(t_C)(U, L) = 1/2$, $\lambda(t_C)(D, R) = 1/2$.

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- Take: $\lambda(t_C)(U, L) = 1/2$, $\lambda(t_C)(D, R) = 1/2$. Then:

$$EV_C(X, t_C) = 1/2(3) + 1/2(0) = 1.5$$

Beliefs about correlations

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- Strategy **Y** is rationalizible for Charlie.
- Take: $\lambda(t_C)(U, L) = 1/2$, $\lambda(t_C)(D, R) = 1/2$. Then:

$$EV_C(X, t_C) = 1/2(3) + 1/2(0) = 1.5$$

$$EV_C(Y, t_C) = 1/2(2) + 1/2(2) = 2$$

Beliefs about correlations

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U	1, 1, 3	1, 0, 3
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U	1, 1, 2	0, 0, 0
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Z	L	R
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- Strategy **Y** is rationalizible for Charlie.
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$$EV_C(Y, t_C) = 1/2(2) + 1/2(2) = 2$$

$$EV_C(Z, t_C) = 1/2(0) + 1/2(3) = 1.5$$

Beliefs about correlations

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- Strategy **Y** is rationalizible for Charlie.
- Take: $\lambda(t_C)(U, L) = 1/2$, $\lambda(t_C)(D, R) = 1/2$.
- **Y** is not rationalizable if Charlie doesn't believe that Ann and Bob correlate their choices. (Left as an exercise).

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  - Team Interactions generate correlations.
- Question: where do (beliefs in) correlations come from?

The Importance of Correlations

- RCBR $\Leftrightarrow$ Playing iteratively rationalizable strategies.
  - For more than two agents: some (beliefs in) correlations are bound to arise.
  - We just saw an example of that.
  - Team Interactions generate correlations.

- Question: where do (beliefs in) correlations come from?
  Answer: A player can think that other players' strategy choices are correlated, because he thinks what they believe about the game is correlated.

Correlations through types

X	L	R
U	1, 1, 3	1, 0, 3
D	0, 1, 0	0, 0, 0

Y	L	R
U	1, 1, 2	0, 0, 0
D	0, 0, 0	1, 1, 2

Z	L	R
U	1, 1, 0	1, 0, 0
D	0, 1, 3	0, 0, 3

$$\lambda(t_C)(U, L) = 1/2$$

$$\lambda(t_C)(D, R) = 1/2$$

Correlations through types

X	L	R
U	1, 1, 3	1, 0, 3
D	0, 1, 0	0, 0, 0

Y	L	R
U	1, 1, 2	0, 0, 0
D	0, 0, 0	1, 1, 2

Z	L	R
U	1, 1, 0	1, 0, 0
D	0, 1, 3	0, 0, 3

- Instead: make strategy choices conditional on types/beliefs/hierarchies.

Correlations through types

X	L	R
U	1, 1, 3	1, 0, 3
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- Instead: make strategy choices conditional on types/beliefs/hierarchies.
  - Suppose $T_A = \{t_A, u_A\}$ and $T_B = \{t_B, u_B\}$ and:

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Correlations through types

t_C	UL	UR	DL	DR
$t_A t_B$	1/2	0	0	0
$t_A u_B$	0	0	0	0
$u_A t_B$	0	0	0	0
$u_A u_B$	0	0	0	1/2

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- We get the same result: Charlie is rational at state

$$(t_C, \bullet), (s_A, s_B, Y)$$

Correlations through types

- Strategy choices conditional on signals.
  - $\rho_A = \{Head \Rightarrow U, Tail \Rightarrow D\}$, $\rho_B = \{Head \Rightarrow L, Tail \Rightarrow R\}$.
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- How much explanatory power?

Correlations through types

- CI: For any type t_i for agent i, the choices $(s_j, s_k, ...)$ for the other agents $(i \neq j, \neq k, ...)$ are independent (i.e. non-correlated), conditional on the type of the other players.

A. Brandenburger and A. Friedenberg. *Intrinsic Correlations in Games*. J.E.T., vol. 141, 2008.

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- In the example:

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- SUFF: For any type t_i for agent i, a type t_j of agent $(j \neq i)$ is sufficient, to determine t_i's beliefs about j's choice.

