March MADness

1

Mean Absolute Deviation

MATERIALS

None

Lesson Overview

Students analyze two data sets displayed on a dot plot that have the same mean, but with different amounts of spread. The concept of deviation is introduced, and students calculate the deviations of each data point from the mean. The *mean absolute deviation* is introduced, and students calculate the mean absolute deviation for each data set. Then, they calculate and interpret the mean absolute deviation for two additional data sets. Finally, students convert non-numerical data from two data sets into numerical data to analyze and interpret it using measures of center and variation.

Grade 8
Measurement and Data

(11) Measurement and data. The student applies mathematical process standards to use statistical procedures to describe data. The student is expected to:

(B) determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean using a data set of no more than 10 data points.

ELPS

1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

Essential Ideas

- Measures of variability in a data set describe how spread out the data is.
- The mean absolute deviation is a measure of variation describing how the data is spread out around the mean of the data set.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: We Are the Champions

Students determine and compare the means of two data sets displayed with dot plots. This activity is designed to engage students to think about important differences between data sets that are not captured by measures of center, such as the mean.

Develop

Activity 1.1: Exploring Variation and Deviation

Using the dot plots from the Getting Started, students investigate the variation in the two data sets, using absolute deviation and mean absolute deviation as measures of variation. Students make decisions about the real-world data by comparing the measures of variation in the two data sets.

Day 2

Activity 1.2: Applying Mean Absolute Deviation

Students analyze and compare two data sets using mean and mean absolute deviation. They complete tables, plot the data on dot plots, and interpret the data using measures of center and variation.

Demonstrate

Talk the Talk: GPA and MAD

Students convert non-numerical data about grades from two data sets into numerical data in order to analyze it using mean and mean absolute deviation. They then write their own interpretations of the data using measures of center and variation.

Getting Started: We Are the Champions

ENGAGE

Facilitation Notes

In this activity, two data sets with the same mean are displayed using dot plots. Students conclude a single measure of center does not always describe similar data sets.

Ask students to work with a partner or in groups to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- How did you determine the mean in this data set?
- How would you describe the spread in this data set?
- How is the spread different in the two data sets?
- How many total points were scored by Tamika?
- How many total points were scored by Lynn?
- Did the girls play the same number of games?
- If you were the coach, who would you choose to start? Why?

Summary

Using a single measure of center, such as mean, is not enough information to properly describe a data set.

Activity 1.1

Exploring Variation and Deviation

DEVELOP

Facilitation Notes

In this activity, students use the dot plots from the Getting Started and investigate the variation in the two data sets, using absolute deviation and mean absolute deviation as measures of variation.

Ask a student to read the introduction aloud and complete Question 1 as a class.

Questions to ask

- What is the shape of each data set?
- Can you identify any clusters in the data sets?
- Can you identify any gaps in the data sets?
- How can two very different data sets have the same mean?
- Why does a mean of 12 make sense for each of the dot plots?

Ask another student to read the information and definition above Question 2 aloud. Have students work with a partner or in groups to complete Questions 2 through 5. Share responses as a class.

Note that students may be inconsistent in subtracting the data value from the mean. Stress that deviation means distance from the mean. Encourage students to think about how they operate with integers, so that they can apply this reasoning to determine how far a data value is from the mean.

Questions to ask

- How can you tell from looking at a data value whether its deviation will be positive or negative?
- If a data value is the same as the mean, what is the value of its deviation from the mean?
- Why is the sum of the deviations from the mean always 0?
- Why is it important to know the distance from each data point to the mean? What does it tell you about the data spread?

Ask a student to read the information and definitions above Question 6 aloud. Have students work with a partner or in groups to complete Questions 6 through 9. Share responses as a class.

Questions to ask

- Is the mean considered the balance point?
- How do the sum of the data points to the left of the mean compare to the sum of the data points on the right side of the mean?
- What is the difference between the absolute deviation and the mean absolute deviation?
- How is the absolute deviation calculated?
- How is the mean absolute deviation calculated? Why is it important?

Summary

The deviation, the absolute deviation, and the mean absolute deviation can be used to get an idea of the spread of the data values in a data set.

Activity 1.2
Applying Mean Absolute Deviation

Facilitation Notes

In this activity, students design a statistical question to fit given data sets, create a dot plot, and calculate the absolute deviation.

Ask students to work with a partner or in groups to complete Question 1. Share responses as a class.

Questions to ask

- Can your question be answered using a measure of center?
- What information is needed to answer your question?
- Will completing the table of values in Question 3 help to answer your question?
- How will calculating the absolute deviation be helpful?

Differentiation strategy

The entire class can share all questions and decide and agree upon answering the same statistical question, or each group can pose their own question to answer.

Ask students to work with a partner or in groups to complete Questions 2 and 3. Share responses as a class.

