One or More Xs to One Y

3

Defining Functional Relationships

MATERIALS

None

Lesson Overview

The terms *relation* and *function* are defined. Relations are represented as mappings, sets of ordered pairs, tables, sequences, contexts, graphs, and equations. Students begin by analyzing mappings, sets of ordered pairs, tables, and sequences and determine whether these relations are functions according to the definition. They then determine whether different real-world contexts represent functions. Next, students analyze graphs and use the vertical line test to determine whether the various graphs represent functions. Students determine whether equations are functions by substituting values for x into the equation, and then determining if any x-values can be mapped to more than one y-value. Students solidify their understanding of functions by completing a graphic organizer with the definition of *function*, a problem situation, a table, and a sketch of a graph of a function.

Grade 8
Proportionality

(5) The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

(G) identify functions using sets of ordered pairs, tables, mappings, and graphs.

ELPS

1.A, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.G, 3.H, 4.A, 4.B, 4.C, 4.D, 4.F, 4.K, 5.E

Essential Ideas

- A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs. The first coordinate in an ordered pair in a relation is the input, and the second coordinate is the output.
- A function is a relation which maps each input to one and only one output. Relations that are not functions will have more than one output for each input.
- A scatter plot is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.
- The vertical line test is a visual method of determining whether a relation represented as a graph is a function.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: What's My Rule?

Students are given completed tables with ordered pairs and are asked to determine the equation that generated those ordered pairs. Then students create their own table of ordered pairs based on an equation they generate and give it to a partner to determine the equation.

Develop

Activity 3.1: Functions as Mappings from One Set to Another

Students are introduced to the terms *mapping, set, relation, input, output, function, domain*, and *range*. Students analyze sets of ordered pairs (or mappings, tables, and sequences) to determine if they are functions according to the definition and then identify their domains and ranges.

Day 2

Activity 3.2: Functions as Mapping Inputs to Outputs

Relations are represented as contexts. Students test whether the contexts are functions by analyzing the contexts using the definition of a function.

Activity 3.3: Determining Whether a Relation Is a Function

Students are given relations represented as mappings, tables, sequences, ordered pairs, and contexts. They determine which relations are functions and explain their reasoning.

Day 3

Activity 3.4: Functions as Graphs

Students are introduced to scatter plots and the vertical line test. They use the vertical line test to determine whether relations represented as graphs are functions. Then students use the cards from the previous lesson's sorting activity in conjunction with the vertical line test to determine which relations are functions.

Activity 3.5: Functions as Equations

Students test whether equations represent functions by substituting values for x into each equation and then determining if any x-value can be mapped to more than one y-value. While one example where x is mapped to two values proves the equation does not represent a function, students cannot test an infinite number of points to prove it is a function; graphing the equation and using the vertical line test is suggested in these cases.

Demonstrate

Talk the Talk: Function Organizer

Students write a definition for *function* in their own words and then create a problem situation that can be represented by a function. Students represent this function using a table and a graph. This work is captured in a graphic organizer.

Getting Started: What's My Rule?

Facilitation Notes

In this activity, students are given completed tables with ordered pairs and are asked to determine the equation that generated those ordered pairs. Then students create their own table of ordered pairs based on an equation they generate and give it to a partner to determine the equation.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses with the class.

Questions to ask

- Are the x-values in the table consecutive integers?
- Can you use first differences to determine the slope?
- How can you determine the slope from a table of values?
- How many times did you have to calculate slope to know the table represented a linear relationship?
- Do any of the tables represent a non-linear relationship? If so, how can you tell?
- How did you determine the value of b?
- Did you realize that b was already provided in three of the tables as the point $(0, y)$?
- Can more than one equation describe the relationship between the independent variable x and the dependent variable y in a table of values that represents a linear relationship?

Differentiation strategies

- To scaffold support for Question 1 part (c), the main point is that students realize the table does not represent a linear relationship. If you choose to spend more time on the question, provide more data points to help students recognize the pattern in the table. Another option would be to graph the points and see that it makes a V-shape. Students could practice writing linear equations for each side of the V-shape. Dealing with absolute-value equations is not the focus of this lesson; they will be addressed in Lesson 4, Activity 3.
- To extend the activity, have students graph the points and determine the equation from the graph to check their work.

Summary

A table of values can be described using a rule or an equation.

DEVELOP

Activity 3.1
Functions as Mappings from One Set to Another

Facilitation Notes

In this activity, students are introduced to the terms *mapping, set, relation, input, output, function, domain*, and *range*. Students analyze sets of ordered pairs (or mappings, tables, and sequences) to determine if they are functions according to the definition and then identify their domains and ranges.

Ask a student to read the information and definitions aloud. Discuss as a class. Have students work with a partner or in a group to complete Questions 1 through 3. Share responses with the class.

Questions to ask

- Does each *x*-value map to one and only one *y*-value?
- Does each *y*-value map to one and only one *x*-value?
- What is an example of an *x*-value that maps onto more than one *y*-value?
- What is an example of a *y*-value that maps onto more than one *x*-value?
- When you create a mapping, should the *x*-values be listed from least to greatest? Why or why not?
- When you create a mapping, should the *y*-values be listed from least to greatest? Why or why not?
- Is it easier to write a set of ordered pairs from a mapping or from a table of values? Why?

