3

Not All Statements Are Made Equal

Modeling Linear Inequalities

MATERIALS

None

Lesson Overview

Students begin with a scenario and table that can be modeled by a linear inequality with a positive rate of change. They then analyze a graph that models the situation. Students use that graph to solve inequalities and graph the solution set on a number line. Next, the term *solve an inequality* is defined, and students write and solve inequalities algebraically, taking into account the context of the problem situation. They then analyze an inequality with a negative rate of change to make sense of how the sign of the solution to the inequality is affected. Lastly, students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation and ones that require the Distributive Property.

Algebra 1
Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(C) write linear equations in two variables given a table of values, a graph, and a verbal description.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ELPS

1.A, 1.C, 1.D, 1.E, 1.H, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.F, 4.K, 5.E

Essential Ideas

- A linear inequality context can be modeled with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement.
- Solutions to linear inequalities can be determined both graphically and algebraically; they can be expressed using a number line or inequality statement.
- The steps to solving a linear inequality algebraically are the same steps to solve a linear equation, except that when solving a linear inequality with a negative rate of change, the inequality sign of the solution must be reversed to accurately reflect the relationship.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Fundraising Function

A scenario is given that can be represented by a function in the form $f(x) = ax + b$. Students write and analyze the function that describes the scenario, identifying the independent and dependent quantities, the rate of change, and the y-intercept.

Develop

Activity 3.1: Modeling Linear Inequalities

Students continue working with the Fundraising Function scenario. They analyze a graph that models the situation and use it to solve inequalities. Students graph the solution set for each inequality on a number line.

Activity 3.2: Solving Two-Step Linear Inequalities

The term *solve an inequality* is defined. A Worked Example demonstrates how to solve an inequality algebraically. Students practice writing and solving inequalities and choosing the most accurate answer in the context of the problem situation. They then determine that the method to solve an inequality does not change regardless of the form of the inequality.

Day 2

Activity 3.3: Reversing Inequality Signs

A scenario is provided that can be represented by an inequality with a negative slope. Students write a function that describes the scenario and identify key characteristics of the function in terms of the problem situation. They use the graph of the function to solve the inequality before solving the inequality algebraically. They compare the graphic solution to the algebraic solution. Students complete a table of inequalities relating $h(m)$ and m to demonstrate that multiplication or division of a negative number reverses the inequality symbol when solving an inequality.

Activity 3.4: Solving Other Linear Inequalities

Students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation and ones that require the Distributive Property.

Demonstrate

Talk the Talk: It's About Solutions, More or Less

Students solve four inequalities and graph each solution on a number line. They also identify constraints that make different inequality statements true.

Getting Started: Fundraising Function

ENGAGE

Facilitation Notes

In this activity, a scenario is given that can be represented by a function in the form $f(x) = ax + b$. Students write and analyze the function that describes the scenario, identifying the independent and dependent quantities, the rate of change, and the y-intercept.

Ask a student to read the introduction aloud. Discuss as a class.

Questions to ask

- Explain the scenario in your own words.
- How are the key features of the function identifiable in the scenario given?

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What function family is the function associated with? How do you know?
- When the function written in the form of $f(x) = ax + b$, what do the values of a and b represent?
- What is the significance of $25 in this scenario?
- If you sell one box of popcorn, what is the total sales credit? Two boxes? Three boxes?
- How many boxes must you sell to have $70 in sales credit?
- Describe the graph of this function.

Summary

A scenario can be represented by a function that describes the rate of change and the y-intercept.

DEVELOP

Activity 3.1
Modeling Linear Inequalities

Facilitation Notes

In this activity, students continue working with the Fundraising Function scenario. They analyze a graph that models the situation and use it to solve inequalities. Students graph the solution set for each inequality on a number line.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategy

To assist all students, suggest they get in the habit of extending their arrows to the end of the number line. This will help them list other solutions to the inequality represented by the shaded region. It will also be important in the lesson on compound inequalities when students determine overlapping and non-overlapping shaded regions.

Questions to ask

- How could Question 1 part (b) be represented using function notation?
- Is the $25 credit included in the $1600? How can you tell?
- What is another way to solve Question 1 part (b)?
- How are the number line and the graph related?
- If Alan earns less than $1600, how many boxes could he have sold? What phrase includes all answers? List some specific values.
- If Alan earns more than $1600, how many boxes could he have sold? What phrase includes all answers? List some specific values.
- What would a point represent if it was located by drawing a horizontal line through the y-value of 1000 from the y-axis until it intersected the graphed function?
- Could Alan realistically sell more than 420 boxes of popcorn? Why or why not?
- Does the graph include the b-value equal to $4\frac{1}{2}$?
- Does the number line graph include the b-value equal to $4\frac{1}{2}$?
- Could Alan sell $4\frac{1}{2}$ boxes of popcorn?