Questions to ask

- What is the mean height for the NBA players?
- What is the mean height for the 6th-grade players?
- What is the MAD for the NBA players?
- What is the MAD for the 6th-grade players?

Summary

Measures of center and variation can be used in real-life situations to answer relevant questions.

DEMONSTRATE

Talk the Talk: GPA and MAD

Facilitation Notes

In this activity, students convert a non-numeric data set into a numeric data set to determine the mean absolute deviation and answer questions relevant to the problem scenario.

Ask a student to read the introduction aloud. Discuss different methods of representing the data as numeric data and complete Question 1 as a class.

Questions to ask

- Is there more than one way to change this into numeric data?
- What is another way to change this into numeric data?

Ask students to work with a partner or in groups to complete Questions 2 through 4. Share responses as a class.

Questions to ask

- How did you calculate the deviation from the mean for Luca's grades?
- How did you calculate the absolute deviation for Luca's grades?
- How did you calculate the deviation from the mean for Eric's grades?
- How did you calculate the absolute deviation for Eric's grades?
- Which student has better grades?

Summary

Non-numeric data can be converted into numeric data and compared using deviation from the mean and mean absolute deviations.

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March MADness

Mean Absolute Deviation

WARM UP

Determine the absolute value of each number.

1. $|-4|$
2. $|12.5|$
3. $|-1.09|$
4. $|4\frac{2}{3}|$

LEARNING GOALS

- Determine the absolute deviations of data points in a data set.
- Give quantitative measures of variation, including mean absolute deviation, for a data set.
- Use the mean absolute deviation as a measure of variation to describe and interpret data.
- Compare data sets using variation and the mean absolute deviation.
- Summarize numerical data sets in relation to their context.

KEY TERMS

- deviation
- absolute deviation
- mean absolute deviation

The interquartile range is used as a measure of variation when the median is the measure of center. How can you measure the variation when mean is the measure of center?

Warm Up Answers

1. 4
2. 12.5
3. 1.09
4. $4\frac{2}{3}$

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Answers

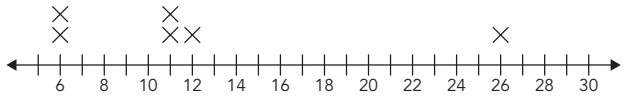
1. The mean is 12. On average, each player scored about 12 points per game over the last six games.
2. Answers will vary.
3. Lynn's scores are very close to 12. Tamika's mean of 12 is caused by her high and low scores moving the balance point of the data to 12.

Getting Started

We Are the Champions

Coach Harris's basketball team is advancing to the district championship. Tamika and Lynn are possible starters for the game. Dot plots for each player's scoring over the past six games are shown.

Number of Points Scored by Tamika



Number of Points Scored by Lynn

1. Determine the mean of each data set. Explain what this number tells you.

2. How are the two data sets similar and different?

3. Explain why the two data sets have the same mean.

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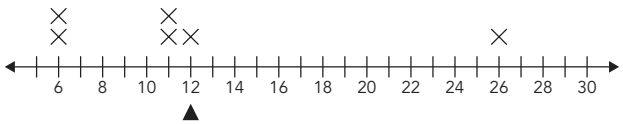
ACTIVITY
1.1

Exploring Variation and Deviation

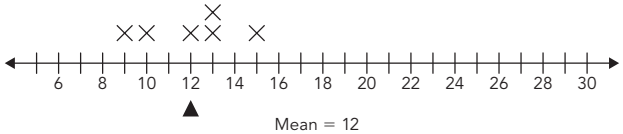
Previously, you examined the dot plots of two basketball players–Tamika and Lynn. Coach Harris needs to choose between Tamika and Lynn as starters for the game.

Number of Points Scored by Tamika



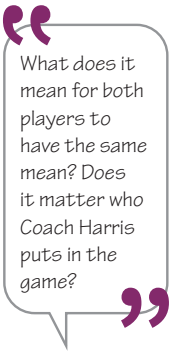
Mean = 12

Number of Points Scored by Lynn



Mean = 12

1. Based on the dot plots, which player do you think Coach Harris should choose?



When analyzing a data set, measures of center give you an idea of where the data is centered, or what a typical data value might be. There is another measure that can help you analyze data. Measures of variation describe the spread of the data values. Just as there are several measures of central tendency, there are also several measures of variation.

The **deviation** of a data value indicates how far that data value is from the mean. To calculate the deviation, subtract the mean from the data value:

deviation = data value − mean

Answer

1. Answers will vary.

2. Describe the deviations. Record your results in the tables.

Tamika	
Points Scored	Describe the Deviation from the Mean
11	
11	
6	
26	
6	
12	

Lynn	
Points Scored	Describe the Deviation from the Mean
15	
12	
13	
10	
9	
13	

3. What is the meaning if a deviation is positive? Is negative? Is 0?

4. What do you notice about the deviations for each player?

Answers

2.

