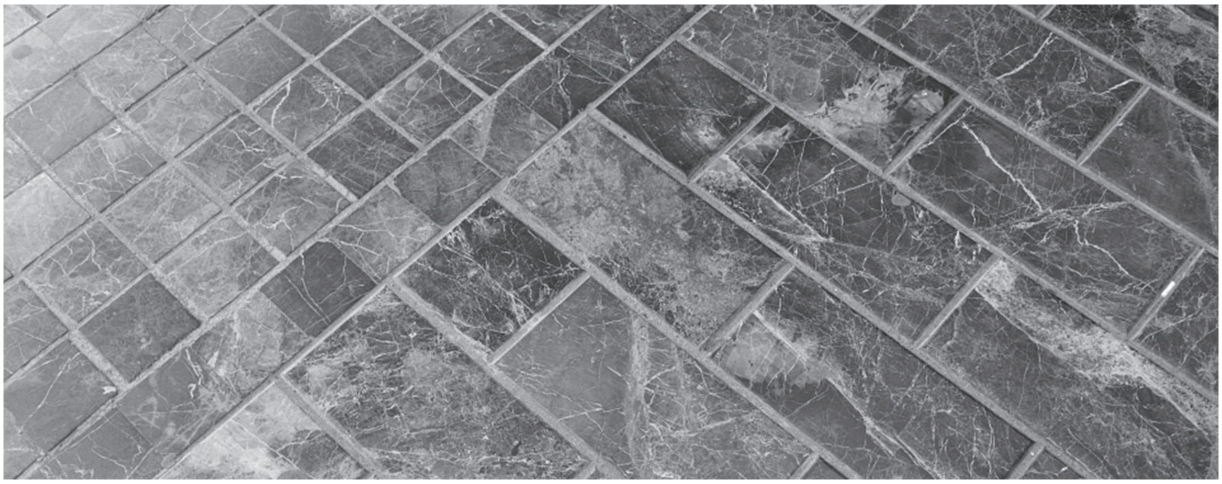
TOPIC 3

Systems of Equations and Inequalities



Multiple lines (a system of equations) can define all sorts of regions (systems of inequalities). Imagine graphing this tile pattern!

Systems of Equations and Inequalities

Topic 3 Overview

How is *Systems of Equations and Inequalities* organized?

In this topic, students begin with writing a system of linear equations to represent scenarios and learning to solve systems algebraically using substitution. Next, students write systems of linear equations and solve them graphically. Students have experience with this concept from middle school, and this topic reminds them of what they already know. Next, they work with a system of linear equations written in standard form. This builds upon what they learned about standard form in Topic 1, *Linear Functions,* and prepares them to solve systems using the linear combinations method. They analyze consistent systems of equations, which have either one solution or infinite solutions, and inconsistent systems, which have no solution. Students then move on to solve systems of linear equations using the linear combinations method. They first analyze and solve systems where two equations share a variable whose coefficient in one equation is the additive inverse of its coefficient in the other; then students learn to transform one or both equations to use this method. Many of the systems are in context, and students are asked to verify and interpret their solution in terms of the problem situation.

Students then consider linear inequalities in two variables. They extend their knowledge of solutions to understand that the solution to an inequality in two variables is half of a plane. They then graph two linear inequalities on the same plane and identify the solution set as the intersection of the corresponding half-planes.

Finally, students synthesize their understanding of systems by encountering several problems that can be solved using either a system of linear equations or a system of linear inequalities. Students decide which type of system is required and choose a reasonable and efficient solution strategy. They are asked to show their work and report their solutions in terms of the problem situation.

What is the entry point for students?

Coming into this topic, students know that every point on the graph of an equation represents a value that make the equation true. In grade 8, they learned that the point of intersection of two graphs provides x- and y-values that make both equations true. Students have written systems of linear equations and have solved them graphically. That knowledge is a springboard for this topic.

Students have also solved one-variable inequalities and graphed their solutions on a number line. Their work with two-variable inequalities in this topic will connect to their understanding of inequalities in one variable and their knowledge of graphing a line on the coordinate plane. The new knowledge in this topic is developed by combining the knowledge of these two different concepts from previous grades.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Systems of Equations and Inequalities* if they can:

- Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane.
- Write an equation, an inequality, or a system of linear equations or inequalities that best models a problem.
- Graph systems of equations on a coordinate grid with appropriate labels and scales.
- Solve a system of two linear equations in two variables approximately (graphically) and exactly (algebraically).
- Solve a system of linear equations algebraically using substitution and the linear combinations method.
- Interpret the meaning of the solution to a system of linear equations in terms of a problem situation.
- Identify linear systems that have no solution and explain why they have no solution.
- Identify linear systems that have infinite solutions and explain why they have infinite solutions.
- Write and graph an inequality and a system of inequalities in two variables on a coordinate plane.
- Understand that the solution of a system of linear inequalities is the intersection of the shaded regions of both inequalities.
- Interpret the solution to an inequality in two variables graphed on a coordinate plane.

Why is *Systems of Equations and Inequalities* important?

Systems of equations and inequalities provide students with a way to compare and contrast similar real-world situations. They allow students to consider and represent constraints on a real-world situation. Knowing how to solve systems of linear equations prepares students to solve systems that include nonlinear equations. In later courses, students may encounter more advanced methods, such as matrices or Cramer's Rule, to solve systems of equations with more than two variables and more than two equations.

How do the activities in *Systems of Equations and Inequalities* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so productive mathematical process standards are evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Throughout this topic, students model real-world situations using equations and inequalities. They reason about the coefficients as they rewrite systems to use substitution or the linear combinations methods to solve. Student examine the structure of the equations to recognize when systems represent problems with one solution, no solution, or infinite solutions. Students use the structure of inequalities to determine which half-plane represents the solution set. Finally, students reason about situations to determine which type of system each represents and which solution strategy is the most efficient to use.

Materials Needed

None

New Tools and Notation

The linear combinations method is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. To use this method, the corresponding coefficients of a given variable in a system must be additive inverses.

For example, to solve the system of equations shown, both equations can be rewritten as equivalent equations with coefficients that are additive inverses. Then, the two equations can be added together resulting in an equation with one variable to solve.

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

$$\begin{aligned} -5(4x + 2y) &= -5(3) \\ 4(5x + 3y) &= 4(4) \end{aligned}$$

$$\begin{aligned} -20x - 10y &= -15 \\ 20x + 12y &= 16 \\ \hline 2y &= 1 \\ y &= \tfrac{1}{2} \end{aligned}$$

$$\begin{aligned} 4x + 2\left(\tfrac{1}{2}\right) &= 3 \\ 4x + 1 &= 3 \\ 4x &= 2 \\ x &= \tfrac{1}{2} \end{aligned}$$

Learning Together

ELPS: 1.A, 1.C, 1.D, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Lesson	Lesson Name	TEKS	Days	Highlights
1	The County Fair: Using Substitution to Solve Linear Systems	A.2I A.3F A.3G A.5C	2	Students use the substitution method to solve systems of linear equations. They use substitution to solve systems of linear equations including those with no solution or with infinite solutions. Students define variables, write systems of equations, solve systems, and interpret the meaning of the solution in terms of the problem context. In the last activity they are given four systems of linear equations and solve each system using the substitution method.
2	Double the Fun: Introduction to Systems of Equations	A.2A A.2C A.2I A.3F A.3G A.5C	2	Students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They graph the linear equation using intercepts, and then analyze a second graph with the independent and dependent variables reversed. A new relationship between the quantities is then provided, and students write the equation expressing the relationship. Finally, they graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario creating a *system of linear equations*. A system of linear equations is defined. Students solve the system both graphically and using technology, checking the solution by substituting the values back in to the original equations. Next, they are provided three related scenarios in which they write systems of equations in general form and solve the systems graphically and algebraically using the substitution method. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions. The related terms *consistent systems* and *inconsistent systems* are defined.

Lesson	Lesson Name	TEKS	Days	Highlights
3	The Elimination Round: Using Linear Combinations to Solve a System of Linear Equations	A.2I A.5C	3	Students are given a problem scenario and use reasoning to determine the two unknowns. They then write a system of linear equations in standard form to represent a problem situation. Students analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They practice the linear combinations method with systems in which the coefficients of one variable are additive inverses. Next, Worked Examples guide students to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system, and then they solve two problems in context, one with fractional coefficients. The lesson concludes with students addressing when it is appropriate to use the graphing, substitution, or linear combinations methods.
4	Throwing Shade: Graphing Inequalities in Two Variables	A.2H A.3D	2	Students explore a linear inequality in two variables through a scenario. They write an inequality, complete a table of values, graph the coordinate pairs from the table, and determine which parts of the graph are solutions to the inequality. Students then formalize the process of graphing inequalities through practice without context; they graph the corresponding equation of an inequality as a boundary line, determine whether the line should be solid or dashed, and identify which half plane to shade by testing the point (0,0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs. They then solve a problem in context where they use a table of values to write and graph a linear inequality and refer to the inequality and/or its graph to respond to questions. Finally, students summarize the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.

Lesson	Lesson Name	TEKS	Days	Highlights
5	Working with Constraints: Systems of Linear Inequalities	A.2H A.3D A.3H	3	Students represent a scenario with a system of linear inequalities and graph the system. Overlapping shaded regions identify the possible solutions to the system. Students then practice graphing several systems of inequalities and representing the solution set. A different scenario is given that students model with a system of linear inequalities. They then graph the system, determine two different solutions, and algebraically prove that the solutions satisfy both constraints defined by the system. Finally, students match systems, graphs, and possible solutions of systems that have identical terms with different inequality symbols.
6	Working the System: Solving Systems of Equations and Inequalities	A.2H A.2I A.3D A.3H A.5C	2	Students solve problems in context requiring a system of linear equations. While most problems can be modeled by a system of two equations, they are guided through the process of solving a system of four equations, and another context can be modeled by a system of three equations. Students have the opportunity to solve the systems using any method and sometimes must respond in the format of an email or proposal. Solutions involve making a decision based upon inputs that lie before or after the point of intersection, thus requiring solutions written as inequalities.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1
TEKS: A.2I, A.3F, A.3G, A.5C
LESSON 1
The County Fair
GETTING STARTED
ACTIVITY 1

Day 2
LESSON 1 continued
ACTIVITY 2
ACTIVITY 3
TALK THE TALK

Day 3
MATHia
Use LiveLab and Reports to monitor students' progress

Day 4
TEKS: A.2A, A.2C, A.2I, A.3F, A.3G, A.5C
LESSON 2
Double the Fun
GETTING STARTED
ACTIVITY1
ACTIVITY 2

Day 5
LESSON 2 continued
ACTIVITY 3
TALK THE TALK

Day 6
MATHia
Use LiveLab and Reports to monitor students' progress

Day 7
TEKS: A.2I, A.5C
LESSON 3
The Elimination Round
GETTING STARTED
ACTIVITY 1

Day 8
LESSON 3 continued
ACTIVITY 2
ACTIVITY 3

Day 9
LESSON 3 continued
ACTIVITY 4
TALK THE TALK

Day 10
MATHia
Use LiveLab and Reports to monitor students' progress

Day 11
MID-TOPIC ASSESSMENT

Day 12
TEKS: A.2H, A.3D
LESSON 4
Throwing Shade
GETTING STARTED
ACTIVITY 1
ACTIVITY 2

Day 13
LESSON 4 continued
ACTIVITY 3
TALK THE TALK

Day 14
MATHia
Use LiveLab and Reports to monitor students' progress

Day 15
TEKS: A.2H, A.3D, A.3H
LESSON 5
Working with Constraints
GETTING STARTED
ACTIVITY 1

Day 16
LESSON 5 continued
ACTIVITY 2
ACTIVITY 3

Day 17
LESSON 5 continued
ACTIVITY 4
TALK THE TALK

Day 18
MATHia
Use LiveLab and Reports to monitor students' progress

Day 19
TEKS: A.2H, A.2I, A.3D, A.3H, A.5C
LESSON 6
Working the System
GETTING STARTED
ACTIVITY 1
ACTIVITY 2

Day 20
LESSON 6 continued
ACTIVITY 3
ACTIVITY 4
TALK THE TALK

Day 21
MATHia
Use LiveLab and Reports to monitor students' progress

Day 22
END OF TOPIC ASSESSMENT

Assessments

There are two assessments aligned with this topic: Mid-Topic Assessment and End of Topic Assessment.

Module 2: Exploring Constant Change

TOPIC 3: SYSTEMS OF EQUATIONS AND INEQUALITIES

In this topic, students begin by writing systems of linear equations and solving them graphically and algebraically using substitution. They then move on to solve systems of linear equations using the linear combinations method. Students consider linear inequalities in two variables and learn that their solutions are represented as half-planes on a coordinate plane. They then graph two linear inequalities on the same plane and identify the solution set as the intersection of the corresponding half-planes. Finally, students synthesize their understanding of systems by encountering problems that can be solved by using either a system of equations or a system of inequalities.

Where have we been?

Coming into this topic, students know that every point on the graph of an equation represents a value that makes the equation true. They have learned that the point of intersection of two graphs provides *x*- and *y*-values that make both equations true. Students have written systems of linear equations and have solved them graphically.

Where are we going?

Knowing how to solve systems of linear equations prepares students to solve systems that include nonlinear equations. In later courses, students may encounter more advanced methods, such as matrices or Cramer's Rule, to solve systems of equations with more than two variables and more than two equations.

Linear Combinations

The linear combinations method is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable.

For example, to solve the system of equations shown, both equations can be rewritten as equivalent equations with coefficients that are additive inverses. Then, the two equations can be added together to eliminate one of the variables. After solving for the remaining variable, substitution can be used to determine the value of the other variable.

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

$$\begin{aligned} -5(4x + 2y) &= -5(3) \\ 4(5x + 3y) &= 4(4) \end{aligned}$$

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$$\begin{aligned} 4x + 2\left(\tfrac{1}{2}\right) &= 3 \\ 4x + 1 &= 3 \\ 4x &= 2 \\ x &= \tfrac{1}{2} \end{aligned}$$

$$\left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$

Optimizing

A coffee company produces just two flavors of coffee: cinnamon creme and regular dark roast. The company expects a demand of at least 100 bags of cinnamon creme and 80 bags of dark roast each day. Yet, no more than 200 bags of cinnamon creme and 170 bags of dark roast can be made every day. To satisfy a shipping contract, a total of at least 200 bags of coffee must be shipped each day.

If each bag of cinnamon creme sold results in a $2 loss, but each bag of dark roast sold produces a $5 profit, how many bags of each should be made every day to maximize profits?

Companies solve problems like this every day, and they do so using systems of equations and inequalities.

Talking Points

Systems of equations is an important topic to know about for college admissions tests.

Here is a sample question:

If (x, y) is a solution to the system of equations, what is the value of $x - y$?

$$\begin{aligned} 2x - 3y &= -14 \\ 3x - 2y &= -6 \end{aligned}$$

Multiplying the first equation by 3 and the second equation by −2 gives

$$\begin{aligned} 6x - 9y &= -42 \\ -6x + 4y &= 12 \end{aligned}$$

Then, adding the equations gives

$$\begin{aligned} -5y &= -30 \\ y &= 6 \end{aligned}$$

The value of y can be substituted in one of the equations to get the value of x.
The solution is (4, 6), so $x - y = -2$.

Key Terms

consistent system
Systems that have one or many solutions are called consistent systems.

inconsistent system
Systems with no solution are called inconsistent systems.

half-plane
The graph of a linear inequality in two variables is a half-plane, or half of a coordinate plane.

boundary line
A boundary line, determined by an inequality, divides the plane into two half-planes and the inequality symbol indicates which half-plane contains all the solutions.

1

The County Fair

Using Substitution to Solve Linear Systems

MATERIALS

None

Lesson Overview

Students use the substitution method to solve systems of linear equations. They use substitution to solve systems of linear equations including those with no solution or with infinite solutions. Students define variables, write systems of equations, solve systems, and interpret the meaning of the solution in terms of the problem context. In the last activity they are given four systems of linear equations and solve each system using the substitution method.

Algebra 1
Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

(G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

ELPS

1.A, 1.C, 1.D, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- The substitution method is a process for solving a system of equations. It is an alternative method to graphing, especially when the solution is difficult to read from a graph.
- To use the substitution method, it is useful if at least one equation is written in slope-intercept form.
- When a system has no solution, the equation resulting from the substitution step has no solution.
- When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.
- Problem situations can be expressed using systems of equations and solved for unknown quantities using substitution methods.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Goats, Chickens, and Pigs

Students are given a context with three unknowns. Because they have not yet encountered 3-variable problems, they must employ problem-solving strategies and reasoning skills in addition to algebraic methods, such as defining variables and writing equations. The problem-solving strategies required to solve this problem hint to the informal use of substitution, which is the strategy for solving systems of two equations that will be addressed in this lesson.

Develop

Activity 1.1: Introduction to Substitution

Students learn to solve a system of linear equations using substitution. They write and graph a system of linear equations that has one equation written in standard form and the other in slope-intercept form. Because the decimal solution cannot be read accurately from the graph, students use substitution to solve the system.

Day 2

Activity 1.2: Substitution with Special Systems

In this activity, students write and use substitution to solve a system of linear equations with no solution and a system of linear equations with infinitely many solutions. Connections are made between solving these systems graphically and solving them algebraically.

Activity 1.3: Solving Systems by Substitution

Students write systems of linear equations to represent real-world situations, use substitution to solve them, and interpret the solutions.

Demonstrate

Talk the Talk: The Substitution Train

In this activity, students solve systems of linear equations using the substitution method and check their answers algebraically.

ENGAGE

Getting Started: Goats, Chickens, and Pigs

Facilitation Notes

In this activity, students are given a context with three unknowns. Because they have not yet encountered 3-variable problems, they must employ problem-solving strategies and reasoning skills in addition to algebraic methods, such as defining variables and writing equations. The problem-solving strategies required to solve this problem hint to the informal use of substitution, which is the strategy for solving systems of two equations that will be addressed in this lesson.

Ask a student to read the scenario aloud. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- General problem-solving strategies, such as drawing diagrams, creating notation to show trades, or remembering steps.
- Representing the trades using equations, and using substitution to make comparisons.

Differentiation strategy

Have students present their solution strategies, being selective in the order they are presented. Begin with diagram models and build to more abstract algebraic models. Ask questions to make connections among the models.

Questions to ask

- Did you use 3 different variables in this situation?
- Explain how you could use substitution to solve this problem.
- How can you tell from your equations if it is a fair deal or not?
- If one farmer trades 4 goats for 5 chickens and a second farmer trades 3 goats for 5 chickens is this fair?
- Which farmer should include 1 additional goat in the deal to make it fair?

Summary

Systems of equations can involve more than two variables and can be solved using different reasoning strategies.

Activity 1.1
Introduction to Substitution

DEVELOP

Facilitation Notes

In this activity, students learn to solve a system of linear equations using substitution. They write and graph a system of linear equations that has one equation written in standard form and the other in slope-intercept form. Because the decimal solution cannot be read accurately from the graph, students use substitution to solve the system.

Ask a student to read the introduction and the given scenario aloud. Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Questions to ask

- Why did it make sense to write this equation in standard form?
- What information in the problem situation helped you determine the coefficient of x in the equation written in standard form?
- What information in the problem situation helped you determine the coefficient of y in the equation written in standard form?
- What information in the problem situation helped you determine the constant in the equation written in standard form?
- Is the second equation $y = 8x$ or $x = 8y$? Explain.
- How much will 1 pound of onions and 8 pounds of potatoes cost?
- What information in the problem situation helped you determine the upper and lower bounds?
- Where is the solution to the linear system of equations located on the graph?

Ask a student to read the information and definition aloud. Analyze and discuss the Worked Example and complete Questions 7 through 9 as a class.

Differentiation strategy

To scaffold support, have students interact with the Worked Example.

- Have them label the equations first and second equation according to the Worked Example.
- For step 1, have them draw a box around $8x$ in the equation $y = 8x$ to make it explicit what they are substituting for y.
- After going through the Worked Example, have them redo the steps in the margin by following the directions to solidify the process in their minds.
- Have students write the answer as an ordered pair. This step is automatic for students when reading an answer from a graph. They should be comfortable writing answers in ordered pair notation regardless of the method used to solve the system of equations.

Questions to ask

- Why is it easier to use the equation in slope-intercept form rather than the equation in standard form?
- Why does it make more sense to substitute $8x$ into the first equation for y rather than substituting $\frac{y}{8}$ into the first equation for x?
- Does x represent the pounds of potatoes or the pounds of onions?
- Why does it make sense to substitute $8x$ for y in the context of the problem?
- Why is there only one variable in the equation after completing the substitution?
- Why does it make more sense to substitute the answer for x into the equation $y = 8x$ rather than $1.25x + 1.05y = 30$?
- How do you check to make sure your solution is correct?
- What are the advantages to using the substitution method rather than graphing to determine the solution to a system of equations?

Differentiation strategy

To extend the activity, discuss the flexibility and power of the Multiplication Property of Equality. Consider the equation $1.25x + 1.05y = 30$. This equation can be multiplied by 100 to rewrite the decimals as integers, leading to $125x + 105y = 3000$. Discuss these facts:

- The graph of the original line and the new line are the same. Have students demonstrate this through showing both equations have the same x- and y-intercepts or that they have the same slope and y-intercept.
- Although the equation is multiplied by 100, the other equation in the system does not need to be multiplied by 100. Because we demonstrated that multiplying the equation by 100 on both sides creates an equivalent equation, there is no need to change the other equation in the system.

Summary

The substitution method can be used to solve a system of equations. It is an alternative method to graphing, especially when the solution is difficult to read from a graph.

Activity 1.2
Substitution with Special Systems

Facilitation Notes

In this activity, students write and use substitution to solve a system of linear equations with no solution and a system of linear equations with infinitely many solutions. Connections are made between solving these systems graphically and solving them algebraically.

Have students work with a partner or in a group to complete Questions 1 through 9. Share responses as a class.

Questions to ask

- Is this equation written in standard form or slope-intercept form?
- Can this equation also be used to represent the amount of money that Adrian will earn in terms of the amount of hours he works?
- Is the first equation $y = 7x$ or $x = 7y$?
- How can 90 minutes be expressed in hours?
- Why is the second equation written differently than the first equation if both Samson and Adrian are getting paid the same amount per hour?
- Is the second equation written in standard form or slope-intercept form?
- If Samson works until noon, how many hours did he work?
- If Adrian works until noon, how many hours did he work?
- How much money do they lose if they take a half-hour lunch break?
- How much longer would Adrian have to work to make the same money as Samson?
- Why does it make more sense to substitute $7x$ into the second equation for y rather than substituting $\frac{y}{7}$ into the second equation for x?
- If you substituted $\frac{y}{7}$ into the second equation for x, do you think you would get the same answer?
- When you subtract $7x$ from both sides of the equation, what happens?
- What does the equation $0 = -10.5$ tell you about the problem situation?
- Looking at the slope in the two equations, what would you expect the graph of this linear system to look like?
- Where is the solution to the linear system of equations located on the graph?
- How can you tell from equations written in slope-intercept form that a linear system has no solution?

Have students work with a partner or in a group to complete Questions 10 through 16. Share responses as a class.

Questions to ask

- Is this equation written in standard form or slope-intercept form?
- Do both equations look the same?
- Do these equations describe the same line?
- What will the graph of this system of equations look like?

- How many values satisfy both equations if they are the same equation?
- Is the result a true or false mathematical statement?
- Is $0 = 0$ a true or false mathematical statement?
- Is any combination of student and adult tickets that sum to 5 a solution?

Misconceptions

Students sometimes misinterpret *infinitely many solutions* with the idea that any or all numbers are solutions to a system. Clarify this misunderstanding by having students provide ordered pairs that satisfy the equations $x + y = 5$ and $4x + 4y = 20$. Have them graph the ordered pairs to see that they all lie on the same line. Contrast this with ordered pairs that are not solutions to the system and do not lie on the line.

Summary

When a system has no solution, the equation resulting from the substitution step has no solution. When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

Activity 1.3
Solving Systems by Substitution

Facilitation Notes

In this activity, students write systems of linear equations to represent real-world situations, use substitution to solve them, and interpret the solutions.

Differentiation strategy

This lesson includes five problems. Create five stations in the classroom, with each station providing the information for one problem. Have students cycle through the stations, completing each problem within given time constraints.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask for Question 1

- How many adults and children bought admission tickets?
- What information in the problem situation helped you determine the attendance equation?
- Do all of the terms in this first equation represent numbers of people?
- What expression did you use to represent the money generated from the number of adult admissions?
- What expression did you use to represent the money generated from the number of child admissions?

- Do all of the terms in the second equation represent an amount of money?
- Does it make a difference which variable, x or y, we solve for?
- Looking at the two equations, what would you expect the graph of this linear system to look like?
- Where is the solution to the linear system of equations located on the graph?
- How would you write this solution as an ordered pair?
- How can you tell from the equations that this linear system has a solution?

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

Questions to ask for Question 2

- Which equation makes more sense in this situation, $y = x + 20$ or $x = y + 20$?
- Are all of the terms in one of the equations associated with money?
- Is 4000 seats used to write either equation?
- Did you solve this system of linear equations for the value of x or the value of y?
- Is it easier to solve this equation for the value of x or the value of y? Explain.
- How is substitution used to solve this system of equations?
- How would you write this solution as an ordered pair?

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

Questions to ask for Question 3

- Which equation makes more sense in this situation, $x + y = 20$ or $x + y = 100$?
- Are all of the terms in one of the equations associated with the number of questions on the test?
- Are all of the terms in one of the equations associated with the number of points on the test?
- Did you solve this system of linear equations for the value of x or the value of y?
- Is it easier to solve this equation for the value of x or the value of y?
- How is substitution used to solve this system of equations?
- How would you write this solution as an ordered pair?

Questions to ask for Question 4

- Which equation makes more sense in this situation, $2x + 2y = 48$ or $2x + 2y = 40$?
- Is it easier to solve one of the equations for the value of x or the value of y?

- Did you solve this system of linear equations for the value of x or the value of y?
- How is substitution used to solve this system of equations?
- How would you write this solution as an ordered pair?

Questions to ask for Question 5

- What information in the problem situation helped you to determine the equation describing the first job offer? What is the constant? What is changing?
- Is the equation written in standard form or slope-intercept form?
- What information in the problem situation helped you to determine the equation describing the second job offer? What is the constant? What is changing?
- How would you write this solution as an ordered pair?
- What is a good reason for Alex to take the first job offer?
- What is a good reason for Alex to take the second job offer?
- What is a reason Alex should not take the first job offer?
- What is a reason Alex should not take the second job offer?

Summary

The substitution method can be used to solve systems of linear equations that represent real-world situations.

DEMONSTRATE

Talk the Talk: The Substitution Train

Facilitation Notes

In this activity, students solve systems of linear equations using the substitution method and check their answers algebraically.

Have students complete Question 1 individually. Share responses as a class.

Questions to ask

- How did you determine which equation to use first to isolate for one variable?
- How would you write your solution as an ordered pair?
- How did you check your answers?

Summary

The substitution method can be used to solve systems of linear equations, regardless of whether they are written in standard form or slope-intercept form.

The County Fair

1

Using Substitution to Solve Linear Systems

Warm Up

Analyze each system of equations. What can you conclude about the value of y in each?

1. $\begin{cases} x = 12 \\ y = x + 22 \end{cases}$
2. $\begin{cases} x = 0 \\ y = x - 45 \end{cases}$
3. $\begin{cases} x = y \\ y = 2x - 10 \end{cases}$
4. $\begin{cases} x = y + 3 \\ y = 2x - 10 \end{cases}$

Learning Goals

- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using substitution.
- Interpret a solution to a system of linear equations in terms of the problem situation.
- Solve real-world and mathematical problems with two linear equations in two variables.

Key Terms

- system of linear equations
- standard form of a linear equation
- substitution method

Suppose you graph a system of equations, but the point of intersection is not clear from the graph? How can you determine the solution to the system?

Warm Up Answers

1. $y = 34$
2. $y = -45$
3. $y = 10$
4. $y = 4$

Answers

1. This is not a fair trade. Farmer Lyndi's animals are worth more than Farmer Simpson's animals. To make this a fair trade, Farmer Simpson should include 1 more goat.

GETTING STARTED

Goats, Chickens, and Pigs

At the county fair, farmers bring some of their animals to trade with other farmers. To make all trades fair, a master of trade oversees all trades. Assume all chickens are of equal value, all goats are of equal value, and all pigs are of equal value.

- In the first trade of the day, 4 goats were traded for 5 chickens.
- In the second trade, 1 pig was traded for 2 chickens and 1 goat.
- In the third trade, Farmer Lyndi put up 3 chickens and 1 pig against Farmer Simpson's 4 goats.

1. Is this a fair trade? If not, whose animals are worth more? How could this be made into a fair trade?

ACTIVITY

1.1

Introduction to Substitution

In this lesson, you will explore *systems of equations*. When two or more linear equations define a relationship between quantities, they form a **system of linear equations**.

The **standard form of a linear equation** is $Ax + By = C$, where A, B, and C are constants and A and B are not both zero.

Janet was helping her mother make potato salad for the county fair and was asked to go to the market to buy fresh potatoes and onions. Sweet onions cost $1.25 per pound, and potatoes cost $1.05 per pound. Her mother told her to use the $30 she gave her to buy these two items.

