MATH 3110 - Fall 2018

Homework 3

Due: Friday, September 14th

Note the following:

(a) Homework is due at the beginning of class.

(b) Use only one side of each sheet of paper and staple them together.

(c) State the problem before writing the solution.

(d) SHOW your work. Even if it's true but you did not show it, you will receive only very little credit.

(e) Late homework will NOT be accepted.

Exercise 1 (5 points):

Mark each statement TRUE or FALSE. In **any case, justify your answer**.

(a) The product $\begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ is undefined.

(b) The product $\begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ is undefined.

(c) A vector $\vec{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\vec{x} = \vec{b}$ has at least one solution.

(d) If the columns of a $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $A\vec{x} = \vec{b}$ is consistent for each $\vec{b} \in \mathbb{R}^m$.

(e) If the coefficient matrix A has a pivot position in every row, then the equation $A\vec{x} = \vec{b}$ is inconsistent.

Exercise 2 (5 points):

Consider the following linear system:

$$\text{(E)} \qquad \begin{aligned} x_1 - 2x_2 + 3x_3 &= 0 \\ -2x_1 + 2x_2 &= 0 \\ 4x_1 - 2x_2 + 3x_3 &= 10 \end{aligned}$$

(1) Write the vector equation and the matrix equation that correspond to the linear system (E).

(2) Consider the matrix equation $A\vec{x} = \vec{b}$ that corresponds to (E). Solve the equation by using the corresponding augmented matrix. Write the solution as a vector.

(3) Let A be the coefficient matrix of the system (E) (note that A is not the augmented matrix).

(a) Let $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Is $\vec{b}$ in the span of the columns of A? Justify your answer.

(b) Show that not all $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbb{R}^3$ is in the span of the columns of A.

(c) Do the columns of A span $\mathbb{R}^3$? Justify.

Exercise 3 (4 points):

Mark each statement TRUE or FALSE. In **any case, justify your answer**.

(a) A homogeneous equation is always consistent.

(b) The homogeneous equation $A\vec{x} = \vec{0}$ has a nontrivial solution if the it has a free variable.

(c) If $\vec{v}_h$ is a solution of $A\vec{x} = \vec{0}$ and if $\vec{p}$ is a solution of $A\vec{x} = \vec{b}$ (with $\vec{b} \neq \vec{0}$), then $\vec{p} + \vec{v}_h$ is a solution of $A\vec{x} = \vec{b}$.

(d) If $A\vec{x} = \vec{b}$ is consistent, then the solution set of $A\vec{x} = \vec{b}$ is obtained by translating the solution set of $A\vec{x} = \vec{0}$.

Exercise 4 (6 points):

Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 2 & -3 \\ -1 & 1 & 0 \end{bmatrix}$ be a 3×3 matrix.

(1) (a) Show, by using the reduction of the corresponding augmented matrix, that the homogeneous equation $A\vec{x} = \vec{0}$ has a nontrivial solution.

(b) Write the solutions of $A\vec{x} = \vec{0}$ in parametric vector form.

(c) write the solution set of $A\vec{x} = \vec{0}$ as a span of some vector(s) (give the vector(s)). Describe it geometrically.

(b) Let $\vec{b} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$.

(a) Give a solution $\vec{p}$ of the equation $A\vec{x} = \vec{b}$.

(b) By using 1-b), give a solution $\vec{w}$ of $A\vec{x} = \vec{b}$ such that $\vec{w} = \vec{p} + \vec{v}_h$ where $\vec{v}_h$ is a solution of $A\vec{x} = \vec{0}$.