MATH 3110 - Fall 2018

Homework 8

Due: Wednesday, November 28th

Note the following:

(a) Homework is due at the beginning of class.

(b) Use only one side of each sheet of paper and staple them together.

(c) State the problem before writing the solution.

(d) SHOW your work. Even if it's true but you did not show it, you will receive only very little credit.

(e) Late homework will NOT be accepted.

EXERCISE I (6 points):

Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 4 & 1 \\ -1 & -1 & 2 \end{bmatrix}$ be a 3×3 matrix.

(a) Show that $\vec{x} = \begin{bmatrix} -3 \\ 3 \\ -3 \end{bmatrix}$ is an eigenvector of A.

(b) Show that $\lambda = 2$ is an eigenvalue of A.

(c) Find a basis for the eigenspace of A corresponding to $\lambda = 2$.

EXERCISE II (10 points):

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & -1 & -4 \\ 2 & 2 & 5 \end{bmatrix}$ be a 3×3 matrix.

(a) Find the characteristic polynomial of A.

(b) Show that $\lambda = 1$ is an eigenvalue of A of multiplicity 2 (Hint: Show that $(\lambda - 1)^2$ is a factor of the characteristic polynomial of A).

(c) What are all the eigenvalues of A?

(d) Show that each eigenspace of A has dimension that is equal to the multiplicity of each corresponding eigenvalue.

(e) Deduce that A is diagonalizable.

(f) Diagonalize A.

EXERCISE III (4 points):

Is the matrix A in the following cases diagonalizable? Justify your answer.

(a) $A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$.

(b) $A = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix}.$