As students work, look for

Confusion between the discrete and continuous aspects of the graphical representations of the problem situation on both the coordinate plane and the number line. The domain of the situation can only be positive integers whereas the domains in both graphical representations appear to include all real numbers.

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

Questions to ask

- What concerns do you have about solving these problems graphically?
- What inequality sign is associated with the phrase *at least*? Is this situation associated with an open or closed circle on a number line?
- What inequality sign is associated with the phrase *less than*? Is this situation associated with an open or closed circle on a number line?
- What sign is associated with the word *exactly*? Is this situation associated with an open or closed circle on a number line?

Summary

Regions of a graphed function can be represented on a number line as solutions to an inequality.

Activity 3.2

Solving Two-Step Linear Inequalities

Facilitation Notes

In this activity, the term *solve an inequality* is defined. A Worked Example demonstrates how to solve an inequality algebraically. Students write and solve inequalities and choose the most accurate answer in the context of the problem situation. They conclude that an inequality, regardless of its form, is solved in the same way as an equation, by using Properties of Equality and number properties.

Ask a student to read the introduction. Discuss the Worked Example and answer Question 1 as a class.

Questions to ask

- Why was the $\geq$ symbol used?
- Is it okay to switch the sides of the inequality so that it is written as $1100 \geq 3.75b + 25$? Why not?
- How would you express $1100 \geq 3.75b + 25$ reading from right to left instead of left to right?

Differentiation strategy

To assist all students, suggest a consistent strategy that follows the Worked Example where the algebraic expression with the variable is placed in front of the inequality symbol and the $f(x)$-value is placed behind the inequality symbol. This way, the inequality symbol required matches the wording in the question.

For example: Alan's total sales, $f(b)$, need to be at least $1100.

At least means $\geq$.

Correct: $3.75b + 25 \geq 1100$ Incorrect: $1100 \geq 3.75b + 25$

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

Differentiation strategies

To scaffold support,

- Model for students how to ask themselves whether values higher or lower than $600 and $1500 make sense in the context. Guide them to write inequalities based on their decisions.
- Have students check their answers by graphing each solution on a number line.
- Have students create a graphic organizer listing $<$, $\leq$, $>$, and $\geq$ and phrases that imply the use of these inequality symbols.

As students work, look for

- Incorrect selection of inequality symbols. Sometimes the choice of $>$ or $<$ and whether $=$ should be used is counterintuitive to students based upon the wording of the problem.
- Confusion whether or not to round up or down to the nearest integer based upon the context.

Questions to ask

- If Alan sells 153 boxes, what would be his total sales?
- If Alan sells 154 boxes, what would be his total sales?
- How did you know what inequality symbol to use to solve this problem?
- If Alan sells 393 boxes, what would be his total sales?
- If Alan sells 394 boxes, what would be his total sales?
- How do you know when to round up or round down to the nearest integer?

Summary

Problems in context can be represented and solved using inequalities. Solving an inequality is similar to solving an equation; both involve using Properties of Equality and number properties.

Activity 3.3
Reversing Inequality Signs

Facilitation Notes

In this activity, a scenario is provided that can be represented by an inequality with a negative slope. Students write a function that describes the scenario, and identify key characteristics of the function in terms of the problem situation. They then use a graph of the function to solve an inequality. The remainder of the activity may flow two different ways depending upon whether students remember from middle school that the inequality symbol must be reversed when multiplying or dividing an inequality by a negative value. If students know this rule, the algebraic solution to the inequality verifies the graphical solution. They then complete a table with inequality statements that justify this rule. If students do not know this rule, the algebraic solution to the inequality contradicts the graphical representation. They then complete a table with inequality statements based upon the correct graphical approach, and use the table results to make sense of why multiplication and division by a negative value requires reversing the inequality symbol.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- What sign is associated with hiking down a mountain?
- What function family is associated with this situation?
- If the function is written in the form of $f(x) = ax + b$, what do a and b represent?
- Why might a student write the function as $h(m) = 4800 - 20m$ rather than $h(m) = -20m + 4800$?
- Is the slope negative or positive?
- Would the graph of this function pass through the origin? Why or why not?
- What equation is used to determine the x-intercept?
- How is the graph in this problem different from the graph of the fundraising function?

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

As students work, look for

- Placement of the oval on the graph.
- Estimates that are close to 80.
- Algebraic solutions of $m < 80$ that conflict with the graphical solution of $m > 80$.

