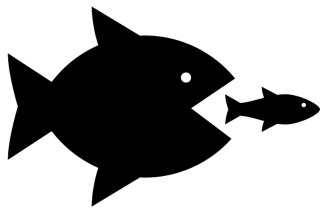
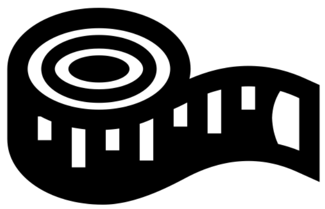
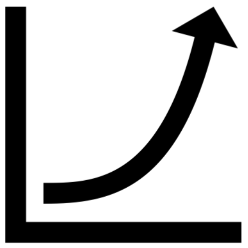
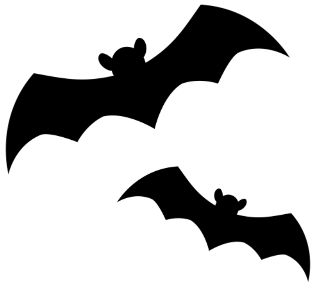








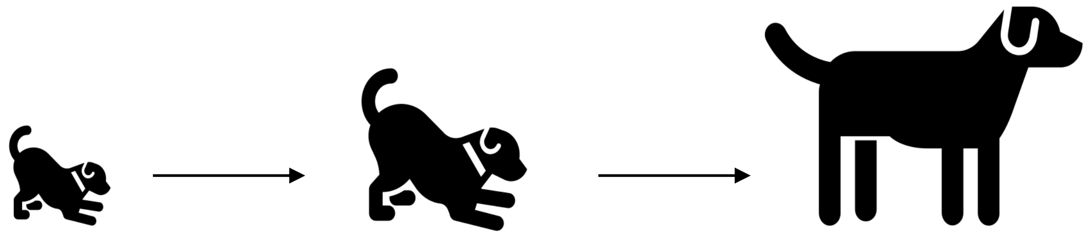
Size-based Models

Why size?

- Length and weight metrics are often much easier to collect
- There is variability in size-at-age
  - Within an age class
  - Across a time series
- Size correlates with value
- Size correlates with physiological and behavioral processes

How to account for size?

- Directly via growth model
- **Probability of moving from one size class to another**
- Generally more complicated than age-based
  - One year from now, organism will be one year older (or dead?)
  - One year from now, organism will be ? cm larger



The Markov process

- Consists of three basic elements
  1. Where can we be? (states)
     - $\boldsymbol{s} = \{s_1, \dots, s_N\}$
  2. Where do we start? (initial distribution)
     - $\boldsymbol{\pi}_0 = \{\pi_1, \dots, \pi_N\}$
  3. Where do we go from here? (probability transition matrix)
     - $P_{NxN} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,N} \\ \vdots & \ddots & \vdots \\ p_{N,1} & \cdots & p_{N,N} \end{bmatrix}$

$$\begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \\ p_{3,1} & p_{3,2} & p_{3,3} \end{bmatrix}$$

