Quaternion Example.

Point $P = (0, 1.5, 0.5)$ $\theta = 90°$, $u = [0, 1, 0]$

Where is P'?

$P' = T_u \cdot P$

a). $q = (s, v) \Rightarrow (s, V_x, V_y, V_z)$

Convert u, θ to Quaternion representation

$s = \cos \frac{\theta}{2} \Rightarrow \cos 45 = \sqrt{2}/2$.

$u = [0, 1, 0] \rightarrow$ Note unit vector.

$\therefore v = (\sin \theta/2 \, [0, 1, 0]^T)$.

$\Rightarrow (\sqrt{2}/2 \, (0, 1, 0)) \Rightarrow [0, \sqrt{2}/2, 0]$

$q = (\sqrt{2}/2, 0, \sqrt{2}/2, 0)$.

— Quaternion representation of u & θ

$q_P = (0, 0, 1.5, 0.5)$

$q_{P'} = q \cdot q_P \cdot q^{-1}$, $q_1 \cdot q_2 = (s_1 s_2 - V_1 \cdot V_2, s_1 V_2 + s_2 V_1 + V_1 \times V_2)$

B. $\therefore q \cdot q_P = (\sqrt{2}/2, 0, \sqrt{2}/2, 0) \cdot (0, 0, 1.5, 0.5)$

$\Rightarrow (0 - 3\sqrt{2}/4, (0, 3\sqrt{2}/4, \sqrt{2}/4) + (0, 0, 0) + (\sqrt{2}/4, 0, 0))$

$\Rightarrow q \cdot q_P = (-3\sqrt{2}/4, \sqrt{2}/4, 3\sqrt{2}/4, \sqrt{2}/4)$

since u is a unit vector, $q^{-1} = (s, -v)$

$q^{-1} = (\sqrt{2}/2, 0, -\sqrt{2}/2, 0)$.

C. $q q_P \cdot q^{-1} = (-3\sqrt{2}/4, \sqrt{2}/4, 3\sqrt{2}/4, \sqrt{2}/4) \cdot (\sqrt{2}/2, 0, -\sqrt{2}/2, 0)$

$\therefore q_{P'} = (-3/4 - (-3/4)), (0, 3/4, 0) + (1/4, 3/4, 1/4) + (1/4, 0, -1/4)$

$q_{P'} = (0, 1/2, 3/2, 0)$

$P' = (1/2, 3/2, 0) \Rightarrow (0.5, 1.5, 0)$