MATH 3110 - Fall 2018

Homework 4

Due: Wednesday, September 26th

Note the following:

(a) Homework is due at the beginning of class.

(b) Use only one side of each sheet of paper and staple them together.

(c) State the problem before writing the solution.

(d) SHOW your work. Even if it's true but you did not show it, you will receive only very little credit.

(e) Late homework will NOT be accepted.

Question 1:

Determine by inspection whether the following vectors are linearly independent, justify.

(a) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ -5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ -12 \end{bmatrix}$

Question 2:

Find the value(s) of h for which the following vectors are linearly dependent:

$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ h \end{bmatrix}.$

Question 3:

Let $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5$ be vectors in $\mathbb{R}^5$ with $\vec{v}_2 = \vec{v}_1 - 3\vec{v}_3$. Show that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\}$ is linearly dependent.

Question 4:

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -2x_1 + 3x_2 - x_3 \\ x_1 + 4x_3 \\ x_2 + x_3 \end{bmatrix}$$

(a) Give the standard matrix A for T.

(b) Is T onto $\mathbb{R}^3$? Justify.

(c) Is T one-to-one? Justify.

Question 5:

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Let $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$. Suppose $T(\vec{x}) = x_1\vec{v}_1 + x_2\vec{v}_2$. Determine the standard matrix A of T.