$$\boldsymbol{\pi}_0 = \{\pi_1, \pi_2, \pi_3\}$$

$$\pi_1 = 1$$

$$\pi_2 = 0$$

$$\pi_3 = 0$$

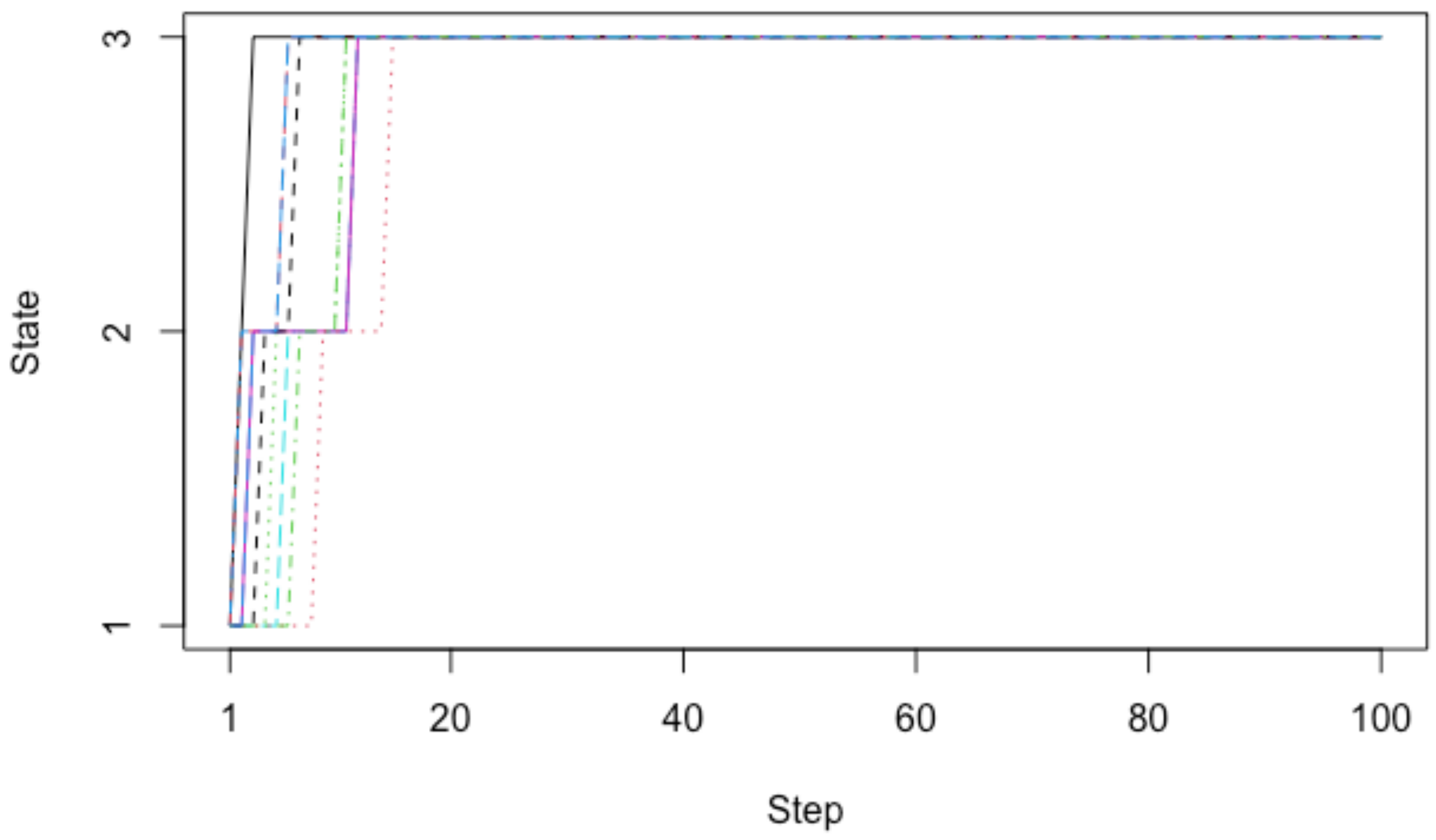
$$\boldsymbol{s} = \{s_1, s_2, s_3\}$$

$$s_1 = 0 - 10 \text{ cm}$$

$$s_2 = 10 - 20 \text{ cm}$$

$$s_3 = 20 - 30 \text{ cm}$$

$$\begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.0 & 0.8 & 0.2 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$


State
3
2
1
1
20
40
60
80
100
Step


What about survival and
reproduction?

Leslie Matrix redux

$$\begin{pmatrix} S_1 P_{1,1} + S_0 f_1 & S_0 f_2 & S_0 f_3 & \cdots & S_0 f_{Y-1} & S_0 f_Y \\ S_1 P_{1,2} & S_2 P_{2,2} & 0 & \cdots & 0 & 0 \\ S_1 P_{1,3} & S_2 P_{2,3} & S_3 P_{3,3} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 & \vdots \\ \vdots & \vdots & \vdots & \ddots & S_{Y-1} P_{Y-1,Y-1} & 0 \\ S_1 P_{1,Y} & S_2 P_{2,Y} & \cdots & \cdots & S_{Y-1} P_{Y-1,Y} & S_Y P_{Y,Y} \end{pmatrix}$$

Hilborn, R. and Walters, C.J. eds., 2013. *Quantitative fisheries stock assessment: choice, dynamics and uncertainty*. Springer Science & Business Media.

P_{ij}: probability of growing from size class i into size class j

S_i: survival probability of size class i

f_i: fecundity of size class i