1. **Write an equation in standard form that relates the number of pounds of potatoes and the number of pounds of onions that Janet can buy for $30. Use x to represent the number of pounds of onions, and y to represent the number of pounds of potatoes that Janet can buy.**

2. **Janet's mother told her that the number of pounds of potatoes should be 8 times greater than the number of pounds of onions in the salad. Write an equation in x and y that represents this situation.**

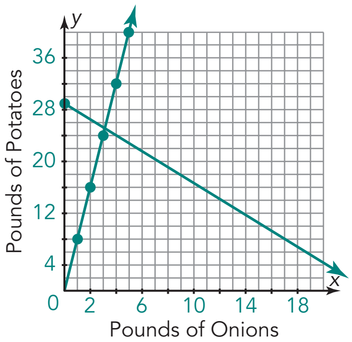
3. **Will 1 pound of onions and 8 pounds of potatoes satisfy both equations? Explain your reasoning.**

Answers

1. The equation in standard form is $1.25x + 1.05y = 30$.
2. $y = 8x$
3. No. Eight pounds of potatoes and one pound of onions does not satisfy both equations. Substituting the ordered pair (1, 8) into both equations makes the first equation false and the second equation true.

Answers

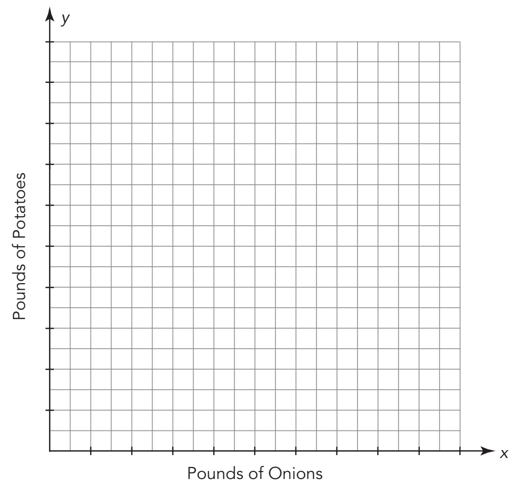
4. See table below.



5. No. The answer cannot be determined from the graph because the point of intersection does not fall exactly on the grid lines.
6. An estimate of the solution might be (3, 25).

4. Create graphs of both equations. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval



5. Can you determine the exact solution of this linear system from your graph? Explain your reasoning.

6. Estimate the point of intersection from your graph.

4.

Variable Quantity	Lower Bound	Upper Bound	Interval
Pounds of Onions	0	20	2
Pounds of Potatoes	0	40	2

In many systems, it is difficult to determine the solution from the graph. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

Worked Example

Let's consider the system you wrote.

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 1: To use the substitution method, begin by choosing one equation and isolating one variable. This will be considered the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation.

Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$.

Write the new equation.

$$1.25x + 1.05y = 30$$
$$1.25x + 1.05(8x) = 30$$

You have just created a new equation with only one unknown.

Step 3: Solve the new equation.

$$1.25x + 8.40x = 30$$
$$9.65x = 30$$
$$x \approx 3.1$$

Therefore, Janet should buy approximately 3.1 pounds of onions.

Now, substitute the value for x into $y = 8x$ to determine the value of y.

$$y = 8(3.1) = 24.8$$

Therefore, Janet should buy approximately 24.8 pounds of potatoes.

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept of the line.

Ask yourself:

Keep in mind what the value represents.

LESSON 1: The County Fair • 401

Answers

7. $1.25(3.1) + 1.05(24.8) \approx 30$
   $3.875 + 26.04 \approx 30$
   $29.915 \approx 30$
   $24.8 = 8(3.1)$
   $24.8 = 24.8$
8. The solution is (3.1, 24.8). For \$30, Janet should purchase about 3.1 pounds of onions and 24.8 pounds of potatoes. Rounded to the nearest pound, Janet should purchase 3 pounds of onions and 25 pounds of potatoes.
9. The two solutions are approximately equal.

NOTES

7. Check that the solution is correct. Show your work.

8. What is the solution to the system? What does it represent in terms of the problem situation?

9. Compare your solution using the substitution method to the solution on your graph. What do you notice?

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ACTIVITY **1.2**

Substitution with Special Systems

Samson and Adrian are helping to set up the booths at the fair. They are each paid $7 per hour to carry the wood that is needed to build the various booths. Samson arrives at 7:00 A.M. and begins working immediately. Adrian arrives 90 minutes later and starts working.

1. **Write an equation that gives the amount of money that Samson will earn, *y*, in terms of the number of hours he works, *x*.**

2. **How much money will Samson earn after 90 minutes of work?**

3. **Write an equation that gives the amount of money Adrian will earn, *y*, in terms of the number of hours since Samson started working, *x*.**

4. **How much money will each student earn by noon?**

Answers

1. $y = 7x$
2. Samson will earn $10.50 after 90 minutes of working.
3. $y = 7(x - 1.5) = 7x - 10.5$
4. Samson will earn $35 by noon, and Adrian will earn $24.50.

Answers

5. No. Adrian will not earn as much money as Samson because he started 90 minutes later. No matter how many hours they work, Adrian will always be working 1.5 hours less.
6. $y = 7x$
   $y = 7x - 10.5$
7. Because the slopes are the same, the lines are parallel. This means that the system has no solution because the lines will not intersect.

8a. $7x = 7x - 10.5$

8b. $0 = -10.5$

8c. No. Zero does not equal -10.5.

5. Will Adrian ever earn as much money as Samson? Explain your reasoning.

6. Write a system of linear equations for this problem situation.

Think about:

How is this similar to solving linear equations with no solution or with infinite solutions?

7. Analyze the system of linear equations. What do you know about the solution of the system by observing the equations? Explain your reasoning.

Let's see what happens when we solve the system algebraically.

8. Since both equations are written in slope-intercept form as expressions for *y* in terms of *x*, substitute the expression from the first equation into the second equation.

a. Write the new equation.

b. Solve the equation for *x*.

c. Does your result for *x* make sense? Explain your reasoning.

9. What is the result when you algebraically solve a linear system that contains parallel lines?

On Monday night, the fair is running a special for the the local schools: if tickets are purchased from the school, you can buy student tickets for $4 and adult tickets for $4. You buy 5 tickets and spend $20.

10. Write an equation that relates the number of student tickets, *x*, and the number of adult tickets, *y*, to the total amount spent.

11. Write an equation that relates the number of student tickets, *x*, and the number of adult tickets, *y*, to the total number of tickets purchased.

12. Write both equations in slope-intercept form.

13. Analyze the system of linear equations. What do you know about the solution of the system by looking at the equations?

Answers

9. The result is a false mathematical statement, such as $0 = -10.5$.
10. $4x + 4y = 20$
11. $x + y = 5$
12. $y = 5 - x$
    $y = 5 - x$
13. The equations are the same, so there will be infinite solutions.

Answers

14a. $5 - x = 5 - x$

$0 = 0$

14b. Yes. This makes sense. No matter the value of x, the mathematical sentence will always be true. So, x can be any number.

15. It is not possible to determine the number of each ticket purchase; any combination of student and adult tickets that sum to 5 is possible.

16. The result is a mathematical statement that is always true, such as $x = x$ or $0 = 0$.

NOTES

Let's see what happens when you solve the system algebraically.

14. Since both equations are now written in slope-intercept form as expressions for *y* in terms of *x*, substitute the expression from the first equation into the second equation.

a. Write the new equation and solve the equation for *x*.

b. Does your result for *x* make sense? Explain your reasoning.

15. How many student tickets and adult tickets did you purchase?

16. What is the result when you algebraically solve a linear system that contains two lines that are actually the same line?

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ACTIVITY **1.3**

Solving Systems by Substitution

Write and solve a system of equations to solve each problem.

1. The admission fee for the fair includes parking, amusement rides, and admission to all commercial, agricultural, and judging exhibits. The cost for general admission is \$7, and the price for children under the age of 5 is \$4. There were 449 people who attended the fair on Thursday. The admission fees collected amounted to \$2768.

a. Write a system of equations in standard form for this situation. Use *x* to represent the number of people 5 and over, and use *y* to represent the number of children under 5 years of age.

b. Without solving the system of linear equations, interpret the solution.

c. Solve the system of equations using the substitution method. Then interpret the solution of the system in terms of the problem situation.

Answers

1a. $x + y = 449$

$7x + 4y = 2768$

1b. The solution for the linear system will give the number of people 5 and over and the number of children under 5 years of age who attended the county fair on Thursday.

1c. (324, 125); There were 324 people 5 and over and 125 children under the age of 5 at the county fair on Thursday.

Answers

2a. $2800x + 1200y = 236{,}000$

$x = y + 20$

2b. The solution will represent the cost, in dollars, of the main-level tickets and the upper-level tickets needed to make the targeted total sales of $236,000.

2c. (65,45) In order to make the targeted total sales, the cost of main-level seating will be $65 and the cost of upper-level seating will be $45.

2. The business manager for a band must make $236,000 from ticket sales to cover costs and make a reasonable profit. The auditorium where the band will play has 4000 seats, with 2800 seats on the main level and 1200 on the upper level. Attendees will pay $20 more for main-level seats.

a. Write a system of equations with *x* representing the cost of the main-level seating and y representing the cost of the upper-level seating.

b. Without solving the system of linear equations, interpret the solution.

c. Solve the system of equations using the substitution method. Then interpret the solution of the system in terms of the problem situation.

3. **Ms. Ross told her class that tomorrow's math test will have 20 questions and be worth 100 points. The multiple-choice questions will be 3 points each, and the open-ended response questions will be 8 points each. Determine how many multiple-choice and open-ended response questions will be on the test.**

   a. **Write a system of equations. Describe your variables.**

   b. **Without solving the system of linear equations, interpret the solution.**

   c. **Solve the system of equations using the substitution method. Then interpret the solution of the system in terms of the problem situation.**

Answers

3a. Let x represent the number of multiple-choice questions and y represent the number of open-ended response questions.

$x + y = 20$

$3x + 8y = 100$

3b. The solution will represent the number of multiple-choice questions and the number of open-ended response questions on the 100-point test.

3c. (12, 8); There will be 12 multiple-choice questions and 8 open-ended response questions on the test.

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Answers

4. Let x be the cost of student tickets and let y be the cost of adult tickets.

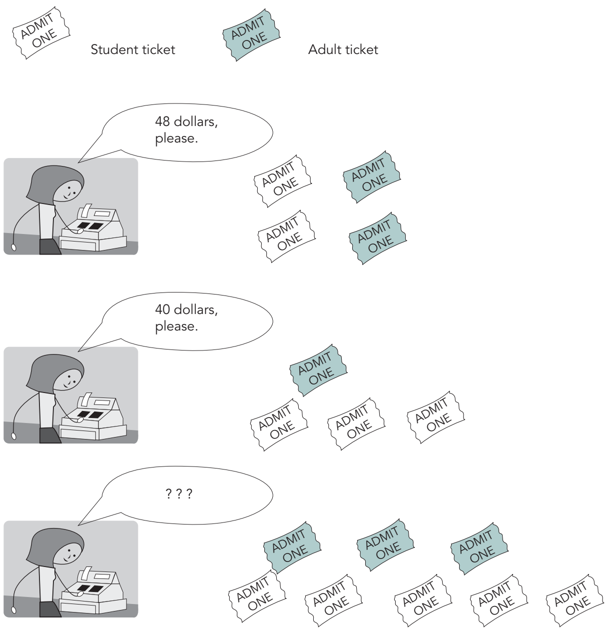
   $2x + 2y = 48$

   $3x + y = 40$

   (8, 16); Student tickets cost $8 each and adult tickets cost $16 each.

   The total cost of three adult and five student tickets will be $88.

4. Ashley is working as a cashier at the sports arena. What should she tell the next person in line?



Write and solve a system of equations that represents the problem situation. Define the variables. Then determine the cost of each type of ticket. Finally, state the amount Ashley charges the third person.

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5. **Alex is applying for positions at two different electronic stores in neighboring towns. The first job offer is a \$200 weekly salary plus 5% commission on sales. The second job offer is a \$75 weekly salary plus 10% commission.**

   a. **Write a system of equations that represents the problem situation. Define the variables. Then solve the system of linear equations and interpret the solution in terms of the problem situation.**

   b. **What is the difference in the weekly pay between stores if Alex sells \$3000?**

   c. **What is the difference in the weekly pay if he sells \$4225?**

   d. **Which job offer would you recommend Alex take? Explain your reasoning.**

Alex's sales targets for each job would be between \$1500 and \$3000 weekly. Each manager tells Alex the same thing: "Some weeks are better than others, depending on the time of year and the new releases of technology."

Answers

5a. Let x represent the amount of weekly sales, and let y represent the total salary for the week.

$y = 200 + 0.05x$

$y = 75 + 0.10x$

(2500, 325); If Alex sells \$2500 worth of electronics per week, he will earn \$325 at either electronics store.

5b. He would earn \$350 at the first store and he would earn \$375 at the second store. He would earn \$25 more at the second store.

5c. He would earn \$411.25 at the first store and he would earn \$497.50 at the second store. He would earn \$86.25 more at the second store.

5d. Sample answers. I would recommend Alex take the first job offer. The guaranteed weekly pay is higher and if most weeks of sales will be around \$3000, there is not that much difference in total pay between the two. But if it is a slow week, his pay would be higher. Or, I would recommend Alex take the second job offer. There is a greater potential to earn more money.

Answers

1a. $2x + 3(5x) = 34$
$2x + 15x = 34$
$17x = 34$
$x = 2$
$y = 5(2)$
$y = 10$
Check: $2(2) + 3(10) = 34$
$4 + 30 = 34$
$34 = 34$ ✓
The solution is (2, 10).

1b. $4x + 2 = 3x - 2$
$x + 2 = -2$
$x = -4$
$y = 4(-4) + 2$
$y = -16 + 2$
$y = -14$
Check: $-14 = 3(-4) -2$
$-14 = -12 - 2$
$-14 = -14$ ✓
The solution is (−4, −14).

1c. $3x + 2(2x - 5) = 4$
$3x + 4x - 10 = 4$
$7x - 10 = 4$
$7x = 14$
$x = 2$
$y = 2x - 5$
$y = 2(2) - 5$
$y = 4 - 5$
$y = -1$
Check: $3(2) + 2(-1) = 4$
$6 + (-2) = 4$
$4 = 4$ ✓
$2(2) - (-1) = 5$
$4 + 1 = 5$
$5 = 5$ ✓
The solution is (2, −1).

1d. $y = -3x + 8$
$6x + 2(-3x + 8) = 10$
$6x + (-6x) + 16 = 10$
$16 \neq 10$
There is no solution.

NOTES

TALK the TALK

The Substitution Train

1. Determine the solution to each linear system by using the substitution method. Check your answers algebraically.

a. $\begin{cases} 2x + 3y = 34 \\ y = 5x \end{cases}$

b. $\begin{cases} y = 4x + 2 \\ y = 3x - 2 \end{cases}$

c. $\begin{cases} 3x + 2y = 4 \\ 2x - y = 5 \end{cases}$

d. $\begin{cases} 3x + y = 8 \\ 6x + 2y = 10 \end{cases}$

Assignment

Write

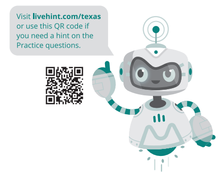
Explain how to use the substitution method to solve systems of linear equations.

Remember

When a system has no solution, the equation resulting from the substitution step has no solution.

When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

Practice



1. Serena is trying to become more environmentally conscious by making her own cleaning products. She researches different cleaners and decides to make furniture polish using olive oil and lemon juice. She wants to make enough to fill two 24-ounce bottles.
   a. Write an equation in standard form that relates the amount of olive oil and lemon juice to the total amount of mixture Serena wants to make. Use x to represent the amount of lemon juice and y to represent the amount of olive oil.
   b. The recommendation for the mixture is that the amount of olive oil be twice the amount of lemon juice. Write an equation in terms of x and y as defined in part (a) that represents this situation.
   c. Use substitution to solve the system of equations. Check your answer.
   d. What does the solution of the system represent in terms of the mixture?
   e. The best price Serena can find for lemon juice is \$0.25 per ounce. The best price she can find for olive oil is \$0.39 per ounce. She buys a total of 84 ounces of lemon juice and olive oil, and spends \$29.40. Write equations in standard form for this situation. Use x to represent the amount of lemon juice she buys, and use y to represent the amount of olive oil she buys.
   f. Solve the system of equations you wrote using the substitution method. Check your answer. Describe the solution in terms of the problem situation.
2. In an effort to eat healthier, Bridget is tracking her food intake by using an application on her phone. She records what she eats, and then the application indicates how many calories she has consumed. One day, Bridget eats 10 medium strawberries and 8 vanilla wafer cookies as an after-school snack. The caloric intake from these items is 192 calories. The next day, she eats 20 medium strawberries and 1 vanilla wafer cookie as an after-school snack. The caloric intake from these items is 99 calories.
   a. Write a system of equations for this problem situation. Define your variables.
   b. Without solving the system of linear equations, interpret the solution.
   c. Solve the system of equations using the substitution method. Check your work.
   d. Interpret the solution of the system in terms of the problem situation.
   e. Bridget's friend Monica also has a calorie counting application on her phone. The two friends decide to compare the two programs. Bridget eats 1 banana and 5 pretzel rods, and her application tells her she consumed 657 calories. Monica eats 1 banana and 5 pretzel rods, and her application tells her she consumed 656 calories. The girls want to know how many calories are in each food. Write a system of equations for this problem. Define your variables.
   f. Solve the system of equations using the substitution method. Interpret your answer in terms of the problem.

Assignment Answers

Write

Sample answer.
The substitution method is a process of solving a system of equations by substituting an equal expression for a variable in one equation. To use the substitution method, begin by choosing one equation and isolating one variable. Substitute the expression equal to the isolated variable into the second equation to create a new equation with only one unknown. Solve the new equation and substitute that value into the first equation to solve for the second variable.

Practice

1a. $x + y = 48$

1b. $y = 2x$

1c. The solution is (16, 32).

1d. Serena will need to use 16 ounces of lemon juice and 32 ounces of olive oil to make 48 ounces of the furniture polish mixture. The mixture will contain twice as much olive oil as lemon juice.

1e. Amount: $x + y = 84$
Price: $0.25x + 0.39y = 29.40$

1f. The solution is (24, 60). Serena bought 24 ounces of lemon juice and 60 ounces of olive oil. She spent \$29.40.

2a. Let x represent the number of calories in one strawberry and y represent the number of calories in one vanilla wafer cookie.
$10x + 8y = 192$
$20x + y = 99$

2b. The solution represents the number of calories in one strawberry and the number of calories in one vanilla wafer cookie.

2c. The solution is (4, 19).

2d. Each strawberry has 4 calories, and each vanilla wafer cookie has 19 calories.

2e. Let x represent the number of calories in a banana, and let y represent the number of calories in a pretzel rod.
$x + 5y = 657$
$x + 5y = 656$

2f. There is no solution to this system.
The applications must have a different number of calories allotted to a banana, a pretzel rod, or both.

Assignment Answers

3a. $n + q = 13$
$0.05n + 0.25q = 2.05$
(6, 7); James has 6 nickels and 7 quarters.

3b. $t + f = 28$
$2t + 4f = 100$
(6, 22); There will be 6 2-point questions and 22 4-point questions.

3c. $a + b = 38$
$2a + 3b = 82$
(32, 6); The team scored 32 2-point baskets and 6 3-point baskets.

4a. (1, 7)

4b. no solution

4c. $(\frac{1}{2}, -\frac{2}{5})$

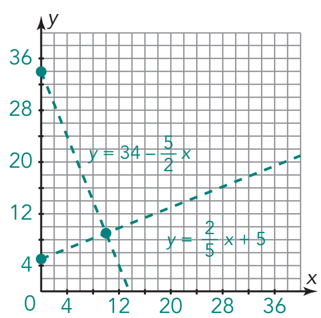
4d. infinite solutions

Stretch

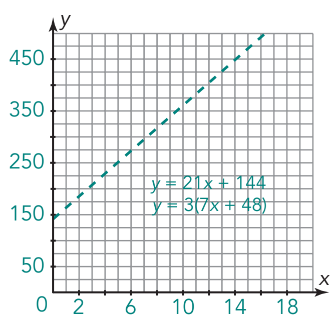
Answers will vary.

Review

1a. The solution is (10, 9).



1b. There are infinitely many solutions. The lines are the same.



3. Write a system of linear equations to represent each situation. Then solve the system using substitution. Interpret the solution of the system in terms of the problem situation.
   a. James has 13 coins. The coins are nickels and quarters. The coins have a total value of \$2.05. Let n represent the number of nickels, and let q represent the number of quarters.
   b. Ms. Snyder is giving a 28-question test that is made up of 2-point questions and 4-point questions. The entire test is worth 100 points. Let t represent the number of 2-point questions, and let f represent the number of 4-point questions.
   c. The basketball team scored 82 points from 2-point and 3-point baskets. They make 38 baskets altogether. Let a represent the number of 2-point baskets, and let b represent the number of 3-point baskets.
4. Use the substitution method to determine the solution of each system of linear equations. Check your solutions.
   a. $\begin{cases} 9x + y = 16 \\ y = 7x \end{cases}$
   b. $\begin{cases} 3x + \frac{1}{2}y = -3.5 \\ y = -6x + 11 \end{cases}$
   c. $\begin{cases} y = -5x \\ 21x - 7y = 28 \end{cases}$
   d. $\begin{cases} 2x + 4y = -32 \\ y = -\frac{1}{2}x - 8 \end{cases}$

Stretch

Create a system of linear equations with solution (2, 5). Solve the system using substitution to verify your system has the given solution.

Review

1. Graph each system of linear equations to determine the solution to the system.
   a. $y = 34 - \frac{5}{2}x$ and $y = \frac{2}{5}x + 5$
   b. $y = 21x + 144$ and $y = 3(7x + 48)$
2. The population growth (in thousands) for a small town near Bay City can be represented by the expression $x + \frac{4}{5}(x + 315)$, where x represents the number of years since 2005. The population growth (in thousands) for a neighboring town can be represented by the expression $2x - \frac{1}{5}(x - 630)$, where x represents the number of years since 2005. When will the populations of the two towns be the same?
3. Two neighboring towns are not having population growth. In fact, they both have been losing population since 1995. The population decline for one of the towns (in thousands) can be represented by the expression $-\frac{2}{5}(x - 500)$, where x represents the number of years since 1995. The population decline for the other town (in thousands) can be represented by the expression $-\frac{1}{2}x + \frac{1}{10}(x + 2000)$, where x represents the number of years since 1995. When will the populations of the two towns be the same?
4. Solve each equation.
   a. $8(2m + 7) = 10(m + 11)$
   b. $-3(y + 20) = -9y$

2. The two populations will never be equal because the equation formed from setting the two expressions equal to each other does not have a solution.
3. The populations of both towns will always be the same because the equation formed from setting the two expressions equal to each other is true for all values of x.

4a. $m = 9$

4b. $y = 10$

2

Double the Fun

Using Graphing to Solve Systems of Equations

MATERIALS

None

Lesson Overview

Students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They graph the linear equation using intercepts, and then analyze a second graph with the independent and dependent variables reversed. A new relationship between the quantities is then provided, and students write the equation expressing the relationship. Finally, they graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario creating a system of linear equations. Students solve the system both graphically and using technology, checking the solution by substituting the values back in to the original equations. Next, they are provided three related scenarios in which they write systems of equations in general form and solve the systems graphically and algebraically using the substitution method. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions. The related terms *consistent systems* and *inconsistent systems* are defined.

Algebra 1
Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

(C) write linear equations in two variables given a table of values, a graph, and a verbal description.

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

(G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

ELPS

1.A, 1.C, 1.D, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- The standard form of a linear equation is $Ax + By = C$ where A, B, and C are constants and A and B are not both zero. Linear functions written in standard form can be graphed using the x- and y-intercepts.
- Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane.
- A linear system of equations is two or more linear equations that define a relationship between quantities. The solution of a linear system is an ordered pair that makes both equations in the system true.
- Lines that do not intersect describe a system of equations in which each linear equation has the same slope and there is no solution.
- Lines intersecting at a single point describe a system of equations in which each linear equation has a different slope and there is one solution.
- Lines intersecting at an infinite number of points describe a system of equations in which each linear equation is the same equation and there are an infinite number of solutions.
- Consistent systems of equations are systems that have one or many solutions. Inconsistent systems of equations are systems that have no solutions.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Ticket Tabulation

Students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They explain why there is more than one possible solution.

Develop

Activity 2.1: Analyzing the Graph of an Equation in Standard Form

Students graph the equation from the previous activity using intercepts. They analyze a second graph with the independent and dependent variables reversed.

Activity 2.2: Determining the Solution to a System of Linear Equations

Students are given additional information about the scenario from the previous activity, and they write an equation to represent the new relationship. They then graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario creating a system of linear equations. Students then approximate the solution to the system of equations using the graph they drew, determine a solution using technology, and calculate a solution algebraically using the substitution method.

Day 2

Activity 2.3: Systems with No Solution, One Solution, or an Infinite Number of Solutions

Students are provided three related scenarios in which they write systems of equations in general form and solve the systems graphically and algebraically using the substitution method. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions.

Demonstrate

Talk the Talk: Beating the System

The related terms *consistent systems* and *inconsistent systems* are defined. Students contrast consistent and inconsistent systems of linear equations in terms of their *y*-intercepts, number of solutions, and description of the graph.

ENGAGE

Getting Started: Ticket Tabulation

Facilitation Notes

In this activity, students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They explain why there is more than one possible solution.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Variables used to represent number of students and number of adults.
- Various methods to determine a solution to the equation.
- Negative values presented as solutions to the situation.

Questions to ask

- How does your equation compare to your classmates' equations?
- Was this equation more difficult to solve than other equations you have written? Why?
- In what form is this equation written?
- What are the independent and dependent quantities?
- What is the general form of the equation representing this situation?
- Does standard form or general form make more sense for this situation? Why?
- If no student tickets were sold, how many adult ticket sales would it take for the athletic association to reach its goal?
- If no adult tickets were sold, how many student ticket sales would it take for the athletic association to reach its goal?
- Does this equation describe the number of tickets sold or the amount of money raised?
- If 200 adult tickets were sold, how much money would still need to be raised for the athletic association to reach its goal?
- If 200 adult tickets were sold, how many student tickets would need to be sold for the athletic association to reach its goal?
- Is there an infinite number of correct answers? Why or why not?
- How would the answers you have listed appear on a graph of the situation?

Differentiation strategy

To extend the activity, create a poster with a coordinate plane and have students post dot stickers as their answers. Discuss how solutions can be written as ordered pairs. Analyze characteristics of the line formed.

Summary

Some situations are better represented by an equation in standard form than an equation in general form.

DEVELOP

Activity 2.1
Analyzing the Graph of an Equation in Standard Form

Facilitation Notes

In this activity, students graph the equation from the previous activity using intercepts. They then analyze a second graph with the independent and dependent variables reversed.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

Differentiation strategy

To scaffold support, have students replace the x at the end of the x-axis with s and the y at the end of the y-axis with a.

As students work, look for

- Confusion about which variable represents the independent quantity and which represents the dependent quantity.
- Mental math to determine the intercepts.
- Identification of the slope from the graph or by changing the form of the equation.

Questions to ask

- What quantity is measured on the x-axis? On the y-axis?
- What are the coordinates of the x-intercept? The y-intercept?
- Why is the slope negative?
- How is the mathematical function different than the function modeling the real world situation?
- Are all real numbers the domain and range for any linear function? Why or why not?
- Is every point on the graph of the line relevant to this problem situation?
- Select a point on the line and explain what it represents in terms of this problem situation.
- Is the value $3000 represented on the graph? If so, how?

Misconceptions

- Students may think the domain and range of the real-world situation include all positive real numbers. Address the fact that the number of tickets must be whole numbers, and that there is an upper limit to the number of each type of ticket because the total sales must be exactly $3000.
- Students may think the dependent variable is the amount of money raised. Clarify this misconception by addressing the units of coordinate pairs, and that the graph represents the fact that as the number of one type of ticket increases, the other decreases.

Have students work with a partner or in a group to complete Questions 6 through 12. Share responses as a class.

Questions to ask

- How can both graphs be correct?
- What quantity is measured on the x-axis? On the y-axis?
- Why is the slope of this line also negative?
- What is the relationship between the intercepts on the two different graphs? Why does this happen?
- How are the points on the two graphs related?

Differentiation strategy

To extend the activity, create a poster with a coordinate plane and have students post dot stickers representing other points on Felino's graph. Discuss how their process is different from the last graph they created.

Summary

When a line is written in standard form, either quantity can be considered the independent or dependent variable. Identifying and graphing the x- and y-intercepts is an efficient method of graphing a line written in standard form.

Activity 2.2

Determining the Solution to a System of Linear Equations

Facilitation Notes

In this activity, students are given additional information about the scenario from the previous activity, and they write an equation to represent the new relationship. They then graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario creating a system of linear equations. Students then approximate the solution to the system of equations using the graph they drew, determine a solution using technology, and calculate a solution algebraically using the substitution method.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

- To scaffold support, have students place the coefficient of 1 in front of the variables to see the structure of this equation is the same as the structure of the previous equation.
- To extend the activity, have students graph this line using sticker dots on the poster graphs from the previous activities.