Ask a student to read the definitions aloud. Analyze the Worked Examples as a class.

Misconception

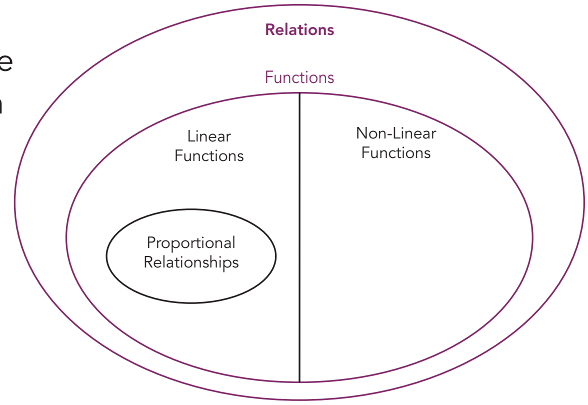
Students are aware of the term *range*, the highest value minus the lowest value, when dealing with 1-variable statistics. Compare and contrast how the term *range* is used when dealing with functions. In this case, it refers only to the *y*-values, and it is listed as an interval rather than a single number.

Differentiation strategies

Have students take notes on the definitions.

- Create a list of related terms: *ordered pair*, (*x*, *y*), (*input*, *output*) as well as *domain* {inputs} and *range* {outputs}.
- Discuss the fact that calculators can only deal with functions because they are programmed to provide only one output for every input.

- Create a simple Venn diagram to show that functions are a special case of relations. Add the other terms as they are addressed in this lesson and the next lesson. You may want to provide students a template for the Venn diagram.



Questions to ask

- Does a function map each input to one and only one output?
- Does a function map each output to one and only one *input*?
- Are all relations considered to be functions?
- Are all functions considered to be relations?
- What is an example of a relation that is a function? Not a function?
- What is an example of a function that is a relation? Not a relation?
- Are all relations linear?
- What is an example of a linear relation?
- What is an example of a non-linear relation?
- Are linear relations also functions?

Have students work with a partner or in a group to complete Questions 4 through 7. Share responses with the class.

Questions to ask

- What is the difference between the domain and the range?
- Is the domain associated with x-values or y-values?
- Is the range associated with x-values or y-values?
- Can the domain and the range of a function contain the same values?
- How do you decide if the sequence is a function?
- In a sequence, what is the difference between a term number and a term value?
- Do the term numbers of a sequence describe the domain or the range? What about the term values?
- Why is every sequence considered a function?

Differentiation strategy

To scaffold support for sequences, place a table around the sequence of values (similar to the warm up in the previous lesson). Some students may prefer to write the table vertically rather than horizontally. Remind them that the top row, the independent variable, becomes the left column of a vertical table.

Summary

A function is a rule that maps each input to one and only one output. The domain is the set of all inputs and the range is the set of all outputs.

Activity 3.2

Functions as Mapping Inputs to Outputs

Facilitation Notes

In this activity, relations are represented as contexts. Students test whether the contexts are functions by analyzing the contexts using the definition of a function.

Ask a student to read the introduction aloud and discuss Question 1 as a class. Have students work with a partner or in a group to complete Questions 2 through 8. Share responses with the class.

Differentiation strategies

- To scaffold support, suggest that students draw mappings to represent each context.
- To extend the activity, have students create their own contexts as examples and nonexamples of functions.

Questions to ask

- Does the note she wrote go to one and only one person or more than one person?
- Is the football game viewed in one and only one home or more than one home?
- Does each puppy have one and only one home or more than one home?
- Does each player wear one uniform with one specific number?
- Is one zip code mapped to one and only one person or many people?
- Is one movie viewed by one and only one person or viewed by many people?
- Does Tara have one and only one job each day or more than one job each day?
- Is Janelle's text message sent to one and only one person or more than one person?
- Is there a way this context could be adjusted to create a function?

Summary

Functional relationships can be represented as a context that maps each input value to one and only one output value.

Activity 3.3
Determining Whether a Relation Is a Function

Facilitation Notes

In this activity, students are given relations represented as mappings, tables, sequences, ordered pairs and contexts. They determine which relations are functions and explain their reasoning.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses with the class.

Differentiation strategy

Have partners complete the even numbers or odd numbers, then explain their solutions to each other.

Questions to ask

- Does the mapping show each input value mapped onto one and only one output value?
- Does the table of values show each input value mapped onto one and only one output value?
- Are all sequences considered functions? Why?
- Do the set of ordered pairs map each input value to one and only one output value?
- Are the morning announcements read to one and only one student or many students?
- Does each student going through the cafeteria line select one and only one lunch option from the menu?

Summary

Relations can be represented as mappings, tables, sequences, ordered pairs, and contexts. A relation is a function when it maps each input value to one and only one output value.

Activity 3.4
Functions as Graphs

Facilitation Notes

In this activity, students are introduced to scatter plots and the vertical line test. They use the vertical line test to determine whether relations represented as graphs are functions. Then students use the cards from the previous lesson's sorting activity in conjunction with the vertical line test to determine which relations are functions.