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Two routes to explain correlations

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Extrinsic

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R.J. Aumann. *Correlated equilibrium as an expression of bayesian rationality*. *Econometrica*, 55(1-18), 1987.

Two routes to explain correlations

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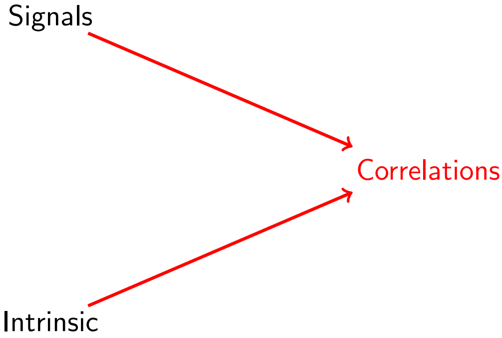

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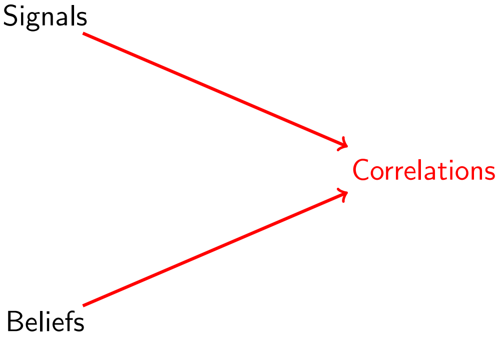

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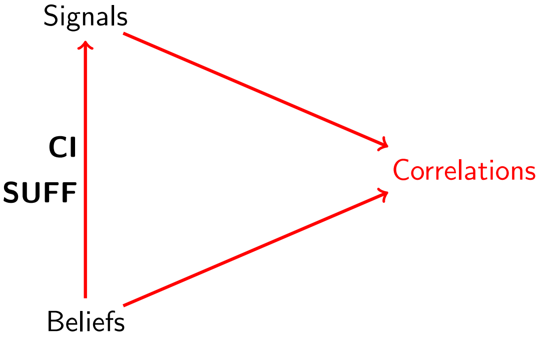

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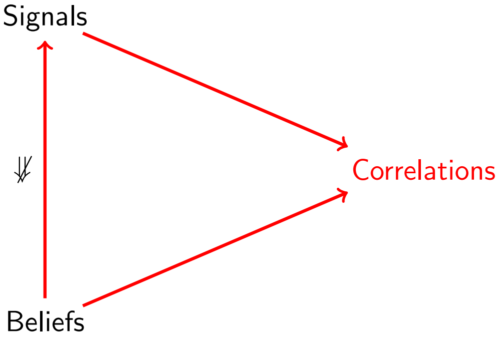

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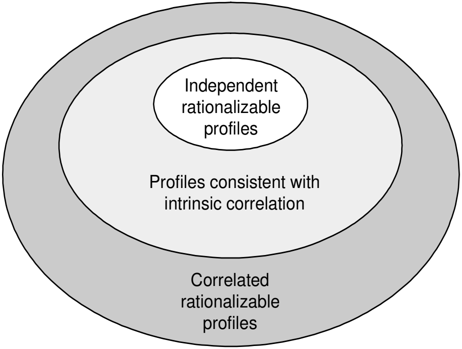

Figure 2.2

A. Brandenburger and A. Friedenberg. *Intrinsic Correlations in Games*. J.E.T., vol. 141, 2008.

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  More general questions:
  - What does it means to say that a team is rational at a given state?
  - And what about correlations in non-probabilistic models of beliefs?

Additional requirements on team work.

What is a team?

1. Group identification.
   - Information about who's in and who's out.
   - Reasoning as group members.
   - Shared goal.
     - Group preference / utilities.
2. Shared commitments.
   - Shared intentions.
   - Sanctions for lapsing?
   - Shared praise[blame] for success[failure]?
3. Common knowledge (beliefs?) of the above?