Differentiation strategies

To scaffold support,

- Suggest that students draw the horizontal line at $y = 3200$, the vertical line at $x = 80$, and label the point (80, 3200) to help understand the graphical solution process and have boundaries for the placement of the oval.
- Help them determine the correct graphical solution based on the context, and direct them to use those results to complete the table in Question 7. Address errors in the algebraic process when discussing Question 7.

Questions to ask

- Should the oval be drawn above or below the line $y = 3200$? Why?
- Why does the oval include values greater than 80 for m?
- What inequality is used to verify the solution?
- Are your steps to solve this inequality different from those used to solve the inequality for the fundraiser function? If so, how?
- If your algebraic and graphical solution methods result in different inequalities, which one do you think is the correct one? Why?

Complete Question 7 as a class.

Questions to ask

- What does a row in the table represent?
- Why are the inequalities symbols opposite for $h(m)$ and its corresponding m-value?
- What is the connection between $h(m)$ and m having opposite inequality symbols and the fact that the function is decreasing?
- If your answer to Question 5 is not correct, how can you adjust your algebra steps to solve the inequality correctly?
- How do you know when to reverse the inequality symbol and when to keep it as is?

Summary

When both sides of an inequality are multiplied or divided by a negative number, the sign of the resulting inequality is reversed.

Activity 3.4

Solving Other Linear Inequalities

Facilitation Notes

In this activity, students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation and ones that require the Distributive Property. The methods for solving inequalities are compared with those for solving equations.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What is the ultimate goal of both solution paths?
- Did Allison and John arrive at different answers or the same answer?
- Did Allison deal with a variable or constant term first? Which one?
- Why do you think she chose that as her first step?
- Did John deal with a variable or constant term first? Which one?
- Why do you think he chose that as his first step?
- Why did the inequality sign change in John's student work, but didn't change in Allison's student work?
- Do you prefer to always have the variable written before the equals sign or always have the variable have a positive coefficient? Why?
- Whose solution path would you prefer to use and why?
- What are other possible first steps to solve this equation?

Differentiation strategies

- To scaffold support, have students rewrite Allison's and John's methods so that the difference in their first steps is more explicit.

Allison

$$\begin{array}{r}5x + 2 \geq 3x - 10 \\ \underline{-3x \qquad -3x \quad} \\ 2x + 2 \geq -10\end{array}$$

John

$$\begin{array}{r}5x + 2 \geq \quad 3x - 10 \\ \underline{-5x \qquad\quad -5x \qquad} \\ 2 \geq -2x - 10\end{array}$$

- To extend the activity, demonstrate methods to check the accuracy of answers.
  - Graph the solution inequality on a number line, identify a value in the solution set, then substitute that value in the original inequality to verify that it results in a true inequality statement.
  - Suggest students always deal with $x = 0$ to check their answer. Refer to the solution inequality to determine whether zero is a solution or not. If zero is a solution, then substituting in zero should result in a true inequality statement. This method will be expanded on when working in the coordinate plane where students can use the origin to check accuracy of solutions.

Misconception

Students may incorrectly rewrite John's inequality solution, $-6 \leq x$, as $x \leq -6$ because they overgeneralize the process used for equations where $6 = x$ can be rewritten as $x = 6$. To correct this error, suggest they read the entire inequality statement, including the inequality symbol, from right to left. In this case, $-6 \le x$ would be read as x is greater than or equal to -6 and is written as $x \ge -6$. Do not use phrasing such as *reversing the inequality symbol*, which is reserved for multiplying or dividing by a negative value; students are simply rewriting an already correct inequality statement.

Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

Questions to ask

- Did Curran and Ajani arrive at different answers or the same answer?
- Did each example of student work involve a sign change? Why?
- Why did the inequality sign change in Ajani's second step, but didn't change until Curran's fourth step?
- Whose solution path is longer? Why?
- Which solution method do you prefer? Why?

Summary

There are multiple solution paths when solving more complex linear inequalities. All correct solution paths involve the same steps as solving a linear equation, as well as reversing the sign of the inequality symbol when both sides of the inequality are multiplied or divided by a negative number.

DEMONSTRATE

Talk the Talk: It's About Solutions, More or Less

Facilitation Notes

In this activity, students solve four inequalities and graph each solution on a number line. They also identify the constraints required to make true inequality statements.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

- Confusion dealing with the negative sign with the fraction.
- Errors regarding inequality sign changes when multiplying or dividing by a negative number.
- Issues dealing with the variable on the right side of the inequality symbol.

Differentiation strategies

To scaffold support,

- Suggest a strategy for abstract problems such as these. First, have students use values such as $A = 3$ and $B = 4$. Then, suggest they try positive numbers, negative numbers, 0, 1, -1 and fractions for C.
- Suggest a range and interval to number the number lines.

