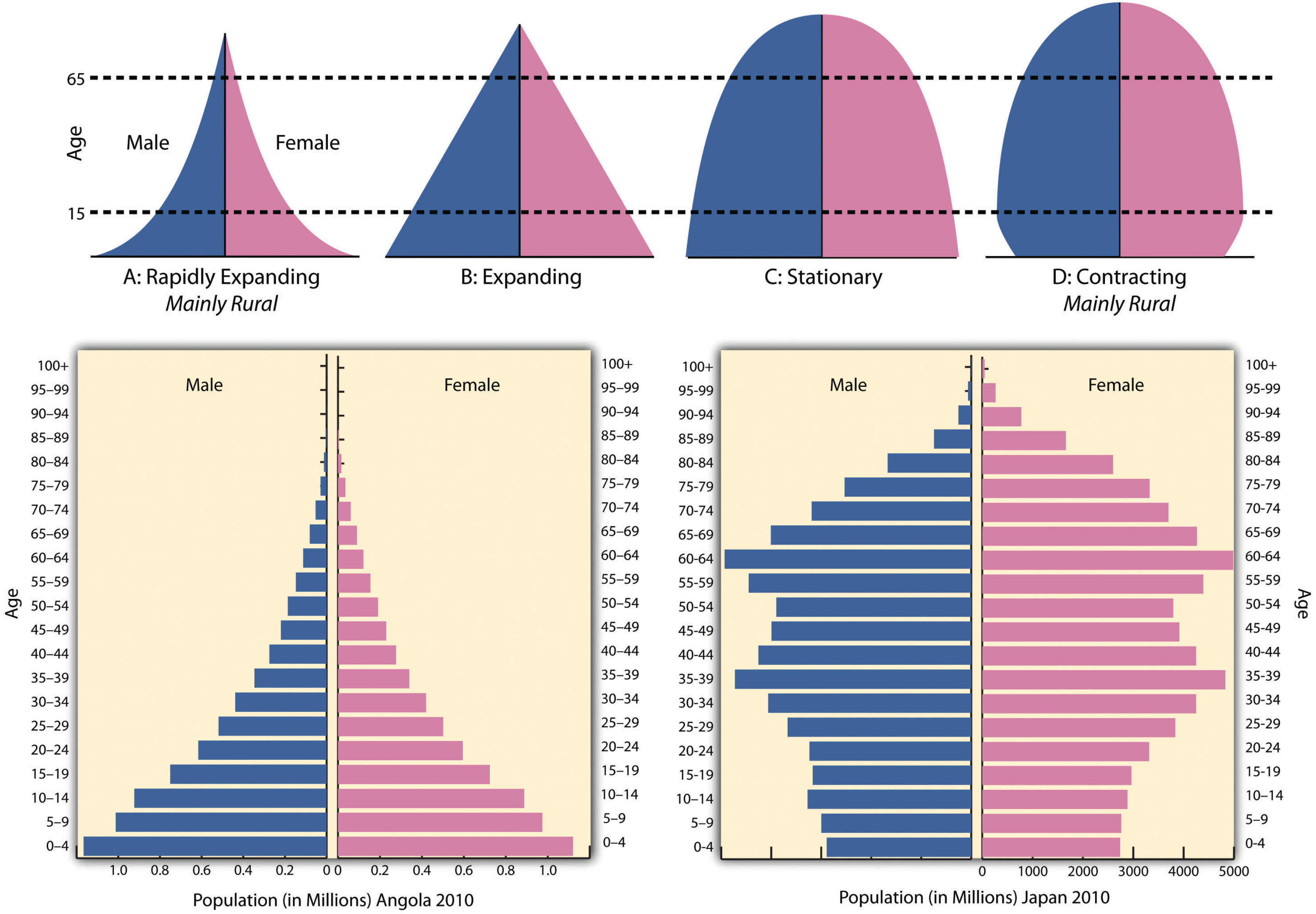


Cohort Dynamics

Age Structure

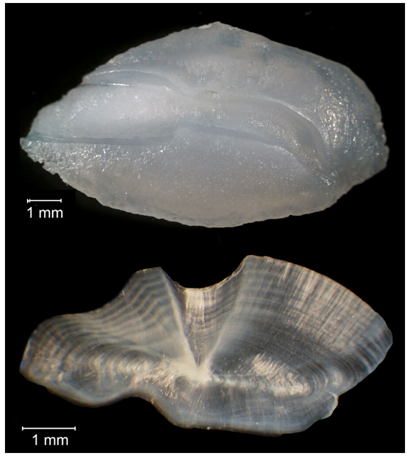
Cohort: a group of animals that shares a trait e.g. birth year

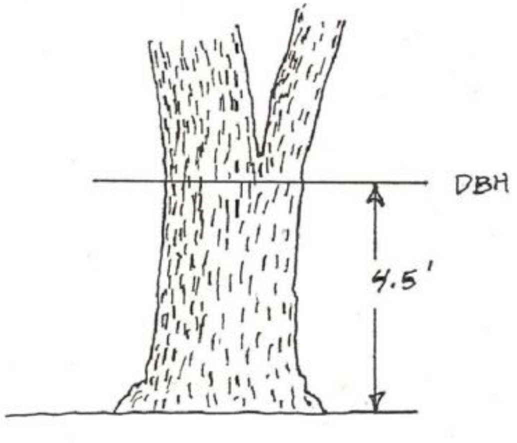
Age-structured assessment

- Growth
- Condition