Ask a student to read the information and definitions aloud. Discuss the Worked Example as a class. Complete Questions 1 and 2 as a class.

Misconception

When students see a scatter plot, they often want to connect the points to form a line graph. Discuss the fact that a relationship has to be determined among the data points before deciding if it makes sense to connect them or not.

Differentiation strategy

To scaffold support, it may help for students to slide their pencil across the graph to complete the vertical line test.

Questions to ask

- What does the scatter plot of a function look like?
- Are any of the input values mapped onto more than one output value?
- Does each x-value on the scatter plot relate to one and only one y-value?
- Do any of the x-values on the scatter plot relate to more than one y-value?
- Do any of the y-values on the scatter plot relate to more than one x-value?
- Did you write the data points as ordered pairs to determine if it was a function?
- What determines if a scatter plot represents a function?
- What are the ordered pairs from the Worked Example that demonstrate that the scatter plot does not represent a function?

Have students work with a partner or in a group to complete Questions 3 and 4. Share responses with the class.

Questions to ask

- Does any vertical line drawn on the graph of the relation intersect it only once or does it intersect it more than once?
- If the vertical line drawn on the graph of the relation intersects it only once, does it pass the test or fail the test?

- If a relation passes the vertical line test, does that mean the relation is a function or does that mean the relation is not a function?
- If a relation fails the vertical line test, does that mean the relation is a function or does that mean the relation is not a function?
- Does the card sort contain more functions or more non-functions?
- Can curved graphs represent functions? Explain.
- Why isn't the circle considered a function?
- Are all lines functions? Are all scatter plots functions?
- How many of the cards represent functions? Non-functions?

Differentiation strategy

To extend the activity, have students draw examples and non-examples of functions. Present them as flashcards to the class and have students use thumbs up or thumbs down to signify whether they represent functions or not.

Summary

The vertical line test is a visual method used to determine whether a relation represented as a graph is a function.

Activity 3.5

Functions as Equations

Facilitation Notes

In this activity, students test whether equations represent functions by substituting values for x into each equation and then determining if any x-value can be mapped to more than one y-value. While one example where x is mapped to two values proves the equation does not represent a function, students cannot test an infinite number of points to prove it is a function; graphing the equation and using the vertical line test is suggested in these cases.

Analyze the Worked Example as a class. Have students work with a partner or in a group to complete Questions 1 and 2. Share responses with the class.

Differentiation strategy

Suggest that students' three ordered pairs include a positive integer, zero, and a negative integer for the x-values.

Questions to ask

- What values were used for x?
- Was each x-value mapped to one and only one y-value?
- What would happen if negative numbers were used for x?
- What does the graph that represents this equation look like?
- Is this a linear or non-linear equation?
- Are all linear equations considered functions?
- How would you graph this equation?
- How would you enter this equation on a graphing calculator?
- Is the graph of this equation a horizontal or vertical line?
- Is there a second value equal to 4?
- If the y-value is -2, what is the x-value?
- Which equation describes an absolute value function?
- Are horizontal lines considered functions? Vertical lines?
- What do you think the graph of Question 1 part (d) looks like?

Differentiation strategy

To extend the activity, demonstrate how the circle must be graphed as two separate curves (a top curve and a bottom curve) on the graphing calculator because calculators can only graph functions. Enter $y_1 = \text{R}(1 - x^2)$ and $y_2 = -\text{R}(1 - x^2)$ using Zoom 5: ZSquare. Students should not be expected to solve for y and do this themselves.

Summary

Equations are functions if every x-value can be mapped to only one y-value and when the graph of the equation passes the vertical line test.

DEMONSTRATE

Talk the Talk: Function Organizer

Facilitation Notes

In this activity, students complete a graphic organizer for the concept of a function using multiple representations such as context, a graph, and a table.

Have students work with a partner or in a group to complete Question 1. Share responses with the class.

Questions to ask

- What is a relation?
- Are all functions also relations? Are all relations also functions?
- What makes your problem situation a function?

- Did you check that positive x-values, negative x-values and zero as an x-value all provide just one y-value?
- How did you know that the sketch of your graph would look as it does?
- What is an equation to represent this problem situation?
- Is your function a linear function? How can you tell?
- Is your function a proportional relationship? How can you tell?

Summary

A function is a relation which maps each input to one and only one output. Functions can be represented using a context, table, set of ordered pairs, graph and equation.

NOTES

One or More Xs to One Y

3

Defining Functional Relationships

WARM UP

Evaluate each expression given the set of values {1, 6, 12, 25}.

1. $5x$

2. $\frac{1}{2}x + 1$

3. $x - 8$

LEARNING GOALS

- Describe a functional relationship in terms of a rule which assigns to each input exactly one output.
- Determine whether a relation (represented as a mapping, set of ordered pairs, table, sequence, graph, equation, or context) is a function.

KEY TERMS

- mapping
- set
- relation
- input
- output
- function
- domain
- range
- scatter plot
- vertical line test

Throughout middle school, you have investigated different types of relationships between variable quantities: additive, multiplicative, proportional, and non-proportional. What are functional relationships?