Questions to ask

- What inequalities require the reversal of the inequality sign?
- What operations and properties are used to solve the inequality?
- Is division or multiplication by a negative number used to solve the inequality?
- What is another way to solve this inequality?
- How did you determine the values on the tick marks on each number line?
- Is the statement true for all values of C? Only positive values of C? Only negative values of C? Only when $C = 0$?

Summary

There are multiple solution paths when solving more complex linear inequalities. All correct solution paths involve the same steps as solving a linear equation, as well as reversing the sign of the inequality symbol when both sides of the inequality are multiplied or divided by a negative number.

NOTES

3 Not All Statements Are Made Equal

Modeling Linear Inequalities

Warm Up

Solve each statement for x.

1. $x + 1 = 6$

2. $x - 2 > 5$

3. $\frac{x}{3} \leq -1$

4. $-x > 4$

Learning Goals

- Write and solve inequalities.
- Analyze a graph on a coordinate plane to solve problems involving inequalities.
- Interpret how a negative rate affects how to solve an inequality.

Key Term

- solve an inequality

You have used horizontal lines on a graph and Properties of Equality with equations to solve problems that can be modeled with linear equations. Can you use these same methods to solve problems involving linear inequalities?

Warm Up Answers

1. $x = 5$
2. $x > 7$
3. $x \leq -3$
4. $x < -4$

ELL Tip

Before students begin the lesson, review the four basic inequality symbols and ensure students' understanding of each sign. Create a chart on the board and list the four symbols as the column headers: $<$, $>$, $\leq$, and $\geq$. Ask students to provide several words that represent each symbol. For example, for *less than or equal to*, students may use *fewer*, *under*, *no more than*, and *at most*. Provide examples of inequalities using the four symbols and ask students to translate each inequality into words.

Answers

1. $f(b) = 3.75b + 25$

2a. The independent quantity is the number of boxes sold, and the dependent quantity is the total sales, in dollars.

2b. The slope is 3.75. It represents the cost $3.75 for each box of popcorn.

2c. The y-intercept is 25. This represents the $25 credit toward the total sales each troop member receives.

GETTING STARTED

Fundraising Function

Alan's camping troop is selling popcorn to earn money for an upcoming camping trip. Each camper starts with a credit of $25 toward his sales, and each box of popcorn sells for $3.75.

1. Write a function, $f(b)$, to show Alan's total sales as a function of the number of boxes of popcorn he sells, b.

Ask yourself:

How will you represent $25 credit in your function?

2. Analyze the function you wrote.

a. Identify the independent and dependent quantities and their units.

b. What is the slope of the function? What does it represent in this problem situation?

c. What is the y-intercept? What does it represent in this problem situation?

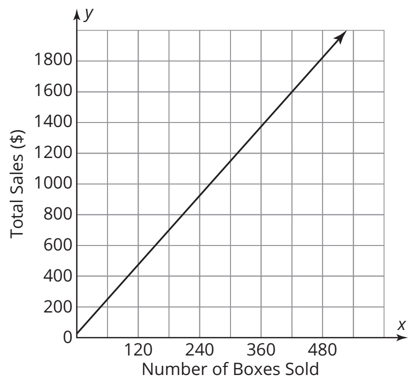
ELL Tip

Review the terms *independent quantity* and *dependent quantity*. Ask students to define the terms *independent* and *dependent* in a non-mathematical context, and then ask them to connect the terms in the context of the quantities in the given function. Review the problem in the section and clarify any additional misunderstandings.

ACTIVITY **3.1**

Modeling Linear Inequalities

The graph shown represents the function you wrote in the previous activity, $f(b) = 3.75b + 25$.

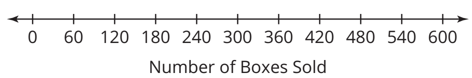


1. Suppose Alan has a sales goal of $1600.

a. Draw a horizontal line from the *y*-axis until it intersects with the graphed function to determine the point on the graph that represents $1600 in total sales.

b. How many boxes would Alan have to sell to make $1600 in total sales? Explain your reasoning.

c. Use the number line to represent the number of boxes sold if the total sales is equal to $1600.

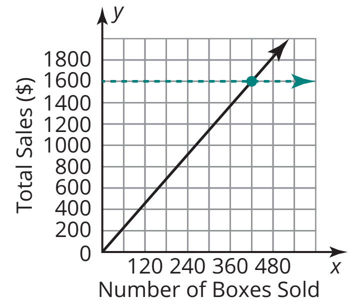


Think about:

Will the points you graph on the number line be open or closed?

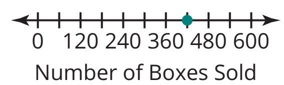
Answers

1a.