Tamika	
Points Scored	Describe the Deviation from the Mean
11	1 less than the mean
11	1 less than the mean
6	6 less than the mean
26	14 more than the mean
6	6 less than the mean
12	12 is the mean

Lynn	
Points Scored	Describe the Deviation from the Mean
15	3 more than the mean
12	12 is the mean
13	1 more than the mean
10	2 less than the mean
9	3 less than the mean
13	1 more than the mean

3. A positive deviation means that the data point is more than the mean. A negative deviation means that the data point is less than the mean. A deviation of 0 means that the data point is equal to the mean.

4. Answers will vary.

5. Carly claims that the sum of the deviations for a data set will always be 0. Do you agree? Why or why not?

The sum of all the deviations less than 0 is equal to the sum of the deviations greater than 0. Because the mean is the balance point, the sums of data points on either side of the balance point are equal to each other.

In order to get an idea of the spread of the data values, you can take the absolute value of each deviation and then determine the mean of those absolute values. The absolute value of each deviation is called the **absolute deviation**. The **mean absolute deviation** (MAD) is the mean of the absolute deviations.

6. Record the absolute deviations for the points scored in the tables.

Tamika		
Points Scored	Deviation from the Mean	Absolute Deviation
11	−1	
11	−1	
6	−6	
26	14	
6	−6	
12	0	

Lynn		
Points Scored	Deviation from the Mean	Absolute Deviation
15	3	
12	0	
13	1	
10	−2	
9	−3	
13	1	

Answers

5. Sample answer. Carly is correct. Since the mean is a balance point, the values less than the mean balance the values greater than the mean.

6.

Tamika		
		Absolute Deviation
11	−1	1
11	−1	1
6	−6	6
26	14	14
6	−6	6
12	0	0

Lynn		
		Absolute Deviation
15	3	3
12	0	0
13	1	1
10	−2	2
9	−3	3
13	1	1

1. Deviation is found by subtracting the mean from the data.
2. The absolute deviation is the absolute value of the deviation.
3. The mean absolute deviation is the mean of the absolute deviations.

ELL Tip

There are three related terms in this section: *deviation*, *absolute deviation*, and *mean absolute deviation*. Have students draw a flow chart graphic organizer of how one term is found and then used in another. The steps should be:

Answers

7. Tamika: $4\frac{2}{3}$ points; Lynn: $1\frac{2}{3}$ points
8. Tamika typically scores within $4\frac{2}{3}$ points away from 12 points in a game. Lynn typically scores within $1\frac{2}{3}$ points away from 12 points in a game.
9. Answers will vary.

7. Calculate the mean absolute deviation for the points scored for each player.

8. What does the mean absolute deviation tell you about the points scored by each player?

9. If you were Coach Harris, which player would you choose to play in the championship game? Justify your decision.

ACTIVITY 1.2

Applying Mean Absolute Deviation

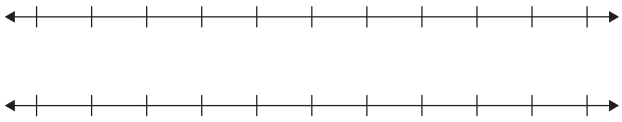
Answers

1. Answers will vary.
2. See dot plots below.

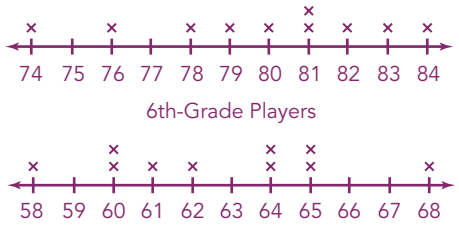
The tables on the next page show the heights in inches of ten NBA basketball players and ten 6th-grade basketball players.

1. Write a statistical question you can answer by analyzing the data.

2. Create a dot plot for each data set.



2.



3. **Complete each table. Then, compare the data sets and interpret your results.**

NBA Players		
Height (in.)	Describe the Deviation from the Mean	Absolute Deviation
79		
74		
78		
81		
81		
76		
84		
80		
82		
83		

6th-Grade Players		
Height (in.)	Describe the Deviation from the Mean	Absolute Deviation
68		
64		
60		
58		
62		
65		
64		
60		
61		
65		

Answer

3. NBA players:

NBA Players	
Describe the Deviation from the Mean	Absolute Deviation
0.8 less than the mean	0.8
5.8 less than the mean	5.8
1.8 less than the mean	1.8
1.2 greater than the mean	1.2
1.2 greater than the mean	1.2
3.8 less than the mean	3.8
4.2 greater than the mean	4.2
0.2 greater than the mean	0.2
2.2 greater than the mean	2.2
3.2 greater than the mean	3.2

6th-grade players:

6th-Grade Players	
Describe the Deviation from the Mean	Absolute Deviation
5.3 more than the mean	5.3
1.3 more than the mean	1.3
2.7 less than the mean	2.7
4.7 less than the mean	4.7
0.7 less than the mean	0.7
2.3 more than the mean	2.3
1.3 more than the mean	1.3
2.7 less than the mean	2.7
1.7 less than the mean	1.7
2.3 more than the mean	2.3

Answers

1. The letters can be assigned numeric values. Possible values: A = 4, B = 3, C = 2, D = 1, F = 0.
2. Luca's mean: 3.2; Eric's mean: 3.4. The means indicate that each student has about a B-average grade.

