MATH 3110 - Fall 2018

PRACTICE 2

Exercise I:

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a 3×3 matrix and suppose that $\det(A) = -4$. Let P be a 3×3 invertible matrix. Compute the determinant of the following matrices:

(a) $B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{31} & 2a_{32} & 2a_{33} \\ a_{21} + 3a_{11} & a_{22} + 3a_{12} & a_{23} + 3a_{13} \end{bmatrix}$

(b) $D = 2A$

(c) $E = A^n$, where n is a positive integer

(d) $F = PAP^{-1}$

Exercise II:

(1) Let $\mathbb{P}_3$ be the set of polynomials of degree at most 3. The set $\mathbb{P}_3$ with the zero polynomial $p(t) = 0$, with the usual addition and scalar multiplication is a vector space.

(a) Let $H = \{p(t) \in \mathbb{P}_3 \mid \deg(p(t)) = 3\} \cup \{0\}$, that is, the zero polynomial is included in H and the nonzero elements of H are exactly of degree 3. Is H a subspace of $\mathbb{P}_3$? Justify your answer.

(b) Let $S = \{3t^2 + t - 2, -t^2 + 2, 6t^2 - 2t - 7\}$ be a set polynomials in $\mathbb{P}_3$. Is S linearly independent? Justify your answer.

(2) Let $M_{2\times 2}(\mathbb{R})$ be the set of all 2×2 matrices (with real number entries), that is

$$M_{2\times 2}(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \text{ in } \mathbb{R} \right\}$$

The set $M_{2\times 2}(\mathbb{R})$ with the zero matrix $\mathbf{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, with the usual addition and scalar multiplication is a vector space. Let $H = \left\{ \begin{bmatrix} a & 0 \\ c & d \end{bmatrix} \mid a, c, d \text{ in } \mathbb{R} \right\}$ be a subset of $M_{2\times 2}(\mathbb{R})$.

(a) Give a nonzero element of H. Give an element of $M_{2\times 2}(\mathbb{R})$ which is not in H.

(b) Is H a subspace of $M_{2\times 2}(\mathbb{R})$? Justify your answer.

Exercise III:

Let $H = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \mid a = -c \text{ and } b = c \right\}$ be a subset of $\mathbb{R}^4$.

(a) Give a nonzero vector $\vec{u}$ that is in H ($\vec{u} \in H$), and a vector $\vec{v}$ in $\mathbb{R}^4$ that is not in H ($\vec{v} \notin H$).

(b) Find a matrix A such that $H = \text{Nul}(A)$.

Exercise IV:

Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}$ be vectors in $\mathbb{R}^4$, and let $H = \text{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$.

(a) Show that $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$ is a basis for H.

(b) Is $\vec{u} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ in H. Justify your answer.

(c) Compute the $\mathcal{B}$-coordinate of $\vec{v} = \begin{bmatrix} -9 \\ -4 \\ 5 \end{bmatrix}$.

Exercise V:

Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ -x_1 + 3x_2 \\ x_1 + x_2 \end{bmatrix}$$

Let A be the standard matrix of T.

(a) Determine the matrix A.

(b) Is there a nonzero vector in $\text{Ker}(T)$? Justify.

(c) Give a basis for $\text{im}(T)$. What is the dimension of $\text{im}(T)$?

Exercise VI:

Let $A = \begin{bmatrix} 5 & -2 & 3 \\ -1 & 0 & -1 \\ -5 & 7 & 2 \end{bmatrix}$

(a) Is $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ in $\text{Nul}(A)$. Justify.

(b) Give a basis for $\text{Nul}(A)$.

(c) Is $\vec{v} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ in $\text{Col}(A)$? Justify.

(d) What is the dimension of $\text{Col}(A)$.