

# Bioenergetics and Individual Growth





Bioenergetics  $\Rightarrow$ : conversion of energy from food into body mass

Individual growth  $\Rightarrow$ : change in body mass



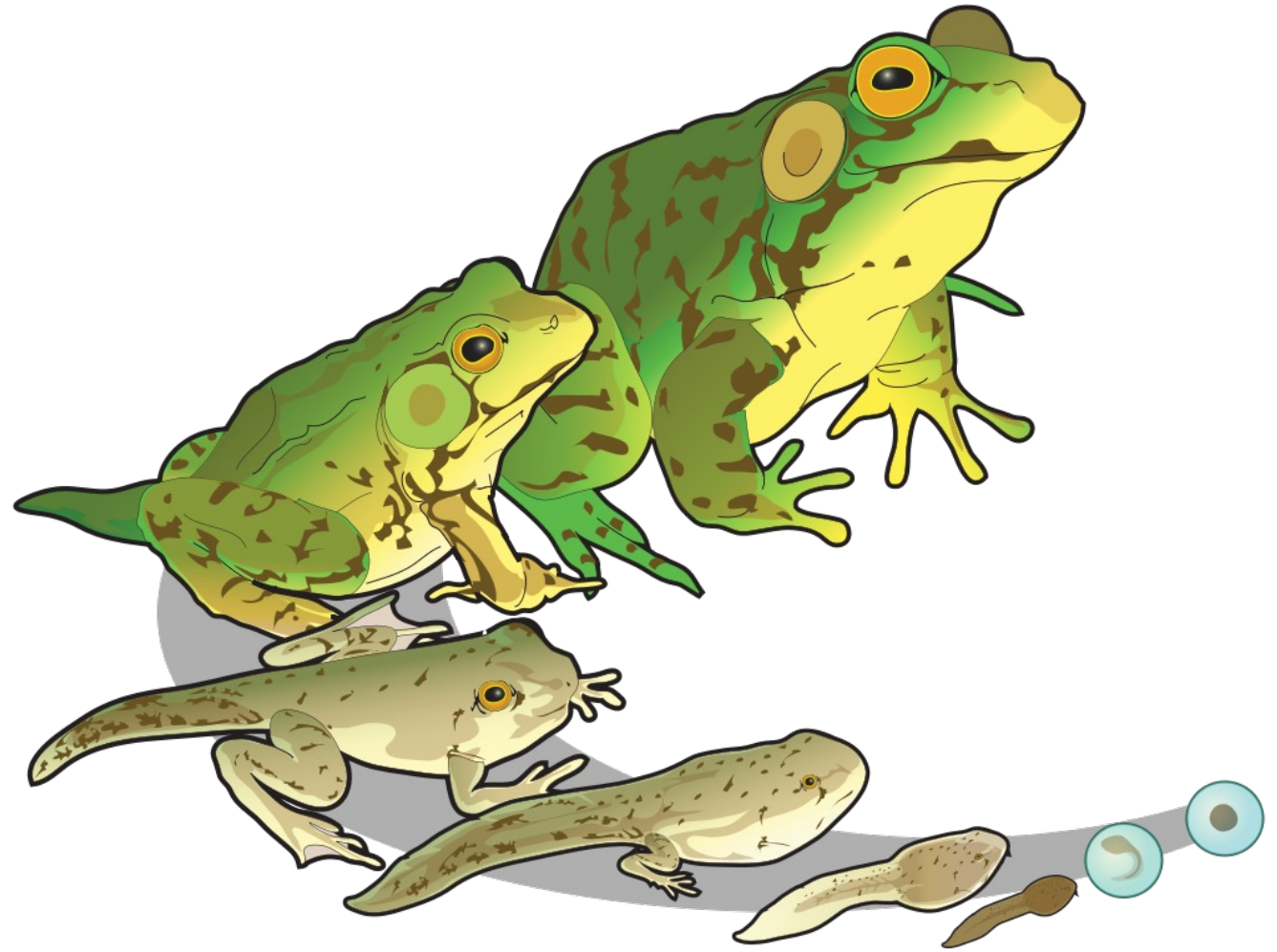
# Modeling Bioenergetics

- Thermodynamics: energy transformations and exchanges
- Overall energy budget
  - Dynamic Energy Budget (DEB): allocation to structure, reserves, and reproductive capacity
- Often a balance of respiration with energy intake
- How living cells produce and store energy (ATP)
- Ultimately models of growth

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# Modeling Growth

- Changes in individual weight
- Population biomass and structure
- Vulnerability to harvest
- Energy requirements
- Trophic changes
- Habitat changes





