Eric Paulz
ECE 4930-004

HW1

1. Derive a reliability function R(t) for each system.

   a. System A

$$R_{parallel}(t) = 1 - \prod_{i=1}^{n}\big(1 - R_i(t)\big)$$

$$R_A(t) = 1 - [(1 - R_{series}(t))(1 - R_{series}(t))]$$

$$R_A(t) = 1 - [(1 - (R_{cpu}(t) * R_{mem}(t)))(1 - (R_{cpu}(t) * R_{mem}(t)))]$$

$$R_A(t) = 1 - [1 - (R_{cpu}(t) * R_{mem}(t))]^2$$

$$R_A(t) = 1 - [1 - \left(e^{-\lambda_{cpu}t} * e^{-\lambda_{mem}t}\right)]^2$$

$$R_A(t) = 1 - [1 - \left(e^{-(\lambda_{cpu}t + \lambda_{mem}t)}\right)]^2$$

   b. System B

$$R_{series}(t) = \prod_{i=1}^{n} R_i(t)$$

$$R_B(t) = (R_{parallel}(t))(R_{parallel}(t))$$

$$R_B(t) = \big[1 - (1 - R_{cpu}(t))(1 - R_{cpu}(t))\big][1 - (1 - R_{mem}(t))(1 - R_{mem}(t))]$$

$$R_B(t) = \big[1 - (1 - e^{-\lambda_{cpu}t})(1 - e^{-\lambda_{cpu}t})\big]\big[1 - (1 - e^{-\lambda_{mem}t})(1 - e^{-\lambda_{mem}t})\big]$$

$$R_B(t) = \big[1 - (1 - e^{-\lambda_{cpu}t})^2\big]\big[1 - (1 - e^{-\lambda_{mem}t})^2\big]$$

2. Determine which system is more reliable. ($MTBF_{cpu}$ = 9hrs, $MTBF_{mem}$ = 6 hrs)

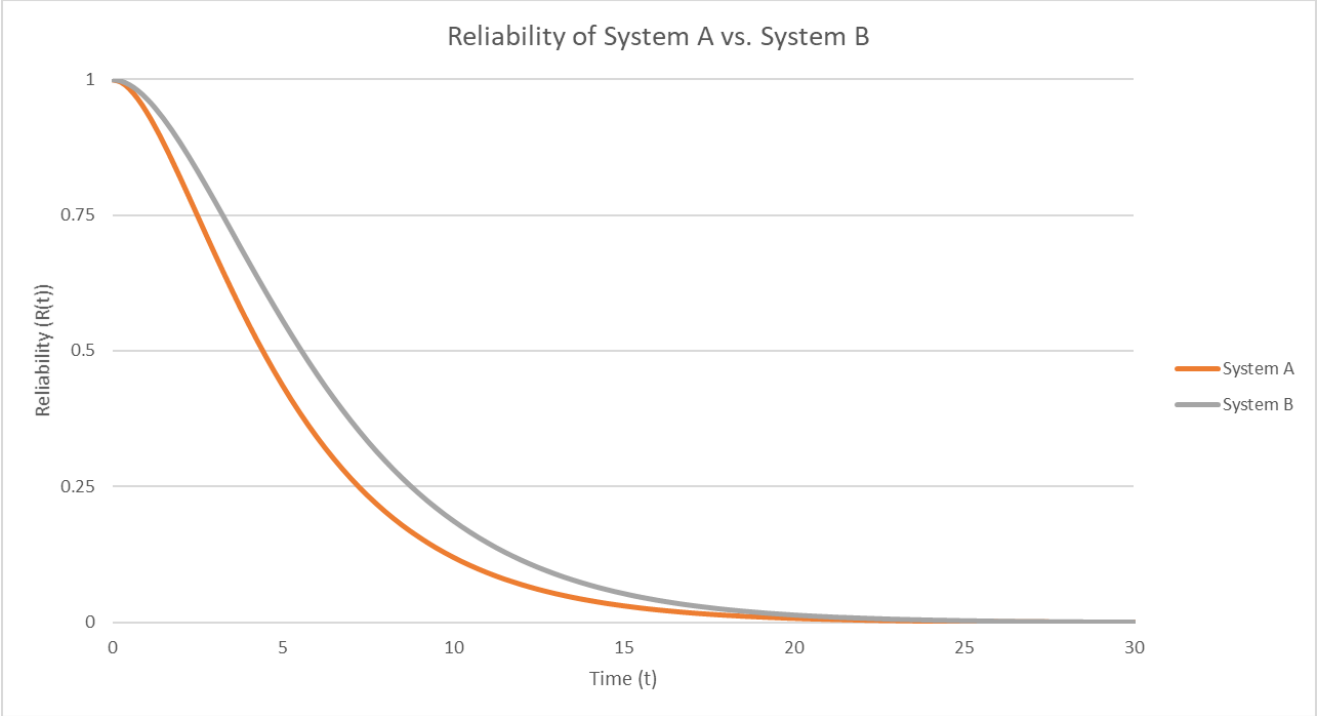

Figure 1: Plot of reliability functions from System A and System B; $\lambda_{cpu} = \frac{1}{9}$, $\lambda_{mem} = \frac{1}{6}$

After replacing λ_{cpu} with $\frac{1}{9}$ and λ_{mem} with $\frac{1}{6}$ in the appropriate places, the plot of the two reliability equations is shown in Figure 1. We can see that the reliability of System B decreases slightly slower than that of System A. Therefore, we can say that System B is the more reliable of the two based on the given MTBF values.