- Population projections
- Recruitment
- Mortality

Size-based assessment

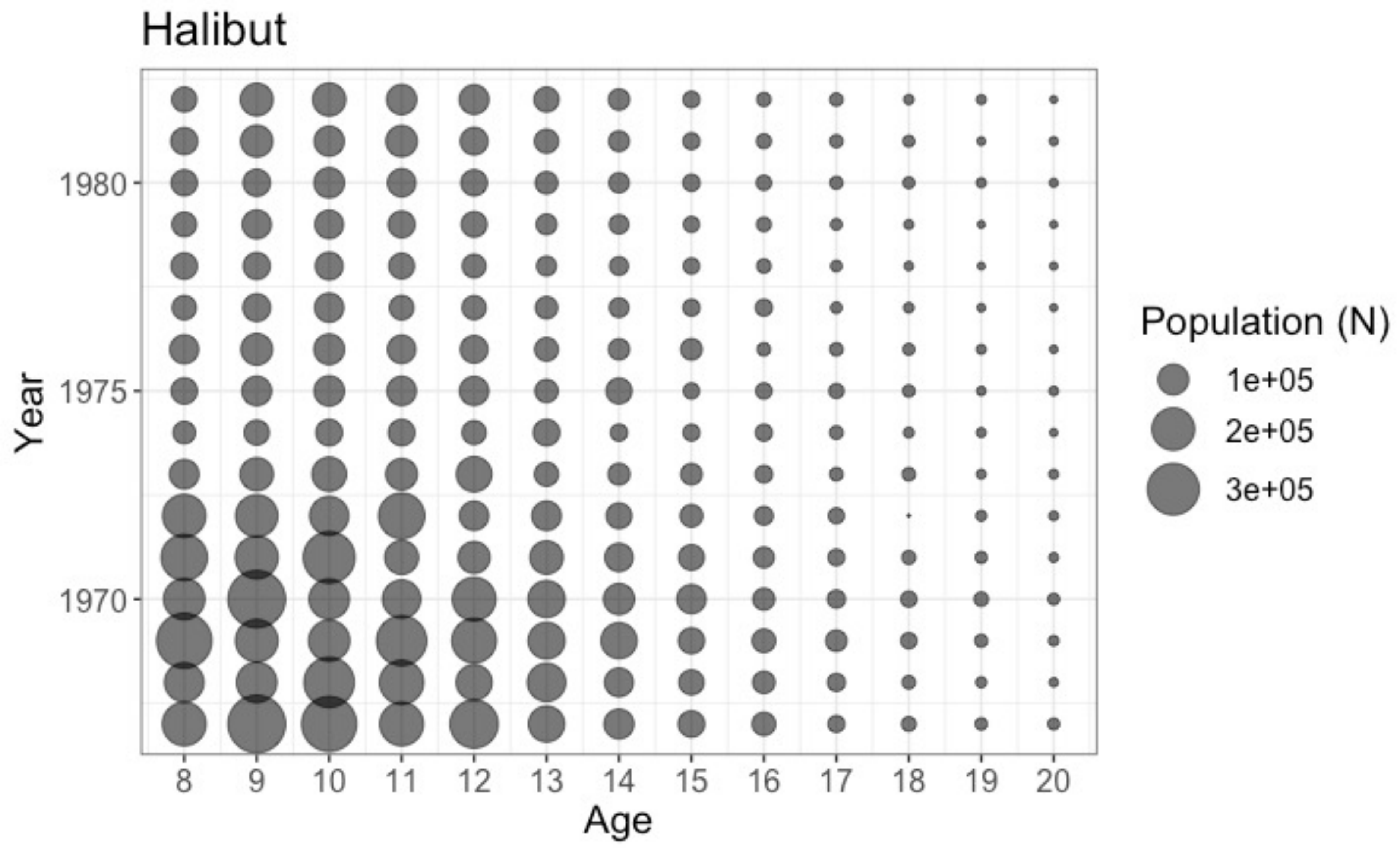
- Fundamental trait
- Determines physiological rates
- Typical of geographic range
- Major factor in community interactions
- Economic value correlates with size





