Population Growth

That the increase of population is necessarily limited by the means of subsistence,
That population does invariably increase when the means of subsistence increase, and,
That the superior power of population is repressed by moral restraint, vice and misery.

-- T.R. Malthus

Why model population growth and mortality?

- To understand how and why populations change
- To provide a framework for asking and answering questions
- To define productivity
- To provide objective predictions about how changes take place and about how populations are likely to respond under management

Population models

- Geometric
  - Discrete rates of changes in population size with fixed equally spaced time steps
- Exponential
  - Instantaneous rates of change
- Logistic
  - Instantaneous rates of change with density dependent feedback

$$N_{t+1} = N_t R$$

$$N_t = N_{t-1} R$$

$$N_{t+1} = (N_{t-1} R) R = N_{t-1} R^2$$

$$\vdots$$

$$N_{t+1} = N_0 R^{t+1}$$

Annual rate of change

$$\frac{\Delta N_t}{\Delta t} = N_{t+1} - N_t$$

$$= RN_t - N_t$$

$$= (R - 1)N_t$$

Incremental rate of change

$$\frac{\Delta N_t}{\Delta t} = \frac{N_{t+h} - N_t}{h}$$

$$= \frac{R^h N_t - N_t}{h}$$

$$= \frac{(R^h - 1)}{h} N_t$$

Incremental rate of change

h = ?

vs.

From discrete to continuous

$$\frac{dN}{dt} = \lim_{h\to 0}\frac{\Delta N_t}{\Delta t}$$

$$= \lim_{h\to 0}\frac{R^h N_t - N_t}{h}$$

$$= \lim_{h\to 0}\frac{(e^{rh}-1)}{h}N_t = \lim_{h\to 0}\frac{\frac{d}{dh}(e^{rh}-1)}{\frac{d}{dh}h}N_t = \lim_{h\to 0}\frac{re^{rh}}{1}N_t$$

