Tools for Formal Epistemology: Doxastic Logic, Probability, and Default Logic

– ESSLLI 2023 –

Problem Set (Default logic)

Task

Complete as many of the exercises as you like. (Exercise 7 will require more work than others.)

Exercise 1 (Inference graphs)

Draw inference graphs depicting the following (fixed priority) default theories:

(a) $\Delta_1 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where:
$\mathcal{W} = \{B, C \supset D\}$;
$\mathcal{D} = \{\delta_1, \delta_2, \delta_3\}$ with $\delta_1 = B \to C$, $\delta_2 = D \to E$, and $\delta_3 = E \to \neg B$;
$< = \emptyset$

(b) $\Delta_2 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where:
$\mathcal{W} = \{A\}$;
$\mathcal{D} = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}$ with $\delta_1 = A \to B$, $\delta_2 = B \to C$, $\delta_3 = C \to D$, $\delta_4 = B \to \neg D$, and $\delta_5 = A \to D$;
$\delta_3 < \delta_4 < \delta_5$

(c) $\Delta_3 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where:
$\mathcal{W} = \{A, B\}$;
$\mathcal{D} = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ with $\delta_1 = A \to C$, $\delta_2 = B \to \neg C$, $\delta_3 = C \to D$, $\delta_4 = \top \to \neg D$;
$< = \emptyset$

(d) $\Delta_4 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where:
$\mathcal{W} = \{A, B, (D \wedge B) \supset E\}$;
$\mathcal{D} = \{\delta_1, \delta_2, \delta_3\}$ with $\delta_1 = A \to C$, $\delta_2 = C \to D$, and $\delta_3 = E \to \neg C$;
$< = \emptyset$

(e) $\Delta_5 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where:
$\mathcal{W} = \emptyset$
$\mathcal{D} = \{\delta_1, \delta_2\}$ with $\delta_1 = A \to A$, $\delta_2 = \top \to B$;
$< = \emptyset$

(f) $\Delta_6 = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where:
$\mathcal{W} = \emptyset$;
$\mathcal{D} = \{\delta_1, \delta_2, \delta_3\}$ with $\delta_1 = \top \to A$, $\delta_2 = A \to \neg A$, and $\delta_3 = \top \to B$;
$\delta_1 < \delta_2$

Recall that an atomic formula A from the hard information $\mathcal{W}$ is to be treated as $\top \supset A$. Also, recall that there can be only one node representing the true proposition $\top$.

Exercise 2 (Interpretation)

Come up with a plausible interpretation for the default theories Δ_2 and Δ_3. (Hint: You might want to think about the Tweety Triangle and the Nixon Diamond.)

Exercise 3 (Stable scenarios)

Determine the stable scenarios of the default theories Δ_1–Δ_6 from Exercise 1.

Exercise 4 (Proper scenarios)

Find all proper scenarios of the default theories from Exercise 1, using approximating sequences.

Exercise 5 (Extensions)

Determine the extensions of default theories from Exercise 1.

Example: The extension of the Tweety Triangle (Δ_1 in the slides) is

$$\mathcal{E} = Th(\mathcal{W} \cup Conclusion(\{\delta_2\})) = Th(\mathcal{W} \cup \{\neg F\}) = Th(\{P, P \supset B\} \cup \{\neg F\}) = Th(\{P, P \supset B, \neg F\})$$

Exercise 6 (Exclusion)

(a) How would you represent the information from the Tweety Triangle in a default theory with an empty priority ordering? What's lost (or gained)?

(b) The addition of exclusionary default rules and the corresponding notion of exclusion prompted us to modify the notion of stable scenarios. One important adjustment is still missing from the slides. What is it?

Exercise 7 (Proofs)

Prove some of the facts listed in Section 6 of Lecture 4.

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