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Average circuit eigenvalue sampling on NISQ devices

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Average circuit eigenvalue sampling

Introduced by Steven T. Flammia in arXiv:2108.05803. Allows us to estimate the Pauli error channels of the individual gates on a quantum computer.

We present the following:

- A step-by-step implementation for NISQ devices.
- Simulation and real-hardware results.

Averaged circuits

A Clifford gate can be twirled to **isolate the Pauli noise** around it. The **$\mathcal{G}$-twisted twirl** of a noisy gate $\mathcal{G}$ is defined as:

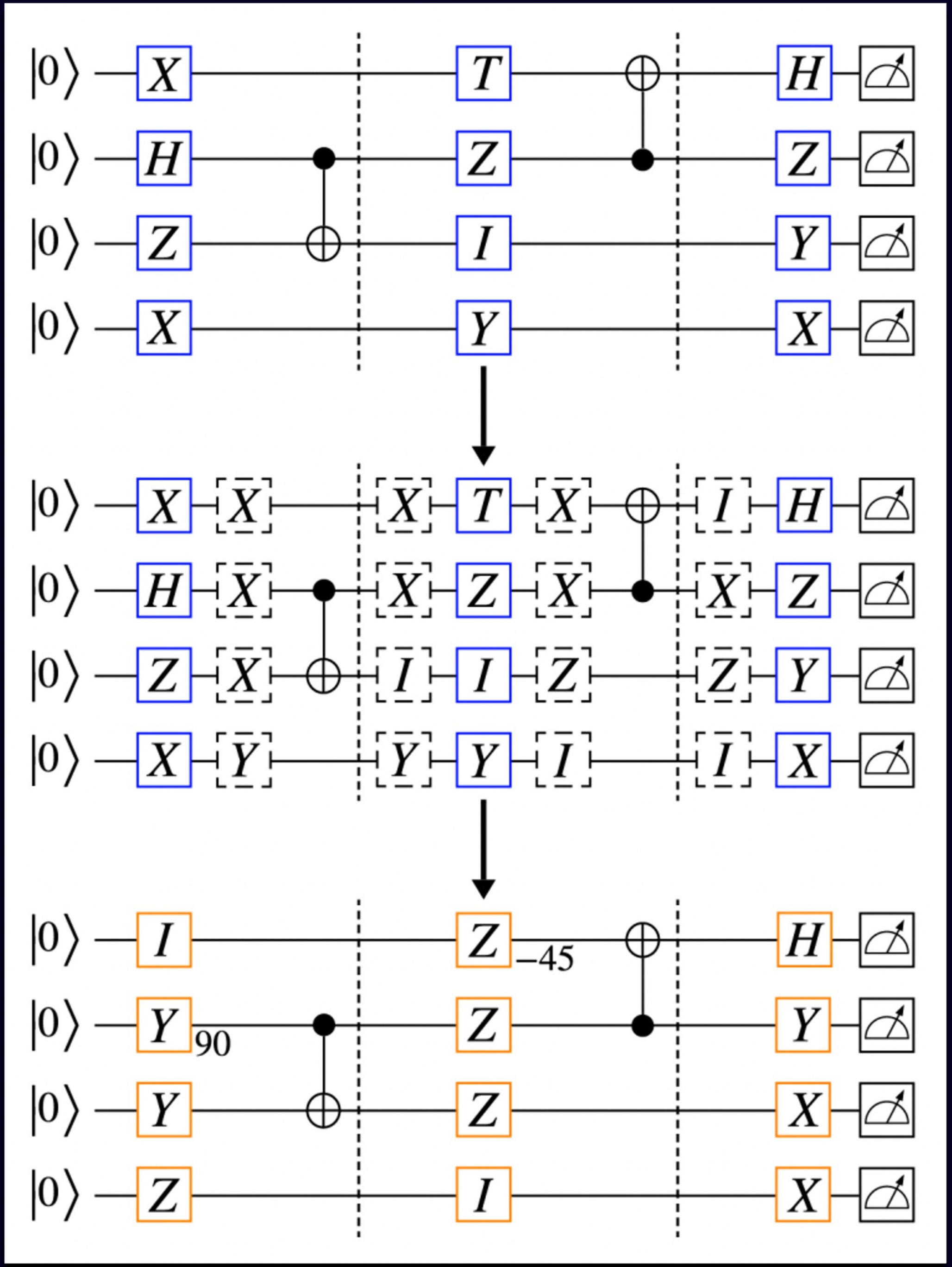
$$\tilde{\mathcal{G}}^{P}(\rho) = \frac{1}{4^n}\sum_{a} P_{a'}^{\dagger}\,\tilde{\mathcal{G}}\left(P_a \rho P_a^{\dagger}\right)P_{a'} = \mathcal{G}(\mathcal{E}^{P}(\rho))$$