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Correlations in UTI

Type spaces for Team Interactions, once again:

Correlations in UTI

Definition

A type space for a team interaction TI is a tuple:

$$\mathcal{T} = \langle S, \{T_i\}_{\in \mathcal{A}}, \Omega \rangle$$

- $T_i = \{k \in M : i \in k\}$ is a set of types for player i.
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In *ex interim* type spaces for games the set of profiles is part of the uncertainty domain.Uncertainty bears on the choices (and types) of others.

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- The uncertainty domain S is such that $\mathfrak{P} \subseteq S$.
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In *ex interim* type spaces for games the set of profiles is part of the uncertainty domain.

In team interaction the natural move is to make the uncertainty bear on choices, types and protocols.

Correlations in UTI

Correlations arise from the Team Authority requirement:

$$\text{If } t_i = k \text{ then } \alpha_i = \alpha_i^k(s)$$

$\lambda_E(I_E)(I_O)$	M_O	H_O
$\bullet, M, MM$		
$\bullet, M, MH$		
$\vdots$		
$\bullet, H, MM$		
$\bullet, H, MH$		
$\vdots$		

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Example:

	Hard Work	Minimal Work
Hard Work	3, 3 (3)	0, 4 (2)
Minimal Work	4, 0 (2)	1, 1 (1)

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$\vdots$		
$\bullet, H, MM$	0	
$\bullet, H, MH$	0	
$\vdots$		

$\lambda_E(I_E)(C_O)$	M_O	H_O
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$$\text{If } t_i = k \text{ then } \alpha_i = \alpha_i^k(s)$$

(Part of) an ex-interim type space for this TI:

$\lambda_E(I_E)(I_O)$	M_O	H_O
$\bullet, M, MM$		0
$\bullet, M, MH$		0
$\vdots$		
$\bullet, H, MM$	0	
$\bullet, H, MH$	0	
$\vdots$		

$\lambda_E(I_E)(C_O)$	M_O	H_O
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CPA not an innocuous assumption!

- Agreement Theorem:

  If two people have the common priors, and their posteriors for an event A are common knowledge, then these posterior have to be the same.

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The Annals of Statistics
1976, Vol. 4, No. 6, 1236–1239

AGREEING TO DISAGREE[1]

BY ROBERT J. AUMANN

Stanford University and the Hebrew University of Jerusalem

Two people, 1 and 2, are said to have *common knowledge* of an event E if both know it, 1 knows that 2 knows it, 2 knows that 1 knows is, 1 knows that 2 knows that 1 knows it, and so on.

THEOREM. *If two people have the same priors, and their posteriors for an event A are common knowledge, then these posteriors are equal.*

If two people have the same priors, and their posteriors for a given event A are common knowledge, then these posteriors must be equal. This is so even though they may base their posteriors on quite different information. In brief, people with the same priors *cannot agree to disagree*.

We publish this observation with some diffidence, since once one has the appropriate framework, it is mathematically trivial. Intuitively, though, it is not

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- Agreement Theorem: Back to Lecture

An digression on CI and SUFF

Suppose a being in whose power to predict your choices you have enormous confidence. [...]

R. Nozick. *Newcomb's Problem and Two Principles of Choice*. 1969.

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Suppose a being in whose power to predict your choices you have enormous confidence. [...] There are two boxes, (A) and (B). (B1) contains 1000 [euros]. (B2) contains either 1 million [euros] (M), or nothing.

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1. taking what is in both boxes
2. taking only what is in the second box.

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Furthermore, [the following is common knowledge]:

- If the being predicts you will take what is in both boxes, he does not put the 1 M in the second box.
- If the being predicts you will take only what is in the second box, he does put the 1M in the second box.

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The situation is as follows. First the being makes its prediction. Then it puts the 1M in the second box, or does not, depending upon what it has predicted. Then you make your choice. What do you do?

An digression on CI and SUFF

	A = 1M	A = 0
1 Box	1M	0
2 Boxes	1M + 100	100

An digression on CI and SUFF

	A = 1M	A = 0
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	A = 1M	A = 0
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2 Boxes	0	1/2

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- No uncertainty on types here.
- SUFF is trivially satisfied (always for two agents).
- CI is violated.

Back to Lecture