NOTES

TALK the TALK

GPA and MAD

Sometimes you can change non-numerical data into numeric data in order to analyze it. Consider, for example, the report cards shown. Grades for the courses are assigned to the categories A, B, C, D, and F, with A being the highest grade.

Luca	
Science	B
Cultural Literacy	A
Music	C
Math	A
English	B

Eric	
Math	A
English	B
Cultural Literacy	C
Science	A
Music	A

1. Explain how you can change the report card data into numeric data.

2. Determine the mean of each data set. What does each mean tell you?

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NOTES

3. Determine the mean absolute deviation for each data set.

Luca			Eric		
Data Value	Describe the Deviation from the Mean	Absolute Deviation	Data Value	Describe the Deviation from the Mean	Absolute Deviation

4. Interpret each of the mean absolute deviations.

Answers

3. Luca: 0.64; Eric: 0.72
4. Answers will vary.

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Assignment

LESSON 1: March MADness

Write

Complete each sentence with the correct term.

Absolute deviation	Mean absolute deviation	Measures of variation	Deviation

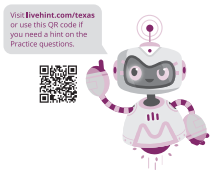
1. _____ describe(s) the spread of the data values.
2. _____ indicates how far the data value is from the mean.
3. _____ is the absolute value of each deviation.
4. _____ is the average, or mean, of the absolute deviations.

Remember

To calculate the mean absolute deviation:

- Determine the mean of the data.
- Subtract the mean from each data value. These are the deviations.
- Record the absolute value of each deviation. These are the absolute deviations.
- Determine the mean of the absolute deviations. This is the mean absolute deviation.

Practice



Calculate the mean absolute deviation for each data set.

1. Data set: 4, 5, 9, 4, 8; Mean = 6
2. Data set: 7, 11, 8, 35, 14; Mean = 15
3. Data set: 60, 65, 66, 67, 67, 65; Mean = 65
4. Data set: 22, 26, 29, 23, 26, 21, 28, 24, 25, 26; Mean = 25
5. Data set: 180, 210, 155, 110, 230, 90, 400, 35, 190, 0, 10, 100, 90, 130, 200; Mean = 142
6. Data set: 55, 74, 90, 20, 47, 59, 26, 83, 77, 62, 58, 33, 57, 44, 31; Mean = 54.4

Stretch

1. Create a data set of 5 numbers that has a mean absolute deviation of 1. Explain how you arrived at your solution.
2. Create a data set of 6 numbers that has a mean absolute deviation of 10. Explain how you arrived at your solution.

Assignment Answers

Write

1. Measures of variation
2. Deviation
3. Absolute deviation
4. Mean absolute deviation

Practice

1. 2
2. 8
3. $1\frac{2}{3}$
4. 2
5. $76\frac{2}{15}$
6. 16.72

Stretch

1. Answers will vary.
2. Answers will vary.

Assignment Answers

Review

1. The median is 37. The mean is approximately 38.2.
2. The median is 7.5.

3a. −116, −35, 0, 32, 92, 115

3b. −5, −2, 0, 27, 31, 90

4a. $1\frac{3}{10}$

4b. $1\frac{2}{3}$

Review

1. The rate at which crickets chirp is affected by the temperature. In fact, you can estimate the outside temperature by counting cricket chirps. As a homework assignment, Mr. Ortega asks each of his students to count the number of chirps they hear in 15 seconds at 8:00 PM. The results are shown.

   36, 37, 41, 39, 35, 39, 35, 39, 42, 37, 40, 35, 36, 37, 42, 35, 37, 37, 38, 42, 41, 37, 41

   Determine the median and mean number of cricket chirps heard in 15 seconds.

2. Patrick recorded the number of emails he sent over two weeks: 11, 5, 6, 9, 10, 5, 4, 2, 9, 10. What is the median of his data?

3. Order the integers in each group from least to greatest.
   a. 0, 115, −35, 32, −116, 92
   b. −2, 31, −5, 27, 0, 90

4. Determine each difference.
   a. $2\frac{4}{5} - 1\frac{1}{2}$
   b. $3 - 1\frac{1}{3}$

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