# Parameter Types

## POPULATION

- Describes the entire population
  - Average height of everyone in this room
  - Variance – how much values vary around the mean on average

## SYSTEM

- Instantaneous rates of change
  - Mortality rate
  - Population growth ( $r, K$ )
  - Individual growth ( $k, L_{\infty}$ )



# Estimation

## DESIGN-BASED

- Follows a statistical sampling design
- No assumptions made about the population
- Variance reflects the design

## MODEL-BASED

- Assumptions are made regarding how the system works
- Variance reflects the structure of the chosen model(s)

# Mathematical models

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Parametric vs. Non-parametric



Mechanistic vs. Empirical



von Bertalanffy growth equation (VBGE) =  
parametric and mechanistic

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## Ludwig von Bertalanffy (1901-1972)

"Why does an organism grow at all, and why, after a certain time, does its growth come to a stop?"

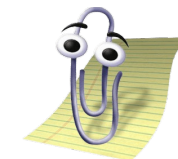





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# Where does the VBGE come from?

- Anabolism  $\Rightarrow$ : the synthesis of molecules
- Catabolism  $\Rightarrow$ : the breakdown of molecules
- Net synthesis: Anabolism - Catabolism

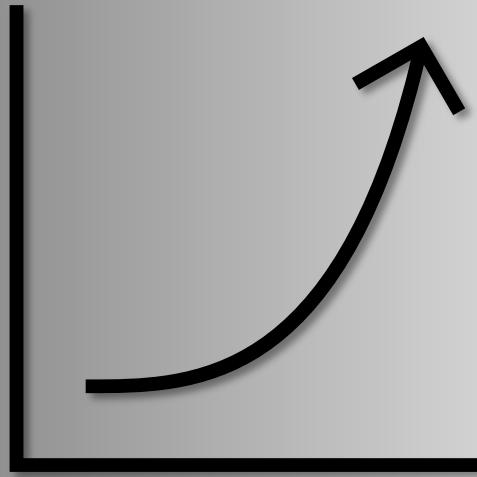


Note: **ANA**dromous fish go **UP**river to spawn,  
**CATA**dromous fish go **DOWN**river to spawn


$$\frac{dw}{dt} = As - Kw$$

- Growth is enormously complex, but follows the law of allometry i.e. physiological rates are a power function of body mass:  $\xi(w) = aw^b$
- $s$  = surface area =  $w^{2/3}$
- $w$  = weight
- $A$  and  $S$  = coefficients
- Anabolism  $\sim$  metabolic rate  $\sim$  respiration  $\sim$  surface area
- Catabolism  $\sim$  total mass  $\sim$  weight

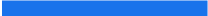
# Weight-Length Relationship



$$w = qL^3$$

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WE OFTEN MEASURE LENGTH  
RATHER THAN WEIGHT


$$\frac{dw}{dt} = As - Kw$$

$$\frac{dw}{dt} = Aw^{2/3} - Kw$$

$$\frac{dw}{dt} = qAL^2 - qKL^3$$

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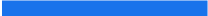
Several steps to solve