Misconceptions

- With the addition of another equation, students may confuse what the variables and their coefficients represent. In the equations $s + a = 450$ or $1s + 1a = 450$, s and a continue to represent the number of student and adult tickets respectively. Each coefficient of 1 keeps a count of the number of tickets, in other words, each ticket counts as 1.
- When interpreting the point of intersection, students often note only one of quantities or one of the equations. Stress the importance of a complete response, such as "When 300 student tickets and 150 adult tickets are sold, a total of 450 tickets with a goal of $3000 is reached."

Questions to ask

- In this situation, what is the meaning of the expression $s + a$?
- What are the coordinates of the y-intercept? The x-intercept?
- What is the significance of the point of intersection of the line on the graph and the graph of the equation $s + a = 450$ with respect to the problem situation?
- What are the coordinates of the point of intersection of the line on the graph and the graph of the equation $s + a = 450$?
- Interpret the meaning of the point of intersection.
- What is the relationship between the point of intersection on both graphs?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

As students work, look for

Sign errors when rewriting the equations in standard form to general form.

Questions to ask

- What are the advantages and disadvantages of using standard form?

- Is it better to use the original equations in standard form or the equations rewritten in general form to check the accuracy of your solution?
- How do you know if your answer is correct after you complete the substitution?

Differentiation strategy

To extend the activity, discuss an alternative method in which students complete a linear regression using the two intercepts to write the equation in general form.

Summary

Two linear equations that share a relationship between quantities are a system of linear equations. If the graphs of the two linear equations intersect, their point of intersection is a solution to the system of linear equations.

Activity 2.3

Systems with No Solution, One Solution, or an Infinite Number of Solutions

Facilitation Notes

In this activity, students are provided three related scenarios in which they write systems of equations in general form and solve the systems graphically and algebraically using the substitution method. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Differentiation strategies

- To assist all students, after they determine the equations for Marcus and Philip, write the equations on the board for reference throughout the activity. Add Tonya's equation once it is encountered for comparison purposes.
- To scaffold support, remind students of how to set up the substitution method learned in the previous lesson.

  Method 1: To determine when the functions have the same amount of money, some students may understand that the expressions representing the money must be set equal to one another.

$$10x + 25 = 10x + 40$$

Method 2: For other students, the term *substitute* will resonate them, and it may make more sense for them to write one equation first and then substitute an equivalent expression for y as a second step.

$$y = 10x + 40$$
$$10x + 25 = 10x + 40$$

Questions to ask

- How much money did Marcus begin with? How much money did Phillip begin with?
- How much is each person saving each week?
- Why won't Marcus and Phillip ever have the same amount of money saved?
- Are the graphs of the lines increasing or decreasing? How do you know?
- If the slopes of two linear equations are equal, how would you describe the behaviors of their graphs?
- How is the algebraic solution of the system determined?
- How many different variables are in the equation once the substitution step is completed?
- If the solution to a system of equations is a false statement such as $25 = 40$, what does this mean with respect to this problem situation?
- Where on the coordinate plane is the graphic solution to the system?
- If the point of intersection is the solution to the system of equations, what does it mean if there is no point of intersection?

Have students work with a partner or in a group to complete Question 7. Share responses as a class.

Questions to ask

- How is the comparison between Marcus and Philip different than the comparison between Tonya and Philip?
- Do Tonya and Phillip ever have the same amount of money saved?
- What is the slope in each equation?
- Are the graphs of the lines increasing or decreasing? How do you know?
- If the slopes of two linear equations are not equal, what does this imply graphically?
- Where on the coordinate plane is the graphic solution to the system?
- How is the algebraic solution of the system determined?
- If the point of intersection is the solution to the system of equations, what does it mean if the point of intersection is on the y-axis?

Have students work with a partner or in a group to complete Questions 8 through 11. Share responses as a class.

Misconceptions

- Students may think that the graph of Tonya's situation is no longer a line since there are two different rates. Discuss that fact that the rate is measured in dollars per week, so Tonya's two deposits should be combined to determine one weekly rate.
- Students may overgeneralize and think the solutions $40 = 40$ and $25 = 40$ signify the same solution type. Question the significance of a resulting true and false statement in terms of possible solutions. Refer to the original equations to verify the solutions.
- Students may confuse *infinite solutions* with *all real numbers*. Ask students to identify coordinate pairs that are solutions. Discuss the fact that all the points that lie on the line are solutions, but all coordinate pairs on the coordinate grid are not solutions.

Questions to ask

- How does Phillip's equation change? How does Tonya's equation change?
- If the solution to a system of equations is a true statement such as $40 = 40$, what does this mean with respect to the problem situation?
- Is there a single point of intersection? Why not?
- If two lines are described by the same equation, what are the graphical implications?
- Are the graphs of the lines parallel? How do you know?
- Is Phillip saving more money than Tonya after the shopping spree? How do you know?

Differentiation strategies

To assist all students,

- Suggest they use double arrows to show that there are two of the same lines graphed.
- Ask them to identify several coordinate pair answers when there are infinite solutions to solidify the fact that all of the infinite solutions lie on the line.

Summary

A system of linear equations can be solved algebraically or graphically and may have no solution, one solution, or an infinite number of solutions.

Talk the Talk: Beating the System

DEMONSTRATE

Facilitation Notes

In this activity, the related terms *consistent systems* and *inconsistent systems* are defined. Students contrast consistent and inconsistent systems of linear equations in terms of their *y*-intercepts, number of solutions, and descriptions of their graphs.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

Confusion why the definition of consistent systems states that the system has one or *many* solutions rather than one or *infinite* solutions. A consistent system of two *linear* equations has one or infinite solutions; however, this may not be the case for systems that include other functions. For example, a consistent system that includes a linear and quadratic function may have one or two solutions. Because the definition of consistent systems applies to other functions, rather than just a system of two linear functions, the phrasing needs to be general enough to include all cases. If students are confused as to whether the response should be infinite solutions or many solutions for a system of linear equations, suggest the response *infinitely* many solutions.

Questions to ask

- How are the steps to solve an equation in one variable and a system of linear equations in two variables related? How are they different?
- How are the algebraic properties of equality used to solve an equation in one variable and a system of linear equations in two variables related?
- How is solving a system of linear equations related to solving an equation in one variable with the variable on both sides of the equation?
- How are the solutions to an equation in one variable and a system of linear equations in two variables different?
- Can the solution to an equation in one variable be graphed on a coordinate plane? On a number line?
- Can an equation in one variable have no solution? What does this look like?
- Can an equation in one variable have an infinite number of solutions? What does this look like?
- Can an equation in one variable have one solution? What does this look like?
- What is the difference between a consistent system and an inconsistent system of equations?

- Why does the column title state that the y-intercepts can be the same or different? What does the graph of a consistent system with one solution with the y-intercepts the same look like?

Differentiation strategy

To extend the activity, provide each student two note cards, and have each of them create an example of a consistent system of linear equations and inconsistent system of linear equations. Next, have each student exchange note cards with a partner and identify the solution type for each system. Follow up by having the titles *inconsistent systems* and *consistent systems* on the board, and have each student post their examples under the appropriate title. Discuss patterns, then separate the consistent systems into two types: one solution or infinite solutions. Have students provide an additional row on the table to provide examples of each type of solution.

Summary

Consistent systems of equations have one or many solutions, and inconsistent systems of equations have no solutions. A system of two linear equations has one, infinite, or no solutions.

2

Double the Fun

Using Graphing to Solve Systems of Equations

Warm Up

Determine an ordered pair that represents a solution to each equation.

1. $4x + 7y = 24$

2. $5x - 2y = -6$

3. $\frac{1}{2}x + \frac{3}{4}y = 10$

Learning Goals

- Write equations in standard form.
- Determine the intercepts of an equation in standard form.
- Use intercepts to graph an equation.
- Write a system of equations to represent a problem context.
- Solve systems of linear equations exactly and approximately.
- Interpret the solution to a system of equations in terms of a problem situation.
- Use slope and y-intercept to determine whether the system of two linear equations has one solution, no solution, or infinite solutions.

Key Terms

- consistent systems
- inconsistent systems

You have examined different linear functions and solved for unknown values. How can you solve problems that require two linear functions? How many solutions exist when you consider two functions at the same time?

Warm Up Answers

Sample answers.

1. (6, 0)
2. (0, 3)
3. (20, 0)

Answers

1. $5s + 10a = 3000$

   s = number of students

   a = number of adults

2. Sample answer.

   The athletic association can sell 400 student tickets and 100 adult tickets.

3. Sample answer.

   No, there is more than one possible combination of student/adult tickets sold that will add up to \$3000.

GETTING STARTED

Ticket Tabulation

The Marshall High School Athletic Association sells tickets for the weekly football games. Students pay \$5 and adults pay \$10 for a ticket. The athletic association needs to raise \$3000 selling tickets to send the team to an out-of-town tournament.

1. Write an equation to represent this situation.

2. What combination of student and adult ticket sales would achieve the athletic association's goal?

3. Compare your combination of ticket sales with your classmates'. Did you all get the same answer? Explain why or why not.

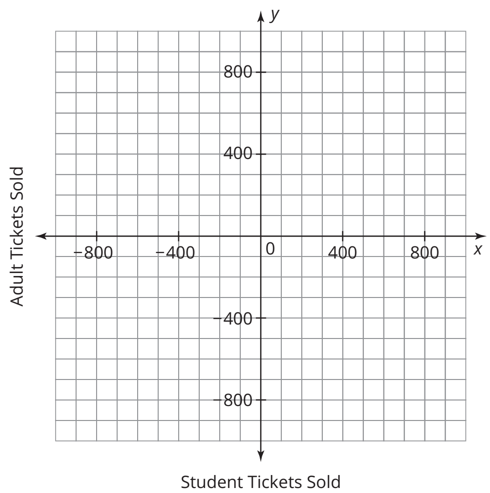
ACTIVITY **2.1**

Analyzing the Graph of an Equation in Standard Form

Consider the goal of the athletic association described in the previous activity. Let s represent the number of student tickets sold, and let a represent the number of adult tickets sold. Written in standard form, the equation that represents the situation is $5s + 10a = 3000$.

One efficient way to graph a linear function in standard form is to use x- and y-intercepts. You can calculate the x-intercept by substituting $y = 0$ and solving for x. You can calculate the y-intercept by substituting $x = 0$ and solving for y.

1. Use the x-intercept and y-intercept to graph the equation.



2. Determine the domain and range of each.

a. the mathematical function

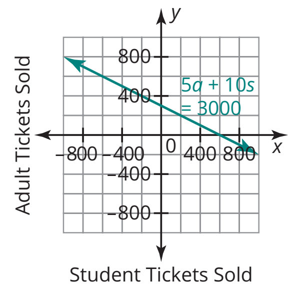
b. the function modeling the real-world situation

Ask yourself:

Which quantity is represented on each axis?

Answers

1.



2a. The domain is all real numbers.

The range is all real numbers.

2b. The domain is all whole numbers such that $0 \le s \le 600$.

The range is all whole numbers such that $0 \le a \le 300$.

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Answers

3. The x-intercept is (600, 0). The x-intercept represents the number of student tickets that must be sold to reach \$3000 if no adult tickets are sold.

   The y-intercept is (0, 300). The y-intercept represents the number of adult tickets that must be sold to reach \$3000 if no student tickets are sold.

4. The slope is $-\frac{1}{2}$. This means that for every 1 adult ticket sold, 2 less student tickets need to be sold.

5. I can use any point on the graphed line within the domain and range that has an x- and y-value that are whole numbers to determine a possible combination of ticket sales that will meet the goal of \$3000.

6. Felino's graph is correct. He reversed the independent and dependent variables, letting the x-axis represent the number of adult tickets sold and the y-axis represent the number of student tickets sold, and he reversed the x- and y-intercepts when graphing the line.

3. Explain what each intercept means in terms of the problem situation.

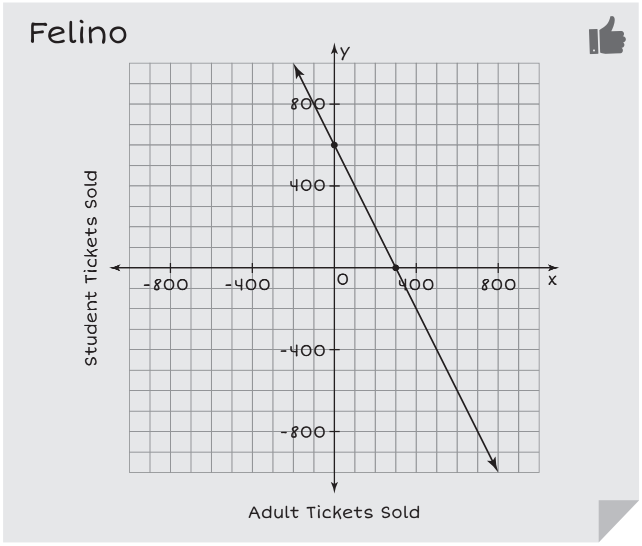
4. Identify the slope of the function. Interpret its meaning in terms of the problem situation.

Ask yourself:

What does each point on the graph of an equation represent?

5. How can you use the graph to determine a combination of ticket sales to meet the goal of \$3000?

6. Felino graphed the equation $5s + 10a = 3000$ in a different way. Explain why Felino's graph is correct.



7. **Use Felino's graph to describe the domain and range of each.**

   a. **the mathematical function**

   b. **the function modeling the real-world situation**

8. **Explain what each intercept means in terms of the problem situation.**

9. **Identify the slope of the function. Interpret its meaning in terms of the problem situation.**

10. **Compare the domain and range of the two functions that model the real-world situation. What do you notice?**

11. **Compare the *x*-intercepts and the *y*-intercepts of the two graphs. What do you notice?**

12. **Is there a way to determine the total amount of money collected from either graph? Explain why or why not.**

11. The intercepts are switched. The *x*-intercept of the first graph is the *y*-intercept of the second graph, and the *y*-intercept of the first graph is the *x*-intercept of the second graph.

12. The group raises \$3000 for any ordered pair that lies on either graph.

Answers

7a. The domain of the mathematical function is all real numbers.

The range of the mathematical function is all real numbers.

7b. The domain of the function modeling the real-world situation is all whole numbers such that $0 \leq a \leq 300$.

The range of the function modeling the real-world situation is all whole numbers such that $0 \leq s \leq 600$.

8. The *x*-intercept is (300, 0). The *x*-intercept represents the number of adult tickets that must be sold to reach \$3000 if no student tickets are sold.

The *y*-intercept is (0, 600). The *y*-intercept represents the number of student tickets that must be sold to reach \$3000 if no adult tickets are sold.

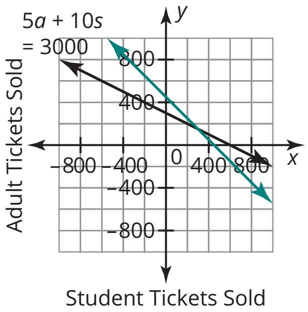
9. The slope is -2. This means that for every two student tickets sold, 1 less adult ticket needs to be sold.

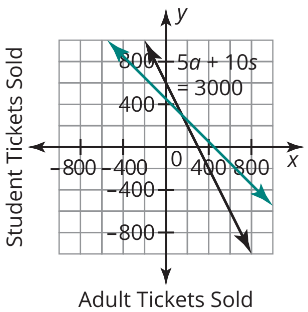
10. The domain and range are switched. The domain of the first function that models the real-world situation is the range of the second function that models the real-world situation, and the range of the first function that models the real-world situation is the domain of the second function that models the real-world situation.

Answers

1. $s + a = 450$

2.





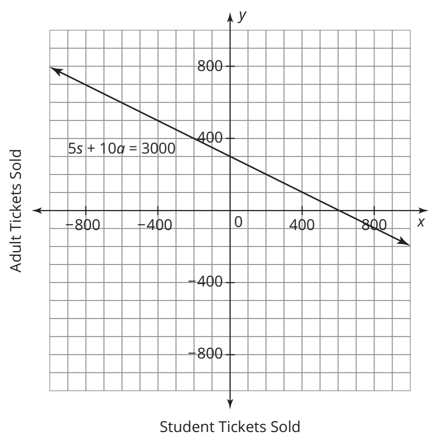
ACTIVITY **2.2**

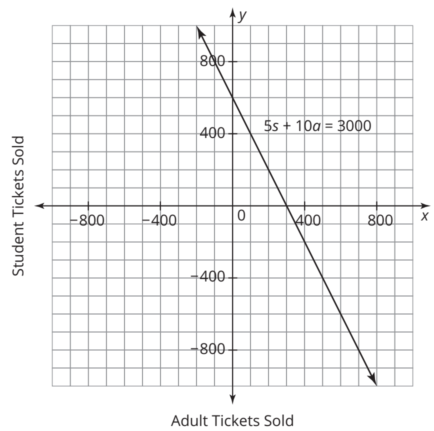
Determining the Solution to a System of Linear Equations

The athletic director of the Marshall High School Athletic Association says that 450 total tickets were sold to the home game.

1. Write an equation that represents this situation. Let *s* represent the number of student tickets sold, and let *a* represent the number of adult tickets sold.

The coordinate planes shown already contain the function that models the earnings from ticket sales.





2. Use *x*- and *y*-intercepts to graph the function modeling the total number of tickets sold on each coordinate plane.

3. If the athletic association reached its goal of $3000 in ticket sales, how many of each type of ticket was sold? Is there more than one solution?

Remember:

According to the situation, 450 tickets were sold to the game.

4. Use technology to locate the exact point of intersection. Explain the process you used.

5. Justify algebraically that your solution is correct.

Answers

3. There should be 300 student tickets and 150 adult tickets sold. There is only one solution because there is only one point of intersection between the graphed lines.
4. Place the equations of the lines in general form. Enter them into the graphing calculator. Follow the prompts to determine a point of intersection.
5.

$$\begin{aligned} 5s + 10a &= 3000 \\ 5(300) + 10(150) &= 3000 \\ 1500 + 1500 &= 3000 \\ 3000 &= 3000 \end{aligned}$$

$$\begin{aligned} s + a &= 450 \\ 300 + 150 &= 450 \\ 450 &= 450 \end{aligned}$$

Answers

1. Let x represent the time in weeks.

   Let y represent the amount of money saved in dollars.

   $$\begin{cases} y = 25 + 10x \\ y = 40 + 10x \end{cases}$$

2. Marcus and Phillip will never have the same amount of money saved.

3a. The slopes are the same. This means that both friends save the same amount each week, which is $10.

3b. The y-intercept in Phillip's equation is greater than the y-intercept in Marcus's equation. This means that Phillip started with more money than Marcus.

ACTIVITY **2.3**

Systems with No Solution, One Solution, or an Infinite Number of Solutions

Marcus and Phillip are in the Robotics Club. They are both saving money to buy materials to build a new robot.

Marcus opens a new bank account. He deposits $25 that he won at a robotics competition. He also plans on depositing $10 a week that he earns from tutoring. Phillip decides he wants to save money as well. He already has $40 saved from mowing lawns over the summer. He plans to also save $10 a week from his allowance.

1. Write equations to represent the amount of money Marcus saves and the amount of money Phillip saves.

2. Use your equations to predict when Marcus and Phillip will have the same amount of money saved.

You can prove your prediction by solving and graphing a system of linear equations.

3. Analyze the equations in your system.

a. How do the slopes compare? Describe what this means in terms of this problem situation.

b. How do the *y*-intercepts compare? Describe what this means in terms of this problem situation.

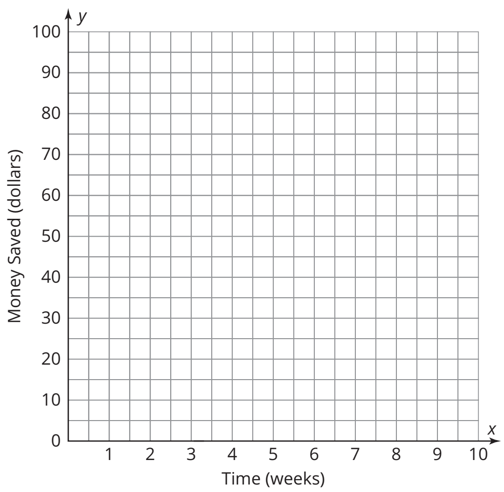
4. **Determine the solution of the system of linear equations algebraically and graphically.**

   a. **Use the substitution method to determine the intersection point.**

   b. **Does your solution make sense? Describe what this means in terms of the problem situation.**

   c. **Predict what the graph of this system will look like. Explain your reasoning.**

   d. **Graph both equations on the coordinate plane.**



Answers

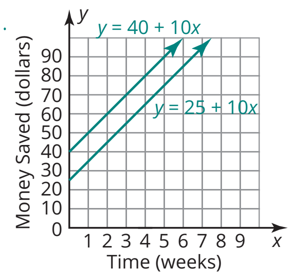
4a. $25 + 10x = 40 + 10x$

$25 \neq 40$

4b. No. My solution does not make sense because $25 \neq 40$. This means that there is no solution.

4c. The graph will be two parallel lines.

4d.



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Answers

5a. The graphs are the same distance apart from one another which means they are parallel.

5b. No. The graphs will never intersect because they are parallel, so there is no solution.

6. Sample answer.

Yes. My prediction was correct because Marcus will never have as much money as Phillip. Algebraically, the solution of $25 \neq 40$ using the substitution method proved that there is no solution. Graphically, the two parallel lines proved there is no solution.

7a. $\begin{cases} y = 40 + 4x \\ y = 40 + 10x \end{cases}$

5. Analyze the graph you created.

a. Describe the relationship between the graphs.

b. Does this linear system have a solution? Explain your reasoning.

6. Was your prediction in Question 2 correct? Explain how you algebraically and graphically proved your prediction.

Tonya is also in the Robotics Club and has heard about Marcus's and Phillip's savings plans. She wants to be able to buy her new materials before Phillip, so she opens her own bank account. She is able to deposit $40 in her account that she has saved from her job as a waitress. Each week she also deposits $4 from her tips.

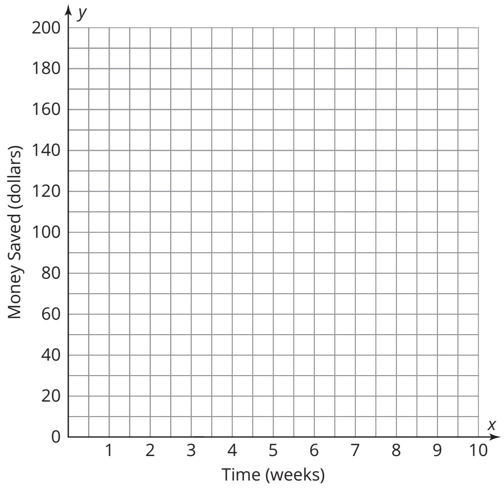
7. Use equations and graphs to determine when Tonya and Phillip have saved the same amount of money.

a. Write a linear system to represent the total amount of money Tonya and Phillip have saved after a certain amount of time.

Remember:

Don't forget to define your variables!

b. Graph the linear system on the coordinate plane.



c. Do the graphs intersect? If so, describe the meaning in terms of this problem situation.

Phillip and Tonya went on a shopping spree this weekend and spent all their savings except for $40 each. Phillip is still saving $10 a week from his allowance. Tonya now deposits her tips twice a week. On Tuesdays she deposits $4 and on Saturdays she deposits $6.

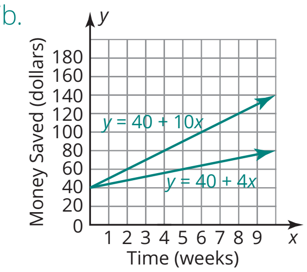
8. Phillip claims he is still saving more each week than Tonya.

a. Do you think Phillip's claim is true? Explain your reasoning.

b. How can you prove your prediction?

Answers

7b.



7c. Yes. The graphs intersect at (0, 40). This means that they both start out with the same amount, which is $40.

8a. No. I do not think Phillip's claim is true because Phillip and Tonya are now saving the same amount each week.

8b. I can prove my prediction by writing a new system of linear equations and solving them algebraically and graphically.

ELL Tip

Students may not be familiar with the term *shopping spree*. Define the word *spree* as a *sustained period of unrestrained activity*. Discuss other examples of *sprees* besides *shopping sprees*, such as an eating *spree*, a laughing *spree*, or a scoring *spree*. Finally, review the problem scenario for Question 8 and ensure students' understanding of *shopping spree* in the context of the problem.

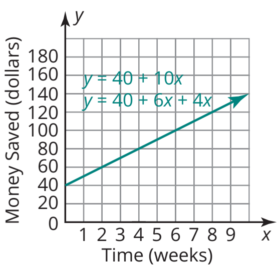
Ask students to identify what the prefix *pre-* means in the word *predict*. Follow up with additional examples of words with the prefix *pre-*, including *pretest, preview,* and *precooked*. Define these words and then ask students to explain why *predict* means to *state what should happen in the future*, in the context of making predictions about the outcome of the problems based on the given information.

Answers

9a. Tonya's equation is now $y = 40 + 6x + 4x$ because of her extra savings per week. Phillip's equation is the same.

$$\begin{cases} y = 4 + 10x \\ y = 40 + 6x + 4x \end{cases}$$

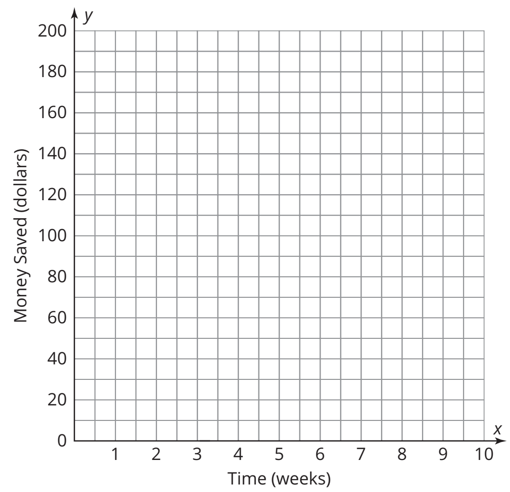
9b.



9. Prove your prediction algebraically and graphically.

a. Write a new linear system to represent the total amount of money each friend has after a certain amount of time.

b. Graph the linear system on the coordinate plane.



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10. Analyze the graph.

a. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

b. Algebraically prove the relationship you stated in part (a).

c. How does this solution prove the relationship? Explain your reasoning.

11. Was Phillip's claim that he is still saving more than Tonya a true statement? Explain why or why not.

Answers

10a. The graphs are the same. This means that Tonya and Phillip will always have the same amount of money.

10b. $40 + 10x = 40 + 6x + 4x$

$$40 = 40$$

10c. Determining that $40 = 40$ means there are an infinite number of solutions. Therefore, the graphs must be the same.

11. No. Phillip is not saving more than Tonya. He is saving the same amount as Tonya.

Answers

1. In both cases, you must isolate a variable using algebraic properties of equality. For a system of equations, steps must also be completed to solve for the second variable. For both an equation in one variable and a system of linear equations in two variables, there can either be one solution, an infinite number of solutions, or no solution.
2. Number of Solutions: one solution; infinite solutions; no solutions

   Description of Graph: lines intersect; lines are the same; lines are parallel
3. The *x*- and *y*-values of the point of intersection of the two graphs makes both equations true.

NOTES

TALK the TALK

Beating the System

1. How does solving a linear system in two variables compare to solving an equation in one variable?

A system of equations may have one unique solution, no solution, or infinite solutions. Systems that have one or many solutions are called **consistent systems.** Systems with no solution are called **inconsistent systems.**

2. Complete the table.

System of Two Linear Equations	Consistent Systems		Inconsistent Systems
Description of *y*-Intercepts	*y*-intercepts can be the same or different	*y*-intercepts are the same	*y*-intercepts are different
Number of Solutions			
Description of Graph			

3. Explain why the *x*- and *y*-coordinates of the points where the graphs of a system intersect are solutions.

Assignment

LESSON 2: Double the Fun

Write

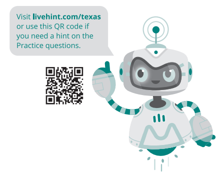
Define each term in your own words.

1. consistent systems
2. inconsistent systems

Remember

When two or more equations define a relationship between quantities, they form a system of linear equations. The point of intersection of a graphed system of linear equations is the solution to both equations. A system of linear equations can have one solution, no solution, or infinite solutions.