1b. 420 boxes; I drew a vertical line down from where the horizontal line intersected the graph and saw that it intersected the *x*-axis at 420.

1c.

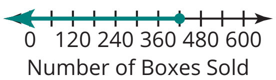


Answers

2a. It represents all the numbers of boxes sold, *b*, that would earn Alan \$1600 or less.

2b. $f(b) \leq 1600$ or $b \leq 420$

2c.

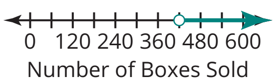


2d. No, only positive integers less than or equal to 420 make sense in the problem situation because Alan cannot sell negative or fractional boxes of popcorn

3a. It represents all the numbers of boxes sold, *b*, that would earn Alan more than \$1600.

3b. $f(b) > 1600$ or $b > 420$

3c.



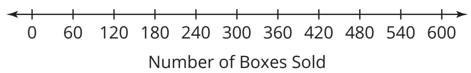
3d. No, only positive integers greater than 420 make sense in the problem situation because Alan cannot sell fractional boxes of popcorn.

2. Analyze the region of the graph that lies below the horizontal line you drew, up to and including the point intersected by the line.

a. What does this region of the graph represent?

b. Write an inequality to represent this region.

c. Use the number line to represent the number of boxes that are solutions to the inequality you wrote.



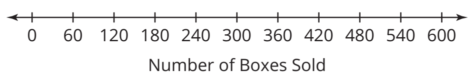
d. Do all the solutions make sense in context of the problem? Explain your reasoning.

3. Analyze the region of the graph that lies above the horizontal line you drew, not including the point intersected by the line.

a. What does this region of the graph represent?

b. Write an inequality to represent this region.

c. Use the number line to represent the number of boxes that are solutions to the inequality you wrote.



d. Do all the solutions make sense in context of the problem? Explain your reasoning.

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4. **Explain the difference between the open and closed circles on your number lines.**

Ask yourself:

How does determining the intersection point help you determine your answers?

5. **Use the graph to answer each question. Write an equation or inequality statement for each.**

   a. **How many boxes would Alan have to sell to earn at least $925?**

   b. **How many boxes would Alan have to sell to earn less than $2050?**

   c. **How many boxes would Alan have to sell to earn exactly $700?**

Answers

4. The open circle means that the point is not included in the solution.

   The closed circle means that the point is included in the solution.

5a. Alan would have to sell at least 240 boxes. $b \geq 240$ or $3.75b + 25 \geq 925$

5b. Alan would have to sell fewer than 540 boxes. $b < 540$ or $3.75b + 25 < 2050$

5c. Alan would have to sell exactly 180 boxes. $b = 180$ or $3.75b + 25 = 700$

Answer

1. Alan needs to sell at least 287 boxes of popcorn. Alan can't sell partial boxes of popcorn. If he sold 286 boxes, that would not be enough to earn at least $1100.

ACTIVITY **3.2**

Solving Two-Step Linear Inequalities

To **solve an inequality** means to determine the values of the variable that make the inequality true.

Another way to determine the solution set of any inequality is to solve it algebraically. The objective when solving an inequality is similar to the objective when solving an equation: You want to isolate the variable on one side of the inequality symbol.

To make the first deposit on the trip, Alan's total sales, $f(b)$, need to be at least $1100.

Worked Example

You can set up an inequality and solve it to determine the number of boxes Alan needs to sell.

$$f(b) \geq 1100$$
$$3.75b + 25 \geq 1100$$

Solve the inequality in the same way you would solve an equation.

$$3.75b + 25 \geq 1100$$
$$3.75b + 25 - 25 \geq 1100 - 25$$
$$3.75b \geq 1075$$
$$\frac{3.75b}{3.75} \geq \frac{1075}{3.75}$$
$$b \geq 286.66\ldots$$

Think about:

How accurate does your answer need to be?

1. **How many boxes of popcorn does Alex need to sell to make the first deposit? Explain your reasoning.**

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2. **Write and solve an inequality for each. Show your work.**

   a. **What is the greatest number of boxes Alan could sell and still not have made $600 in sales?**

   b. **At least how many boxes would Alan have to sell to make $1500 in sales?**

3. **The Worked Example showed how to solve an inequality of the form, $y = ax + b$. How does your method change if the inequality is in a different form?**

Answers

2a. $3.75b + 25 < 600$

$b < 153.33...$

Alan could sell at most 153 boxes of popcorn.

2b. $3.75b + 25 \geq 1500$

$b \geq 393.33...$

Alan would need to sell at least 394 boxes.

3. Because an inequality is solved in the same way as an equation, by using Properties of Equality and number properties, the method to solve an inequality does not change if it is written in a different form.