Warm Up Answers

1. {5, 30, 60, 125}
2. $\{1\frac{1}{2}, 4, 7, 13\frac{1}{2}\}$
3. {−7, −2, 4, 17}

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Answers

1a. $y = 4x + 12$ or $4(x + 3)$

1b. $y = -2x$

1c. $y = |x| - 1$

1d. $y = \frac{1}{2}x + 2$

2. Answers will vary.

Getting Started

What's My Rule?

Rules can be used to generate sequences of numbers. They can also be used to generate (x, y) ordered pairs.

1. Write an equation to describe the relationship between each independent variable *x* and the dependent variable *y*. Explain your reasoning.

a.

x	y
−6	−12
−3	0
0	12
3	24

b.

x	y
1	−2
5	−10
−1	2
−10	20

c.

x	y
−10	9
−2	1
0	−1
5	4

d.

x	y
0	2
4	4
5	4.5
20	12

2. Create your own table and have a partner determine the equation you used to build it.

ACTIVITY 3.1

Functions as Mappings from One Set to Another

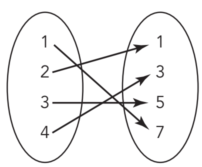
As you learned previously, ordered pairs consist of an *x*-coordinate and a *y*-coordinate. You also learned that a series of ordered pairs on a coordinate plane can represent a pattern. You can also use a *mapping* to show ordered pairs. A **mapping** represents two sets of objects or items. Arrows connect the items to represent a relationship between them.

When you write the ordered pairs for a mapping, you are writing a *set* of ordered pairs. A **set** is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

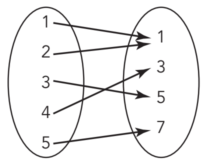
Use braces, { }, to denote a set.

1. Write the set of ordered pairs that represent a relationship in each mapping.

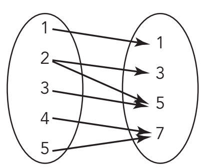
a.



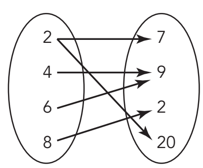
b.



c.



d.



2. Create a mapping from the set of ordered pairs.

a. **{(5, 8), (11, 9), (6, 8), (8, 5)}** **b.** **{(3, 4), (9, 8), (3, 7), (4, 20)}**

Answers

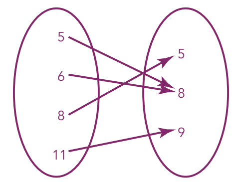
1a. {(1, 7), (2, 1), (3, 5), (4, 3)}

1b. {(1, 1), (2, 1), (3, 5), (4, 3), (5, 7)}

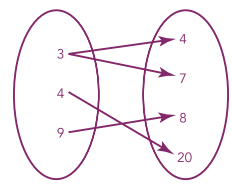
1c. {(1, 1), (2, 3), (2, 5), (3, 5), (4, 7), (5, 7)}

1d. {(2, 7), (2, 20), (4, 9), (6, 9), (8, 2)}

2a.



2b.



Answers

3a. {(−10, −20), (−5, −10), (0, 0), (5, 10), (10, 20)}

3b. {(20, −10), (10, −5), (0, 0), (10, 5), (20, 10)}

3. Write the set of ordered pairs to represent each table.

a.

Input	Output
−10	−20
−5	−10
0	0
5	10
10	20

b.

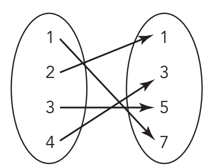
x	y
20	−10
10	−5
0	0
10	5
20	10

The mappings and ordered pairs shown in Questions 1 through 3 form *relations*. A **relation** is any set of ordered pairs or the mapping between a set of *inputs* and a set of *outputs*. The first coordinate of an ordered pair in a relation is the **input**, and the second coordinate is the **output**. A **function** maps each input to one and only one output. In other words, a function has no input with more than one output. The **domain** of a function is the set of all inputs of the function. The **range** of a function is the set of all outputs of the function.

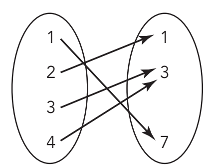
Notice the use of set notation when writing the domain and range.

WORKED EXAMPLE

In each mapping shown, the domain is {1, 2, 3, 4}.



The range is {1, 3, 5, 7}.



The range is {1, 3, 7}.

Each mapping represents a function because no input, or domain value, is mapped to more than one output, or range value.

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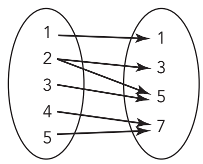
WORKED EXAMPLE

In the mapping shown, the domain is {1, 2, 3, 4, 5} and the range is {1, 3, 5, 7}.



This mapping does not represent a function.

NOTES

4. State why the relation in the Worked Example shown is not a function.

5. State the domain and range for each relation in Questions 2 and 3. Then, determine which relations represent functions. If the relation is not a function, explain why not.

Answers

4. The relation is not a function because the domain value of 2 has two outputs, 3 and 5.

5. Question 2(a):
Domain: {5, 6, 8, 11}
Range: {5, 8, 9}

The ordered pairs represent a function. Every input has one and only one output value.