In a circuit, this looks like the following:

$$\text{-------}\ \tilde{\mathcal{G}}\ \text{-------} \rightarrow \text{-------}\ P_a\ \text{-------}\ \tilde{\mathcal{G}}\ \text{-------}\ P_{a'}\ \text{-------}$$

Where $P_{a'} = \mathcal{G}(P_a) = \mathcal{G}P_a\mathcal{G}^{\dagger}$ and P_a is chosen at random.

Original $\tilde{C}$



Ensemble of circuits $\tilde{C}^P$

arxiv:2010.00215

Eigenvalue sampling

Given a noisy averaged implementation $\tilde{\mathcal{C}}^P$ of a circuit $\mathcal{C}$, we define the **circuit eigenvalue** of this circuit with respect to some Pauli P_a as

$$\tilde{\mathcal{C}}^P(P_a) = \Lambda_{c,a}\mathcal{C}^P(P_a) = \Lambda_{c,a}P_{a'}$$

$$\Lambda_{c,a} = \frac{1}{2^n}\mathrm{Tr}\left(P_{a'}\tilde{\mathcal{C}}^P(P_a)\right)$$

However, we can't send a Pauli directly through a circuit, but we can send its eigenvectors.

$$\Lambda_{c,a} = \frac{1}{2^n}\left[\mathrm{Tr}\left(P_{a'}\tilde{\mathcal{C}}^P(\rho_+)\right) - \mathrm{Tr}\left(P_{a'}\tilde{\mathcal{C}}^P(\rho_-)\right)\right]$$

Circuit and gate eigenvalues

We can characterize the individual **gate eigenvalues** from the circuit eigenvalues as:

$$\tilde{\mathcal{C}}^P(P_a) = \prod_{k=1}^{M} \lambda_{k,a_k} \mathcal{C}(P_a)$$

Where the circuits have M gates and each gate eigenvalue is:

$$\tilde{\mathcal{G}}_i^P(P_{a_i}) = \lambda_{i,a_i} \mathcal{G}(P_{a_i}) = \lambda_{i,a_i} P_{a_j}$$

Then, the circuit and gate eigenvalues are related by:

$$\Lambda_{\mathcal{C},a_1} = \prod_{k=1}^{M} \lambda_{k,a_k}$$

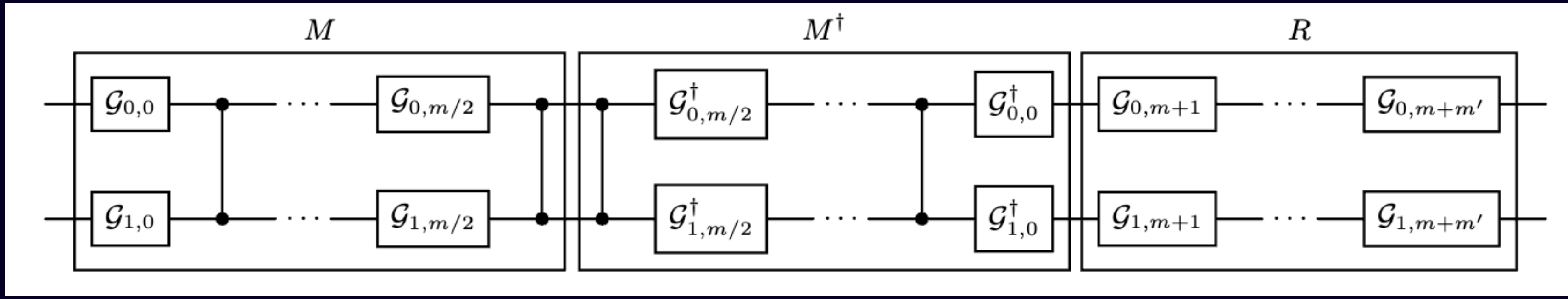
Solving the model

- For every circuit and input Pauli, we define an index $\mu = \left(\mathcal{C}_k, a_{k_i}\right)$. We do the same for gates, defining an index $\nu = \left(\mathcal{G}_k, a_{k_i}\right)$.
- We then define $\Lambda_\mu = e^{-b_\mu}$ and $\lambda_\nu = e^{-x_\nu}$.
- To get the gate eigenvalues, we solve $A\vec{x} = \vec{b}$.