$$w = qL^3$$

$$\frac{dw}{dL} = 3qL^2$$

$$1 = \frac{dL}{dw} 3qL^2$$

$$\frac{dw}{dt} \frac{dL}{dw} 3qL^2 = qAL^2 - qKL^3$$


$$\frac{dw}{dt} \frac{dL}{dw} 3qL^2 = qAL^2 - qKL^3$$

$$\frac{dL}{dt} = E - KL$$

$$L_t = L_\infty (1 - e^{-K(t-t_0)})$$



$$L = L_{inf} \left( 1 - \exp(-k(t - t_0)) \right)$$



$$t_0 = -1.5$$



-2  0



$$k = 0.4$$



0.01  1



$$L_{inf} = 100$$



50  100

5



-2

0

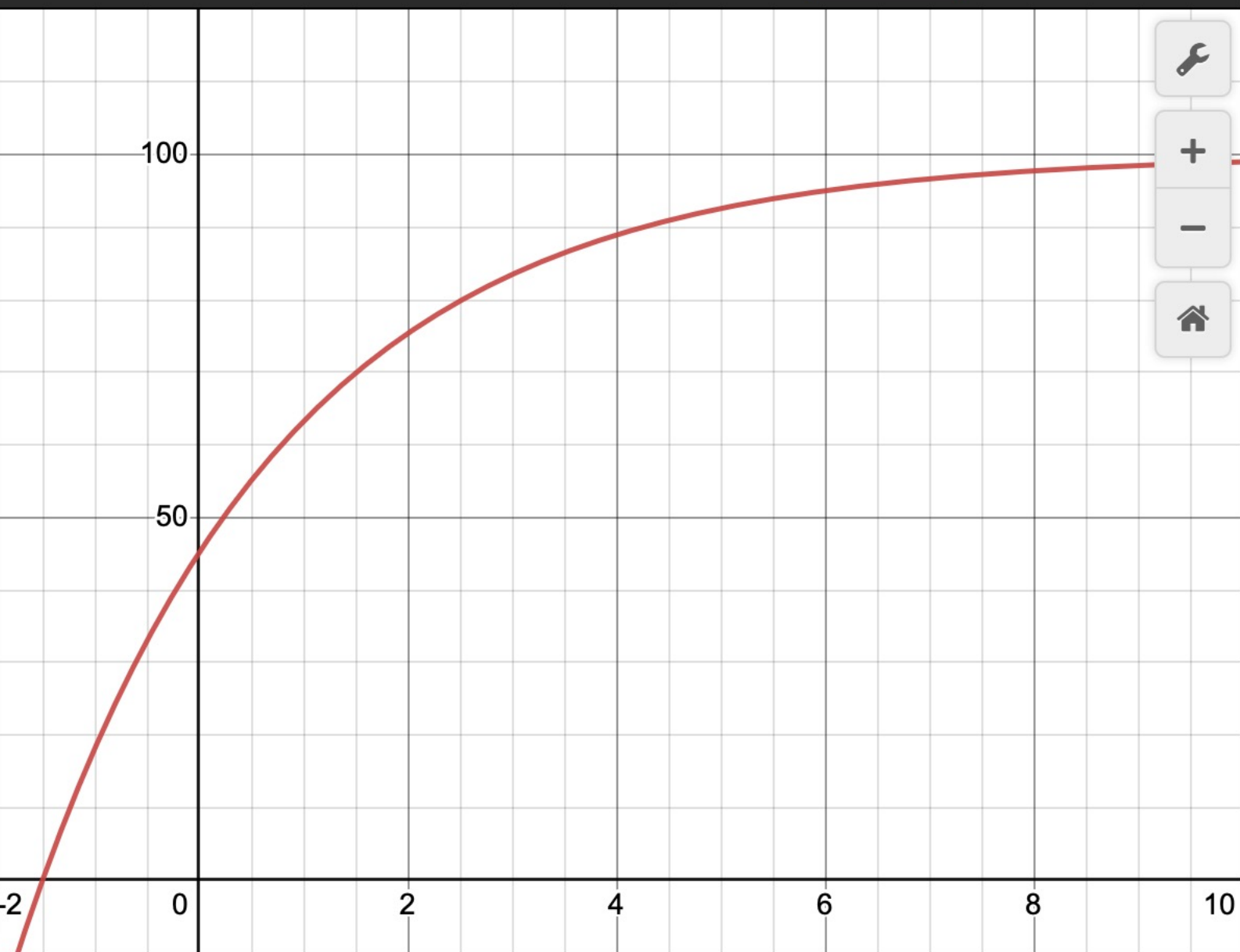
2

4

6

8

10





# Gompertz Equation








- Rate of absolute mortality/decay falls exponentially with increasing size

$$\frac{dL}{dt} = rL \ln \left( \frac{k}{L} \right)$$

$$L_t = L_{\infty} e^{-e^{-k(t-t_i)}}$$



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Name	Form	Model $dM/dt$
Linear		$r$
Exponential		$rM$
Power law		$rM^\beta$
Monomolecular		$r(K - M)$
Three-parameter logistic		$rM\left(1 - \frac{M}{K}\right)$
Four-parameter logistic*		$r(M - L)\left(\frac{K - M}{K - L}\right)$
Gompertz		$rM\left(\ln \frac{K}{M}\right)$

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Table 1 from Paine et al 2012



# Model Assumptions

- Homogeneous error variance
- Constant relationship of length in time  
Slow-fast dynamics
- How adequate is our model?  
Is it representative of our system?  
Are the parameters estimable from our data?