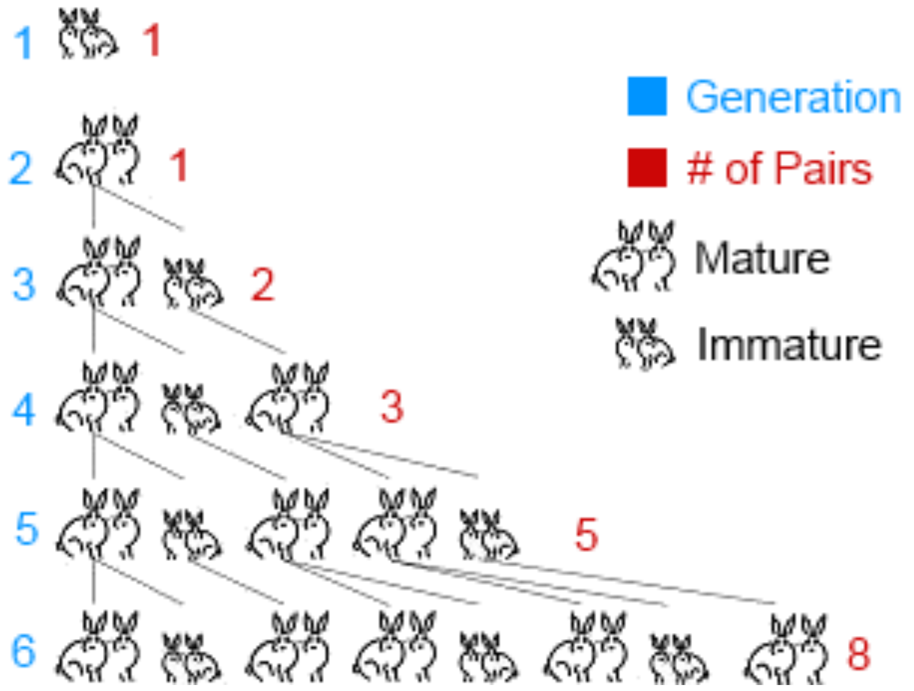
How do we get data?