Question 2(b):
Domain: {3, 4, 9}
Range: {4, 7, 8, 20}

The ordered pairs do not represent a function. The input 3 has two different outputs, 4 and 7.

Question 3(a):
Domain: {−10, −5, 0, 5, 10} Range: {−20, −10, 0, 10, 20}

This table represents a function. Every input has one and only one output.

Question 3(b):
Domain: {0, 10, 20}
Range: {−10, −5, 0, 5, 10}

This table does not represent a function. The input of 20 has two outputs, −10 and 10. The input of 10 has two outputs, −5 and 5. To be a function, no input can have more than one output.

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Answers

6. Emil shows the input 4 having two outputs. Therefore, this cannot be an example of a function.

7a. (1, 2) (2, 4) (3, 6) (4, 8) (5, 10) This sequence represents a function. Each input, or term number in the sequence, has one output, the term. Domain: {1, 2, 3, 4, 5} Range: {2, 4, 6, 8, 10}

7b. (1, 1) (2, 0) (3, 1) (4, 0) (5, 1) This sequence represents a function. Each input, or term number in the sequence, has one output, the term. Domain: {1, 2, 3, 4, 5} Range: {0, 1}

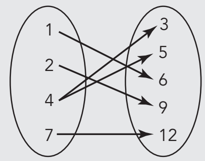
7c. (1, 0) (2, 5) (3, 10) (4, 15) (5, 20) This sequence represents a function. Each input, or term number in the sequence, has one output, the term. Domain: {1, 2, 3, 4, 5} Range: {0, 5, 10, 15, 20}

Think about the mappings as ordered pairs.

6. Review and analyze Emil's work. Explain why Emil's mapping is not an example of a function.

Emil

My mapping represents a function.



7. Determine if each sequence represents a function. Explain why or why not. If it is a function, identify its domain and range. Create a mapping to verify your answer.

a. 2, 4, 6, 8, 10, …

Remember that a sequence has a term number and a term value.

b. 1, 0, 1, 0, 1, …

c. 0, 5, 10, 15, 20, …

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ACTIVITY 3.2 Functions as Mapping Inputs to Outputs

You have determined if sets of ordered pairs represent functions. In this activity you will examine different situations and determine whether they represent functional relationships.

Read each context and decide whether it fits the definition of a function. Explain your reasoning.

1. *Input:* **Sue writes a thank-you note to her best friend.**
   Output: **Her best friend receives the thank-you note in the mail.**

2. *Input:* **A football game is being telecast.**
   Output: **It appears on televisions in millions of homes.**

3. *Input:* **There are four puppies in a litter.**
   Output: **One puppy was adopted by the Smiths, another by the Jacksons, and the remaining two by the Fullers.**

4. *Input:* **The basketball team has numbered uniforms.**
   Output: **Each player wears a uniform with her assigned number.**

5. *Input:* **Beverly Hills, California, has the zip code 90210.**
   Output: **There are 34,675 people living in Beverly Hills.**

6. *Input:* **A sneak preview of a new movie is being shown in a local theater.**
   Output: **65 people are in the audience.**

Answers

1. Yes. Sue's one note goes to one place.
2. No. The football game is mapped to more than one home.
3. Yes. Each puppy has one and only one home.
4. Yes. Each player wears one uniform with one specific number.
5. No. One zip code is mapped to many people.
6. No. One movie is mapped to 65 people.

Answers

7. Yes. On any given day of the week, Tara has just one job.
8. No. One text message is mapped to 41 people.

7. ***Input:*** **Tara works at a fast food restaurant on weekdays and a card store on weekends.**

Output: **Tara's job on any one day.**

8. ***Input:*** **Janelle sends a text message to everyone in her contact list on her cell phone.**

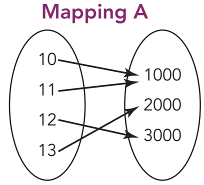
Output: **There are 41 friends and family on Janelle's contact list.**

ACTIVITY 3.3

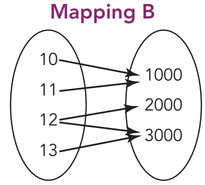
Determining Whether a Relation Is a Function

Analyze the relations in each pair. Determine which relations are functions and which are not functions. Explain how you know.

1. **Mapping A**



Mapping B



Answers

1. Mapping A is a function.

   Mapping B is not a function.

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2. Table A

Input	Output
–2	4
–1	1
0	0
1	1
2	4

Table B

x	*y*
2	–4
1	–1
0	0
1	1
2	4

3. Sequence A
7, 10, 13, 16, 19, …

Sequence B
10, 30, 10, 30, 10, …

4. Set A
{(2, 3), (2, 4), (2, 5), (2, 6), (2, 7)}

Set B
{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1)}

5. Scenario A
Input:
The morning announcements are read over the school intercom system during homeroom period.

Output:
All students report to homeroom at the start of the school day to listen to the announcements.

Scenario B
Input:
Each student goes through the cafeteria line.

Output:
Each student selects a lunch option from the menu.