$$= rN_t$$

Per capita rate of change

$$\frac{1}{N}\frac{dN}{dt} = r$$

Continuous growth

$$\int_{N_0}^{N_t} \frac{1}{N} dN = \int_0^t r dt$$

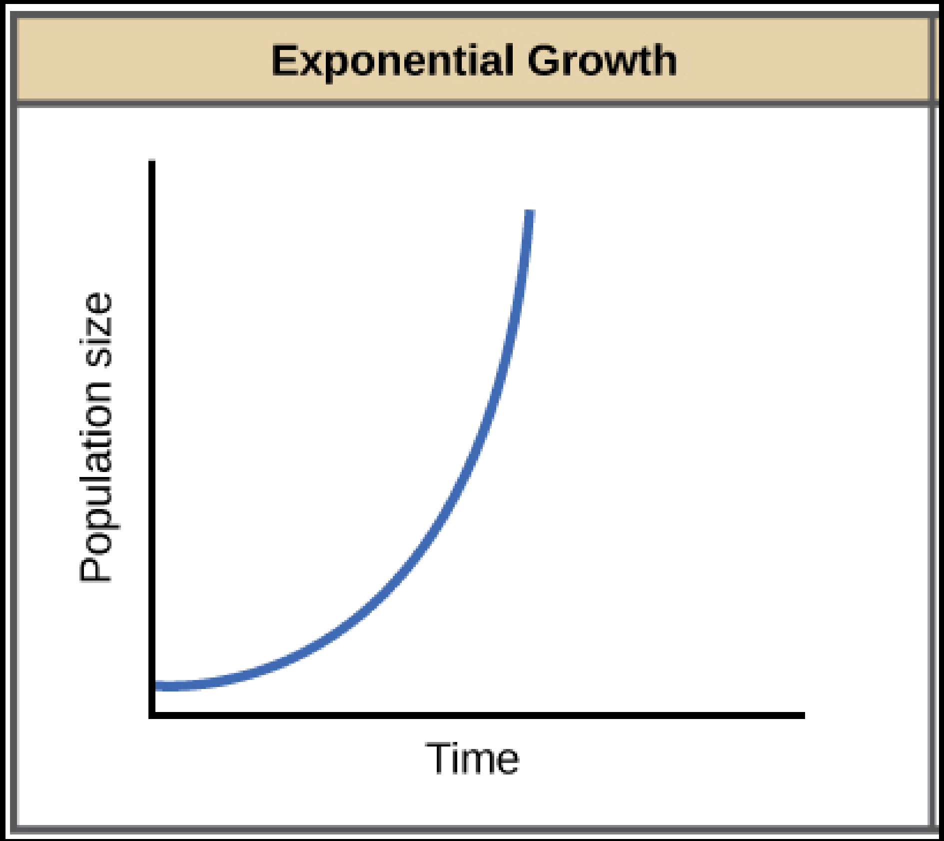
$$\ln(N) \,|_{N_0}^{N_t} = rt|_0^t$$

$$\ln\left(\frac{N_t}{N_0}\right) = rt - r0 = rt$$

$$\frac{N_t}{N_0} = e^{rt}$$

$$N_t = N_0 e^{rt}$$

Malthusian growth



Annual vs. instantaneous rates of change

$$N_t = N_0 R^t \qquad N_t = N_0 e^{rt}$$

$$R^t = e^{rt}$$

$$R = e^r$$

$$\ln(R) = \ln(e^r) = r \ln(e) = r$$

Annual vs. instantaneous rates of change

$$\exp(r) = R$$

$$\exp(0.05) = 1.05$$

$$\exp(0.00) = 1.00$$

$$\exp(-0.05) = 0.95$$

Human population growth

- Since 2000 the world population has been growing at about 1.3% per year (R = 1.013)
- In 2000 the world population was at about 6 billion people
- The year 2023 corresponds to t=23 and r ≈ R-1 = 0.013, so:

$$N_{2023} = N_{2000}\exp(0.013 * 23)$$

$$= 6\exp(0.013 * 23)$$

$$= 8.09$$

Human population growth

- When will it hit 10 billion humans?

$$N_t = 10 = 6\exp(0.013 * t)$$

$$\frac{10}{6} = \exp(0.013 * t)$$

$$\ln\left(\frac{10}{6}\right) = 0.013 * t$$

$$t \approx 39$$

Unlimited growth

- Discrete vs. instantaneous rates of change
- An instantaneous rate is the rate that is happening at any precise point in time
- Instantaneous measures are useful for analytical approaches using calculus
- Discrete methods can be used to approximate rates of change on a computer
- Exponential growth is perhaps the simplest model of how populations change