Practice



1. Mr. Johanssen gives his class 50-question multiple choice tests. Each correct answer is worth 2 points, while a half point is deducted for each incorrect answer. If the student does not answer a question, that question does not get any points.
   a. A student needs to earn 80 points on the test in order to keep an A grade for the semester. Write an equation in standard form that represents the situation. Identify 3 combinations of correct and incorrect answers that satisfy the equation.
   b. Determine the x- and y-intercepts of the equation and use them to graph the equation. Explain what each intercept means in terms of the problem situation.
2. Wesley owns a dairy farm. In the morning, it takes him 0.3 hour to set up for milking the cows. Once he has set up, it takes Wesley 0.2 hour to milk each cow by hand. He is contemplating purchasing a milking machine in hopes that it will speed up the milking process. The milking machine he is considering will take 0.4 hour to set up each morning and takes 0.05 hour to milk each cow.
   a. Write a system of linear equations that represents the total amount of time Wesley will spend milking the cows using the two different methods.
   b. Graph both equations on a coordinate plane.
   c. Estimate the point of intersection. Explain how you determined your answer.
   d. What does the point of intersection represent in this problem situation?
   e. Verify your answer to part (c) by solving the system algebraically.
   f. Does the solution make sense in terms of this problem situation? Explain your reasoning.
   g. Is this system of equations consistent or inconsistent? Explain your reasoning.
3. Identify whether each system is consistent or inconsistent. Explain your reasoning.

   a. $\begin{cases} -3x + 4y = 3 \\ -12x + 16y = 8 \end{cases}$ b. $\begin{cases} 7x + 3y = 0 \\ 14x + 6y = 0 \end{cases}$ c. $\begin{cases} 6x + y = 1 \\ -6x - 4y = -4 \end{cases}$

Assignment Answers

Write

1. A system of linear equations is a consistent system if there is at least one solution that satisfies both equations. A consistent system may have 1 or many solutions.
2. A system of linear equations is an inconsistent system if there is no solution that satisfies both equations.

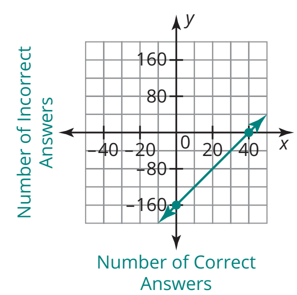
Practice

1a. Let c represent the number of correct answers and w represent the number of incorrect answers.

$2c - \frac{1}{2}w = 80$

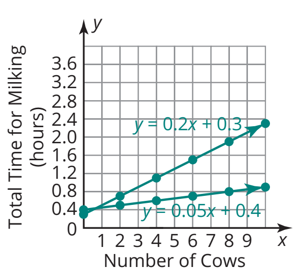
(40, 0), (41, 4), (42, 8)

1b. The x-intercept is (40, 0), and the y-intercept is (0, −160). The x-intercept represents the number of answers the students must get correct to get an 80 if no answers are incorrect. The y-intercept is −160. There cannot be a negative number of incorrect answers, so this indicates that it is impossible to get an 80 on the test if there are 0 correct answers.



2a. $\begin{cases} y = 0.2x + 0.3 \\ y = 0.05x + 0.4 \end{cases}$

2b.



2c. The lines intersect at about (0.7, 0.4).

2d. The intersection point represents when both methods take the same amount of time for the same number of cows.

2e. $x = 0.\overline{6}, y = 0.4\overline{3}$

2f. No. Fractional answers do not make sense for the number of cows.

2g. This system is consistent because it has one unique solution.

3. See answers on the next page.

Assignment Answers

Practice

3a. Inconsistent; no solutions

3b. Consistent; infinite solutions

3c. Consistent; one solution: (0, 1)

Stretch

I can solve the system by graphing the two equations and determining their point of intersection. There are two solutions: (1, 3) and (4.75, 0.75).

Review

1. $x = 12$

   Check:

   $\frac{3}{4}(12) - 11 = 4 + [-\frac{3}{4}(12) + 3]$

   $-2 = -2$

2. $y = \frac{1}{5}x - 7$; $m = \frac{1}{5}$; $b = -7$

3. Set A:
   $y = 4.574x + 5.53$
   $r = 0.9898$

   Set B:
   $y = 0.284x + 5.411$
   $r = 0.23091$

   The regression equation for Set A appears to be the better fit because its correlation coefficient is closer to 1 than the correlation coefficient of the regression equation for Set B.

Stretch

Solve the system of equations shown. Explain your reasoning.

$$\begin{cases} 3x + 5y = 18 \\ y = |x - 4| \end{cases}$$

Review

1. Solve the equation and check your solution.

$$\frac{3}{4}x - 11 = 4 + \left(-\frac{3}{4}x + 3\right)$$

2. Consider the equation $\frac{2}{5}x - 2y = 14$. Write the equation in general form and identify the slope and y-intercept.

3. Determine the linear regression equation for each data set. Which regression equation is the better fit? Explain your reasoning.

Set A

x	y
1	12
2	11
5	30
7	39
10	50

Set B

x	y
12	3
10	9
8	11
5	14
0	0

3 The Elimination Round

Using Linear Combinations to Solve a System of Linear Equations

MATERIALS

None

Lesson Overview

Students are given a problem scenario and use reasoning to determine the two unknowns. They then write a system of linear equations in standard form to represent a problem situation. Students analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They practice the linear combinations method with systems in which the coefficients of one variable are additive inverses. Next, Worked Examples guide students to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system, and then they solve two problems in context, one with fractional coefficients. The lesson concludes with students addressing when it is appropriate to use the graphing, substitution, or linear combinations methods.

Algebra 1
Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

ELPS

1.A, 1.C, 1.D, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- The linear combinations method is a process to solve a system of linear equations by adding two equations together, resulting in an equation in one variable.
- When using the linear combinations method, it is often necessary to multiply one or both equations by a constant to create two equations in which the coefficients of one of the variables are additive inverses.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Comic-Con(line)

Students are given a problem scenario and use reasoning to determine the two unknowns.

Develop

Activity 3.1: Combining Linear Systems

Students write a system of linear equations in standard form to represent the Comic-Con(line) problem. They analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They then solve two systems of linear equations using the linear combinations method where the coefficients of one of the variables are additive inverses.

Day 2

Activity 3.2: Using Additive Inverses to Combine Linear Systems

Students are provided Worked Examples to guide them to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system of equations.

Activity 3.3: Applying the Linear Combinations Method

Students solve a problem in context by writing a system of equations in standard form, applying the linear combinations method, and interpreting the solution in terms of the context.

Day 3

Activity 3.4: Fractions and Linear Combinations

Students solve a problem in context by writing a system of equations in standard form with fractional coefficients. They are provided two strategies to solve the system, one that rewrites the given equations without fractional coefficients, and the other that uses the linear combinations method with fractional coefficients.

Demonstrate

Talk the Talk: There's a Method in My Madness

Students summarize when it is appropriate to use the graphing, substitution, or linear combinations methods.

ENGAGE

Getting Started: Comic-Con(line)

Facilitation Notes

In this activity, students are given a problem scenario and use reasoning to determine the two unknowns.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

- Levels of sophistication used to solve the problem, from guessing and checking, to graphing, to writing and solving a system of equations using substitution.
- Reference to the context to make sure their answers make sense.

Questions to ask

- What is the total number of males and females that joined the Comic Gurus group?
- What is meant by the phrase *females outnumber males*?
- What is the difference between the number of females that joined the Comic Gurus group and the number of males that joined the Comic Gurus group?
- How did you solve the problem?
- Is there another way to solve the problem?
- Does your answer make sense?
- How can you check your answer?

Summary

Some problems in context involve two different relationships between two variables that can be represented using a system of equations. There is more than one method to solve a system of equations.

DEVELOP

Activity 3.1

Combining Linear Systems

Facilitation Notes

In this activity, students write a system of linear equations in standard form to represent the Comic-Con(line) problem. They analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They then solve two systems of linear equations using the linear combinations method where the coefficients of one of the variables are additive inverses.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

To scaffold support,

- Suggest that students use the variables f and m rather than x and y to write and interpret their equations.
- Have students interact with the student work by
  - Inserting additional steps within the shown student work.
  - Numbering the steps.
  - Visualizing the vertical orientation of Mara's work by drawing vertical arrows to make it explicit that the third line of Mara's work is determined by summing like terms in the first two lines of her work. Also, aligning the sum of the terms, $2x$, in the same column as the x's.

$$\begin{array}{l} x + y = 324 \\ x - y = 34 \\ 2x \quad\;\; = 358 \end{array}$$

As students work, look for

Errors translating the context into an equation, such as $f + 34 = m$, rather than $f = m + 34$.

Questions to ask

- Did Chloe and Mara write the same equations? What are the equations?
- How do the equations relate to the scenario?
- Why did Chloe isolate x in the second line of her work?
- How did Chloe come up with the equation $(y + 34) + y = 324$?
- How did Chloe determine the value of x?
- What is the name of the method that Chloe used? Why does it have that name?
- How did Mara get the third line, $2x = 358$?
- How did Mara determine the value of y?
- Do you think Mara's method will always work?
- What is special about the way the problem is set up allows Mara to solve the problem this way?
- In Mara's work, can we say that the coefficients or the y-variables are additive inverses? What does that mean?
- If the coefficients of y were not additive inverses, would you have been able to eliminate the y-variable when you added the equations together?
- What is the total number of males and females that joined the Comic Gurus group?

- How could you express your solution to the system of equations using coordinate notation?
- How can you check that your solution is correct?

Ask a student to read the information following Question 4 aloud. Discuss as a class.

Have student practice the linear combinations method with Question 5. Share responses as a class.

As students work, look for

- Forgetting to solve for the second variable.
- Substitution of the solution of the first variable in the wrong place when solving for the second variable.

Questions to ask

- How do you know what variable will be eliminated?
- How can you tell what variable you will be solving for first?
- Is your strategy any different if the coefficients are fractions? Explain.
- Why does it make more sense to use the linear combinations method rather than substitution when solving these systems?
- What is always the strategy to calculate the value of the second variable?
- How can you tell if your answer is correct?

Differentiation strategies

- To assist all students, help them make sense of the name *linear combinations method* by making the connection that they are combining linear equations.
- To extend the lesson, address the fact that the linear combinations method can be applied to other forms of equations as well. For example, have students solve this system using the linear combinations method.

$$y = 4x + 6$$
$$y = -4x + 22$$

Summary

The linear combinations method is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. For the linear combinations method to work, the coefficients of one of the variables must be additive inverses.

Activity 3.2
Using Additive Inverses to Combine Linear Systems

Facilitation Notes

Students are provided Worked Examples to guide them to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system of equations.

Analyze the Worked Examples and complete Questions 1 through 3 as a class.

Questions to ask

- Which variable adds to zero in this Worked Example?
- What are additive inverses?
- Do all numbers have additive inverses?
- What operation is used to create additive inverses?
- What would have had to happen to rewrite the system with a resulting equation in the other variable?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

As students work, look for

Interpretation of $0 = 0$ as a system with no solutions, rather than an infinite number of solutions. Emphasize that any true statement such as $0 = 0$, $1 = 1$, or $2 = 2$ implies an infinite number of solutions.

Differentiation strategies

- To scaffold support, model the thinking to create additive inverses. Suggest that students write this pre-work before solving the system.

Question 4 part (b):

$$\begin{array}{c} -6 \\ x + 3y = 15 \\ 5x + 2y = 7 \\ +6 \end{array} \quad \begin{array}{c} \text{think: } \frac{-6}{3} = -2 \\ \Longrightarrow \\ \text{think: } \frac{6}{2} = 3 \end{array} \quad \begin{array}{l} -2(x + 3y) = (15)(-2) \\ 3(5x + 2y) = (7)3 \end{array}$$

- To extend the activity, have students solve the systems.

Questions to ask

- Which coefficients are easier to rewrite as additive inverses?
- Describe the steps necessary to rewrite the other coefficients as additive inverses.
- How do you know when no rewrite is necessary to add the equation resulting in a new equation in a single variable?

- When the equations are added and the resulting new equation does not contain any variables, what does this imply about the algebraic solution and the graph of the system of equations?
- When the equations are added and all the like terms of the resulting equation add to zero, what does this imply about the algebraic solution and the graph of the system of equations?
- How do you know a system has an infinite number of solutions?
- Algebraically, what does it look like when a system has an infinite number of solutions?
- Graphically, what does it look like when a system has an infinite number of solutions?
- How do you know a system has no solutions?
- Algebraically, what does it look like when a system has no solutions?
- Graphically, what does it look like when a system has no solutions?

Summary

Sometimes one or both equations in a linear system must be multiplied by a constant to create additive inverses so the resulting new equation contains one variable.

Activity 3.3

Applying the Linear Combinations Method

Facilitation Notes

In this activity, students solve a problem in context by writing a system of equations in standard form, applying the linear combinations method, and interpreting the solution in terms of the context.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Questions to ask

- What expression represents two nights of lodging and four meals?
- What expression represents three nights of lodging and eight meals?
- Looking at both equations, what are the n-coefficients? m-coefficients?
- Will adding the equations together result in an equation with the n- or the m-variable? Why or why not?

- Which coefficients would you prefer to make additive inverses? How?
- Which equation did you use to substitute the value of *n*?
- How do you express your solution to the system of equations using coordinate notation?
- To check your solution algebraically, what do you need to do?

Differentiation strategies

To extend the activity,

- Have students solve the system of equations a second time, this time solving for the other variable first. Compare this process and solution to the first time solving the system.
- Discuss naming conventions when writing ordered pairs for variables other than (*x*, *y*). Typically, variables are listed in alphabetical order. In this case, the solution would be written as (*m*, *n*).

Summary

After writing a system of equations to represent a real-world problem, you can use the linear combinations method to solve the system and interpret the solution in terms of the problem.

Activity 3.4

Fractions and Linear Combinations

Facilitation Notes

In this activity, students solve a problem in context by writing a system of equations in standard form with fractional coefficients. They are provided two strategies to solve the system, one that rewrites the given equations without fractional coefficients, and the other that uses the linear combinations method with fractional coefficients.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

Questions to ask

- What expression represents one-half the number of 5-inch bracelets plus three-fourths the number of 7-inch bracelets?
- Looking at both equations, what are the *x*-coefficients? *y*-coefficients?
- How is this system of linear equations different than other systems of linear equations you have previously solved?
- Do you prefer Karyn's method or Jacob's method?

- Will adding the equations together result in an equation with the x- or the y-variable? Why or why not?
- Is there any way you can rewrite the equation without the fractions? How can you rewrite the equation so the fractions are whole numbers?
- Which coefficients did you make additive inverses? How did you do it?
- Which equation did you use to substitute the value of y?
- How do you express your solution to the system of equations using coordinate notation?
- To check your solution algebraically, what do you need to do?

Summary

The linear combinations method can be used to solve systems of equations with any real number coefficients.

DEMONSTRATE

Talk the Talk: There's a Method in My Madness

Facilitation Notes

In this activity, students summarize when it is appropriate to use the graphing, substitution, or linear combinations methods.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- What characteristic of the graph identifies the solution to a system of linear equations?
- When using the graphing method, where on the graph is the solution to the system of linear equations?
- Which method requires first isolating the variable in one of the linear equations?
- When using the substitution method, what is the purpose of first isolating a variable?
- Does the linear combinations method always involve rewriting one or both equations?
- When using the linear combinations method, how do you know that one or both equations must be rewritten?
- Which method can easily be used to make predictions?
- Which method does not always provide an exact answer?
- Which method is best to use when the equations are both written in standard form?

- Which method requires at least one of the equations to be written in general form?

Differentiation strategy

To extend the activity, have students make posters solving the same system of linear equations using all three methods. Provide systems of equations written in various forms (2 in general form, 2 in standard form, 1 in general form and 1 in standard form). Follow-up by having a class discussion listing pros and cons of each method.

Summary

The graphing method, the substitution method, and the linear combinations method can be used to solve a system of linear equations. Sometimes one method is more efficient than the others based upon the forms of the equations.

NOTES

3

The Elimination Round

Using Linear Combinations to Solve a System of Linear Equations

Warm Up

Determine the additive inverse for each expression.

1. 4
2. x
3. $20x$
4. $-9x$
5. $78.5x$

Learning Goals

- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using linear combinations.
- Interpret the solution to a system of equations in terms of a problem situation.

Key Term

- linear combinations method

You have solved a system of linear equations graphically and algebraically, using the substitution method. What are other algebraic strategies for solving systems of equations?

Warm Up Answers

1. -4
2. $-x$
3. $-20x$
4. $9x$
5. $-78.5x$

Answers

1. There are 179 females and 145 males who joined the group.
2. I can make sure that the total number of members adds to 324 and that the difference between the number of females and the number of males is 34.

GETTING STARTED

Comic-Con(line)

There are a total of 324 people who joined the Comic Gurus group on a social media site. Female group members outnumber males by 34.

1. Use reasoning to determine the number of males and females who joined the Comic Gurus.

2. How can you check that your solution is correct?

ACTIVITY 3.1

Combining Linear Systems

Consider the scenario from the Getting Started. Let's explore an algebraic strategy to determine a solution.

1. Write the system that represents the problem situation. Use *x* to represent the female members of the group, and use *y* to represent the male members of the group. Write the equations in standard form.

Answer

1. $\begin{cases} x + y = 324 \\ x - y = 34 \end{cases}$

ELL Tip

Students may not be familiar with the non-mathematical term *guru*. Define *guru* as *an influential teacher* and write a list of synonyms for *guru* on the board. Examples may include *expert, master, leader*, and *specialist*. Discuss the connection between a *guru* and the term *Comic Gurus*, as it is used in the context of the problem.

432 • TOPIC 3: Systems of Equations and Inequalities

Chloe and Mara used different strategies to solve the system of equations in a similar way. Analyze each student's reasoning.

Chloe

I can use the substitution method to solve this system.

$$\begin{aligned} x + y &= 324 \\ x - y &= 34 \rightarrow x = y + 34 \\ (y + 34) + y &= 324 \\ 2y + 34 &= 324 \\ 2y &= 290 \\ y &= 145 \end{aligned}$$

$$\begin{aligned} x &= 145 + 34 \\ x &= 179 \end{aligned}$$

Mara

You can eliminate one of the quantities by adding the two equations together.

$$\begin{array}{r} x + y = 324 \\ + \; x - y = 34 \\ \hline 2x = 358 \\ x = 179 \end{array}$$

$$\begin{aligned} 179 + y &= 324 \\ y &= 145 \end{aligned}$$

Remember:

As long as you maintain equality you can rewrite equations any way you want.

2. Explain why each student is correct in their reasoning.

3. Examine the structure of the system. What characteristic of the system made Mara's strategy efficient?

4. Identify the solution to the linear system as an ordered pair. Then interpret the solution in terms of this problem situation.

Answers

2. Chloe is correct because substituting an equivalent expression for x in one equation results in an equation in one variable, y, that can be solved. Then, by substituting the value of y into one of the original equations, the value of x can be calculated.

   Mara is correct because the y-variables have coefficients that are additive inverses, so when the equations are added together, the y-variables are eliminated from the equation, resulting in an equation in one variable, x, that can be solved. Then, by substituting the value of x into one of the original equations, the value of y can be calculated.

3. Both equations are in standard form, and one variable has coefficients that are additive inverses.

4. (179, 145)

   179 females and 145 males joined the Comic Gurus group on a social media site.

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Answers

5a. $(-1.5, 4)$

5b. $(-8, -2)$

The algebraic method used by Mara to solve the linear system is called the linear combinations method. The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

5. Solve each system of equations using the linear combinations method.

a. $\begin{cases} -4x + y = 10 \\ -2x - y = -1 \end{cases}$

b. $\begin{cases} -\frac{1}{2}x + 5y = -6 \\ \frac{1}{2}x + y = -6 \end{cases}$

ACTIVITY **3.2**

Using Additive Inverses to Combine Linear Systems

In the system of equations from the previous activity, one of the variables in both equations has coefficients that are additive inverses. What if a system doesn't have variables that are additive inverses? Let's use the strategy of linear combinations to solve other systems.

Worked Example

Consider this system of equations: $\begin{cases} 7x + 2y = 24 \\ 4x + y = 15 \end{cases}$.

$$\begin{aligned} 7x + 2y &= 24 \\ -2(4x + y) &= -2(15) \end{aligned}$$

Multiply the second equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$\begin{array}{r} 7x + 2y = 24 \\ + \ \ -8x - 2y = -30 \\ \hline -x = -6 \\ x = 6 \end{array}$$

Now that the y-values are additive inverses, you can solve this linear system for x.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ 2y &= -18 \\ y &= -9 \end{aligned}$$

Substitute the value for x into one of the equations to determine the value for y.

The solution to the system of linear equations is $(6, -9)$.

1. In the Worked Example, only one equation needs to be rewritten to solve using the linear combinations method. Why?

Remember:

Two numbers with a sum of zero are called additive inverses.

Ask yourself:

How can you solve the system of equations by transforming the first equation instead of the second?

Answer

1. If you multiply the equation $4x + y = 15$ by -2, then the coefficients of the y-terms will be additive inverses.

Answers

2. $\left(\frac{1}{2}, \frac{1}{2}\right)$
3. Sample answer.

   Multiply the equation $4x + 2y = 3$ by 5, and the equation $5x + 3y = 4$ by -4. The coefficients of x would be additive inverses, 20 and -20. Then, solve for y. Substitute $y = \frac{1}{2}$ into one of the equations and solve for x, $x = \frac{1}{2}$.

4a. Sample answer.

   Multiply the second equation by -1 then add the equations to get a resulting equation to determine the x-value.

4b. Sample answer.

   Multiply the first equation by -5 then add the equations to get a resulting equation to determine the y-value.

4c. Sample answer.

   Multiply the first equation by 3 and the second equation by -4 then add the equations to get a resulting equation to determine the y-value.

Now, let's consider a system where both equations need to be rewritten.

Worked Example

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

$$\begin{aligned} 3(4x + 2y) &= 3(3) \\ -2(5x + 3y) &= -2(4) \end{aligned}$$

Multiply each equation by a constant that results in coefficients that are additive inverses for one of the variables

$$\begin{aligned} 12x + 6y &= 9 \\ -10x - 6y &= -8 \end{aligned}$$

Ask yourself:

If you multiply both sides of an equation by the same number, is the equation still true?

2. Determine the solution for the linear system shown in the second Worked Example.

3. How could you have solved this system by creating *x*-values that are additive inverses?

4. Describe the first step needed to solve each system using the linear combinations method. Identify the variable that will be solved for when you add the equations.

a. $\begin{cases} 5x + 2y = 10 \\ 3x + 2y = 6 \end{cases}$

b. $\begin{cases} x + 3y = 15 \\ 5x + 2y = 7 \end{cases}$

c. $\begin{cases} 4x + 3y = 12 \\ 3x + 2y = 4 \end{cases}$

5. **Analyze each system. How would you rewrite the system to solve for one variable? Explain your reasoning.**

a. $\begin{cases} \frac{1}{2}x - 5y = -45 \\ -\frac{1}{2}x + 10y = -20 \end{cases}$

b. $\begin{cases} 4x + 3y = 24 \\ 3x + y = -2 \end{cases}$

c. $\begin{cases} 3x + 5y = 17 \\ 2x + 3y = 11 \end{cases}$

d. $\begin{cases} 6x + 3y = 5 \\ 2x + y = 1 \end{cases}$

e. $\begin{cases} x + 2y = -6 \\ 2x + 4y = -12 \end{cases}$

Answers

5a. Sample answer.

Do not rewrite either equation. The coefficients of the *x*-terms are already additive inverses.

5b. Sample answer.

Multiply the second equation by −3, then add the equations to get a resulting equation to determine the *x*-value.

5c. Sample answer.

Multiply the first equation by 2 and the second equation by −3, then add the equations to get a resulting equation to determine the *y*-value.

5d. Sample answer.

Multiply the second equation by −3, then add the equations to get a resulting equation with no variable terms and is a false statement.

5e. Sample answer.

Multiply the first equation by −2, both the *x*-terms and *y*-terms will be eliminated resulting in a true statement.

Answers

1. $\begin{cases} 2n + 4m = 270 \\ 3n + 8m = 435 \end{cases}$
2. Both equations are written in standard form.

   The coefficients of n and m are different.
3. $n = 105$, $m = 15$

ACTIVITY **3.3**

Applying the Linear Combinations Method

Let It Snow Resort offers two winter specials: the Get-Away Special and the Extended Stay Special. The Get-Away Special offers two nights of lodging and four meals for $270. The Extended Stay Special offers three nights of lodging and eight meals for $435. Determine what Let It Snow charges per night of lodging and per meal.

1. Write the system of linear questions that represents the problem situation. Let *n* represent the cost for one night of lodging at the resort and *m* represent the cost for each meal. Write the equations in standard form.

2. How are these equations the same? How are these equations different?

3. Solve the system comparing the two winter specials.

4. Interpret the solution of the linear system in the problem situation.

NOTES

5. Check your solution algebraically.

6. Is the Extended Stay Special the better deal? Explain why or why not.

Answers

4. Let It Snow Resort charges \$105 per night of lodging and \$15 per meal.

5.
$$2n + 4m = 270$$
$$2(105) + 4(15) = 270$$
$$210 + 60 = 270$$
$$270 = 270$$

$$3n + 8m = 435$$
$$3(105) + 8(15) = 435$$
$$315 + 120 = 435$$
$$435 = 435$$

6. No. Let It Snow Resort charges the same amount for meals and lodging for both specials.

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Answers

1. $\begin{cases} x + y = 84 \\ \frac{1}{2}x + \frac{3}{4}y = 49 \end{cases}$
2. Both students are correct. Karen chose to multiply by the LCD to rewrite the second equation without fractions, then she will have to multiply one of the equations by a constant to create additive inverses for either the x- or y-term. Jacob chose to work with the fractions in the second equation; by multiplying by $-\frac{1}{2}$, when he adds the equations the resulting equation determine the y-value.

ACTIVITY **3.4**

Fractions and Linear Combinations

The School Spirit Club is making beaded friendship bracelets with the school colors to sell in the school store. The bracelets are black and orange and come in two lengths: 5 inches and 7 inches. The club has enough beads to make a total of 84 bracelets. So far, they have made 49 bracelets, which represents $\frac{1}{2}$ the number of 5-inch bracelets plus $\frac{3}{4}$ the number of 7-inch bracelets they plan to make and sell. Determine how many 5-inch and 7-inch bracelets the club plans to make.

1. Let x represent the number of 5-inch bracelets, and let y represent the number of 7-inch bracelets. Write a system of equations in standard form to represent this problem situation.

2. Karyn says that the first step to solve this system is to multiply the second equation by the least common denominator (LCD) of the fractions. Jacob says that the first step is to multiply the first equation by $-\frac{1}{2}$. Who is correct? Explain your reasoning.

ELL Tip

Ensure students understand the acronym *LCD* (*least common denominator*). Ask students to demonstrate their understanding of the term by identifying the *LCD* of several fraction pairs.

3. **Rewrite the equation containing fractions as an equivalent equation without fractions.**

4. **Determine the solution to the system of equations by using linear combinations and check your answer.**

5. **Interpret the solution of the linear system in terms of this problem situation.**

Answers

3. $2x + 3y = 196$
4. (56, 28)

$$x + y = 84$$
$$56 + 28 = 84$$
$$\frac{1}{2}x + \frac{3}{4}y = 49$$
$$\frac{1}{2}(56) + \frac{3}{4}(28) = 49$$
$$28 + 21 = 49$$

5. The School Spirit Club plans to make a total of 56 five-inch and 28 seven-inch friendship bracelets.

Answers

1a. Graphing Method: Graph the two lines and identify the intersection point; however, sometimes only an estimate of the intersection point is possible. This method is most appropriate when the equations are both written in general form, and the values are reasonable to graph.

1b. Substitution Method: Choose one equation and isolate one variable; this will be considered the first equation. Then substitute the expression equal to the isolated variable into the second equation. Solve the new equation with only one variable. Use substitution to solve for the second variable. This method is most appropriate when one of the equations is written in general form.

1c. Linear Combinations Method: Rewrite one or both of the equations so that the coefficients of one of the variables are additive inverses. Add the two equations resulting in an equation in one variable. Solve the new equation with only one variable. Use substitution to solve for the second variable. This method is most appropriate when both equations are written in standard form.

NOTES

TALK the TALK

There's a Method in My Madness

You have used three different methods for solving systems of equations: graphing, substitution, and linear combinations.

1. Describe how to use each method and the characteristics of the system that makes this method most appropriate.

a. Graphing Method:

b. Substitution Method:

c. Linear Combinations Method:

Assignment

LESSON 3: The Elimination Round

Write

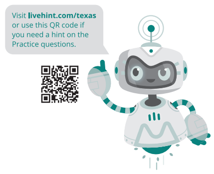
Explain how you would combine the two equations to solve for x and y. Use the following terms in your explanation: *linear combination* and *additive inverses*.

$$3x + 2y = -25$$
$$x - 4y = 5$$

Remember

You can use the linear combinations method to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

Practice



1. The two high schools, Jefferson Hills East and Jefferson Hills West, are taking field trips to the state capital. A total of 408 students from Jefferson Hills East will be going in 3 vans and 6 buses. A total of 516 students from Jefferson Hills West will be going in 6 vans and 7 buses. Each van has the same number of passengers and each bus has the same number of passengers.
   a. Write a system of equations that represents this problem situation. Let x represent the number of students in each van, and let y represent the number of students in each bus.
   b. How are the equations in the system the same? How are they different?
   c. Describe the first step needed to solve the system using the linear combinations method. Identify the variable that will be eliminated as well as the variable that will be solved for when you add the equations.
   d. Solve the system of equations using the linear combinations method. Show your work.
   e. Interpret the solution of the linear system in terms of the problem situation.
   f. Check your solution algebraically.
2. Solve each system of linear equations.

   a. $\begin{cases} 3x + y = 9 \\ 7x + y = 32 \end{cases}$ b. $\begin{cases} 5x - 8y = 25 \\ -x + 4y = -8 \end{cases}$ c. $\begin{cases} \frac{2}{3}x + \frac{1}{4}y = 18 \\ \frac{1}{6}x - \frac{3}{8}y = -6 \end{cases}$ d. $\begin{cases} 5x + 4y = -14 \\ 3x + 6y = 6 \end{cases}$

Stretch

Use linear combinations to solve the given system of three equations in three variables. Show your work.

$$\begin{cases} 3x + y + 3z = -2 \\ 6x + 2y + 9z = 5 \\ -2x - y - z = 3 \end{cases}$$

Assignment Answers

Write

I would apply the linear combinations method and create additive inverses for the coefficients of the x terms. Then I would add the two equations and solve for y. I would then substitute the value for y and solve for x.