$$\Lambda_{\mathcal{C},a_1} = \lambda_{1,a_1} \cdot \ldots \cdot \lambda_{T,a_T}$$
$$\ln \Lambda_{\mathcal{C},a_1} = \ln \lambda_{1,a_1} \cdot \ldots \cdot \lambda_{T,a_T}$$
$$\ln \Lambda_{\mathcal{C},a_1} = \ln \lambda_{1,a_1} + \cdots + \ln \lambda_{T,a_T}$$
$$b_\mu = x_{v_1} + \cdots + x_T$$

Protocol

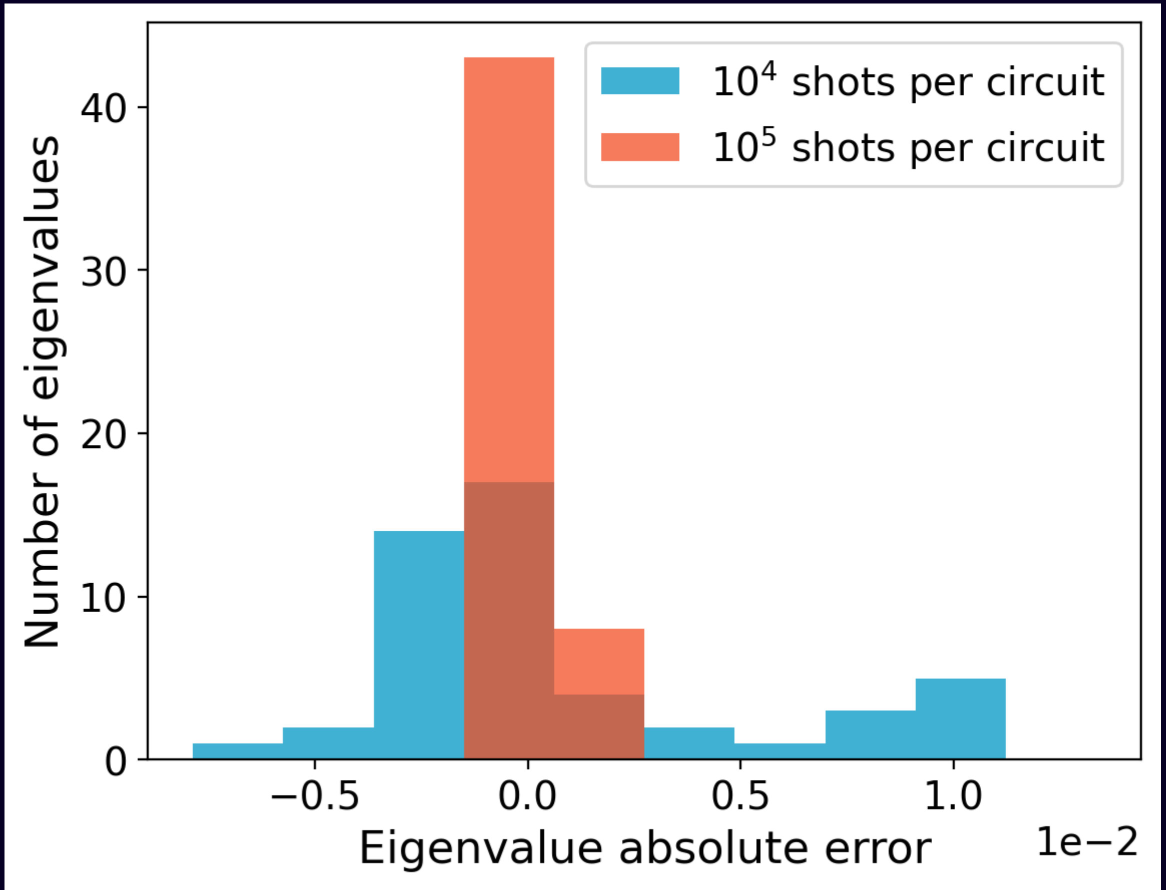
- Define the **gateset** we want to characterize .
- Get a collection of **random circuits** built from gates in the set.

