Theorem I ($L_{ind} \subset L_{em}$): $\forall \sigma, (\Gamma \vdash_{ind} \sigma) \implies (\Gamma \vdash_{em} \sigma)$

Proof:
Proceding by induction on the derivation of σ, we observe that for every rule other than induction, Theorem I immediately holds, since these rules are shared by both L_{ind} and L_{em}.

In the case of induction, we have:

$$\frac{\Gamma, P(0), \forall n(P(n) \to P(S(n))) \vdash_{ind} C}{\Gamma, \forall n(P(n)) \vdash_{ind} C} \text{ (Induction)}$$

By the inductive hypothesis, we deduce:

$$\Gamma, P(0), \forall n[P(n) \to P(S(n))] \vdash_{em} C$$

To prove Theorem I, it suffices to show $\Gamma, \forall n(P(n)) \vdash_{em} C$

First, we present a proof in prose:

1. Suppose $P(0) \wedge \forall n(P(n) \to P(S(n)))$.
2. For the purposes of contradiction, further suppose that $\neg \forall n(P(n))$.
3. Then, by DeMorgan's Quantifier Negation Lemma, $\exists n(\neg P(n))$.
4. By the Infimum property of natural numbers, we then have,

$$\exists n(\neg P(n) \wedge \forall m(m < n \to \neg\neg P(m)))$$

5. Additionally, we conclude from **1** that $\exists n(P(n))$
6. An additional application of the Infimum property on **5** yields

$$\exists n(P(n) \wedge \forall m(m < n \to \neg P(m))$$

7. By **4** and **6**, we let N and M be such that:

$$\neg P(M) \wedge \forall m(m < M \to \neg\neg P(M))$$

$$P(N) \wedge \forall m(m < N \to \neg P(N))$$