Practice

1a. $\begin{cases} 3x + 6y = 408 \\ 6x + 7y = 516 \end{cases}$

1b. Both equations are written in standard form. The coefficients of x and y are different, as are the constant terms.

1c. Answers may vary.

Multiply the first equation by −2. Then, when the two equations are added the resulting equation can be solved for y.

1d. $x = 16$; $y = 60$

1e. The solution (16, 60) means that each van is carrying 16 students and each bus is carrying 60 students.

1f. $3(16) + 6(60) = 408$
$48 + 360 = 408$
$408 = 408$

$6(16) + 7(60) = 516$
$96 + 420 = 516$
$516 = 516$

2a. (5.75, −8.25)
2b. (3, −1.25)
2c. (18, 24)
2d. (−6, 4)

Stretch

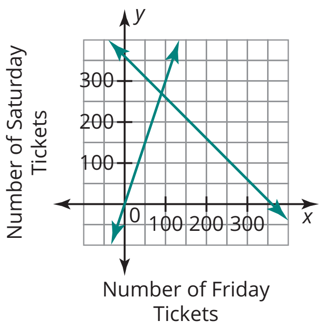
$x = -5$; $y = 4$; $z = 3$

Assignment Answers

Review

1a. $f + s = 360$; $s = 3f$

1b.



1c. Friday: 90 tickets

Saturday: 270 tickets

There is only one solution that satisfies both equations and conditions of the problem.

1d. $f + s = 360$

$s = 3f$

$f + (3f) = 360$

$4f = 360$

$f = 90$

$s = 3f$

$s = 270$

Friday: 90; Saturday: 270 (90, 270)

2a. $y = 76.0625x + 154.3125$

2b. $y = 1675.5625$, The population after 20 years is predicted to be 1676.

Review

1. The drama department sold a total of 360 tickets to their Friday and Saturday night shows. They sold three times as many tickets for Saturday's show than for Friday's show.
   a. Write a system of equations to represent this scenario.
   b. Graph the system of equations on a coordinate plane.
   c. How many tickets were sold for Friday? Saturday? Is there more than one solution?
   d. Justify algebraically that your solution is correct.
2. Analyze the data in the table.
   a. Write the equation of the regression line for the data.
   b. Predict the population after 20 years. Round your answer to the nearest whole number.

Number of years	Population
1	240
2	360
3	280
5	500
6	625
7	830
8	720
9	813
10	900

4

Throwing Shade

Graphing Inequalities in Two Variables

MATERIALS

None

Lesson Overview

Students explore a linear inequality in two variables through a scenario. They write an inequality, complete a table of values, graph the coordinate pairs from the table, and determine which parts of the graph are solutions to the inequality. Students then formalize the process of graphing inequalities through practice without context; they graph the corresponding equation of an inequality as a boundary line, determine whether the line should be solid or dashed, and identify which half plane to shade by testing the point (0,0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs. They then solve a problem in context where they use a table of values to write and graph a linear inequality and refer to the inequality and/or its graph to respond to questions. Finally, students summarize the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.

Algebra 1
Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(D) graph the solution set of linear inequalities in two variables on the coordinate plane.

ELPS

1.A, 1.C, 1.D, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- The graph of a linear inequality is a half-plane, or half of a coordinate plane.
- Shading is used to indicate which half-plane describes the solution to the inequality.
- Dashed and solid lines are used to indicate if the line itself is included in the solution set of an inequality.
- Linear inequalities and their graphs can be used to represent and solve problems in context.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Making a Statement

Students compare five different solution statements that relate the value of *x* to the constant 2. Next, they write a scenario that can be represented by one of the statements, and then modify the scenario so that it can be represented by one of the different statements.

Develop

Activity 4.1: Linear Inequalities in Two Variables

Students explore a linear inequality in two variables through a scenario. They write an inequality, complete a table of values, graph the values from the table, and determine which parts of the graph are solutions to the inequality.

Activity 4.2: Determining the Graphs of Linear Inequalities

Students graph inequalities by graphing each inequality's corresponding equation as a boundary line, determining whether it should be solid or dashed, and identifying which half plane to shade by testing the point (0, 0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs.

Day 2

Activity 4.3: Interpreting the Graph of a Linear Inequality

Students use a table of values to write and graph a linear inequality to model a context. They then use the inequality and/or its graph to respond to questions provided in context. Students also interpret the meaning of points on the line, above the line, or below the line.

Demonstrate

Talk the Talk: There's a Fine Line

Students demonstrate an understanding of the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.

Getting Started: Making a Statement

Facilitation Notes

In this activity, students compare five different solution statements that relate the value of *x* to the constant 2. Next, they write a scenario that can be represented by one of the statements, and then modify the scenario so that it can be represented by one of the different statements.

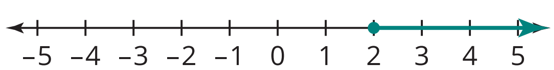
Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

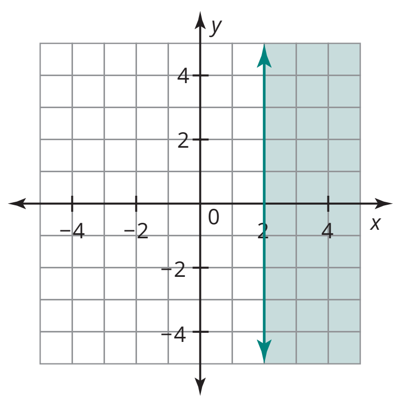
Questions to ask

- Which statement(s) has only one solution?
- Which statement(s) has an infinite number of solutions?
- Which statement(s) has no solution?
- Which solution statement(s) include the value of 0?
- Which solution statement(s) include negative numbers?
- Which solution statement(s) include the value of 2?
- Which solution statement(s) include fractional values?

Differentiation strategy

To extend the activity, ask students to graph the solution set $x \geq 2$. Do not provide additional guidance to see if students choose a number line or coordinate plane to sketch the graph. Then, discuss the appropriateness of each representation and the possible *y*-values when the solution statement is graphed on a coordinate plane.





Summary

When a solution statement compares a variable to a constant, the constant may or may not be included in the solution set.

Activity 4.1
Linear Inequalities in Two Variables

DEVELOP

Facilitation Notes

In this activity, students write an inequality to represent a given scenario. They use the inequality to complete a table of values, graph the data from the table, and then determine which parts of the graph are solutions to the inequality.

Differentiation strategy

To extend the activity, have students work in groups using large poster-sized coordinate planes. Make the activity more open-ended by eliminating many of the scaffolding questions, and have students complete only Question 1 and Question 5. Extend Question 5 so that each student is responsible for creating a table with at least 10 different sets of values. Have them use green and red dot stickers to plot combinations of points that meet/exceed or do not meet/exceed Bena's points-per-game average, respectively. Have all groups display their graphs. Lead a discussion by asking students to explain patterns they observed, noting the boundary line and pulling out the mathematics from the remaining questions. When students are making the graphs, allow them to choose what axis represents the number of 2-point and the number of 3-point shots; however, suggest that they scale their axes by an interval of 1.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- What is a combination of shots in which Bena earns exactly 20 points?
- What is a combination of shots in which Bena exceeds her average of 20 points per game?
- What algebraic expression did you use to represent the number of two-point shots Bena makes?
- What algebraic expression did you use to represent the number of three-point shots Bena makes?
- What does the x-axis represent? What do the x-values represent?
- What does the y-axis represent? What do the y-values represent?
- What are the coordinates of a point representing 4 two-point shots and 1 three-point shot?
- When you graphed the equation, did you use a solid or dashed line? Why?
- What portion of the graph represents the problem situation?
- Why would the inequality $2x + 3y > 20$ be incorrect to represent this situation?

- Why would the inequality $2x + 3y < 20$ be incorrect to represent this situation?

Have students work with a partner or in a group to complete Questions 5 through 11. Share responses as a class.

Questions to ask

- How did you determine the total points in each row of the table of values?
- How did you decide whether to plot the point with an *x* or with a dot?
- How would you describe all of the points on the graph located below the line with respect to the problem situation?
- How would you describe all of the points on the graph located on the line with respect to this problem situation?
- How would you describe all of the points on the graph located above the line with respect to this problem situation?
- Did you shade the region located above the line or below the line? Why?
- Where is Bena's average of 20 points located on the graph?

Summary

The solutions to a linear equality are points either above or below the graph of the line and may or may not include the points on the line.

Activity 4.2

Determining the Graphs of Linear Inequalities

Facilitation Notes

In this activity, the graph of a linear equality is described as a half-plane. Students graph inequalities by graphing each inequality's corresponding equation as a boundary line, determining whether it should be solid or dashed, and identifying which half plane to shade by testing the point (0,0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Different methods to graph the boundary line, such as slope and *y*-intercept, a table of values, or transformations.
- Errors when using a table of values to graph the boundary line. If this occurs, it may be because students used the inequality, rather than the equation, to complete the table.

Misconception

Some students may be confused by language such as shading above or below the line, especially when the line appears nearly vertical to them and *shading left or right* seems to make more sense. Explain that the inequality is written with the variable y isolated, so they are looking for (x, y) pairs where the y-value is greater than $4x - 6$. If students are confused, have them focus on the y-intercept of the boundary line where above and below may have a more obvious correspondence to the terms *greater than* and *less than*.

Questions to ask

- Are the points in the shaded or unshaded region of the graph the solutions to the inequality?
- Do you think the origin, (0, 0), can be used as a test point to determine which region on the graph to shade for other problems?
- What is an example of a problem where the origin, (0, 0), cannot be used as a test point?
- Other than the origin, what is an example of another easy point that could be used as a test point?
- Why would a point located on either the x-axis or the y-axis be an easy point to test?
- How did you know to use a solid line or dashed line?
- How did you know which half-plane to shade?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

As students work, look for

- Whether students are using a test point or the inequality symbol to determine which half-plane to shade.
- Whether students keep Question 5, part (c), written in standard form to graph or if they rewrite it in general form.

Misconception

Students may overgeneralize and think that when the inequality symbol of an inequality written in standard form is greater than, it always means to shade above the boundary line, and when the inequality symbol is less than, it always means to shade below the boundary line. This is not always correct. This is only correct when the y-variable is isolated and written in front of the expression. Address this misconception when discussing Question 5, part (c).

Questions to ask

- Which inequalities are represented using a dashed line? A solid line?
- Which inequalities have graphs that are shaded above the line? Below the line?

- How do you graph the boundary line?
- How do you know to use a solid line or dashed line?
- How do you know which half-plane to shade?

Analyze the Worked Example following Question 5 as a class.

Have students work with a partner or in a group to complete Question 6. Share responses as a class.

Questions to ask

- What steps did you use to determine the inequality that represents the graph?
- How did you determine the equation of the boundary line?
- How is the dashed boundary line represented in your inequality?
- How did you know whether to use a greater than or less than symbol?
- How can you check whether your inequality is correct?

Differentiation strategy

To extend the activity, provide students with a linear equation and ask them to use the equation to draw a graph of each inequality situation. For example given $y = -3x + 5$, ask students to graph $y > -3x + 5$, $y \leq -3x + 5$, $y < -3x + 5$, and $y \geq -3x + 5$.

Summary

When graphing linear inequalities, dashed lines are associated with the symbols $<$ and $>$, whereas solid lines are associated with the symbols $\leq$ and $\geq$. The shaded region above or below the graphed line includes all the points that are solutions to the inequality.

Activity 4.3

Interpreting the Graph of a Linear Inequality

Facilitation Notes

In this activity, students use a table of values to write and graph a linear inequality to model a context. They then use the inequality and/or its graph to respond to questions provided in context. Students also interpret the meaning of points on the line, above the line, or below the line.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

Whether students use the table or graph to write the equation of the boundary line.

Questions to ask

- What points did you use to determine the slope?
- What is the slope of the line?
- What are the intercepts? What do they mean with respect to the context?
- How did you know whether to shade above or below the boundary line?
- What inequality symbol did you use? Why?
- What is another combination of calls to United States and Mexico that Cesar can make?

Have students work with a partner or in a group to complete Questions 3 through 6. Share responses as a class.

As students work, look for

Whether students use the inequality or graph to respond to the questions in context.

Questions to ask

- If Cesar used his phone card for 70 minutes talking to his aunt in Mexico, how can this information be used to determine the remaining minutes on the card?
- Which variable in the inequality represents the number of minutes talking to someone in Mexico? What is the coefficient of this variable?
- Using the inequality, how are the remaining U.S. minutes calculated?
- Using the inequality, how are the remaining Mexico minutes calculated?
- How can you use the inequality to answer this question?
- How can you use the graph to answer this question?

Summary

A real-world problem can be modeled by a linear inequality, and both the inequality and its graph can be used to solve problems in context.

DEMONSTRATE

Talk the Talk: There's a Fine Line

Facilitation Notes

In this activity, students demonstrate an understanding of the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- Which inequality graphs are shaded above the line? Below the line?
- Which inequality graphs are represented using a dashed line? A solid line?
- Which point was used to test the solution?
- How are the solutions to a linear equation represented on a graph?

Summary

The graph of an inequality uses the same line as the graph of the equation. The inequality symbol determines whether or not the line is included in the solution set and which half-plane contains the rest of the solution set.

4

Throwing Shade

Graphing Inequalities in Two Variables

Warm Up

Determine if each point is a solution to $y > x$, $y < x$, or $y = x$.

1. (8, −2)
2. (0, 7)
3. (−1, −1)
4. (−4, −3)
5. (9, 9)
6. (−3, −10)

Learning Goals

- Write an inequality in two variables.
- Graph an inequality in two variables on a coordinate plane.
- Determine whether a solid or dashed boundary line is used to graph an inequality on a coordinate plane.
- Interpret the solutions of inequalities mathematically and in the context of real-world problems.

Key Terms

- half-plane
- boundary line

You have graphed linear inequalities in one variable. What does the graph of a linear inequality in two variables look like? How does it compare to the graph of a linear equation?

Warm Up Answers

1. $y < x$
2. $y > x$
3. $y = x$
4. $y > x$
5. $y = x$
6. $y < x$

Answers

1. Each solution statement compares the variable x to the constant 2. The value 2 is the only solution to $x = 2$. Values greater than 2 are included in the solution sets of $x > 2$ and $x \geq 2$, and values less than 2 are included in the solution sets of $x < 2$ and $x \leq 2$. The value 2 is also included in the solution set of $x \geq 2$ and $x \leq 2$.
2. Answers will vary.

GETTING STARTED

Making a Statement

Consider each solution statement.

$$x = 2$$

$$x < 2 \qquad x \leq 2$$

$$x > 2 \qquad x \geq 2$$

1. Compare the solution statements. What does each one mean?

2. Choose a solution statement and write a scenario to represent it. Then, modify the scenario so the resulting interpretation is one of the other four solution statements.

ACTIVITY **4.1**

Linear Inequalities in Two Variables

1. Coach Purvis is analyzing the scoring patterns of players on his basketball team. Bena is averaging 20 points per game from scoring on two-point and three-point shots.

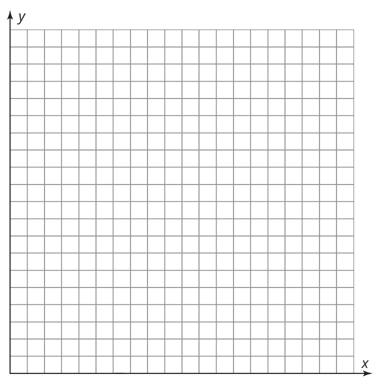
a. If she scores 6 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?

b. If she scores 7 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?

c. If she scores 7 two-point shots and 4 three-point shots, will Bena meet her points-per-game average?

2. Write an equation to represent the number of two-point shots and the number of three-point shots that total 20 points.

3. Graph the equation you wrote on the coordinate plane.



Ask yourself:

How should you label the graph?

Answers

1a. $12 + 6 = 18$. No, Bena will not meet her points-per-game average.

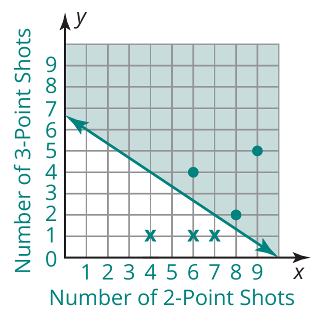
1b. $14 + 6 = 20$. Yes, Bena will meet her points-per-game average.

1c. $14 + 12 = 26$. Bena will exceed her points-per-game average.

2. Let x represent the number of two-point shots and let y represent the number of three-point shots.

$2x + 3y = 20$

3.



ELL Tip

Ask students to share experiences of any sports activities they have been involved in. Prior to Question 4, discuss the idea of sports teams, and ask if they are familiar with the term *district playoffs*. Explain how a *district* is *a particular area, or unit, such as in a school system*. Also discuss how sports teams will often compete in *playoffs*, which are *additional games played to determine the teams that will compete in a championship game*.

Answers

4a. The equation must be rewritten to show that the 2-point baskets and 3-point baskets Bena scores must be equal to or greater than Bena's points-per-game average.

4b. $2x + 3y \geq 20$

5.

Number of Two-Point Shots Scored	Number of Three-Point Shots Scored	Number of Total-Points Scored
4	1	11
6	1	15
7	1	17
8	2	22
6	4	24
9	5	33

6. All the points that exceed Bena's average are above the line, and the points that are do not exceed Bena's average are below the line.

4. Coach Purvis believes that Danvers High School can win the district playoffs if Bena scores at least 20 points per game.

a. How can you rewrite the equation you wrote in Question 2 to represent that Bena must score at least 20 points per game?

b. Write an inequality in two variables that represents this problem situation.

Remember:

An inequality is a statement formed by placing an inequality symbol ($<$, $\leq$, $>$, $\geq$) between two expressions.

5. Complete the table of values. Then, add the ordered pairs in the table to the graph in Question 3. If the number of total points scored does not meet or exceed Bena's points-per-game average, use an "*x*" to plot the point. If the number of total points scored meets or exceeds Bena's points-per-game average, use a dot to plot the point.

Number of Two-Point Shots Scored	Number of Three-Point Shots Scored	Number of Total Points Scored
4	1	
6	1	
7	1	
8	2	
6	4	
9	5	

6. What do you notice about your graph?

ELL Tip

Determine whether students are familiar with the word *exceed*. If not, create a list of synonyms for *exceed*, such as *surpass, outdo, to go past a limit,* and *is greater than*. Review the basketball team problem scenario, and ask students to explain how *exceeds* is used in the context of the scenario in reference to scoring points.

7. What can you interpret about the solutions of the inequality from the graph?

8. Choose a different ordered pair located above the line and a different ordered pair that is located below the graph. How do these points confirm your interpretation of the situation? Explain your reasoning.

9. Shade the side of the graph that contains the combinations of shots that are greater than or equal to Bena's points-per-game average.

10. How do the solutions of the linear equation $2x + 3y = 20$ differ from the solutions of the linear inequality $2x + 3y \geq 20$?

11. Does the ordered pair (6.5, 5.5) make sense as a solution in the context of this problem situation? Explain why or why not.

Like linear equations, linear inequalities take different forms. Each of the linear inequalities in two variables shown represents a different relationship between the variables.

$$ax + by < c \qquad ax + by > c$$

$$ax + by \leq c \qquad ax + by \geq c$$

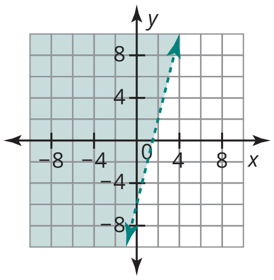
Answers

7. Sample answer.

   The line is the boundary of points that are and are not solutions to the inequality.

8. Answers will vary.
9. See graph in Question 3.
10. The solutions of the linear equation are the points on the line. The solutions of the linear inequality include half of the coordinate plane.
11. No. Bena cannot score any partial baskets so there can only be whole numbers of two-point and three-point shots.

Answers

1. Because the inequality symbol is $>$, the line is not included in the graph. Therefore, it should be represented by a dashed line.
2. 

ACTIVITY 4.2 Determining the Graphs of Linear Inequalities

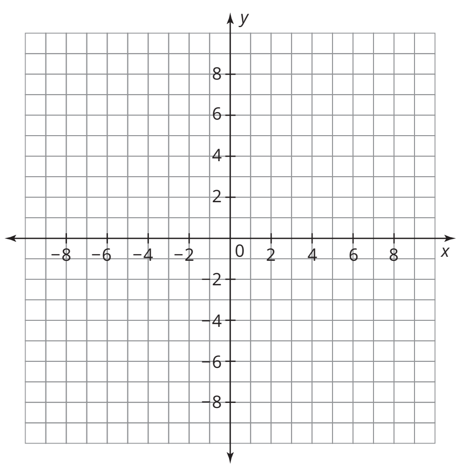
If the inequality symbol is $\leq$ or $\geq$, the boundary line is a solid line because all points on the line are part of the solution set. If the symbol is $<$ or $>$, the boundary line is a dashed line because no point on that line is a solution.

The graph of a linear inequality in two variables is a **half-plane**, or half of a coordinate plane. A **boundary line**, determined by the inequality, divides the plane into two half-planes and the inequality symbol indicates which half-plane contains all the solutions. These solutions are represented by shading the appropriate half-plane.

Consider the linear inequality $y > 4x - 6$. The boundary line that divides the plane is determined by the equation $y = 4x - 6$.

1. Should the boundary line in this graph be a solid line or a dashed line? Explain your reasoning.

2. Graph the boundary line on the coordinate plane shown.



ELL Tip

The term *boundary line* is used throughout the lesson. Discuss the non-mathematical use of the term *boundary line*, such as a line used to separate states and countries, or *boundary lines* on a sports field or court. Discuss the difference between the non-mathematical examples and the application of a *boundary line* when graphing the solution of a linear inequality.

After you graph the inequality with either a solid or a dashed boundary line, you need to decide which half-plane to shade. To make your decision, consider the point (0, 0). If (0, 0) is a solution, then the half-plane that contains (0, 0) contains the solutions and should be shaded. If (0, 0) is *not* a solution, then the half-plane that does not contain (0, 0) contains the solutions and should be shaded.

3. Decide which half-plane to shade.

a. Is (0, 0) a solution? Explain your reasoning.

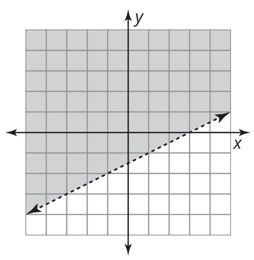
b. Shade the correct half-plane on the coordinate plane.

4. Match each graph to one of the inequalities given. In part (d), graph the inequality that was not graphed in parts (a) through (c).

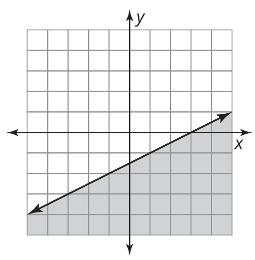
$y \geq \frac{1}{2}x - 3$ $\quad y \leq \frac{1}{2}x - 3$

$y > \frac{1}{2}x - 3$ $\quad y < \frac{1}{2}x - 3$

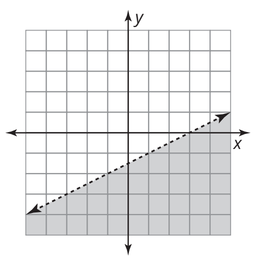
a.



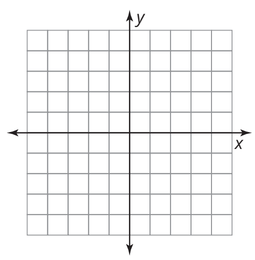
b.



c.



d.



Think about:

It's a good idea to check points in both half-planes to verify your solution.

Answers

3a. $0 > 4(0) - 6$
$0 > -6$

The ordered pair (0, 0) is a solution of the inequality.

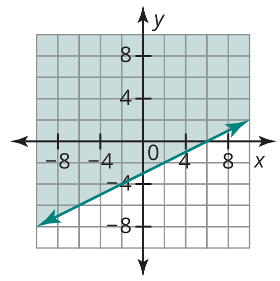
3b. See answer to Question 2.

4a. $y > \frac{1}{2}x - 3$

4b. $y \leq \frac{1}{2}x - 3$

4c. $y < \frac{1}{2}x - 3$

4d.



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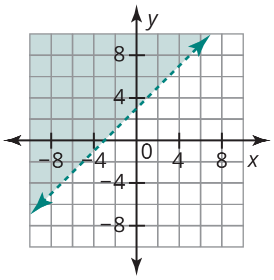
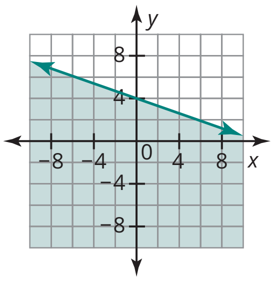
Answers

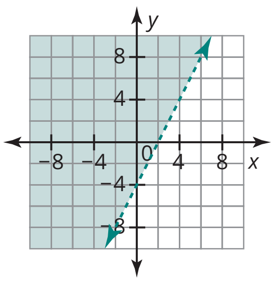
5a.



5b.



5c.

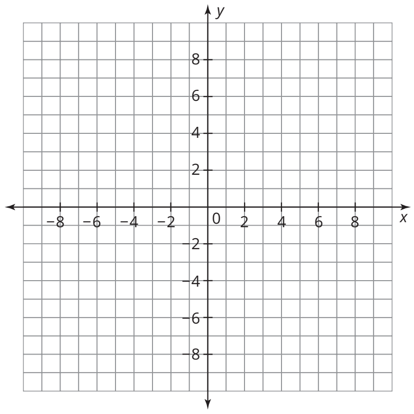


Think about:

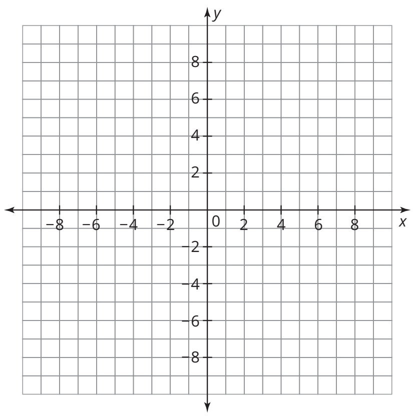
Consider the inequality symbol and which half-plane will be shaded before you test any points.

5. Graph each linear inequality.

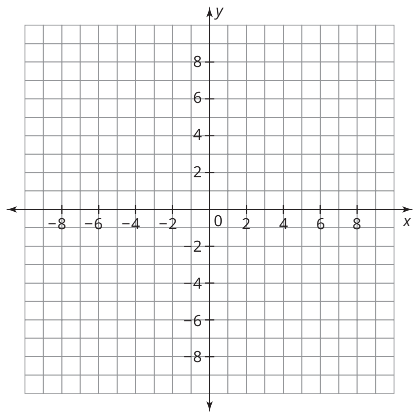
a. $y > x + 3$



b. $y \leq -\frac{1}{3}x + 4$



c. $2x - y < 4$



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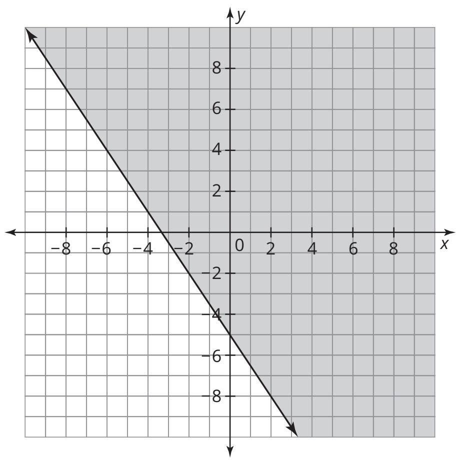
Previously you have written a linear equation given various representations, including two points, one point and the slope, a table of values, or a graph. You can use a similar approach when writing a linear inequality.

Worked Example

Write a linear equality for the graph.



You can use what you have previously learned about the graphs of linear equations to determine that the boundary line is represented by the equation $y = -\frac{3}{2}x - 5$. Now you must decide which inequality symbol should replace the equals sign in the equation.

Since the graph shows a solid boundary line and the half-plane above the line is shaded, use the symbol ≥.

$$y \geq -\frac{3}{2}x - 5$$

Test a point in the solution set to check the linear inequality.

Test the point (0, 0):

$$0 \overset{?}{\geq} -\frac{3}{2}(0) - 5$$

$$0 \geq -5 \checkmark$$

Remember:

The point (0, 0) can be used as a test point unless the boundary line passes through (0, 0).