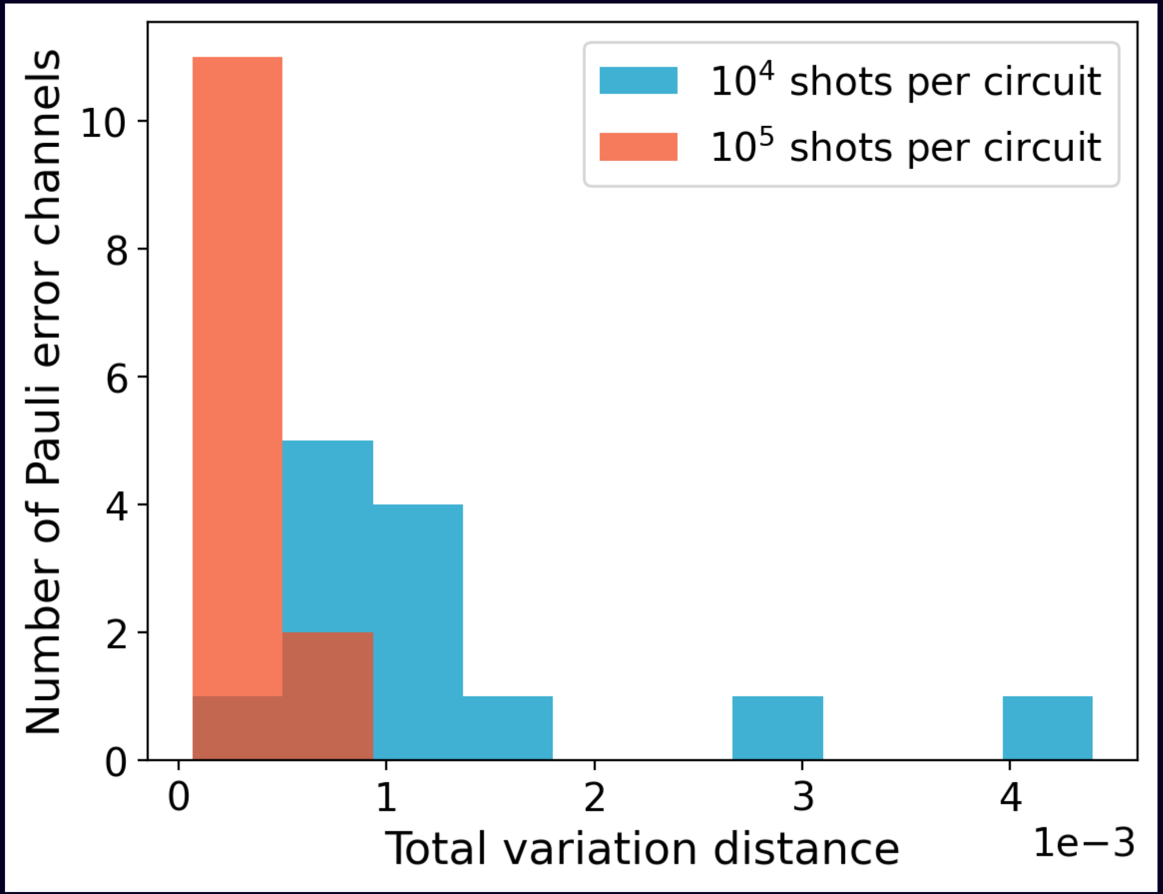


- For each circuit, we get a set of **good Pauli operators** we can send through.
- Construct a **design matrix** from the set of circuits and corresponding Pauli operators.
- Use the difference trick to get the circuit eigenvalues.
- Solve the model!