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Answers

1. $h(m) = -20m + 4800$
2. The independent quantity is the number of minutes hiked, and the dependent quantity is the elevation in feet.

3a. The slope is -20. This represents a decrease of 20 feet every minute.

3b. The y-intercept is 4800. The troop starts their hike at an elevation of 4800 feet.

3c. The x-intercept is 240. The hikers reach the bottom of the mountain in 240 minutes, or 4 hours.

3d. The domain is $0 \le x \le 240$. They begin their descent at 0 minutes and end it 240 minutes later.

ACTIVITY

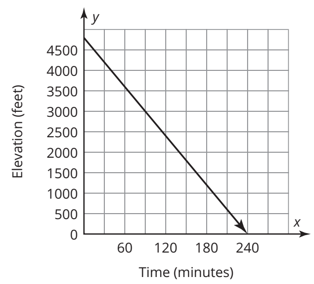
3.3

Reversing Inequality Signs

Sea level has an elevation of 0 feet.

Alan's camping troop hikes down from their campsite at an elevation of 4800 feet to the base of the mountain, which is at sea level. They hike down at a rate of 20 feet per minute.

1. Write a function, *h*(*m*), to show the troop's elevation as a function of time in minutes. Label the function on the coordinate plane.



2. Identify the independent and dependent quantities and their units.

3. Analyze the function. Identify each characteristic and explain what it means in terms of this problem situation.

a. slope

b. *y*-intercept

c. *x*-intercept

d. domain of the function

ELL Tip

Have students circle the verb or verb statement at the beginning of each question in the activity. Discuss what each verb means and have students suggest examples of wording they could use when answering the question.

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4. Use the graph to determine how many minutes passed if the troop is below 3200 feet. Draw an oval on the graph to represent this part of the function and write the corresponding inequality statement.

5. Write and solve an inequality to verify the solution set you interpreted from the graph.

6. Compare and contrast the solution sets you wrote using the graph and the function. What do you notice?

7. Analyze the relationship between the inequality statements representing *h*(*m*) and *m*.

a. Complete the table by writing the corresponding inequality statement that represents the number of minutes for each height.

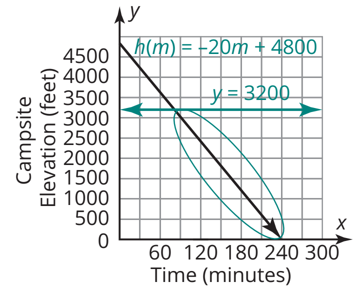
***h*(*m*)**	***m***
$h(m) > 3200$	
$h(m) \geq 3200$	
$h(m) = 3200$	
$h(m) < 3200$	
$h(m) \leq 3200$	

b. Compare each row in the table shown. What do you notice about the inequality signs?

c. Explain your answer from part (b). Use what you know about solving inequalities when you have to multiply or divide by a negative number.

Answers

4.



It appears from the graph that about 80 or more minutes have passed if the troop is below 3200 feet; $m > 80$

5. $-20m + 4800 < 3200$

$m > 80$

6. The solution set should be the same regardless of the strategy used.

7a.

***h*(*m*)**	***m***
$h(m) > 3200$	$m < 80$
$h(m) \geq 3200$	$m \leq 80$
$h(m) = 3200$	$m = 80$
$h(m) < 3200$	$m > 80$
$h(m) \leq 3200$	$m \geq 80$

7b. The inequality signs are reversed in each row.

7c. The function includes a negative coefficient for *x*. When I divide by the negative coefficient to solve the inequality, the sign reversed. That is why the inequality symbol is reversed in every row.

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Answers

1a. First, the expressions containing variables were moved to the left side of the inequality. Next, the constants were moved to the right side of the inequality. Finally, the inequality. Finally, the variable was isolated by dividing by 2.

1b. First, the expression containing variables were moved to the right side of the inequality. Next, the constants were moved to the left side of the inequality. Finally, the variable was isolated by dividing by −2 which reversed the sign of the inequality.

2. The process for solving is basically the same. In both, I have to move all the variables to one side and all the constants to the other side, and then isolate the variable. When I solve an inequality I have to pay attention to the inequality symbol. If I multiply or divide by a negative value the inequality symbol is reversed.

ACTIVITY **3.4**

Solving Other Linear Inequalities

The inequalities that you have solved in this lesson so far have all been two-step inequalities. Let's consider inequalities in different forms.

Allison and John each solved the inequality $5x + 2 \geq 3x - 10$.