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Answers

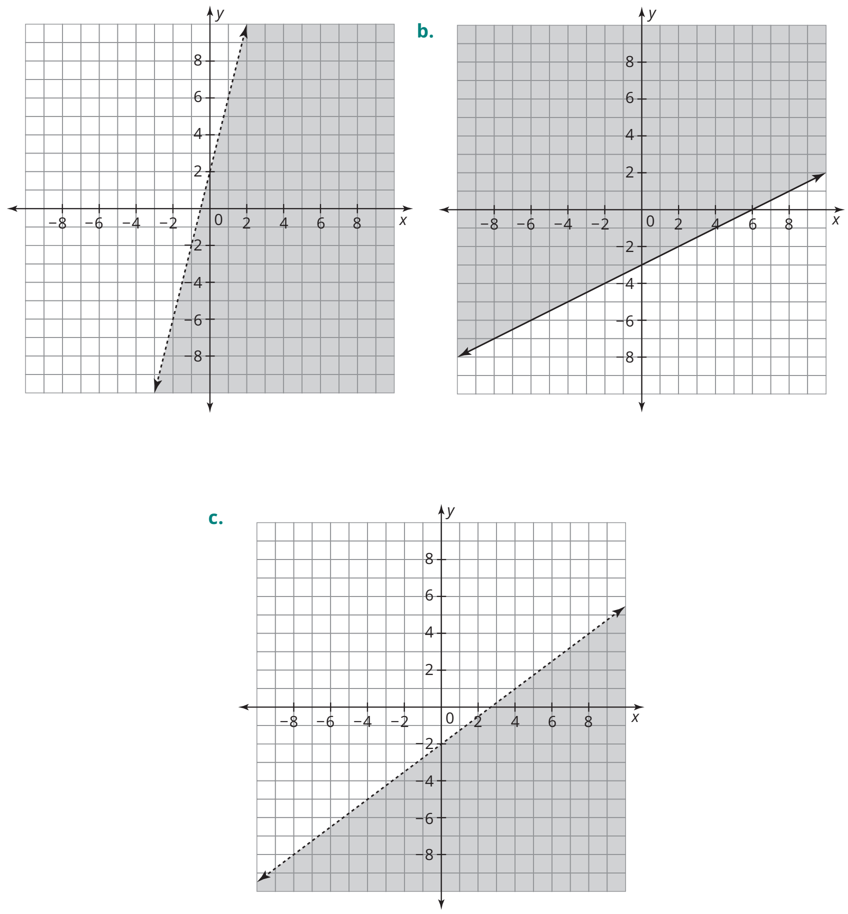
6a. $y < 4x + 2$

6b. $y \geq \frac{1}{2}x - 3$

6c. $y < \frac{3}{4}x - 2$

6. Write a linear inequality for each graph.

a. **b.** **c.**



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ACTIVITY **4.3**

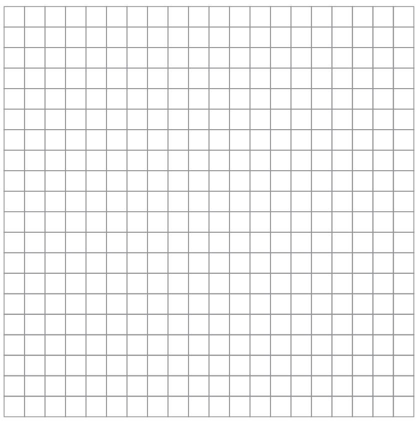
Interpreting the Graph of a Linear Inequality

César has relatives living in both the United States and Mexico. He is given a prepaid phone card worth $50 for his birthday. The table of values shows combinations of minutes for calls within the United States, *x*, and calls to Mexico, *y*, that expend his $50 prepaid phone card.

Length of Calls within United States (minutes)	Length of Calls to Mexico (minutes)
0	100
50	80
140	44
200	20
240	4

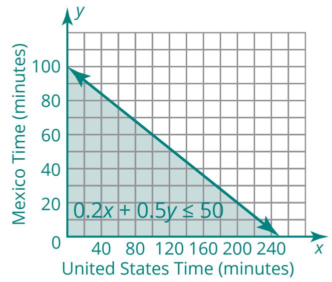
1. Write an inequality modeling the number of minutes César can use for calls within the United States and for calls to Mexico.

2. Graph your inequality on the given coordinate grid. Be sure to label your axes.



Answers

1. $0.2x + 0.5y \le 50$
2.



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Answers

3. César can speak with his cousin in New York for 75 or fewer minutes using his prepaid phone card.
4. No. The point (100, 80) is not in the solution set of the linear inequality.
5. Yes. The point (55, 90) is in the solution set of the linear inequality.

6a. The points on the line represent the maximum number of minutes César can spend on a call to Mexico given the number of minutes spent on a call within the United States using his prepaid phone card.

6b. The points above the line represent times in minutes that are impossible for César to spend on calls either to Mexico or within the United States using his prepaid phone card.

6c. The points below the line represent all the combinations of minutes César can spend on calls to Mexico and within the United States using his prepaid phone card.

3. If César speaks with his aunt in Guadalajara, Mexico, for 70 minutes using his phone card, how long can he speak with his cousin in New York using the same card?

4. Can César call his uncle in San Antonio for 100 minutes and also call his grandmother in Juárez, Mexico, for 80 minutes using his phone card? Explain your reasoning.

5. Can César call his brother in Mexico City, Mexico, for 55 minutes and also call his sister in Denver, for 90 minutes using his phone card? Explain your reasoning.

6. Interpret the meaning of each.

a. points on the line

b. points above the line

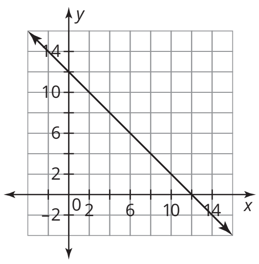
c. points below the line

NOTES

TALK the TALK

There's a Fine Line

Consider the graph of the linear equation $x + y = 12$.



Use the graph to answer each question.

1. **Describe how to graph $x + y < 12$ and choose a point to test this region.**

2. **Describe how to graph $x + y \leq 12$ and choose a point to test this region.**

3. **Describe how to graph $x + y > 12$ and choose a point to test this region.**

Answers

1. I would change the graph to a dashed line and I would shade the half-plane below the line.

   I can use the point (0, 0) to test this region: $0 + 0 < 12$

2. I would keep the graphed line and shade the half-plane below the line.

   I can use the point (0, 0) to test this region: $0 + 0 \leq 12$

3. I would change the graph to a dashed line and I would shade the half-plane above the line.

   I can use the point (10, 10) to test this region: $10 + 10 > 12$

Answer

4. See table below.

NOTES

4. Complete the table.

Equation or Inequality	Description of the Solution Set
$x + y = 0$	
$x + y \geq 0$	
$x + y \leq 0$	
$x + y > 0$	
$x + y < 0$	

Equation or Inequality	Description of the Solution Set
$x + y = 0$	all points that lie on the graphed line $y = -x$
$x + y \geq 0$	all points that lie on the graphed line $y = -x$ and all points contained in the half-plane above the graphed line
$x + y \leq 0$	all points that lie on the graphed line $y = -x$ and all points contained in the half-plane below the graphed line
$x + y > 0$	all points contained in the half-plane above the graphed line $y = -x$
$x + y < 0$	all points contained in the half-plane below the graphed line $y = -x$

Assignment

LESSON 4: Throwing Shade

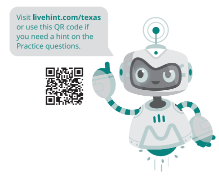
Write

Describe a half-plane in your own words.

Remember

The graph of a linear inequality in two variables is the half-plane that contains all the solutions. If the inequality symbol is $\geq$ or $\leq$, the graph shows a solid boundary line because the line is part of the solution set. If the symbol is $>$ or $<$, the boundary line is a dashed line because no point on the line is a solution.

Practice



1. Jeremy is working two jobs to save money for his college education. He makes \$8 per hour working for his uncle at Pizza Pie bussing tables and \$10 per hour tutoring peers after school in math. His goal is to make \$160 per week.
   a. If Jeremy works 8 hours at Pizza Pie and tutors 11 hours during the week, does he reach his goal?
   b. Write an expression to represent the total amount of money Jeremy makes in a week from working both jobs. Let x represent the number of hours he works at Pizza Pie and y represent the number of hours he tutors.
   c. After researching the costs of colleges, Jeremy decides he needs to make more than \$160 each week. Write an inequality in two variables to represent the amount of money Jeremy needs to make.
   d. Graph the inequality from part (c).
   e. Is the point (0, 0) in the shaded region of the graph? Explain why or why not.
   f. According to the graph, if Jeremy works 5 hours at Pizza Pie and tutors for 10 hours, will he make more than \$160? Explain why or why not.
   g. Due to days off from school, Jeremy will only be tutoring for 6 hours this week. Use the graph to determine the least amount of full hours he must work at Pizza Pie to still reach his goal. Then show that your result satisfies the inequality.
2. Graph each inequality on a coordinate plane.
   a. $x + 3y > 9$
   b. $2x - 6y \leq 15$
   c. $2x + y < 6$
   d. $3x - y \geq 1$

Assignment Answers

Write

Sample answer.
A half-plane is the half of the coordinate plane that contains the solutions to an inequality. The coordinate plane is separated by a line created from the inequality, and the half of the plane that contains the solutions to the inequality is shaded.

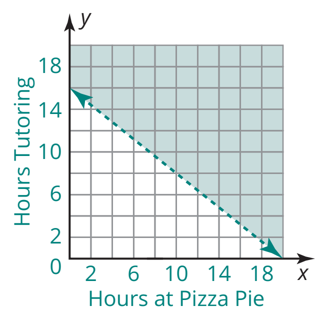
Practice

1a. Yes.

1b. $8x + 10y$

1c. $8x + 10y > 160$

1d.



1e. No. The point (0, 0) is not in the shaded region because it is not a solution to the inequality.

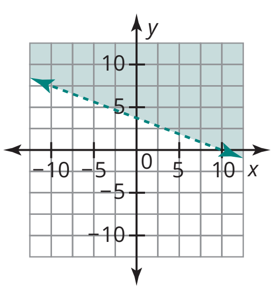
1f. No. The point (5, 10) on the graph is not in the shaded solution region.

1g. Jeremy must work at least 13 hours at Pizza Pie in order to make more than 160 dollars for the week.

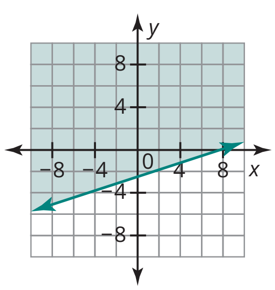
$8(13) + 10(6) > 160$

$164 > 160$

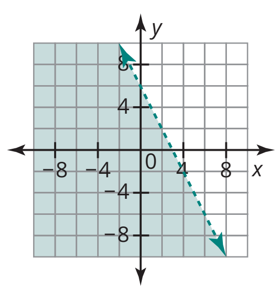
2a.



2b.



2c.

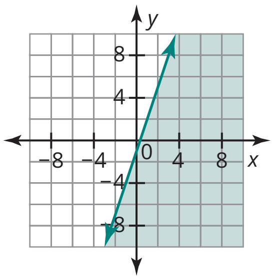


2d. See answer on the next page.

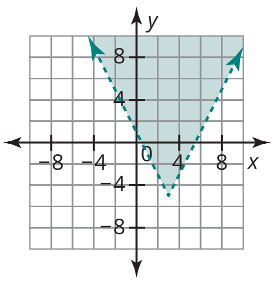
Assignment Answers

Practice

2d.



Stretch



Review

1a. (−1, 2)

1b. (7, −1)

2a. $y = \frac{2}{3}x - \frac{16}{3}$

2b. $y = -4x + 9$

Stretch

Use what you know about absolute value functions to graph the inequality $y > 2|x - 3| - 5$.

Review

1. Solve each system using the Linear Combinations Method.

   a. $\begin{cases} 8x - 6y = -20 \\ -16x + 7y = 30 \end{cases}$ b. $\begin{cases} x + 3 = -7y + 3 \\ 2x - 8y = 22 \end{cases}$

2. Write the equation of the line that has the given slope and passes through the point given.

   a. $m = \frac{2}{3}$; (2, −4) b. $m = -4$; (0.5, 7)

5

Working with Constraints

Systems of Linear Inequalities

MATERIALS

None

Lesson Overview

Students represent a scenario with a system of linear inequalities and graph the system. Overlapping shaded regions identify the possible solutions to the system. Students then practice graphing several systems of inequalities and representing the solution set. A different scenario is given that students model with a system of linear inequalities. They then graph the system, determine two different solutions, and algebraically prove that the solutions satisfy both constraints defined by the system. Finally, students match systems, graphs, and possible solutions of systems that have identical terms with different inequality symbols.

Algebra 1

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(D) graph the solution set of linear inequalities in two variables on the coordinate plane.

(H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

ELPS

1.A, 1.C, 1.D, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are constrained to lie within a certain region.
- The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersecting region satisfies all inequalities in the system.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: A River Runs Through It

Students are presented with a scenario and write two inequalities to represent each of the defined relationships before writing a system of linear inequalities to represent the entire problem situation.

Develop

Activity 5.1: Determining Solutions to Systems of Linear Inequalities

Students graph the inequalities they wrote in the Getting Started. The definition of a *solution of a system of linear inequalities* as the intersection of the solutions to each inequality is provided, and students apply that definition to determine the solution graphically. They discuss the number of possible solutions, as well as interpreting coordinate pairs that are and are not solutions in terms of the context.

Day 2

Activity 5.2: Analyzing Graphs of Systems of Linear Inequalities

Students graph a system of linear inequalities. They then use algebraic methods to determine when a point is a solution to the system. Students explain why the point of intersection of two linear inequalities is not always included in the solution to a system of inequalities. Finally, they solve two systems of linear inequalities that include parallel lines.

Activity 5.3: Applying Systems of Linear Inequalities

Students solve a problem in context requiring a system of linear inequalities. They interpret solutions in terms of the problem situation and use algebra to demonstrate the solutions satisfy both constraints. Because of the decimal coefficients in this problem, students are encouraged to use graphing technology.

Day 3

Activity 5.4: Identifying Systems of Linear Inequalities

Students match each of four systems of linear inequalities to their graph and a solution expressed as a coordinate pair. The systems differ only by their combinations of inequality symbols.

Demonstrate

Talk the Talk: Get to Know the Region

Students determine the region of the graph of the solution set for different systems of linear inequalities. Next they compare the solution of a system of linear inequalities to the solution of a system of linear equations.

ENGAGE

Getting Started: A River Runs Through It

Facilitation Notes

In this activity, students are presented with a scenario and write two inequalities to represent each of the defined relationships before writing a system of linear inequalities to represent the entire problem situation.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Whether or not Chase was counted as one of the adults on the raft. Chase will be on the raft so his weight must be considered, but he will not be paying for the experience.
- Different inequalities that students may possibly write involving money, where c represents the number of children and a represents the number of adults including Chase.

$$50c + 75(a - 1) \geq 150$$
$$50c + 75a - 75 \geq 150$$
$$50c + 75a \geq 225$$

  Discuss their equivalence, what each term represents, and what inequality makes most sense for the problem situation.

Differentiation strategies

- To scaffold support, suggest students label each inequality so they can refer to the units in the problem to help connect coefficients to the correct variables in the appropriate inequality.

$$\text{Weight (pounds): } 100c + 200a \leq 800$$
$$\text{Money Collected (dollars): } 50c + 75(a - 1) \geq 150$$

- To extend the activity, have students modify their system of equations if a represents the number of adults not including Chase.

Questions to ask

- What is the weight capacity of a raft?
- What is the weight capacity of the raft not including Chase?
- What phrasing in the question guided you to use that inequality symbol?
- What is the meaning of each term in your inequality?
- When a represents the number of adult rafters, why does the expression $75(a - 1)$ represent the amount of money generated by adult rafters rather than the expression $75a$?

- When c represents the number of child rafters, why doesn't the expression $50(c - 1)$ represent the amount of money generated by child rafters rather than the expression $50c$?

Summary

Real-world problems can be modeled using a system of linear inequalities. The same variables must be used and defined the same way for all inequalities in the system.

Activity 5.1

Determining Solutions to Systems of Linear Inequalities

DEVELOP

Facilitation Notes

In this activity, students graph the inequalities they wrote in A River Runs Through It. The definition of a *solution of a system of linear inequalities* as the intersection of the solutions to each inequality is provided, and students apply that definition to determine the solution graphically. They discuss the number of possible solutions and interpret coordinate pairs that are and are not solutions in terms of the context.

Ask a student to read the definition aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Misconception

Students may think that satisfying only one goal, either the weight goal or the financial goal, qualifies it as part of the solution to the system.

Questions to ask

- If Chase takes 2 adults on the raft trip, what is the total weight of the adults on the raft trip?
- If Chase takes 2 children on the raft trip, what is the total weight of the children on the raft trip?
- If 2 adults and 2 children go with Chase on the raft trip, will Chase meet his financial goal?
- Why isn't 5 adults considered a solution to this system of inequalities?
- Do you need to meet both goals, the weight goal and the financial goal, to consider the values a solution to the system? Or can you meet just one goal?

- Are the lines representing the system of linear inequalities solid or dashed? Why?
- What are the coordinates of the point of intersection?
- Do negative values on this graph make sense in the problem situation? Why or why not?
- How are the solutions to the system of linear inequalities different than the solutions to the problem situation?

Summary

The solution to a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

Activity 5.2

Analyzing Graphs of Systems of Linear Inequalities

Facilitation Notes

In this activity, students graph a system of linear inequalities. They first determine the solution graphically and are then guided through a process including algebraic methods. Students explain why the point of intersection of two linear inequalities is not always included in the solution to a system of inequalities. Finally, they solve two systems of linear inequalities that include parallel lines.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

Different methods to graph the boundary line.

Misconceptions

- Students may graph both lines and then test the (0,0) point one time, rather than testing (0,0) and shading after each line is graphed. Address why testing (0,0) and shading must occur each time a new line is graphed and the coordinate plane is separated into two different half planes.
- Because the point of intersection is the solution to a system of linear equations, students may assume it is always included as part of the solution to a system of linear inequalities. Question 3 addresses this misconception.

Differentiation strategies

- To scaffold support, provide a Worked Example with numbered steps for reference.
- To assist all students,
  - Insist they extend all lines the entire length and width of the coordinate plane.
  - Have them use a different color pencil to graph each inequality, with each boundary line and its corresponding half-plane having the same color.
  - Make the two methods of determining the solution to a system of inequalities explicit: 1) shade after each line is graphed, and the solution to the system is the intersection of the solutions to each inequality, and 2) graph all lines with or without shading, then test a point from each portion of the graph in the algebraic representation of each inequality in the system. If the point value creates true statements for each inequality, then that portion of the graph is a solution to the system. The entire solution to the system is the sum of all portions of the graph that are solutions.

Questions to ask

- How are the inequality signs in this system different than the inequality signs in the last problem?
- Are the lines representing the system of linear inequalities solid or dashed? Why?
- What are the coordinates of the point of intersection?
- Do your coordinates satisfy the constraint $y > -x + 1$? How do you know?
- Do your coordinates satisfy the constraint $y \leq x + 3$? How do you know?
- How do you know whether the point of intersection is a solution to the system of linear equations?
- How is the algebraic solution to a system of linear equations different than the algebraic solution to a system of linear inequalities?

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

Misconception

Because parallel lines imply no solution for a system of linear equations, students may assume that parallel boundary lines always imply there is no solution for this type of system of linear inequalities. Question 4, part (b) disproves this misconception. To provide an example using non-vertical lines, make both inequality symbols the same for Question 4, part (a).

Questions to ask

- What do you notice about the slopes of the equations?
- If the slopes are the same, what does that tell you about the graph of the equations?
- Do the shaded regions overlap?
- If the shaded regions do not overlap, what does this tell you about the solutions to the system of inequalities?
- Do all systems with parallel lines have no solution?
- How do you test the point (0, 0) in an equation that does not have a *y*-variable?
- Do you think it is possible for a system of linear inequalities to have a solution that is the entire coordinate plane? Why or why not?

Summary

A system of inequalities can be solved graphically or by using a combination of graphing and algebraic methods. For a system of two linear equations, the point of intersection is the solution; however, for a system of linear inequalities, the point of intersection of two boundary lines may or may not be included in the solution. For a system of two linear equations, when the lines are parallel, there is no solution; however, for a system of linear inequalities, when the lines are parallel, the system may or may not have solutions.

Activity 5.3

Applying Systems of Linear Inequalities

Facilitation Notes

In this activity, students solve a problem in context requiring a system of linear inequalities. They interpret solutions in terms of the problem situation and use algebra to demonstrate the solutions satisfy both constraints. Because of the decimal coefficients in this problem, students are encouraged to use graphing technology.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Confusion about what coefficients to use for the equation representing time.
- Errors sketching the graph of the equation for calories by hand due to the decimal values.

- Errors rewriting the equation in general form to enter it in a graphing calculator.
- Difficulty transferring the graph created by technology to the coordinate plane on paper.

Misconception

Because information about the context is provided in table form, students may think that the dependent variable is calories burned per minute because it is the title of the second column. They may also assume the independent quantity is time because it often used as a label for the *x*-axis.

Questions to ask

- What are Jackson's goals?
- What are the relationships that need to be described by inequalities?
- What are the independent and dependent quantities in this situation? What are their units?
- How did you know what coefficients to use for the variables in each equation?
- Explain how you composed your inequalities.
- What units are measured on the axes?
- Are the lines on the graph dashed or solid? Why?
- Describe what the intercepts mean in the context of this situation.
- Where is the value 400 calories evident on the graph?
- Where is the value 45 minutes evident on the graph?
- If Jackson only uses the stair stepper with light effort, will he reach his goal?
- If Jackson only uses the stair stepper with vigorous effort, will he reach his goal?
- If Jackson uses the stair stepper with light effort for half the time and uses vigorous effort for half the time, will he reach his goal?
- How do you know if your solution satisfies both constraints?
- Can your solution have non-integer values? Why or why not?
- What are solutions that do not lie on the boundary line?
- Is it a good idea to use technology to solve this problem?
- How do you enter the inequalities on a graphing calculator?
- Is this problem more difficult to solve than A River Runs Through It? Why or why not?

Differentiation strategies

- To assist all students with graphing the inequalities, suggest they use technology to graph the boundary lines prior to creating the graph on paper.

    - Use technology to determine the point of intersection. Because the point of intersection must be visible on the graph, this is helpful to know prior to scaling the axes.
    - Use the table feature to plot points on the graph. This may be more accurate than trying to use the slope and *y*-intercept that are decimal values on a coordinate plane scaled with large intervals.
- To extend the activity, have students solve similar problems using any two exercise entries from the first column of the table.

Summary

Technology is a useful tool when solving problems with non-integer values.

Activity 5.4
Identifying Systems of Linear Inequalities

Facilitation Notes

In this activity, students match each of four systems of linear inequalities to their graph and a solution expressed as a coordinate pair. The systems differ only by their combinations of inequality symbols.

Ask a student to read the introduction. Discuss Question 1 as a class.

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

Differentiation strategy

To assist all students, suggest they number the top inequality in each system as 1, and the bottom inequality as 2. That way, they can label the boundary lines with their corresponding inequality for reference purposes.

Questions to ask

- How do you know which boundary line corresponds to each inequality?
- What is missing from each graph?
- Were you able to mentally match the systems with their graphs, or did you need to graph each one yourself first?
- Does the solution satisfy both constraints in the system?
- Did you match the possible solutions written as coordinate pairs with the system or the graph? Why is that process easier for you?

Summary

The solution sets of systems of linear inequalities that differ by inequality symbols also differ graphically by intersecting regions.

Talk the Talk: Get to Know the Region

DEMONSTRATE

Facilitation Notes

In this activity, students determine which region of the graph represents the solution set for each given system of linear inequalities. They then compare the solution of a system of linear inequalities to the solution of a system of linear equations.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

- Incorrect interpretations of the inequality symbol when y is on the right of the inequality symbol.
- Rewriting the inequality so that y is on the left of the inequality symbol.
- Sketches to determine the solutions to each inequality.

Questions to ask

- Is it necessary to sketch the graph of each system to identify the solution set? Why?
- What are the coordinates of the intercepts?
- What are the slopes of the two lines?
- Did you shade above or below the first inequality? The second inequality?
- Is the intersecting region in each system different?
- Is there one solution or multiple solutions when the graphs of two inequalities in a system of linear inequalities intersect?
- Is there one solution or multiple solutions when two equations in a system of linear equations intersect?

Summary

The inequality symbols in a system of linear inequalities determine the location of the intersecting region.

NOTES

5

Working with Constraints

Systems of Linear Inequalities

Warm Up

Determine an ordered pair (x, y) that satisfies each inequality.

1. $x + y < 18$
2. $x - y > -7$
3. $2x + 3y \leq -5$
4. $-5x - 2y \geq 10$

Learning Goals

- Represent constraints in a problem situation with systems of inequalities.
- Write and graph systems of linear inequalities.
- Graph the solutions to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- Verify solutions to systems of linear inequalities algebraically.

Key Terms

- constraints
- solution of a system of linear inequalities

You have graphed a linear inequality in two variables and interpreted the solutions. What does the graph of a system of linear inequalities look like, and how can you describe the solution set?

Warm Up Answers

Sample answers.

1. (0, 0)
2. (0, 0)
3. (−1, −2)
4. (0, −5)

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Answers

1. Let a represent the number of adult rafters, and let c represent the number of children under age sixteen.

   $200a + 100c \leq 800$
2. $75(a - 1) + 50c \geq 150$
3. $200a + 100c \leq 800$
   $75(a - 1) + 50c \geq 150$

GETTING STARTED

A River Runs Through It

Chase is an experienced whitewater rafter who guides groups of adults and children out on the water for amazing adventures. The raft he uses can hold 800 pounds of weight. Any weight greater than 800 pounds can cause the raft to sink, hit more rocks, and/or maneuver more slowly.

Chase estimates the weight of each adult as approximately 200 pounds and the weight of each child as approximately 100 pounds. Chase charges adults $75 and children $50 to ride down the river with him. His goal is to earn at least $150 each rafting trip.

Think about:

Does Chase count himself when determining the weight and the cost?

1. Write an inequality to represent the most weight Chase can carry in terms of rafters. Define your variables.

2. Write an inequality to represent the minimum amount of money Chase wants to collect for each rafting trip.

3. Write a system of linear inequalities to represent the maximum weight of the raft and the minimum amount of money Chase wants to earn per trip.

ELL Tip

Students may not be familiar with the terms *whitewater rafting* and *maneuver*. Discuss what is meant by whitewater and what a raft is. Provide a list of synonyms for *maneuver*, such as *move, steer, navigate,* and *guide*. Relate the scenario of the trip to creating linear equations that represent the number of children and adults and their estimated weights for the rafts.

ACTIVITY
5.1 Determining Solutions to Systems of Linear Inequalities

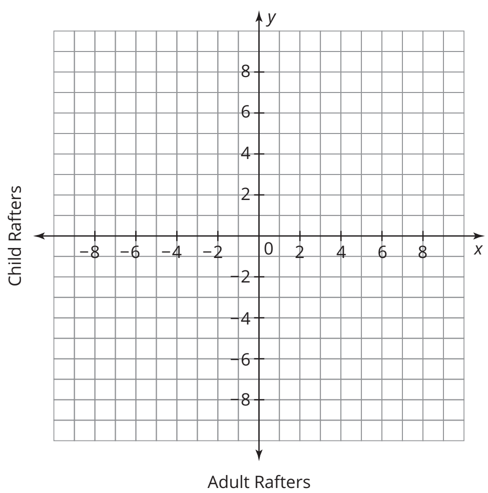
In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region on the graph.

1. Let's consider two trips that Chase guides. Determine whether each combination of rafters is a solution of the system of linear inequalities. Then describe the meaning of the solution in terms of this problem situation.

a. First Trip: Chase guides 2 adults and 2 children.

b. Second Trip: Chase guides 5 adults.

2. Graph the system of linear inequalities.



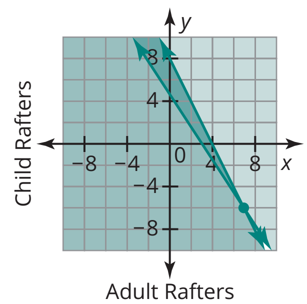
Shade the half-plane of each inequality differently. You can use colored pencils or simply vertical and horizontal lines.

Answers

1a. Yes. This is a solution of this system of linear inequalities because the number of adults and children results in a true statement for both inequalities. If 2 adults and 2 children go with Chase, they are at the weight requirements of the raft and he will earn at least $150.

1b. No. This is not a solution to the system of equations because it does not satisfy the inequality representing the weight requirement.

2.



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Answers

3a. A system of linear inequalities can have many solutions as long as the half-planes overlap.

3b. While the intersection point is a solution to this system of inequalities, it does not make sense in the problem situation. The intersection point (7, −6) would mean that there were 7 adults and −6 children, but there cannot be a negative number of children.

3c. Answers will vary.

3d. Answers will vary. The point (2, 1) does not represent a solution. Although Chase, 1 other adult, and 1 child are within the weight limit for the raft, the money earned is less than $150.