2-qubit simulation

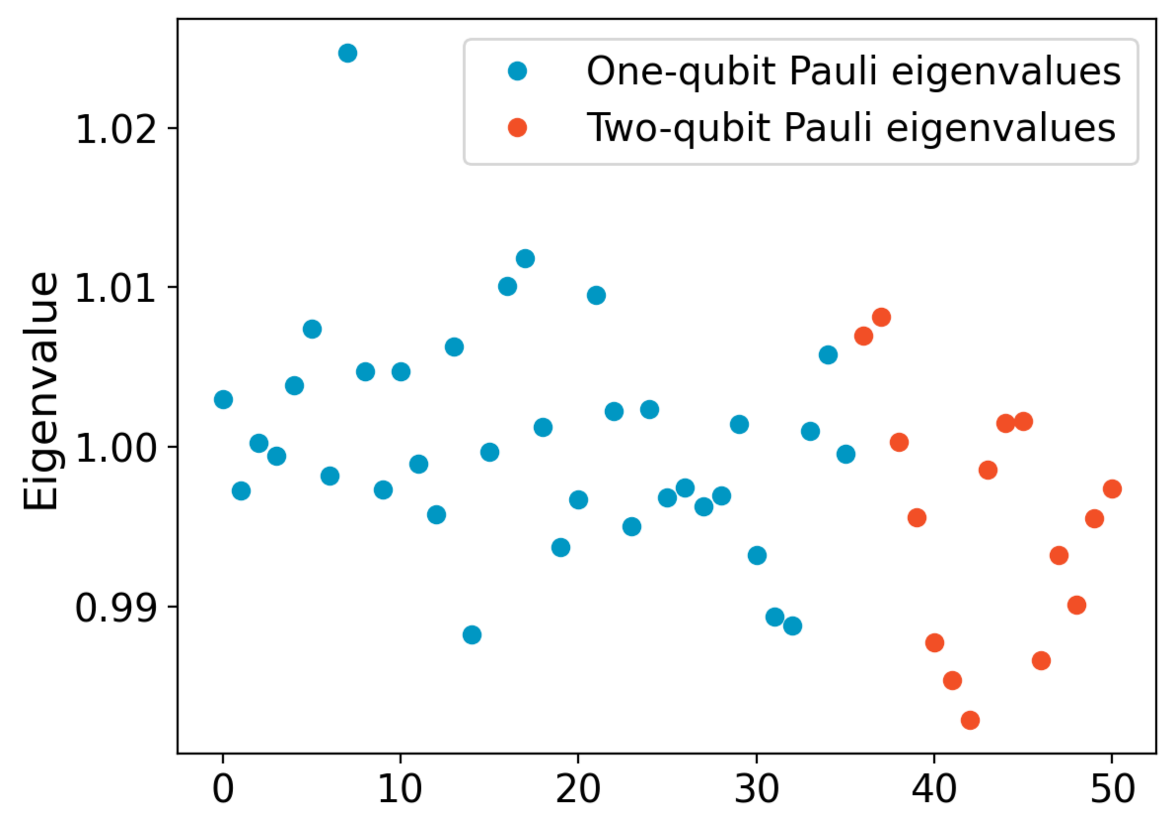
4 mirror + 6 random, 5 circuits, 10 twirls