Answers

2. Table A is a function.
   Table B is not a function.
3. Sequence A is a function.
   Sequence B is a function.
4. Set A is not a function.
   Set B is a function.
5. Scenario A is not a function.
   Scenario B is a function.

Answers

1a. This scatter plot represents a function. Each input value has only one output value.

1b. This scatter plot does not represent a function. The input value $x = 4$ has two output values, 1 and 4.

ACTIVITY
3.4

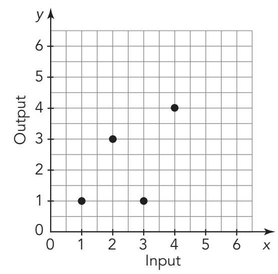
Functions as Graphs

A relation can be represented as a graph.

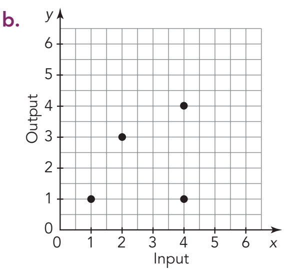
A **scatter plot** is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

1. Determine if each scatter plot represents a function. Explain your reasoning.

a.



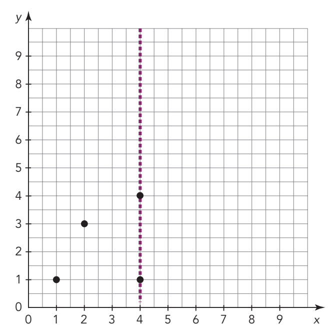
b.



The **vertical line test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

WORKED EXAMPLE

Consider the scatter plot shown.



In this scatter plot, the relation is not a function. The input value 4 can be mapped to two different outputs, 1 and 4. Those two outputs are shown as intersections to the vertical line drawn at $x = 4$.

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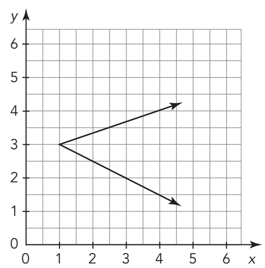
2. Use the definition of function to explain why the vertical line test works.

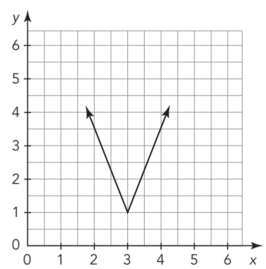
NOTES

3. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.

a.



b.



4. Use the 12 cards that you sorted in the previous lesson. Sort the graphs into two groups: functions and non-functions. Use the letter of each graph to record your findings.

Functions	Non-functions

Answers

2. If a vertical line passes through two points, that means an x-value can be mapped to more than one y-value. This means that the graph does not represent a function.

3a. This graph does not represent a function. A vertical line can be drawn that passes through 2 points in all places except where $x = 1$.

3b. This graph represents a function. When a vertical line is drawn through any portion of the graph, it never passes through more than one point.

4.

Functions	Non-functions
A, B, C, E, G, H, K, L	*D, F, I, J*

NOTES

ACTIVITY

3.5 Functions as Equations

So far, you have determined whether a mapping, context, or a graph represents a function. You can also determine whether an equation is a function.

WORKED EXAMPLE

The given equation can be used to convert yards to feet. Let x represent the number of yards, and let y represent the number of feet.

$$y = 3x$$

To test whether this equation is a function, first, substitute values for x into the equation, and then determine if any x-value can be mapped to more than one y-value. If each x-value has exactly one y-value, then it is a function. Otherwise, it is not a function.

x	$y = 3x$
1	3
3	9
4	12
8	24

In this case, every x-value can be mapped to only one y-value. Each x-value is multiplied by 3. Some examples of ordered pairs are (2, 6), (10, 30), and (5, 15). Therefore, this equation is a function.

It is not possible to test every possible input value in order to determine whether or not the equation represents a function. You can graph any equation to see the pattern and use the vertical line test to determine if it represents a function.

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1. **Determine whether each equation is a function. List three ordered pairs that are solutions to each. Explain your reasoning.**

   a. $y = 5x + 3$ b. $y = x^2$

   c. $y = |x|$ d. $x^2 + y^2 = 1$

   e. $y = 4$ f. $x = 2$

If you do not recognize the graph of the equation, use a graphing calculator to see the pattern.

2. **Explain what is wrong with Taylor's reasoning.**

Taylor

The equation $y^2 = x$ represents a function.

x	y
4	2
9	3
25	5

Answers

1a. This equation is a function. No x-value can be mapped to more than one y-value.

(0, 3) (1, 8) (2, 13)

1b. This equation is a function. No x-value can be mapped to more than one y-value.

(0, 0) (1, 1) (2, 4)

1c. This equation is a function. No x-value can be mapped to more than one y-value.

(0, 0) (1, 1) (−1, 1)

1d. This equation is not a function. When $x = 0$, $y = 1$ and -1.

(0, 1) (0, −1) (1, 0)

1e. This equation is a function. No x-value can be mapped to more than one y-value.

(1, 4) (2, 4) (3, 4)

1f. This equation is not a function. A single x-value, 2, maps to every possible y-value.