$$F = G \frac{m_1 m_2}{d^2}$$

# A Brief Primer on Probability

$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

$$i\hbar \frac{\partial}{\partial t} \psi = \hat{H} \psi$$

$$E = mc^2$$

$$ds \geq 0$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

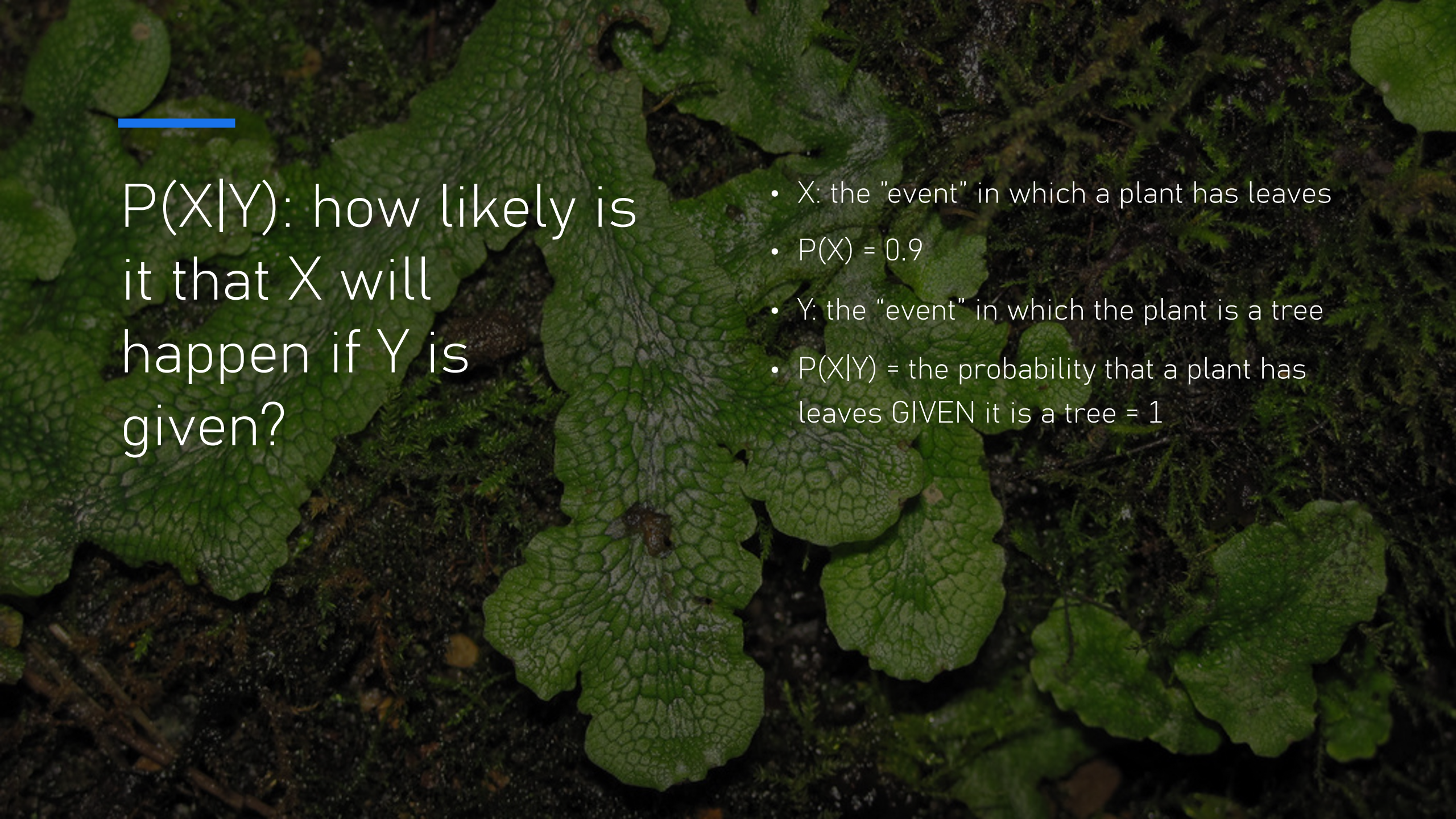
$$\frac{df}{dt} = \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h}$$



$P(X)$ : how likely is it that  $X$  will happen?

- $P(X) = 0$  i.e. it will “almost surely” not happen
- $P(X) = 1$  i.e. it will “almost surely” happen





$P(X|Y)$ : how likely is  
it that  $X$  will  
happen if  $Y$  is  
given?

- $X$ : the "event" in which a plant has leaves
- $P(X) = 0.9$
- $Y$ : the "event" in which the plant is a tree
- $P(X|Y)$  = the probability that a plant has leaves GIVEN it is a tree = 1





# Bayes' Law

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

$$P(\theta|X) \sim P(X|\theta)P(\theta)$$

Likelihood:  $P(X|\theta)$

Prior:  $P(\theta)$