- Direct measurement of length/height/width/mass/diameter
- Age sampled directly
- Age measured as a sub-sample of an easier measurement
    - Relate age to simple measurement with a key


Halibut
Year
1980
1975
1970
8
9
10
11
12
13
14
15
16
17
18
19
20
Age
Population (N)
1e+05
2e+05
3e+05

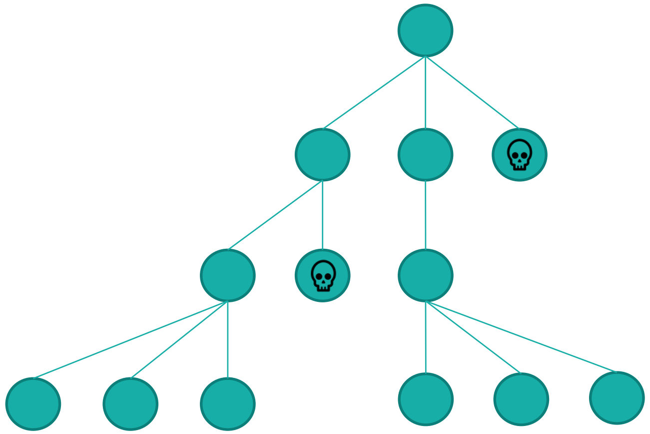
Age-structured models

- Branching processes
- Leslie matrix models
- Virtual population analysis
- Cohort analysis (HW)



Branching process

- Population growth
- Individuals reproduce and die according to probability distributions



$$Z_0 = 1$$

$$Z_1 = 3$$

$$Z_2 = 3$$

$$Z_4 = 6$$

Leslie matrix models

$$\boldsymbol{N}_{t+1} = \boldsymbol{M}\boldsymbol{N}_t$$

$$\boldsymbol{N}_{t+1} = \boldsymbol{M}\boldsymbol{N}_t$$

To build a matrix, the following information must be known from the population:

- n_x, the count of individuals (*n*) of each age class *x*
- s_x, the fraction of individuals that survives from age class *x* to age class *x+1*,
- f_x, fecundity, the per capita average number of female offspring reaching n_0 born from mother of the age class *x*. More precisely, it can be viewed as the number of offspring produced at the next age class b_{x+1} weighted by the probability of reaching the next age class. Therefore, $f_x = s_x b_{x+1}$.

From the observations that n_0 at time *t+1* is simply the sum of all offspring born from the previous time step and that the organisms surviving to time *t+1* are the organisms at time *t* surviving at probability s_x, one gets $n_{x+1} = s_x n_x$. This implies the following matrix representation:

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t+1} = \begin{bmatrix} f_0 & f_1 & f_2 & \ldots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & s_1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & s_2 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & s_{\omega-2} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_t$$

where ω is the maximum age attainable in the population.

$$N_{1,t+1} = S_0 \sum_{a=1}^{\omega} f_a N_{a,t}$$

$$N_{a+1,t+1} = S_a N_{a,t}$$

Leslie matrix models

$$\boldsymbol{N}_t = \boldsymbol{M}^t \boldsymbol{N}_0$$

Eigenvalues and eigenvectors

- For every square matrix M, there exists a vector k and a scalar λ such that:
  - ($\forall\ [M]_n\ \exists\ \vec{k}, \lambda$ s.t.)