(2, −5) (2, 5) (2, 10)

2. Taylor forgot that both 2^2 and $(-2)^2$ are equal to 4. The same is true for the other squares in the table. This means that two x-values map to one y-value, which means that the relation is not a function.

Answers

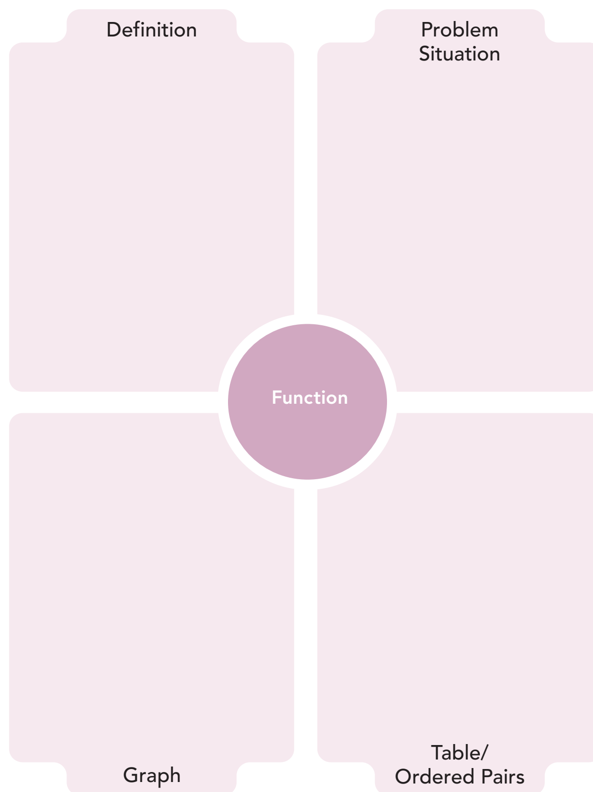
1. Answers will vary.

NOTES

TALK the TALK

Function Organizer

1. **Complete the graphic organizer for the concept of function. Write a definition for *function* in your own words. Then, create a problem situation that can be represented using a function. Finally, create a table of ordered pairs and sketch a graph to represent the function.**



ELL Tip

To help support English Language Learners, provide printed statements for students to add to the graphic organizer. After they have finished the graphic organizer, have students write a short paragraph, explaining what a *function* is, using the facts that they have added to their graphic organizer.

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Assignment

LESSON 3: One or More Xs to One Y

Write

Write the term from the box that best completes each sentence.

scatter plot	output	relation	input	vertical line test
mapping	set	domain	range	function

1. A(n) ____________ is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.
2. The first coordinate of an ordered pair in a relation is the ____________.
3. The second coordinate of an ordered pair is the ____________.
4. A(n) ____________ maps each input to one and only one output.
5. A(n) ____________ is a graph of a collection of ordered pairs.
6. The ____________ is a visual method of determining whether a relation represented as a graph is a function by visualizing whether any vertical lines would intersect the graph of the relation at more than one point.
7. A(n) ____________ shows objects in two sets connected together to represent a relationship between the two sets.
8. A(n) ____________ is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.
9. The ____________ of a function is the set of all inputs of the function.
10. The ____________ of a function is the set of all outputs of the function.

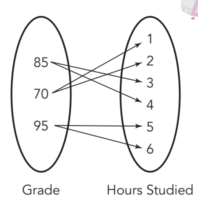
Remember

A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.

A relation is a function when each input value maps to one and only one output value.

Practice

1. A history teacher asks six of her students the number of hours that they studied for a recent test. The diagram shown maps the grades that they received on the test to the number of hours that they studied.
   a. Is the relation a function? If the relation is not a function, explain why not.
   b. Write the set of ordered pairs to represent the mapping.
   c. What does the first value in each ordered pair in part (b) represent? What does the second value in each ordered pair represent?
   d. Create a scatter plot. Does the graph agree with your conclusion from part (a)? Explain your reasoning.



Assignment Answers

Write

1. relation
2. input
3. output
4. function
5. scatter plot
6. vertical line test
7. mapping
8. set
9. domain
10. range

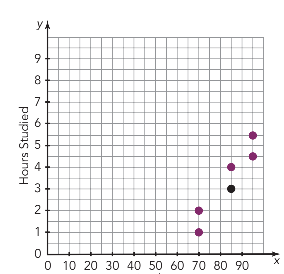
Practice

1a. The mapping does not represent a function. The input 85 has two outputs, 3 and 4. The input 70 has two outputs, 1 and 2. The input 95 has two outputs, 5 and 6. To be a function, no input can have more than one output.

1b. {(85, 3), (85, 4), (70, 1), (70, 2), (95, 5), (95, 6)}

1c. The first value in each ordered pair represents the grade of the student. The second value represents the number of hours the student studied.

1d. Yes. This graph shows that the relation is not a function. A vertical line can be drawn that passes through two points at 70, 85, and 95.

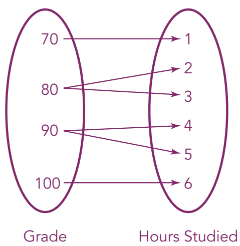


LESSON 3: One or More Xs to One Y • 399

Assignment Answers

Practice

2a.