The **solution of a system of linear inequalities** is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

3. Analyze your graph.

a. Describe the possible number of solutions for a system of linear inequalities.

b. Is the intersection point a solution to this system of inequalities? Why or why not?

c. Identify three different solutions of the system of linear inequalities you graphed. What do the solutions represent in terms of the problem situation?

d. Determine one combination of adults and children that is not a solution for this system of linear inequalities. Explain your reasoning.

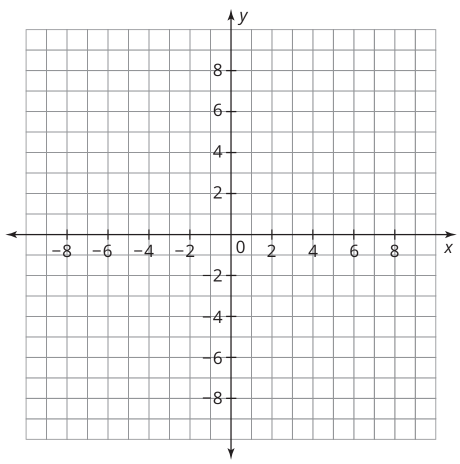
ACTIVITY **5.2**

Analyzing Graphs of Systems of Linear Inequalities

Determine the solution set of the given system of linear inequalities.

$$\begin{cases} x + y > 1 \\ -x + y \leq 3 \end{cases}$$

1. Graph the system of linear inequalities.

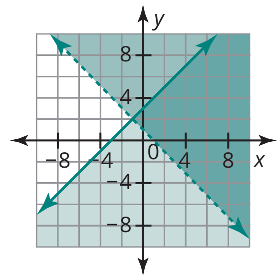


Think about:

Notice the inequality symbols. How does this affect your graph?

Answer

1.



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Answers

2. Answers will vary.
3. Alan is incorrect because the intersection point is not always a solution to the system of linear inequalities. The intersection point for this system, (−1, 2), only works for one of the inequalities, not both, which means it is not a solution. If the inequality symbols are not both "or equal to" then the intersection point is not a solution.

2. Choose a point in each shaded region of the graph. Determine whether each point is a solution of the system. Then describe how the shaded region represents the solution.

Point	$x + y > 1$	$-x + y \leq 3$	Description of location
(−8, 2)	$-8 + 2 > 1$ $-6 > 1$ ✗	$-(-8) + 2 \leq 3$ $10 \leq 3$ ✗	The point is not a solution to either inequality and it is located in the region that is not shaded by either inequality.

3. Alan makes the statement about the intersection point of a system of inequalities. Explain why Alan's statement is incorrect.

Alan
The intersection point is always a solution to a system of inequalities because that is where the two lines meet.

Point	$x + y > 1$	$-x + y \leq 3$	Description of location
(−8, 2)	$-8 + 2 > 1$ $-6 > 1$ ×	$-(-8) + 2 \leq 3$ $10 \leq 3$ ×	The point is not a solution to either inequality and it is located in the region that is not shaded by either inequality.
(2, 8)	$2 + 8 > 1$ $10 > 1$ ✓	$-2 + 8 \leq 3$ $6 \leq 3$ ×	The point is a solution of the first inequality, but not the second. It is located in the region shaded by the first inequality.
(8, 2)	$8 + 2 > 1$ $10 > 1$ ✓	$-8 + 2 \leq 3$ $-6 \leq 3$ ✓	The point is a solution for both inequalities and it is located in the region shaded by both inequalities.
(−2, −8)	$-2 + (-8) > 1$ $-10 > 1$ ×	$-(-2) + (-8) \leq 3$ $-6 \leq 3$ ✓	The point is a solution of the second inequality, but not the first. It is located in the region shaded by the second inequality.

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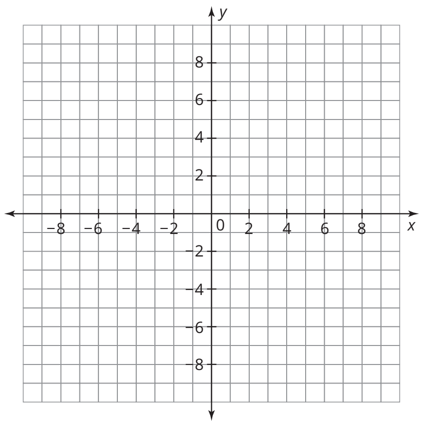
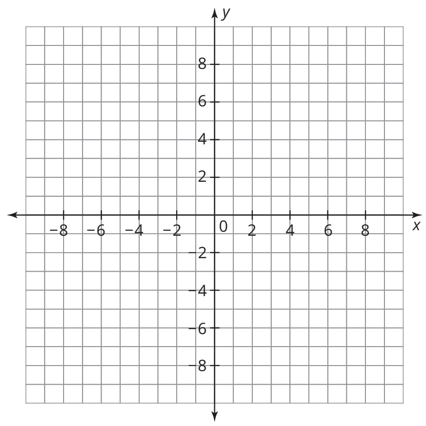
4. Solve each system of linear inequalities by graphing the solution set. Then identify two points that are solutions of the system.

a. $\begin{cases} y > 5x + 3 \\ y < 5x - 3 \end{cases}$

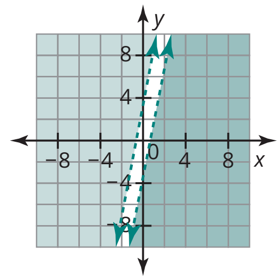


b. $\begin{cases} x \geq -5 \\ x \geq 1 \end{cases}$



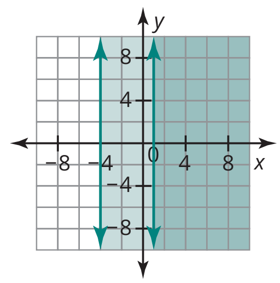
Answers

4a.



The solutions of each inequality in the system do not intersect. Therefore, there are no solutions.

4b.



Answers will vary.

Two possible solutions are (2, 0) and (5, 5).

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Answer

1. Let x represent the number of minutes spent on the stair stepper with light effort and let y represent the number of minutes spent on the stair stepper with vigorous effort.

$$\begin{cases} x + y \leq 45 \\ 6.9x + 10.4y \geq 400 \end{cases}$$

ACTIVITY 5.3

Applying Systems of Linear Inequalities

Jackson and a group of friends decide to use the fitness room after school. A sign on the wall provides the information shown.

Exercise	Calories Burned per Minute
Treadmill—light effort	7.6
Treadmill—vigorous effort	12.4
Stair Stepper—light effort	6.9
Stair Stepper—vigorous effort	10.4
Stationary Bike—light effort	5.5
Stationary Bike—vigorous effort	11.1

Jackson decides to use the stair stepper. He has at most 45 minutes to exercise and he wants to burn at least 400 calories.

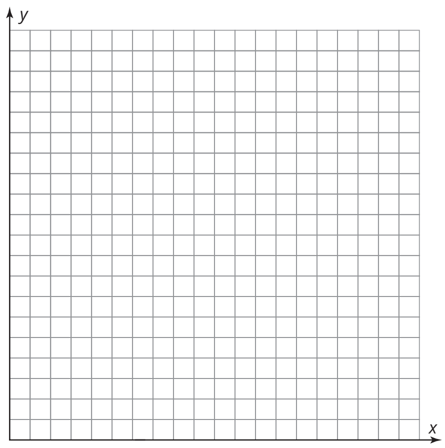
1. Write a system of linear inequalities to represent Jackson's workout. Define your variables.

ELL Tip

Some non-mathematical terms that appear in this activity are *fitness room, treadmill, stair stepper,* and *stationary bike*. Discuss these terms so that students may engage more fully in the activity.

2. Graph the system of inequalities from Question 1 on the coordinate plane. Be sure to label your axes.

Use technology to graph your inequalities and check your answer.



3. Analyze your graph.

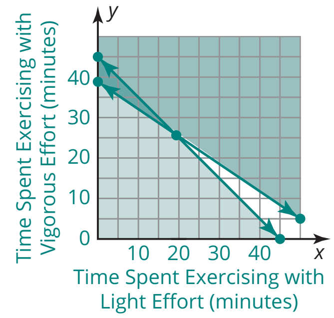
a. Identify two different solutions of the system of inequalities.

b. Interpret your solutions in terms of Jackson's workout.

c. Algebraically prove that your solutions satisfy the system of linear inequalities.

Answers

2.



3a. Two possible solutions are (0, 45) and (5, 40).

3b. The solution (0, 45) means that Jackson can exercise with light effort for 0 minutes and exercise with vigorous effort for 45 minutes and burn at least 400 calories in at most 45 minutes.

The solution (5, 40) means that Jackson can exercise with light effort for 5 minutes and exercise with vigorous effort for 40 minutes and burn at least 400 calories in at most 45 minutes.

3c. $0 + 45 \le 45$

$45 = 45$

$6.9(0) + 10.4(45) \ge 400$

$468 \ge 400$

The solution (0, 45) satisfies the system.

$5 + 40 \le 45$

$45 = 45$

$6.9(5) + 10.4(40) \ge 400$

$450.4 \ge 400$

The solution (5, 40) satisfies the system.

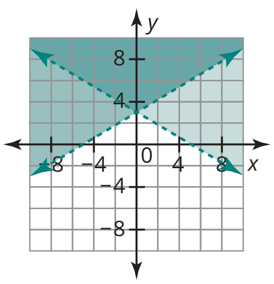
LESSON 5: Working with Constraints • 469

Answers

1. While it is true that the equations in each system are the same, this only defines the boundary lines. The inequalities are not the same. Each system uses a different combination of inequality symbols. While the graphs of the lines are the same, the different inequality symbols means that the intersecting regions will be different so the solutions will not be identical.
2. System A: Graph C; answers will vary for Possible Solutions D.

   Sample answers. (6, 2) and (8, 0)

   System B: see Graph D and Possible Solutions B



System C: Graph B and Possible Solutions C

System D: Graph A and Possible Solutions A

ACTIVITY **5.4**

Identifying Systems of Linear Inequalities

Consider the four systems shown.

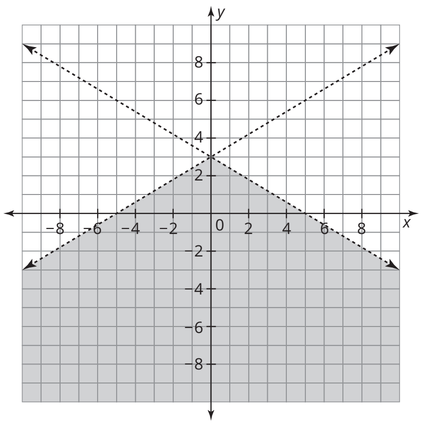
System A	System B	System C	System D
$\begin{cases} y < \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y > \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y > \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y < \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$

1. Analyze Adele's statement and explain why it is incorrect.

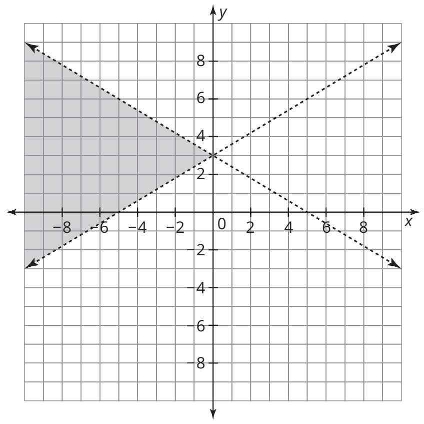
> **Adele**
> Since the equations in each system are the same, the graphs and solutions should all be identical.

2. Match a graph and possible solution to each given system of linear inequalities. Complete the blank graph and partial solution set to make 4 complete sets.

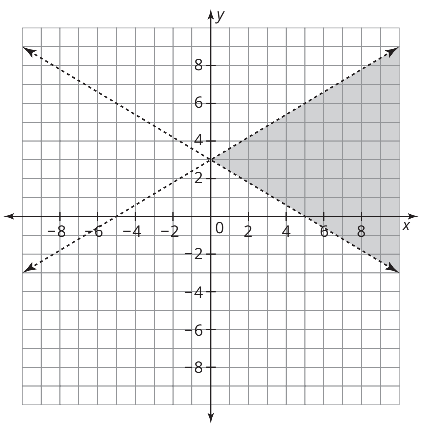
Graph A



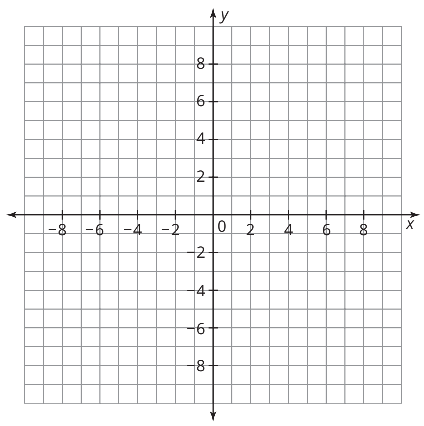
Graph B



Graph C



Graph D



Possible Solutions A	**Possible Solutions B**	**Possible Solutions C**	**Possible Solutions D**
(0, −6) and (4, −4)	(0, 7.5) and (−4, 10)	(−6, 4) and (−10, 8)	_____and_____

LESSON 5: Working with Constraints • 471

Answers

1a. A region below both lines.

1b. A region above both lines.

1c. No solution.

1d. A region between the lines.

2. The solution to a system of linear inequalities and a system of linear equations will satisfy both inequalities or both equations. There may be no solutions to a system of linear inequalities or to a system of linear equations. When two equations in a system of linear equations intersect, there is one unique solution. When the graphs of two inequalities in a system of linear inequalities intersect, there are multiple solutions.

NOTES

TALK the TALK

Get to Know the Region

The solution set to a system of inequalities can be any of four regions on the coordinate plane.

1. Consider each system of linear inequalities and decide which region represents the solution set. Explain your reasoning.

- **A region above both lines.**
- **A region below both lines.**
- **A region between the lines.**
- **No solution.**

a. $y < 8 + 2x$
$3 + 2x > y$

b. $5 + x < y$
$y > 7 + x$

c. $y > 12 - 3x$
$10 - 3x > y$

d. $6 - x < y$
$y < 9 - x$

2. How is the solution to a system of linear inequalities the same as or different from the solution to a system of linear equations?

Assignment

LESSON 5: Working with Constraints

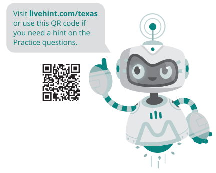
Write

Describe how you know which region, if any, represents the solution to a system of linear inequalities.

Remember

The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

Practice



1. Samuel is remodeling his basement. One part of the planning involves the flooring. He knows that he would like both carpet and hardwood, but isn't sure how much of each he will use. The most amount of flooring area he can cover is 2000 square feet. The carpet is \$4.50 per square foot and the hardwood is \$8.25 per square foot. Both prices include labor costs. Samuel has budgeted \$10,000 for the flooring.
   a. Write a system of inequalities to represent the maximum amount of flooring needed and the maximum amount of money Samuel wants to spend.
   b. One idea Samuel has is to make two rooms—one having 400 square feet of carpeting and the other having 1200 square feet of hardwood. Determine whether this amount of carpeting and hardwood are solutions to the system of inequalities. Explain your reasoning in terms of the problem situation.
   c. Graph this system of inequalities.
   d. Determine the intersection point of the two lines. Is this a solution to this system of inequalities in terms of the problem situation?
   e. Identify two different solutions to the system of inequalities. Explain what the solutions represent in terms of the problem situation.
   f. Determine one combination of amounts of carpet and hardwood that is not a solution for the system of inequalities. Explain your reasoning.
2. Solve each system of linear inequalities.

   a. $\begin{cases} -x + 3y \leq -6 \\ -5x + 3y \geq 6 \end{cases}$ b. $\begin{cases} -x + 2y < 6 \\ 3x + 2y \leq 2 \end{cases}$ c. $\begin{cases} -x + 3y \leq 18 \\ x \leq 3 \end{cases}$

Assignment Answers

Write

Answers will vary.

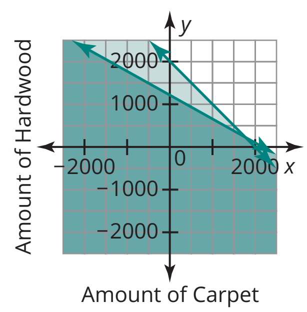
To solve a system of linear inequalities, I graph each inequality individually. I then identify which region represents a solution to both inequalities–the region that has been shaded twice. If there is no region with overlapping shading, then there are no solutions to the system.

Practice

1a. $\begin{cases} x + y \leq 2000 \\ 4.50x + 8.25y \leq 10{,}000 \end{cases}$

1b. No. This is not a solution to this system of inequalities because this amount of carpet and hardwood only results in a true statement for one of the inequalities. This means that although it will not exceed the total amount of square footage available to finish in the basement, the amount of each puts the total cost for flooring over the maximum budget of \$10,000.

1c.



1d. The intersection point of this system of inequalities is $(1733\frac{1}{3}, 266\frac{2}{3})$. Although it does make sense that there can be $1733\frac{1}{3}$ square feet of carpet and $266\frac{2}{3}$ square feet of hardwood, Samuel will most likely have to buy the flooring in whole number values of square feet.

1e. Sample answer.

(1500, 250); this solution means that Samuel can put down 1500 square feet of carpet and 250 square feet of hardwood and not have too much flooring while not going over the budget.

(0, 1000); this solution means that Samuel can put down no carpeting and 1000 square feet of hardwood and not have too much flooring while not going over the budget.

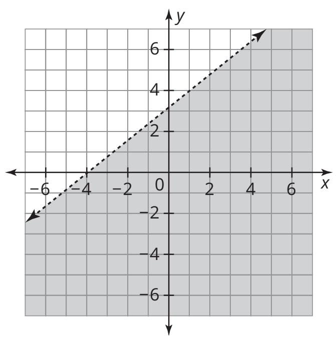
1f.–2c. See answers on the next page.

Stretch

1. Is it possible to create a system of inequalities that has no solutions? If so, create one and explain how the graph would show no solutions. If not, explain why.
2. Is it possible to create a system of two inequalities that has only one solution? If so, create one. If not, explain why.
3. Is it possible to create a system of three inequalities that has only one solution? If so, sketch a graph to show the solution. If not, explain why.

Review

1. Determine whether each equation has one solution, no solution, or infinite solutions.
   a. $24x - 22 = -3(1 - 8x)$
   b. $-3(4a + 3) + 2(12a + 2) = 43$
   c. $4(x + 1) = 6x + 4 - 2x$
2. Graph $3x + y \le 7$ on a coordinate plane.
3. Write a linear inequality for the graph.



4. A new workout gym opens up down the street from your house. Below are their total membership numbers for the first months of business.

Month	January	February	March	April	May
Number of Members	120	190	290	370	450

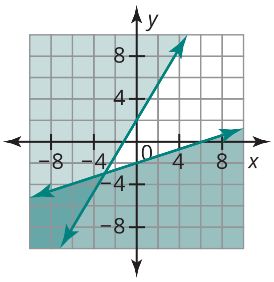
   a. Write the equation of the regression line for the data.
   b. Use the equation to predict the gym's total membership at the end of the year.

Assignment Answers

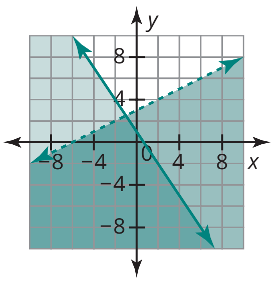
Practice

1f. Sample answer. The point (500, 1000) does not represent a solution. Although Samuel would not have too much flooring, he would go over budget if he bought 500 square feet of carpet and 1000 square feet of hardwood.

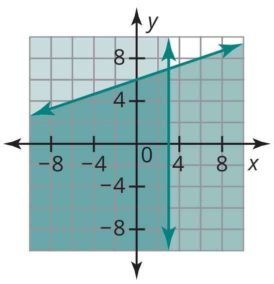
2a.



2b.

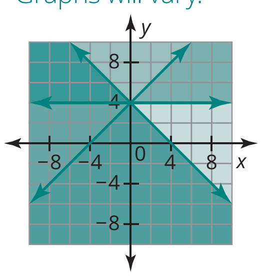


2c.



Stretch

1. Yes. It is possible. There would not be any region where the shading from the two inequalities overlaps.
$$\begin{cases} y \ge -2x + 13 \\ 2x + y \le 6 \end{cases}$$
2. No. It is not possible. All systems of linear inequalities that have a solution, have infinite solutions. There is no way to have the half-planes intersect at only one point. If the inequalities are not linear, then it is possible.
3. Yes. It is possible. Graphs will vary.



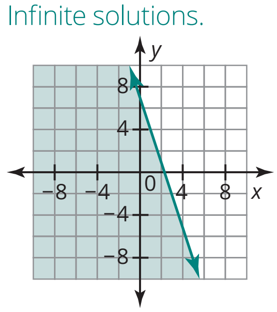
Review

1a. No solution.

1b. One solution.

1c. Infinite solutions.

2.



3. $y < 0.8x + 3$

4a. $y \approx 84x + 32$ (This assumes that January = 1, February = 2, etc.)

4b. 1040 memberships

6

Working the System

Solving Systems of Equations and Inequalities

MATERIALS

None

Lesson Overview

Students solve problems in context requiring a system of linear equations. While most problems can be modeled by a system of two equations, they are guided through the process of solving a system of four equations, and another context can be modeled by a system of three equations. Students have the opportunity to solve the systems using any method and sometimes must respond in the format of an email or proposal. Solutions involve making a decision based upon inputs that lie before or after the point of intersection, thus requiring solutions written as inequalities.

Algebra 1
Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description.

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(D) graph the solution set of linear inequalities in two variables on the coordinate plane.

(H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

ELPS

1.A, 1.C, 1.D, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- Contexts about choosing between two options can sometimes be modeled by a system of linear equations or inequalities.
- The point of intersection of two lines separates the input values, with *x*-values less than and *x*-values greater than the *x*-value of the point of intersection. The solution to a problem in context may be dependent upon where the input values lie relative to the point of intersection.
- Based upon a context, the solution of a system may be represented by inequalities rather than a single coordinate pair.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Systems of Summer Savings

Students write a system of linear equations and a system of linear inequalities to determine the solution to a real-world problem.

Develop

Activity 6.1: Determining the Better Deal

Students write a system of linear equations, with each equation in the form $y = ax + b$, to model a scenario. After determining the point of intersection of the system, students must interpret the meaning of the system and its point of intersection to determine the better deal. Students write a proposal with the better deal, including evidence from their analysis.

Activity 6.2: Determining the Better Buy

Students write a system of linear equations, with each equation in the form $y = a(x - c) + b$, to model a scenario. The best buy must be interpreted using inequalities based upon the input values.

Day 2

Activity 6.3: Solving a System of Linear Inequalities with Four Constraints

Students explore a context that can be modeled using a system of linear equations with four constraints. They write two of the constraints in terms of the sale price given the percent reduction, and represent the other two constraints as vertical lines. Students graph the system and use the graph and algebra to answer questions in context.

Activity 6.4: Determining the Better Job Offer

Students explore a context that can be modeled with several different systems of linear equations. Possible systems include equations based upon years, equations based upon months, equations in the form $y = a(cx) + b$, as well as a system of three equations. Once again, input values must be considered to determine the better job offer.

Demonstrate

Talk the Talk: Which Cab Is More Fab?

Students write a system of linear equations, with each equation in the form $y = ax + b$, to model a scenario. They make a decision as to what is the best option, with their response expressed as inequalities based upon the input values.

ENGAGE

Getting Started: Systems of Summer Savings

Facilitation Notes

In this activity, students write a system of linear equations and a system of linear inequalities to determine the solution to a real-world problem.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- Different interpretations for the system of inequalities:
  1. Using x to represent the number of times Susan uses the pool, $x > 25$ and $2x + 100 \leq 200$, with solution $25 < x \leq 50$.
  2. Using a to represent the number of times Susan uses the pool in July, and b as the number of times Susan uses the pool in August. $a + b > 25$ and $a + b \leq 50$, with solution $25 < a + b \leq 50$.
- Confusion with variable definitions for the equation and inequality.

Questions to ask

- What does the slope represent in this situation?
- What does the y-intercept represent in this situation?
- What algebraic expression best describes Plan A?
- What algebraic expression best describes Plan B?
- What is the significance of the point of intersection?
- What are the coordinates of the point of intersection?
- If Susan goes to the pool 24 times, which plan is the better deal?
- If Susan goes to the pool 25 times, which plan is the better deal?
- If Susan goes to the pool 26 times, which plan is the better deal?

Summary

A scenario can be represented by a system of linear equations and a system of linear inequalities. The solution to each system can then be interpreted in terms of the problem situation.

DEVELOP

Activity 6.1

Determining the Better Deal

Facilitation Notes

In this activity, students write a system of linear equations, with each equation in the form $y = ax + b$, to model a scenario. After determining the point of intersection of the system, students must interpret the meaning of the system and its point of intersection to determine the better deal. Students write a proposal with the better deal, including evidence from their analysis.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategy

To scaffold support, provide a template (with space for defining variables, writing the system of equations, solving the system, and interpreting the solution) to help students organize their work in this activity and in Activities 2 and 4. Provide as little scaffolding as possible so that students can experience the modeling process.

As students work, look for

- Various methods to solve the system of equations.
- Use of technology to graph the system and determine the point of intersection.
- Failure to extend the solution beyond the point of intersection.
- Questions about what makes a better deal, because that is not made explicit.

Questions to ask

- What will your recommendation depend upon?
- How will you know which plan is in the company's best interest?
- What expression represents the cost of making one or more bikes using the first plan?
- What expression represents the cost of designing and building a prototype plus the cost of making the bikes in first plan?
- What expression represents the cost of making one or more bikes using the second plan?
- What expression represents the cost of designing and building a prototype plus the cost of making the bikes in second plan?
- What is the significance of the point of intersection with respect to the problem situation?
- What are the coordinates of the point of intersection?
- What does the solution to this system of equations tell you about the problem situation?
- If the company makes 500 bikes, which plan would you recommend?
- How can you explain your recommendation using inequalities based upon the number of bikes?

Summary

In a real-world context involving determining the better deal between two relationships modeled by a system of linear equations, the intersection point of the lines and the possible input values are helpful in making decisions.

Activity 6.2

Determining the Better Buy

Facilitation Notes

In this activity, students write a system of linear equations, with each equation in the form $y = a(x - c) + b$, to model a scenario. The best buy must be interpreted using inequalities based upon the input values.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Differentiation strategies

- To scaffold support, discuss the context prior to having students write the equations. Provide examples with different numbers of Mb of data and have students solve them using arithmetic to make sense of the context.
- To extend the activity, have students graph the system of equations and interpret the graphs in terms of the context.

As students work, look for

- Difficulty expressing additional Mb of data in an algebraic expression.
- Use of technology to graph the system and determine the point of intersection.

Questions to ask

- What does your recommendation depend upon?
- How do you know which plan is in Demetrius's best interest?
- What expression represents the cost of using more than 2000 Mb of data with the Bouncing Cell Service? Did you use the Distributive Property to write this expression?
- What expression represents the cost of using more than 1500 Mb of data with the Rolling Cell Service? Did you use the Distributive Property to write this expression?

- What is the significance of the point of intersection with respect to the problem situation?
- What does the solution to this system of equations tell you about the problem situation?
- What will the graph of the system of equations look like?

Summary

In a real-world context involving determining the better buy between two relationships modeled by a system of linear equations, the intersection point of the lines and the possible input values are helpful in making decisions.

Activity 6.3

Solving a System of Linear Inequalities with Four Constraints

Facilitation Notes

In this activity, students explore a context that can be modeled using a system of linear inequalities with four constraints. They write two of the constraints in terms of the sale price given the percent reduction, and represent the other two constraints as vertical lines. Students graph the system and use the graph and algebra to answer questions in context.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Expressions for the percent of the discount rather than the sale price.
- Two different correct equations for the same context, such as $s = 0.40r$ and $s = r - 0.60r$.

Questions to ask

- If the glasses Miguel selects are marked with a 60% savings, what percent of the regular price will Miguel pay for the glasses? What if they had a 75% savings?
- Explain why each expression represents a sale price.
- What do you think the graph of this system looks like?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

Differentiation strategies

- To scaffold support, have students use the same color pencil to graph each boundary line and shade the half-plane that is the solution to its corresponding inequality.
- To support all students, have groups of students graph the system on large poster paper grids for comparison and reference during the class discussion.

Questions to ask

- Is the graph what you predicted it would look like? Explain.
- What do the two vertical lines on your graph represent?
- Why do 2 of the lines have the same *y*-intercept?
- Can you graph all 4 of your equations using graphing technology?
- How did you know where to shade?
- What does the shaded region represent? The unshaded region?
- Select a point in the shaded region and interpret it in the context of the problem.
- What do all the points on the same horizontal line represent? The same vertical line?

Have students work with a partner or in a group to complete Questions 6 through 8. Share responses as a class.

Questions to ask

- Did you prefer using the graph or the inequalities to answer the questions? Why?
- Is your graph accurate enough to determine whether Miguel can save $140? Why or why not? How can you algebraically determine this?
- Can all of these questions be answered using algebra? The graph?

Summary

A real-world context can be represented by a system of linear inequalities that has more than two inequalities. The solution is all ordered pairs that satisfy all inequalities in the system.

