

MATH 3110 - Fall 2018

Homework 3

Due: Monday, September 17th

Note the following:

- (a) Homework is due at the beginning of class.
- (b) Use only one side of each sheet of paper and staple them together.
- (c) State the problem before writing the solution.
- (d) SHOW your work. Even if it's true but you did not show it, you will receive only very little credit.
- (e) Late homework will NOT be accepted.

Exercise 1 (5 points):

Mark each statement TRUE or FALSE. In **any case**, justify your answer.

(a) The product $\begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ is undefined.

(b) The product $\begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ is undefined.

- (c) A vector \vec{b} is a linear combination of the columns of a matrix A if and only if the equation $A\vec{x} = \vec{b}$ has at least one solution.
- (d) If the columns of a $m \times n$ matrix A span \mathbb{R}^m , then the equation $A\vec{x} = \vec{b}$ is consistent for each $\vec{b} \in \mathbb{R}^m$.
- (e) If the coefficient matrix A has a pivot position in every row, then the equation $A\vec{x} = \vec{b}$ is inconsistent.

Exercise 2 (5 points):

Consider the following linear system:

$$x_1 - 2x_2 + x_3 = 1$$

(E) $-2x_1 + 2x_2 = 1$

$$4x_1 - 2x_2 - 2x_3 = -5$$

- (1) Write the vector equation and the matrix equation that correspond to the linear system (E).
- (2) Consider the matrix equation $A\vec{x} = \vec{b}$ that corresponds to (E). Solve the equation by using the corresponding augmented matrix. Give a solution and write it as a vector.
- (3) Let A be the coefficient matrix of the system (E) (note that A is not the augmented matrix).

(a) Let $\vec{b} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$. Is \vec{b} in the span of the columns of A ? Justify your answer.

(b) Show that not all $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbb{R}^3$ is in the span of the columns of A .

(c) Do the columns of A span \mathbb{R}^3 ? Justify.

Exercise 3 (4 points):

Mark each statement TRUE or FALSE. In **any case, justify your answer**.

- (a) A homogeneous equation is always consistent.
- (b) The homogeneous equation $A\vec{x} = \vec{0}$ has a nontrivial solution if the it has a free variable.
- (c) If \vec{v}_h is a solution of $A\vec{x} = \vec{0}$ and if \vec{p} is a solution of $A\vec{x} = \vec{b}$ (with $\vec{b} \neq \vec{0}$), then $\vec{p} + \vec{v}_h$ is a solution of $A\vec{x} = \vec{b}$.
- (d) If $A\vec{x} = \vec{b}$ is consistent, then the solution set of $A\vec{x} = \vec{b}$ is obtained by translating the solution set of $A\vec{x} = \vec{0}$.

Exercise 4 (6 points):

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -1 & 1 & 0 \end{bmatrix}$ be a 3×3 matrix.

- (1) (a) Show, by using the reduction of the corresponding augmented matrix, that the homogeneous equation $A\vec{x} = \vec{0}$ has a nontrivial solution.
- (b) Write the solutions of $A\vec{x} = \vec{0}$ in parametric vector form.
- (c) write the solution set of $A\vec{x} = \vec{0}$ as a span of some vector(s) (give the vector(s)). Describe it geometrically.

(b) Let $\vec{b} = \begin{bmatrix} -3 \\ 0 \\ -3 \end{bmatrix}$.

- (a) Give a solution \vec{p} of the equation $A\vec{x} = \vec{b}$.
- (b) By using 1-b), give a solution \vec{w} of $A\vec{x} = \vec{b}$ such that $\vec{w} = \vec{p} + \vec{v}_h$ where \vec{v}_h is a solution of $A\vec{x} = \vec{0}$.