3. What would the MTBF of a memory module need to be to change your decision (if at all)? Explain.
To determine what MTBF for memory modules may make System A more reliable, first I tried doubling the original. Plugging in $\frac{1}{12}$ for λ_{mem} gives the following result:

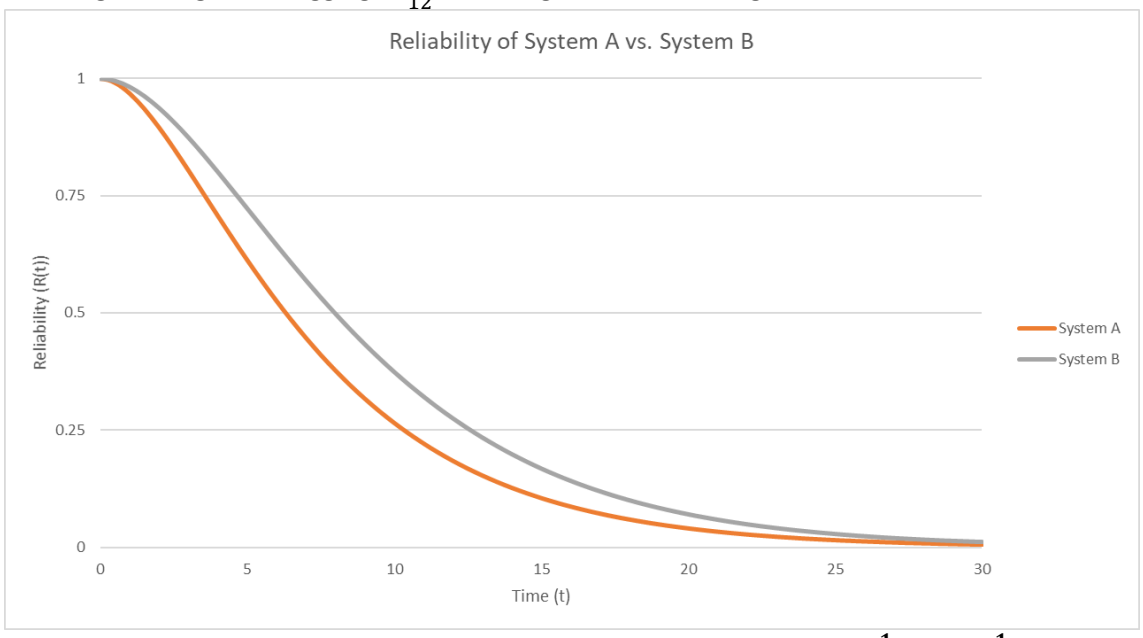

Figure 2: Plot of reliability functions from System A and System B; $\lambda_{cpu} = \frac{1}{9}$, $\lambda_{mem} = \frac{1}{12}$

Since System B is even more reliable than System A in this case, I tried reducing the MTBF for memory modules to 1, plugging in 1 for λ_{mem}, which gives us the following:

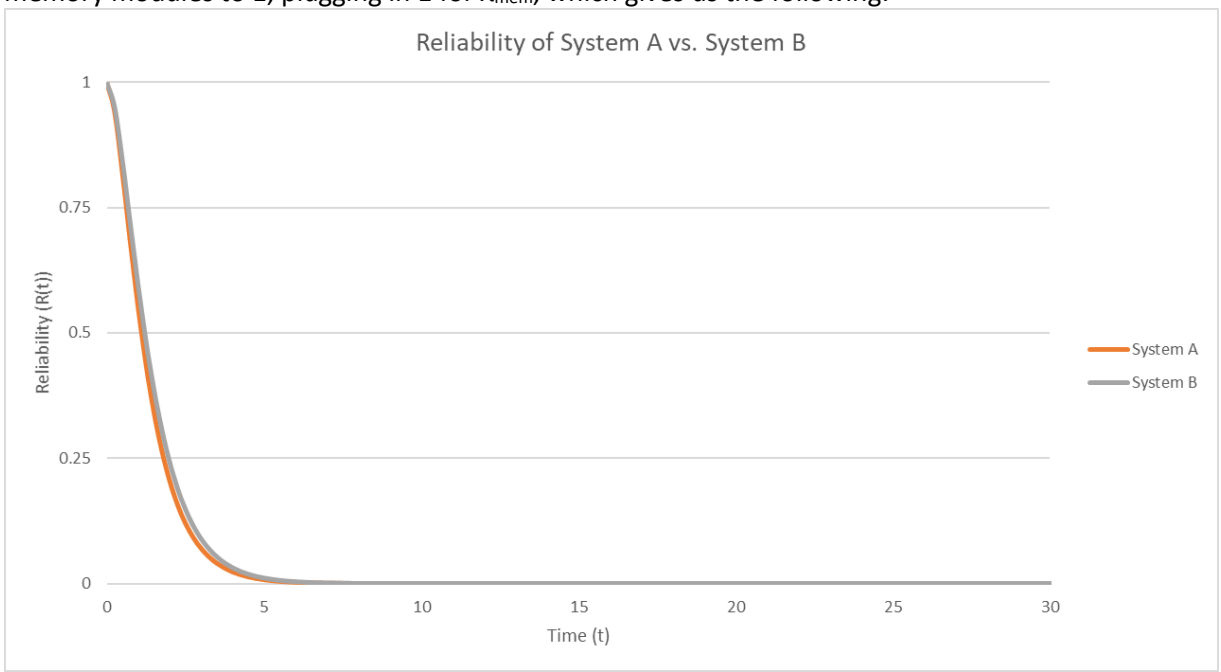

Figure 3: Plot of reliability functions from System A and System B; $\lambda_{cpu} = \frac{1}{9}$, $\lambda_{mem} = 1$

After testing these two extreme alternatives, I still conclude that System B will be more reliable than System A regardless of the MTBF for memory modules.