MATH 3110 - Fall 2018

Homework 6

Due: Wednesday, October 24th

Note the following:

(a) Homework is due at the beginning of class.

(b) Use only one side of each sheet of paper and staple them together.

(c) State the problem before writing the solution.

(d) SHOW your work. Even if it's true but you did not show it, you will receive only very little credit.

(e) Late homework will NOT be accepted.

Exercise I (10 points):

(1) Determine if W is a subspace of $\mathbb{R}^3$ in the following cases. Justify your answer.

(a) $W = \left\{ \begin{bmatrix} 1 \\ a-2b \\ 2a+b \end{bmatrix} \mid a, b \text{ in } \mathbb{R} \right\}$.

(b) $W = \left\{ \begin{bmatrix} -c \\ a-b \\ b+c \end{bmatrix} \mid a, b, c \text{ in } \mathbb{R} \right\}$.

(c) $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a+b+c=-1 \right\}$.

(2) Let $\mathbb{P}_n$ be the set of all polynomials of degree at most n. Let $H = \{p(t) \in \mathbb{P}_n \mid p(0) = 0\}$. Show that H is a subspace of $\mathbb{P}_n$.

Exercise II (10 points):

(1) Find n and m such that $\text{Nul}(A)$ us a subspace of $\mathbb{R}^n$ and $\text{Col}(A)$ is a subspace of $\mathbb{R}^m$ in the following cases

(a) $A = \begin{bmatrix} 0 & 2 & -3 & 7 \\ -1 & 3 & 2 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 4 & 1 \\ -3 & 2 \\ 0 & 6 \end{bmatrix}$

(c) $A = \begin{bmatrix} 5 & -3 & 0 \\ -4 & 2 & -2 \\ 1 & -1 & 0 \end{bmatrix}$

(2) Let $W = \{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \mid a - 2c = -d \text{ and } b + c = 3d \}$. Find a matrix A such that $W = \mathrm{Nul}(A)$.

(3) Let $V = \{ \begin{bmatrix} a - c \\ a + 2b - c \\ 3b + c \\ -2a + 3c \end{bmatrix} \mid a, b, c \text{ in } \mathbb{R} \}$. Find a matrix A such that $V = \mathrm{Col}(A)$.

(4) Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ -2 & 3 & -1 \end{bmatrix}$. Find a nonzero vector $\vec{u}$ in $\mathrm{Nul}(A)$ and a nonzero vector $\vec{v}$ in $\mathrm{Col}(A)$.

(5) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation with standard matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$. Give a spanning set of $\mathrm{Ker}(T)$ and a spanning set of $\mathrm{im}(T)$.

Exercise III (10 points):

(1) Let $A = \begin{bmatrix} 0 & 3 & -6 & 6 \\ 3 & -7 & 8 & -5 \\ 3 & -9 & 12 & -9 \end{bmatrix}$ be a 3×4 matrix. Matrix A is row equivalent to matrix $B = \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Find a bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$.

(2) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 3 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 5 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ be vectors in $\mathbb{R}^4$. Check that $2\vec{v}_1 + \vec{v}_2 = \vec{v}_3$. Find a basis for $H = \mathrm{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$.