MATH 3110 - Fall 2018

Homework 7

Due: Friday, November 16th

Note the following:

(a) Homework is due at the beginning of class.

(b) Use only one side of each sheet of paper and staple them together.

(c) State the problem before writing the solution.

(d) SHOW your work. Even if it's true but you did not show it, you will receive only very little credit.

(e) Late homework will NOT be accepted.

EXERCISE I:

The matrices A and B are row equivalent in the following cases. Determine $\text{rank}(A)$ and $\dim(\text{Nul})(A)$. Find bases for $\text{Col}(A)$, $\text{Row}(A)$, and $\text{Nul}(A)$.

(a) $A = \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 1 & 2 & 0 & 2 & 8 \\ 2 & 4 & -3 & 10 & 9 \\ 3 & 6 & 0 & 6 & 9 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ 0 & 0 & 3 & -6 & 3 \\ 0 & 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

EXERCISE II:

(1) Let A be a 6×8 matrix.

(a) If $\text{rank}(A) = 5$, what is the the dimension of the nulle space of A?

(b) Could A have a 1-dimensional null space? Justify your answer.

(c) What is the largest possible rank of A? Justify your answer.

(d) What is the smallest possible dimension of $\text{Nul}(A)$? Justify your answer.

(2) Let B be a 8×7 matrix.

(a) Could B have a 2-dimensional null space? Justify your answer.

(b) What is the largest possible rank of B? Justify your answer.

(c) What is the smallest possible dimension of $\text{Nul}(B)$? Justify your answer.

EXERCISE III:

(1) Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ be bases for a vector space V. Suppose $\vec{b}_1 = 2\vec{c}_1 - 3\vec{c}_2$, and $\vec{b}_2 = -\vec{c}_1 + 2\vec{c}_2$.

(a) Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$.

(b) Find $[\vec{x}]_{\mathcal{C}}$ where $= -\vec{b}_1 + 2\vec{b}_2$.

(2) Let $\vec{b}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{c}_1 = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, and $\vec{c}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ be vectors in $\mathbb{R}^2$. Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$.

(a) Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$.

(b) Find the change-of-coordinate matrix from $\mathcal{C}$ to $\mathcal{B}$.