Allison

$$5x + 2 \geq 3x - 10$$
$$5x - 3x + 2 \geq 3x - 3x - 10$$
$$2x + 2 \geq -10$$
$$2x + 2 - 2 \geq -10 - 2$$
$$2x \geq -12$$
$$\frac{2x}{2} \geq \frac{-12}{2}$$
$$x \geq -6$$

John

$$5x + 2 \geq 3x - 10$$
$$5x - 5x + 2 \geq 3x - 5x - 10$$
$$2 \geq -2x - 10$$
$$2 + 10 \geq -2x - 10 + 10$$
$$12 \geq -2x$$
$$\frac{12}{-2} \leq \frac{-2x}{-2}$$
$$-6 \leq x$$

1. Describe the process each student used to solve the inequality.

a. Allison

b. John

2. How does the process of solving an inequality with variables on both sides compare to the process of solving an equation with variables on both sides?

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ELL Tip

Ask students to list the types of properties that can be used when solving equations and inequalities. Discuss how the *Distributive Property* relates to the term *distribute*. Create a list of synonyms for *distribute*, such as *to give out*, *hand out*, and *dish out*. Ask students to demonstrate their understanding of the *Distributive Property* by explaining the steps in a simplified example, such as $2(3 + 4) = 2(3) + 2(4)$.

Curran and Ajani each solved the inequality $-4(x - 6) < 22$.

Curran

$$-4(x - 6) < 22$$
$$-4x + 24 < 22$$
$$-4x + 24 - 24 < 22 - 24$$
$$-4x < -2$$
$$\frac{-4x}{-4} > \frac{-2}{-4}$$
$$x > \frac{1}{2}$$

Ajani

$$-4(x - 6) < 22$$
$$\frac{-4(x - 6)}{-4} > \frac{22}{-4}$$
$$x - 6 > -5\frac{1}{2}$$
$$x > \frac{1}{2}$$

3. Describe the process each student used to solve the inequality.

a. Curran

b. Ajani

4. How does the process of solving an inequality using the Distributive Property compare to the process of solving an equation using the Distributive Property?

Answers

3a. First, the Distributive Property was used to remove the parentheses. Next, the constants were moved to the right side of the inequality. Finally the variable was isolated by dividing by −4 which reversed the inequality symbol.

3b. First, both sides of the inequality were divided by −4, which reversed the inequality symbol. Next, the constants were moved to the right side of the inequality, isolating the variable.

4. The process of using the Distributive Property to remove parentheses from either an inequality or an equation is the same.

Answers

1a. $x \le -\frac{21}{2}$

1b. $x > -9$

1c. $x < -1$

1d. $x \ge 3$

2a. The statement is true for all values of A, B, and C.

2b. The statement is only true for values of $C < 0$.

2c. The statement is never true for any values of A or B.

NOTES

TALK the TALK

It's About Solutions, More or Less

1. Solve each inequality and graph the solution on the number line.

a. $-\frac{2}{3}x \ge 7$

b. $32 > 23 - x$

c. $2(x + 6) < 10$

d. $15 - 4x \le 3x - 6$

2. If $A < B$, identify the constraints to make each statement true.

a. $A + C < B + C$

b. $AC > BC$

c. $-A < -B$

Assignment

LESSON 3: Not All Statements are Made Equal

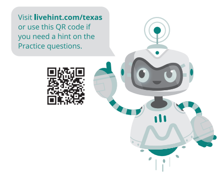
Write

Describe how to solve an inequality in your own words.

Remember

The methods for solving linear inequalities are similar to the methods for solving linear equations. Be sure to reverse the direction of the inequality symbol when multiplying or dividing both sides by a negative number.

Practice



1. Chang-Ho is going on a trip to visit some friends from summer camp. He will use \$40 for food and entertainment. He will also need money to cover the cost of gas. The price of gas at the time of his trip is \$3.25 per gallon.
   a. Write a function to represent the total cost of the trip as a function of the number of gallons used.
   b. Identify the independent and dependent quantities and their units.
   c. Identify the rate of change and the y-intercept. Explain their meanings in terms of the problem situation.
   d. Graph the function representing this situation on a coordinate plane.
   e. Use the graph to determine how many gallons of gas Chang-Ho can buy if he has \$170 saved for the trip. Draw an oval on the graph to represent the solution. Then write your answer in words and as an inequality.
   f. Verify the solution set you interpreted from the graph.
   g. Chang-Ho's mom gives him some money for his trip. He now has a total of \$220 saved for the trip. What is the greatest number of gallons of gas he can buy before he runs out of money? Show your work and graph your solution on a number line.
   h. If Chang-Ho spent more than \$92 on his trip, how much gas could he have bought? Show your work and graph your solution on a number line.
2. Chang-Ho is on his way to visit his friends at camp. Halfway to his destination, he realizes there is a slow leak in one of the tires. He checks the pressure and it is at 26 psi. It appears to be losing 0.1 psi per minute.
   a. Write a function to represent the tire's pressure as a function of time in minutes.
   b. Chang-Ho knows that if the pressure in a tire goes below 22 psi it may cause a tire blowout. What is the greatest amount of time that he can drive before the tire pressure hits 22 psi? Show your work and graph the solution.
3. Solve each inequality for the unknown value.
   a. $13 + 4x > 9$
   b. $3(4 - 5x) > 8x - 149$
   c. $99 - 5d \geq 4d$
   d. $3k - 9 \leq -6k - 225$