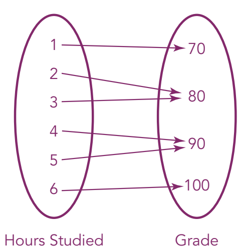


This mapping does not represent a function. The input 90 has two outputs, 5 and 4. The input 80 has two outputs, 3 and 2. To be a function, no input can have more than one output.

2b. The inputs of the relation are 100, 90, 80, and 70.

2c. The outputs of the relation are 6, 5, 4, 3, 2, and 1.

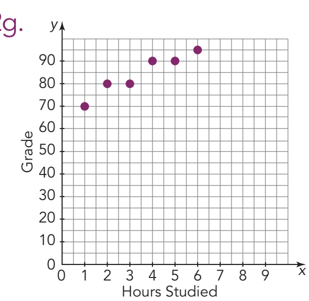
2d.



2e. {(1, 70), (2, 80), (3, 80), (4, 90), (5, 90), (6, 100)}

2f. Yes. This relation is a function. Each input value has only one output value.

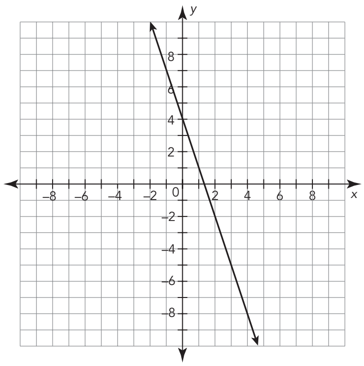
2g.



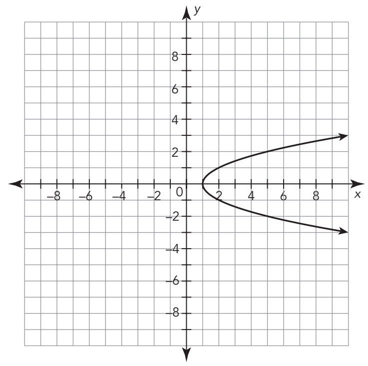
Yes. This graph shows that the relation is a function. A vertical

2. The science teacher created the set of ordered pairs {(100, 6), (90, 5), (80, 3), (70, 1), (90, 4), (80, 2)} to represent six students' grades on the midterm to the number of hours that they had studied. Create a mapping from this set of ordered pairs.
   a. Is the relation a function? If the relation is not a function, explain why not.
   b. List all the inputs of the relation.
   c. List all the outputs of the relation.
   d. Instead of mapping grades to hours studied, the teacher decides to create a new diagram. This diagram maps hours studied to grades. Show the mapping that would result.
   e. Write the set of ordered pairs to represent the mapping in part (d).
   f. Is the relation in part (d) a function? If the relation is not a function, explain why not.
   g. Create a scatter plot. Does the graph agree with your conclusion from part (f)? Explain your reasoning.
3. At the end of the year, a principal decides to create the given mapping.
   Input: the 82 total students in the history class
   Ouput: the final grades they received for the class
   Does this mapping fit the definition of a function? Explain your reasoning.
4. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.

a.



b.



Stretch

Describe how you can tell from an equation whether a function is increasing, decreasing, or constant.

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line drawn through any portion of the graph never passes through more than one point.

3. Yes. Each student only gets one final grade.

4a. This graph represents a function. It passes the vertical line test.

4b. This graph does not represent a function. It does not pass the vertical line test. Every x-value except for $x = 1$ has two y-values.

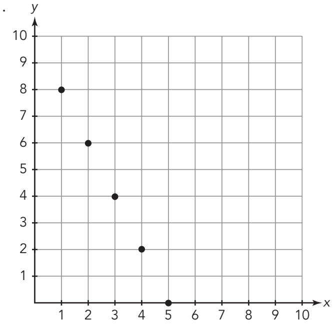
Stretch

If the slope is greater than 0, the function is increasing; if it is less than 0, it is decreasing; if it is equal to 0, the function is constant.

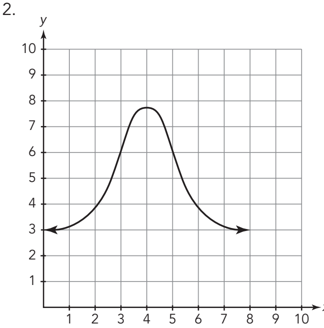
Review

Tell whether each graph is discrete or continuous. Also, tell whether each graph is increasing, decreasing, both, or neither.

1.



2.



Determine the slope and y-intercept of the linear relationship described by each equation.

3. $y = \frac{x}{2} + 5$

4. $y = \frac{x}{4}$

Calculate the slope of the line represented by each table.

5.

x	y
2	-1
3	1.5
4	4
5	6.5

6.

x	y
2	8
4	2
6	-4
9	-13

Assignment Answers

Review

1. discrete, decreasing
2. continuous, both increasing and decreasing
3. Slope: $\frac{1}{2}$, y-intercept: 5
4. Slope: $\frac{1}{4}$, y-intercept: 0
5. Slope: 2.5
6. Slope: -3

LESSON 3: One or More Xs to One Y • 401