Logistic growth

$$\frac{dN}{dt} = rN$$

$$r = (a - bN)$$

$$\frac{dN}{dt} = (a - bN)N$$

$$\frac{dN}{dt} = aN - bN^2 = aN\left(1 - \frac{b}{a}N\right)$$

Logistic growth

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

Logistic growth

$$N_t = \frac{N_0 e^{rt}}{1 - \frac{N_0}{K} + \frac{N_0}{K} e^{rt}}$$

Surplus production

- Quinn and Deriso Chapter 1

$$B_0 = K$$

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) - H_t$$

$$\hat{I}_t = \frac{C_t}{E_t} = qB_t$$

Surplus production

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) - H_t$$

$$B_{t+1} = B_t$$

$$H_t = rB_t\left(1 - \frac{B_t}{K}\right)$$

Surplus production

$$\frac{dB}{dt} = rB_t\left(1 - \frac{B_t}{K}\right) - FB_t$$

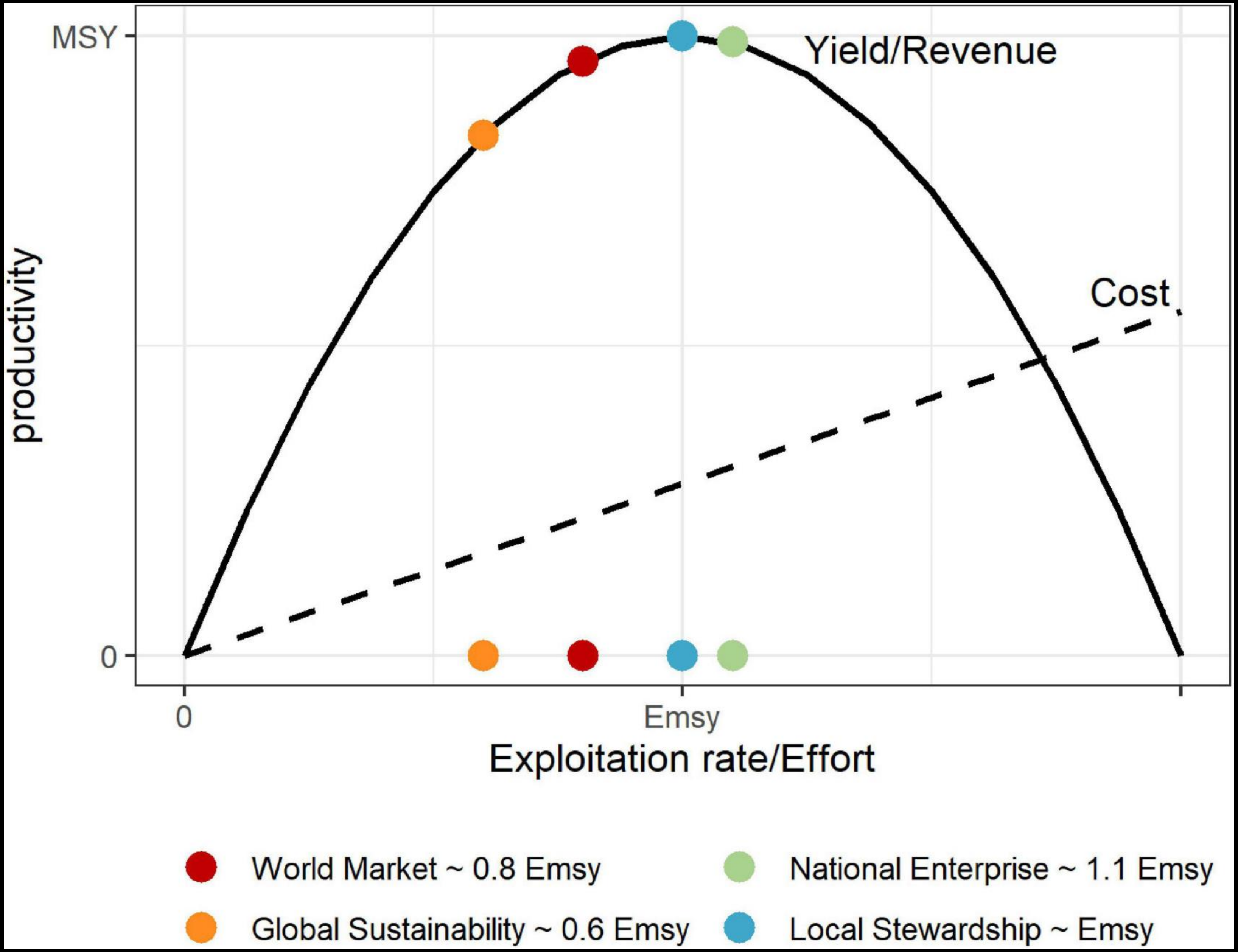
$$F_*B_* = rB_*\left(1 - \frac{B_*}{K}\right)$$

$$\frac{F_*}{r} = 1 - \frac{B_*}{K}$$

$$B_* = K\left(1 - \frac{F_*}{r}\right)$$

Equilibrium yield

$$Y_* = F_* B_* = F_* K \left(1 - \frac{F_*}{r}\right)$$


MSY
Yield/Revenue
Cost
productivity
0
0
Emsy
Exploitation rate/Effort
World Market ~ 0.8 Emsy
National Enterprise ~ 1.1 Emsy
Global Sustainability ~ 0.6 Emsy
Local Stewardship ~ Emsy

Surplus production (Pella-Tomlinson)

$$B_{t+1} = B_t + rB_t \frac{1}{p}\left(1 - \left(\frac{B_t}{K}\right)^p\right) - H_t$$