Assignment Answers

Write

Sample answer.
To solve an inequality means to isolate the variable on one side of the inequality sign to determine what values of the variable will make the inequality a true statement. All correct solution paths involve the same steps as solving a linear equation, as well as reversing the sign of the inequality symbol when both sides of the inequality are multiplied or divided by a negative number.

Practice

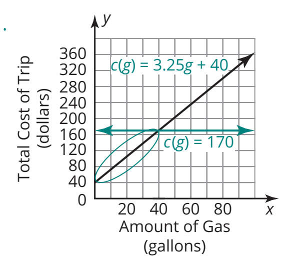
1a. $C(g) = 3.25g + 40$

1b. The independent quantity is the amount of gas used in gallons, and the dependent quantity is the total cost of the trip in dollars.

1c. The rate of change is 3.25. This represents the cost for each gallon of gas.

The y-intercept is 40. This represents the cost of the trip if no gas is used.

1d.



1e. See graph above.

Chang-Ho can buy no more than 40 gallons of gas on the trip.

$g \leq 40$

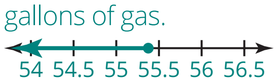
1f. $3.25g + 40 \leq 170$

$g \leq 40$

1g. $3.25g + 40 \leq 220$

$g \leq 55.38$

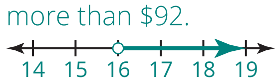
Chang-Ho can buy no more than 55.38 gallons of gas.



1h. $3.25g + 40 > 92$

$g > 16$

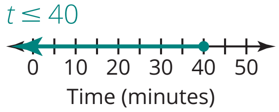
Chang-Ho bought more than 16 gallons of gas if he spent more than \$92.



2a. $p(t) = 26 - 0.1t$

2b. $26 - 0.1t \geq 22$

$t \leq 40$



Chang-Ho can drive a maximum of 40 minutes before the pressure goes below 22 psi.

3a. $x > -1$

3b. $x < 7$

3c. $d \leq 11$

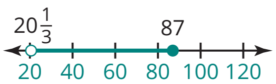
3d. $k \leq -24$

Assignment Answers

Stretch

$100 < 4.50x + 8.50 \leq 400$

$20\frac{1}{3} < x \leq 87$



Review

1.

$\frac{f(t) - f(s)}{t - s} = \frac{-4 - (-5)}{0 - (-4)} = \frac{1}{4}$

The rate of change is $\frac{1}{4}$.

2. Linear function

$f(x) = -\frac{1}{2}x + \frac{7}{2}$

3a. 200 cm^2

3b. $b_1 = \frac{2A - hb_2}{h}$

3c. $b_1 = 50m$

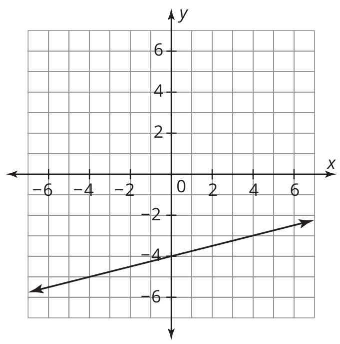
4. $b = \frac{2A}{h}$

Stretch

The Crunch Yum Company orders its nut mixes every month from a distributor. The distributor charges \$4.50 per pound of nut mix. There is a handling fee of \$8.50 for every order. There is free shipping on any order between \$100 and \$400. Write a compound inequality to represent the number of pounds of nuts the company can order and get free shipping. Solve the inequality and graph the solution on one number line.

Review

1. Calculate the average rate of change for the linear function using the rate of change formula. Show your work.



2. Determine whether the table of values represents a linear function. If so, write the function.

x	y
−2	$4\frac{1}{2}$
0	$3\frac{1}{2}$
3	2
6	$\frac{1}{2}$

3. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is its height and b_1 and b_2 are the lengths of each base.
   a. Determine the area of a trapezoid if its height is 10 cm and the lengths of its bases are 22 cm and 18 cm.
   b. Rewrite the equation to solve for b_1.
   c. Determine the length of the other base of a trapezoid if one base measures 10 m, the height is 20 m, and the area of the trapezoid is 600 square meters.
4. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Convert the equation to solve for b.

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