Activity 6.4
Determining the Better Job Offer

Facilitation Notes

In this activity, students explore a more complex context that can be modeled with several different systems of linear equations. Possible systems include equations based upon years, equations based upon months, equations in the form $y = a(cx) + b$, as well as a system of three equations. Once again, input values must be considered to determine the better job offer.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

As students work, look for

- Confusion whether x represents number of weeks or dollars in sales per week.
- Equations in terms of earnings per year or earnings per week.
- In equations in terms of earnings per year, the variable x representing sales per week or sales per year.
- An error in equations in terms of earnings per year where x represents sales per week, where the commission is expressed for a week, $(0.09)x$, instead of for a year, $52(0.09)x$.
- Different methods of including $2000 in the solution process: either solving the system of two equations and using $2000 in the interpretation phase, or including $2000 as an equation in the system of equations and graphs.

Questions to ask

- What will your recommendation depend upon?
- How will you know which job is in Jose's best interest?
- What expression represents Jose's yearly income for Reliable Robotics? Robot Renegades?
- How do you solve this system of equations?
- What is another way to solve this system?
- Is a graph helpful in solving this system?
- What is the significance of the point of intersection with respect to this problem situation?
- How do you use the relationship between the point of intersection and $2000 in sales per week to determine what job is a better offer?

Summary

A real-world context can be interpreted by different systems of equations depending upon the units used. The results of the different systems should all lead to the same solution.

DEMONSTRATE

Talk the Talk: Which Cab Is More Fab?

Facilitation Notes

In this activity, students write a system of linear equations, with each equation in the form $y = ax + b$, to model a scenario. They make a decision as to what is the best option, with their response expressed as inequalities based upon the input values.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- What will your recommendation depend upon?
- How will you know which cab company is most economical?
- What expression represents the charge per mile of the Red Cab Company? Yellow Cab Company?
- What expression represents the charge per mile plus the entry cost for Red Cab Company? Yellow Cab Company?
- What is the significance of the point of intersection with respect to this problem situation?
- If the number of miles to the airport is 15, which company should she choose? Why?
- If the number of miles to the airport is less than 15, which company should she choose? Why?
- If the number of miles to the airport is greater than 15, which company should she choose? Why?

Summary

In a real-world context involving determining the better deal between two relationships modeled by a system of linear equations, both the intersection point of the lines and the possible input values are sometimes necessary in making decisions.

6

Working the System

Solving Systems of Equations and Inequalities

Warm Up

Determine whether the point (1, 7) is a solution to each system.

1. $\begin{cases} 4x - y = -3 \\ -2x + y = 5 \end{cases}$

2. $\begin{cases} x + y > 4 \\ 4x - y < -4 \end{cases}$

3. $\begin{cases} y = -3.5x - 2 \\ y = 4.5x - 10 \end{cases}$

4. $\begin{cases} -2x + y < 8 \\ x - y > -8 \end{cases}$

Learning Goals

- Use various methods of solving systems of linear equations to determine the better buy or the better job offer.
- Solve systems of linear inequalities with more than two inequalities.
- Graph the solutions to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

You have solved systems of linear equations by graphing, by using substitution, and by using linear combinations. You have also graphed systems of linear inequalities to determine possible solutions. How can you use these various methods to reason about real-world problems?

Warm Up Answers

1. The point is a solution to the system.
2. The point is not a solution to the system.
3. The point is not a solution to the system.
4. The point is a solution to the system.

Answer

1. The system of linear equations is

$$\begin{cases} y = 2x + 100 \\ y = 6x \end{cases}$$

The point of intersection for this system is (25, 150), so the two plans cost the same for 25 uses of the pool. This means that Susan will need to use the pool more than 25 times to get the better deal between plans.

The system of linear inequalities is

$$\begin{cases} a + b > 25 \\ a + b \leq 50 \end{cases}$$

where a is the number of times she uses the pool in July and b is the number of times she uses the pool in August.

Any combination of numbers of trips to the pool in July and August should add to more than 25 and no more than 50 in order to get the best deal and stay under budget.

GETTING STARTED

Systems for Summer Savings

A neighborhood pool club offers two membership plans. Plan A includes a seasonal sign-up fee of $100 and charges $2 each time you use the pool. Plan B has no sign-up fee but charges $6 each time you use the pool. Susan chooses Plan A. She has a budget of $200 to spend on pool fees during the months of July and August.

Susan wants to be sure she uses the pool enough times so that the plan she chooses works out to be the better deal between the two plans, but she does not want to go over her budget.

1. Use a system of linear equations and a system of linear inequalities to make a recommendation to Susan as to how often she should use the pool in July and August.

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ACTIVITY
6.1

Determining the Better Deal

The Bici Bicycle Company is planning to make a low price ultra-light bicycle. There are two different plans being considered for building this bicycle. The first plan includes a cost of $125,000 to design and build a prototype bicycle. The combined materials and labor costs for each bike made under the first plan will be $225. The second plan includes a cost of $100,000 to design and build the prototype. The combined materials and labor costs for each bike made under the second plan will be $275.

1. **You recently got a job at Bici Bicycle Company as a financial analyst. Analyze the costs for each proposed bicycle prototype and determine which plan Bici should follow. Provide evidence for your proposal.**

Answer

1. Students' methods will vary.
   The system of linear equations that represents the problem situation is

   $$\begin{cases} y = 125,000 + 225x \\ y = 100,000 + 275x \end{cases}$$

   Using any method, students should determine that $x = 500$. The better plan depends on the number of bicycles made. If the company makes more than 500 bicycles, they should use Plan A. If the company makes fewer than 500 bicycles, they should use Plan B.

ELL Tip

Students may be unfamiliar with the terms *prototype* and *labor costs*. Explain the meaning of these terms so the scenario makes sense to students.

Answer

1. Students' methods will vary.
   The system of linear equations that represents the problem situation is

$$\begin{cases} y = 99.99 + 0.05(t - 2000) \\ y = 79.99 + 0.08(t - 1500) \end{cases}$$

   Using any method, students should determine that $t = 1333.33$. The better plan depends on the amount of data that Demetrius uses each month. If Demetrius uses more than 1333.33 Mb of data per month, he should choose the Bouncing Cell Service. If he uses less than 1333.33 MB of data per month, he should choose the Rolling Cell Service plan.

ACTIVITY **6.2**

Determining the Better Buy

Demetrius is in search of a new cell phone plan. He is considering two different cell phone services from two different providers.

Bouncing Cell Service offers a monthly fee of $99.99 and 2000 Mb of data per month. Once a user exceeds the free monthly data allowance, each additional Mb of data used is $0.05. Rolling Cell Service offers a monthly fee of $79.99 and 1500 Mb of data per month. Once a user exceeds the free monthly data allowance, each additional Mb of data used is $0.08. Demetrius is unsure which plan to choose. He wasn't very careful with his last contract and paid a lot of extra money in charges for data.

1. Write an email to advise Demetrius which plan to choose. Provide evidence in your response.

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ACTIVITY 6.3

Solving a System of Linear Inequalities with Four Constraints

Miguel's eye doctor informed him that he needs glasses. Luckily, the local vision store is having a sale on all eyeglass frames. The advertisement in the window is as shown.

Save 60% to 75% On All Frames
Regularly Priced at
$120–$360

Previously, you solved a system containing two linear inequalities. However, systems can consist of more than two linear inequalities.

1. **Use the advertisement to write two inequalities that represent the regular price of eyeglass frames. Let *r* represent the regular price of the frames.**

2. **Use the same advertisement to write two inequalities that represent the reduced price of the eyeglass frames. Let *s* represent the sales price of the frames in terms of *r*.**

Remember:

When an item is 20% off the regular price, you can think of that item costing 80% of the regular price.

3. **Heather wrote this system of linear inequalities for the problem situation. Explain why it is incorrect.**

Heather

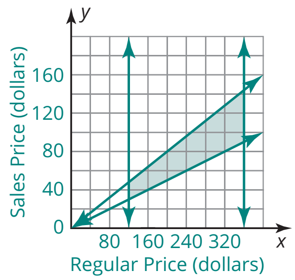
$$\begin{cases} r \geq 120 \\ r \leq 360 \\ s \leq 0.6r \\ s \geq 0.75r \end{cases}$$

Answers

1. $r \geq 120$ and $r \leq 360$
2. $s \geq 0.25r$ and $s \leq 0.4r$
3. Heather correctly wrote the inequalities for the regular price, but she incorrectly wrote the inequalities for the percent off the regular price. Her inequalities mistakenly indicate that the sale price of the eyeglasses is between 25% and 40% off the regular price.

Answers

4.



5. See graph. The solution region resembles a trapezoid.
6. Miguel should expect to spend between about \$80 and \$130 if he purchases glasses that are regularly priced at \$320.
7. The least amount of money Miguel can expect to spend is \$30 if he purchases eyeglasses that are regularly priced at \$120. The greatest amount of money Miguel can expect to spend is a little more than \$140 if he purchases eyeglasses that are regularly priced at \$360.

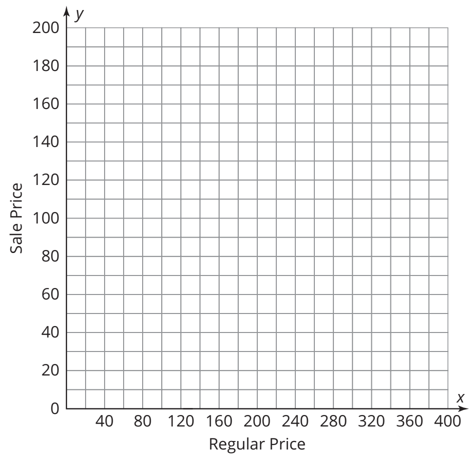
8a. According to my graph, Miguel can expect to spend between \$60 and \$96 for eyeglasses that are regularly priced at \$240. This would be a savings of between \$144 and \$180, which is more than a savings of \$140.

8b. I can substitute the value of *r* as 240 in the greater discount, 75%, and solve for *s*.

$$\begin{cases} s = 0.25(240) \\ s = 60 \end{cases}$$

The least amount Miguel can spend on the purchase of eyeglasses that regularly cost \$240 is \$60. So the greatest amount of money Miguel can save is \$240 − \$60, or \$180.

4. Graph each inequality on the grid.



5. Shade the portion of the graph that satisfies the system of linear inequalities. What shape does the solution region resemble?

6. About how much would Miguel expect to spend if he purchases eyeglasses that are regularly priced at \$320?

Remember:

When graphing a system of linear inequalities, you must determine the portion of the graph that satisfies all the inequalities in the system.

7. Miguel is definitely going to purchase a pair of eyeglasses that are on sale. What is the least amount of money Miguel can expect to spend? What is the greatest amount he can expect to spend?

8. Miguel decides on a pair of eyeglasses that are regularly priced at \$240.

a. Can Miguel expect to save more or less than \$140 off the purchase price of this pair of eyeglasses? Use your graph to determine an approximate answer.

The graph shows you the sale price of the eyeglasses, but how can you determine how much he will save?

b. Use algebra to determine the greatest amount of money Miguel can save by purchasing eyeglasses that are regularly priced at \$240.

ELL Tip

Discuss the definition of *resemble* to assist students in answering Question 5. If necessary, provide a word bank of the names of quadrilaterals that students may choose from.

ACTIVITY 6.4

Determining the Better Job Offer

Jose interviewed for two different sales positions at competing companies. Reliable Robotics has offered Jose a salary of $31,200 per year, plus a 9% commission on his total sales. Robot Renegades will offer him $26,000 per year, plus a 15% commission on his total sales.

Jose isn't sure which offer to accept. He's great at making a sale, but he's just not sure which job will be better in terms of his pay. He is confident that he can make at least $2000 worth of sales each week.

1. Write an email to Jose with your recommendation of which job offers better compensation. Provide evidence in your response.

Answer

1. Sample answer. The system of linear equations that represents the problem situation is

$$\begin{cases} y = 31{,}200 + 52(0.09s) \\ y = 26{,}000 + 52(0.15s) \end{cases}$$

Using any method, students should determine that $s \approx 1666.67$. The better job will depend on Jose's sales each week. If he sells more than $1666.67 each week, he should choose the job with Robot Renegades. If he sells less than $1666.67 each week, he should choose the job with Reliable Robotics. Since Jose states that he is confident he can make $2000 worth of sales each week, then he should choose the job with Robot Renegades.

ELL Tip

Discuss the meaning of the terms *commission* and *compensation* in the context of business so that students may engage more fully in the activity.

Answer

1. Students' methods will vary.
   The system of linear equations that represents the problem situation is

   $$\begin{cases} y = 1.25x + 3.50 \\ y = 1.15x + 5.00 \end{cases}$$

   Using any method, students should determine that $x = 15$. If the number of miles to the airport is 15, the cost will be the same. If the number of miles to the airport is less than 15, using the Red Cab Company is less expensive. If the number of miles to the airport is more than 15, using the Yellow Cab Company is less expensive.

NOTES

TALK the TALK

Which Cab Is More Fab?

The Red Cab Company charges \$3.50 upon entry and an additional \$1.25 per mile driven. The Yellow Cab Company charges \$5 upon entry and an additional \$1.15 per mile driven.

1. **Emma needs to take a cab to the airport. Which company should she use if she wants to minimize the cost? Use any method to solve.**

Assignment

LESSON 6: Working the System

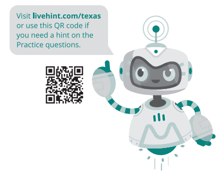
Write

You have used three methods to solve systems: graphing, substitution, and linear combinations. Describe the characteristics you would look for when determining which method to use.

Remember

The solution set to a system of inequalities with more than two constraints can be described as the region where all the graphs overlap.

Practice



1. Antonio wants to subscribe to a service that will allow him to rent DVDs and stream movies online. Movie Madness offers a subscription for \$14.25 a month. With this subscription, Antonio can check out as many DVDs as he wants each month and must pay \$1.40 for each movie he streams online. The Show Must Go On! offers a subscription for \$8.50 a month. With this subscription, Antonio can checkout as many DVDs as he wants each month and must pay \$3.25 for each movie he streams online.
   a. Write a system of linear equations to represent this problem situation.
   b. Analyze the two subscription plans and determine which one is the better deal. Use any or all of the methods you have learned to determine your answer.
   c. Write a short paragraph recommending which subscription Antonio should choose.
   d. Which method do you think provides the quickest way to analyze a system of equations to determine which one is the better deal? Explain your reasoning.
2. The Brunstown Ballet Company needs to rent a venue for their production of the Nutcracker. There are a number of arenas they are considering. The arenas have seating capacities that range from 800 to 1876 seats. The management of the company knows the ticket sales may not be good this year but their goal is to sell between 65% and 90% of the available seats. Whichever arena they choose, one hundred seats must be set aside for the company's donors.
   a. Write a system of inequalities that represents the problem situation. Define your variables.
   b. Graph each inequality on a coordinate plane.
   c. One of the arenas they are considering has 1200 available seats. Determine the minimum and maximum number of seats they would need to sell in order for management to reach their goal.
   d. If the company sold 900 seats, what is the range of seating capacities for the arenas they may have rented?
   e. If they rented an arena that had a 1300-seat capacity and sold 800 tickets, would management reach their goal? Explain your reasoning.

Assignment Answers

Write

Sample answer.

If a system has only smaller whole numbers I would use graphing. If a system has an equation where you could easily get a variable by itself I would use substitution. And if a system has numbers that can be manipulated by multiplication to create coefficients that are additive inverses I would use linear combinations.

Practice

1a. $\begin{cases} y = 14.25 + 1.40x \\ y = 8.50 + 3.25x \end{cases}$

1b. If Antonio streams fewer than 3 movies a month, then The Show Must Go On! subscription is a better deal. If he streams more than 3 movies a month, then the Movie Madness subscription is a better deal.

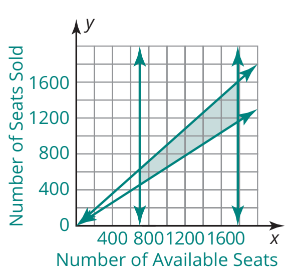
1c. I would recommend that Antonio subscribe to The Show Must Go On! if he anticipates streaming 3 or fewer movies a month. If he plans to stream more than 3 movies a month, I would recommend that he subscribe to Movie Madness.

1d. Answers will vary.

2a. Let a represent the number of available seats and s represent the number of seats sold.

$\begin{cases} a \geq 700 \\ a \leq 1776 \\ s \leq 0.90a \\ s \geq 0.65a \end{cases}$

2b.



2c. The minimum number of seats to sell is 780.

The maximum number of seats to sell is 1080.

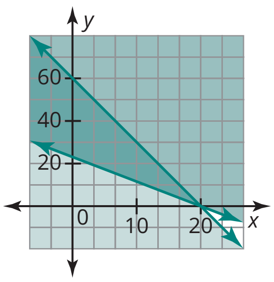
2d. If they sold 900 tickets, then the minimum number of seats in the arena was 1000 and the maximum number of seats was 1384.

2e. No. Management would not reach their goal. The point (1300, 800) does not fall in the solution region, so they would not have sold at least 65% of the seats.

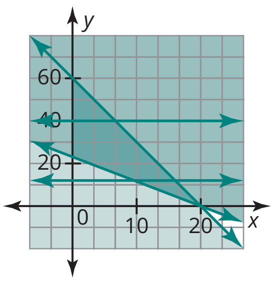
Assignment Answers

Stretch

1. $\begin{cases} 15x + 13y \geq 300 \\ x + \frac{1}{3}y \leq 20 \end{cases}$



2. $\begin{cases} 15x + 13y \geq 300 \\ x + \frac{1}{3}y \leq 20 \\ y \leq 40 \\ x \leq 12 \end{cases}$



3. The maximum amount of baked goods she can make is 40 dozen cupcakes and 6 dozen cookies. She will earn \$610 and it will take her 19 hours and 20 minutes.
4. She could make 24 dozen cupcakes, which would only take her 8 hours and she would make \$312.

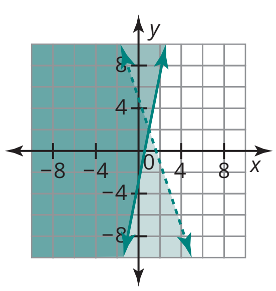
Review

1a. 3

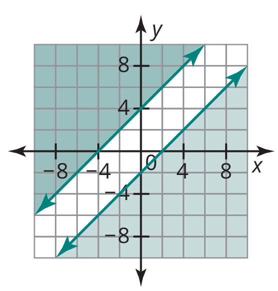
1b. (0, 6) and (−2, 0)

2. $a = \frac{2A}{h} - b$

3a. Answers will vary.
(0, 0) and (−2, 1)



3b. No solutions



4. $y = 7$
5. $y = \frac{1}{5}x + \frac{19}{30}$

Stretch

Isla sells baked goods from her home kitchen. She offers decorated cookies for \$15 per dozen and cupcakes for \$13 per dozen. It takes her an hour to decorate a dozen cookies, but only 20 minutes to decorate a dozen cupcakes. She would like to make at least \$300 per week and not put in more than 20 hours of work per week.

1. Create a system of linear inequalities that fits the situation and graph them.
2. Isla just discovered that she is running out of cake mix for the cupcakes and royal icing for the cookies. She can make a maximum of 40 dozen cupcakes and 12 dozen cookies. What are the new inequalities you need to add to your problem? Add them to your graph.
3. What is the maximum amount of baked goods that she could make? How much will she earn? How long will it take her?
4. What is the least amount of time she could work and still earn \$300? What baked goods would she make?

Review

1. Consider the equation $6x - 2y = -12$.
   a. What is the slope of the equation?
   b. What are the intercepts of the equation?
2. The equation to calculate the area of a trapezoid is $A = \frac{1}{2}(a+b)h$. Rewrite the equation to solve for a.
3. Graph each system of inequalities. Then identify two points that are solutions of the system.
   a. $\begin{cases} y \geq 5x - 3 \\ y < -3x + 5 \end{cases}$
   b. $\begin{cases} y \geq x + 4 \\ x - y \geq 2 \end{cases}$
4. What is the equation for the line that has a slope of 0 and passes through the point (3, 7)?
5. What is the equation for the line that has a slope of $\frac{1}{5}$ and passes through the point $\left(-\frac{2}{3}, \frac{1}{2}\right)$?

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Systems of Equations and Inequalities Summary

KEY TERMS

- standard form of a linear equation
- substitution method
- system of linear equations
- consistent systems
- inconsistent systems
- linear combinations method
- half plane
- boundary line
- constraints
- solution of a system of linear inequalities

LESSON 1

The County Fair

The **standard form of a linear equation** can be written as $ax + by = c$, where a, b, and c are constants, and a and b are not both zero.

Sometimes a system of equations may not be accurately solved using graphs. There is an algebraic method that can be used called the substitution method. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

For example, consider the system of equations.

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Step 1: To use the substitution method, begin by choosing one equation and isolating the variable. This will be considered the first equation. Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation. Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$. Write the new equation: $1.25x + 1.05(8x) = 30$.

Step 3: Solve the new equation.

$$\begin{aligned} 1.25x + 8.40x &= 30 \\ 9.65x &= 30 \\ x &\approx 3.1 \end{aligned}$$

Now, substitute the value for x into $y = 8x$ to determine the value for y.

$$y \approx 8(3.1) \approx 24.8$$

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

When a system has no solution, the equation resulting from the substitution step has no solution. When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

LESSON 2 Double the Fun

When two or more linear equations define a relationship between quantities, they form a **system of linear equations**. The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. One way to predict the solution to a system of equations is to graph both equations and identify the point at which the two graphs intersect.

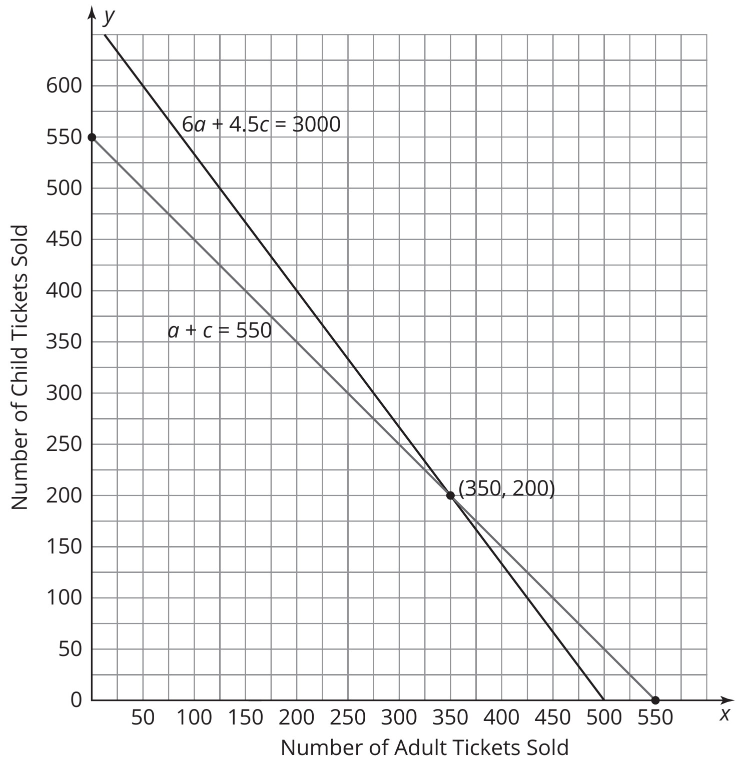
For example, suppose George sells 550 tickets for a spaghetti dinner to raise \$3000 for charity. Adult tickets cost \$6, and child tickets cost \$4.50. To determine how many adult tickets and child tickets were sold, let a represent the number of adult tickets purchased, and let c represent the number of child tickets purchased.

$$\begin{cases} a + c = 550 \\ 6a + 4.5c = 3000 \end{cases}$$

You can use x- and y-intercepts to graph each of the two equations to determine how many of each type of tickets were sold.

The intersection point appears to be (350, 200). There were 350 adult tickets and 200 child tickets sold.

A system of equations may have one unique solution, no solution, or infinite solutions. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.



LESSON 3

The Elimination Round

The **linear combinations method** is a process used to solve a system of equations by adding two equations so they result in an equation with one variable. Once the value of this variable is determined, it can be used to calculate the value of the other variable. In many cases, one or both of the equations may need to be multiplied by a constant so that the coefficients of the term containing either x or y are opposites. Then when the equations are added, the result is an equation in one variable.

For example, consider the system of linear equations shown.

$$\begin{cases} 7x + 2y = 24 \\ 4x + y = 15 \end{cases}$$

$$\begin{aligned} 7x + 2y &= 24 \\ -\,2(4x + y) &= -2(15) \end{aligned}$$

Multiply the bottom equation by a constant that results in opposite coefficients for one of the variables.

$$
\begin{array}{r}
7x + 2y = 24 \\
+ \;\; -8x - 2y = -30 \\
\hline
-x = -6 \\
x = 6
\end{array}
$$

Now that the *y*-values are opposite, you can solve this linear system.

$$
\begin{aligned}
7(6) + 2y &= 24 \\
42 + 2y &= 24 \\
-2y &= -18 \\
y &= -9
\end{aligned}
$$

Substitute the value for *x* into one of the equations to calculate the value for *y*.

The solution to the system of linear equation is $(6, -9)$.

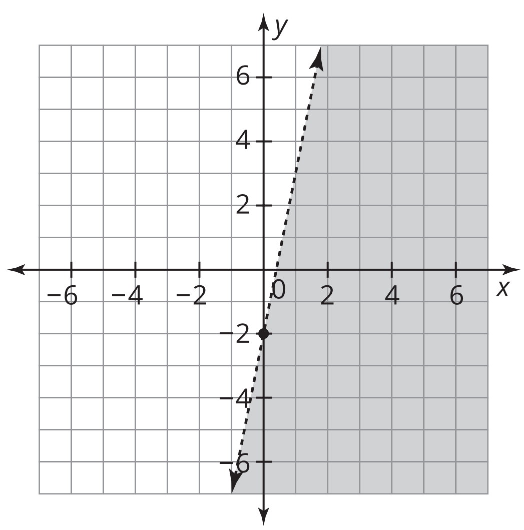
LESSON 4

Throwing Shade

The graph of a linear inequality in two variables is a **half-plane**, or half of a coordinate plane. A **boundary line**, determined by the inequality, divides the plane into two half-planes. The inequality symbol identifies which half-plane contains the solutions. If the symbol is $\le$ or $\ge$, the boundary line is part of the solution and is solid. If the symbol is $<$ or $>$, the boundary line is not part of the solution and is represented with a dashed line. Use $(0, 0)$ as a test point to determine which half of the plane is the solution of the inequality and should, therefore, be shaded.

For example, the graph shows the solution to the inequality $y < 5x - 2$. Since the inequality symbol in the solution is $<$, the shaded half-plane does not include points on the line. Since $(0, 0)$ is not a solution, then the region to the right of the dashed line is the solution set.

$$
\begin{aligned}
&y < 5x - 2 \\
&0 < 5(0) - 2 \\
&0 < -2
\end{aligned}
$$



LESSON 5 Working with Constraints

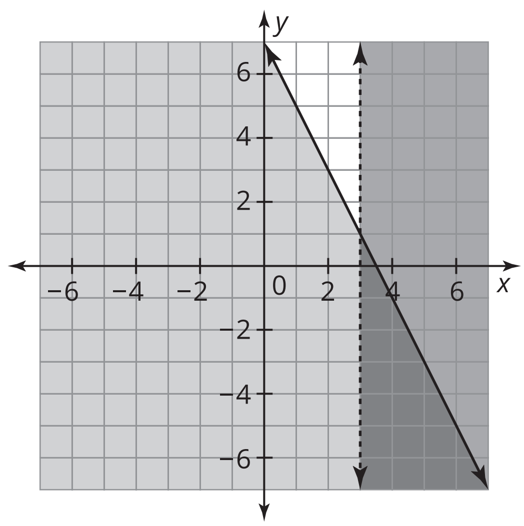
In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region.

The **solution of a system of linear inequalities** is the intersection of the solutions of each inequality. Every point in the intersection region satisfies the system. To determine the solution set of the system, graph each inequality on the same coordinate plane. The region that overlaps is the solution to the system.

For example, the overlapping region of the graphs of the inequalities $2x + y \leq 7$ and $x > 3$ is the solution to the system.

$$\begin{cases} 2x + y \leq 7 \\ x > 3 \end{cases}$$

Two points that are solution of the system are (4, −5) and (5, −8)



LESSON 6

Working the System

Substitution, linear combinations, and graphing are three methods for determining the solution of a linear system of equations. Use substitution when a variable in one equation can easily be isolated or when the equations are in general form. Use the linear combinations method when the coefficients of like terms are opposites or can be easily made into opposites by multiplication. Use the graphing method when the numbers are convenient to graph or if an exact solution is not needed.

To solve a system of linear inequalities with more than two inequalities, graph each linear inequality and shade the correct region that contains the solution sets that satisfy all the inequalities in the system. Also, determine all the points of intersection for the boundary lines that make the vertices of the solution region. Use a closed point if the point is part of the solution region, and use an open dot if the point is not part of the solution region but is a point of intersection.

The graphs show the solution to a system of four inequalities.

$$\begin{cases} x \geq -4 \\ y > -2 \\ y \leq 6 \\ y > 4x + 3 \end{cases}$$

A solution of the system of inequalities is (–2, 2).