MATH 3110 - Fall 2018

PRACTICE 1

Exercise I:

Consider the following linear system

$$\text{(E)} \qquad \begin{aligned} x_1 + 2x_2 - 4x_3 &= 0 \\ -2x_2 - 4x_2 + 8x_3 &= 0 \\ 3x_1 + 6x_2 - 12x_3 &= 0 \end{aligned}$$

(a) Write the matrix equation corresponding to (E).

(b) Let A be the coefficient matrix of (E). Write the solution of the equation $A\vec{x} = \vec{0}$ in parametric vector form. Describe it geometrically.

(c) Let $\vec{b} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$. Is $\vec{b}$ in the span of the columns of A? Justify your answer.

Exercise II:

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_3 \\ -2x_1 + x_2 - x_3 \\ 2x_2 + 6x_3 \end{bmatrix}$$

(a) Let A be the standard matrix of T. Does the equation $A\vec{x} = \vec{b}$ have a solution for every $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in $\mathbb{R}^3$? Justify your answer.

(b) Is T onto $\mathbb{R}^3$ Justify your answer.

(c) Is T one-to-one? Justify your answer.

Exercise III:

Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$.

(a) Compute AB by using the row-column rule.

(b) Is $A^T B^T$ defined? Justify your answer.

EXERCISE IV:

Find the inverse of the following matrices: $A = \begin{bmatrix} 1 & -3 \\ 4 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$