$$\boldsymbol{Mk} = \lambda\boldsymbol{k}$$

$$\boldsymbol{Mk} - \lambda\boldsymbol{k} = \boldsymbol{0}$$

$$(\boldsymbol{M} - \lambda\boldsymbol{I})\boldsymbol{k} = \boldsymbol{0}$$

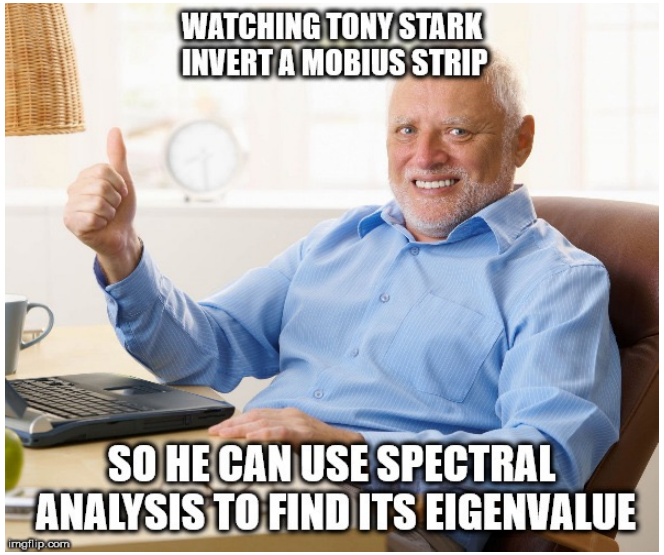
$$\Downarrow$$

$$|\boldsymbol{M} - \lambda\boldsymbol{I}| = 0$$

λ: eigenvalue

$\boldsymbol{k}$: eigenvector

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$



$$|\boldsymbol{M} - \lambda \boldsymbol{I}|$$

$$\begin{bmatrix} S_0 f_1 & S_0 f_2 \\ S_1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} S_0 f_1 - \lambda & S_0 f_2 \\ S_1 & -\lambda \end{bmatrix}$$

$$\begin{vmatrix} S_0 f_1 - \lambda & S_0 f_2 \\ S_1 & -\lambda \end{vmatrix} = -\lambda(S_0 f_1 - \lambda) - S_1 S_0 f_2 = 0$$

$$\lambda^2 - S_0 f_1 \lambda - S_0 S_1 f_2 = 0$$

$$\lambda = \frac{S_0 f_1 \pm \sqrt{S_0^2 f_1^2 + 4 S_0 S_1}}{2}$$

ONLY ONE POSITIVE EIGENVALUE

Leslie matrix model

- $\lambda < 1$ population is decreasing
- $\lambda = 1$ population is unchanging
- $\lambda > 1$ population is increasing

Virtual Population Analysis (VPA)

- Reconstructing a population from harvest data
- “Virtual” because it is not observed but reconstructed
- Current population is prior population minus harvest: $N_{a+1} = N_a - H_a$
- Prior population is current population plus harvest: $N_a = N_{a+1} + H_a$
- Population is the sum of all harvest since since it was age a: $N_a = \sum_{x=a}^{A} H_x$
- VPA (Gulland, 1965)

Baranov catch equation

$$N_{a+1} = N_a e^{-Z}$$

$$Z = H + M$$

$$C_a = N_a \frac{H}{Z}(1 - e^{-Z})$$

Correcting for harvest and natural mortality

$$C_a = N_a \frac{H_a}{Z_a}(1 - e^{-Z_a})$$

$$N_a = \frac{C_a}{\frac{H_a}{Z_a}(1 - e^{-Z_a})}$$

(STEP 1)

Working backwards through harvest history

$$C_{a-1} = N_{a-1} \frac{H_{a-1}}{Z_{a-1}} (1 - e^{-Z_{a-1}})$$

$$N_a = N_{a-1} e^{-Z_{a-1}}$$

$$\frac{C_{a-1}}{N_a} = \frac{H_{a-1}}{Z_{a-1}} (e^{Z_{a-1}} - 1)$$ **(STEP 2)**

VPA

$$N_a = \frac{C_a}{\frac{H_a}{Z_a}\left(1 - e^{-Z_a}\right)} \qquad \textbf{(STEP 1)}$$

$$\frac{C_{a-1}}{N_a} = \frac{H_{a-1}}{Z_{a-1}}\left(e^{Z_{a-1}} - 1\right) \qquad \textbf{(STEP 2)}$$