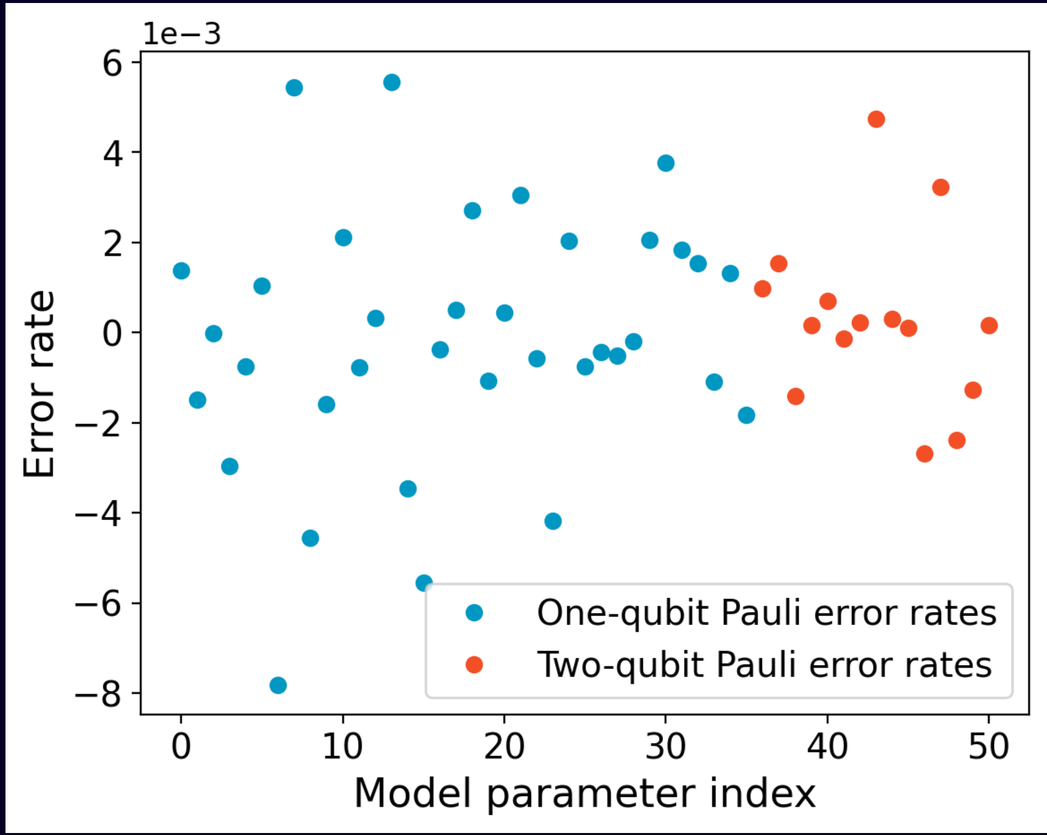




2-qubit IBM Algiers

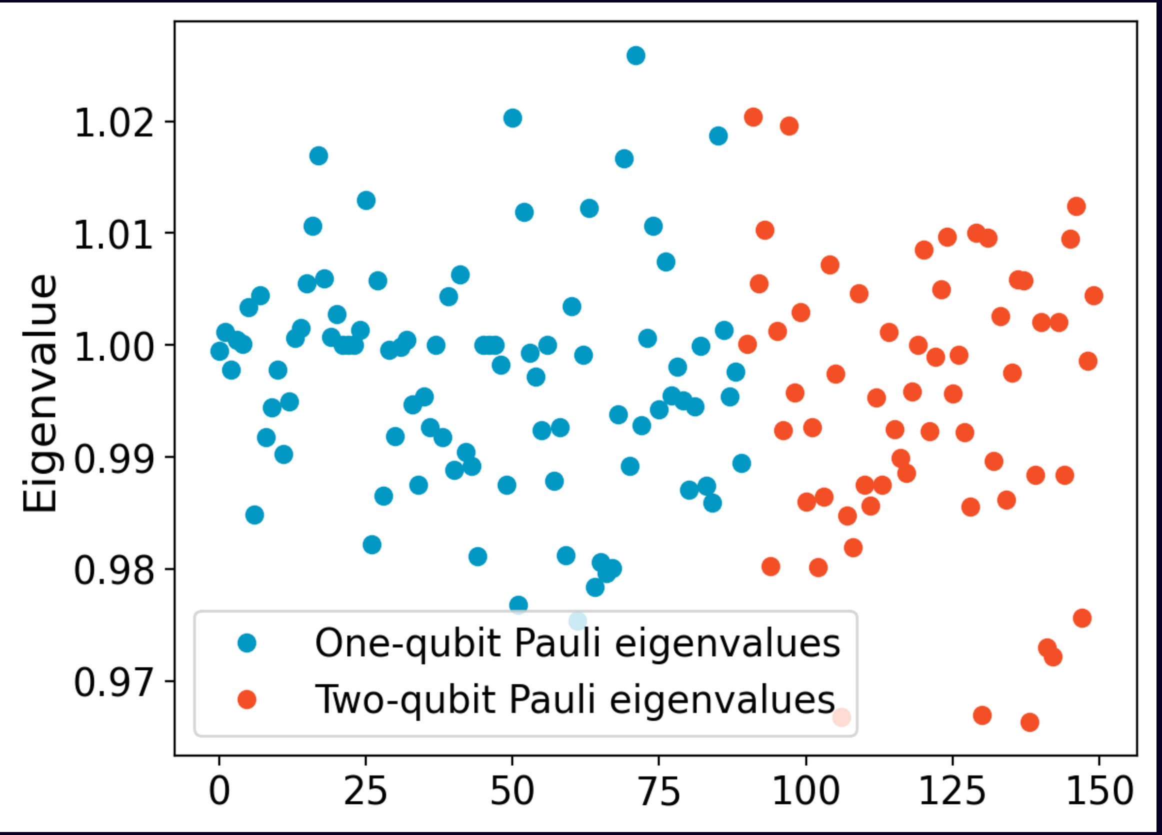
4 mirror + 6 random, 5 circuits, no twirls

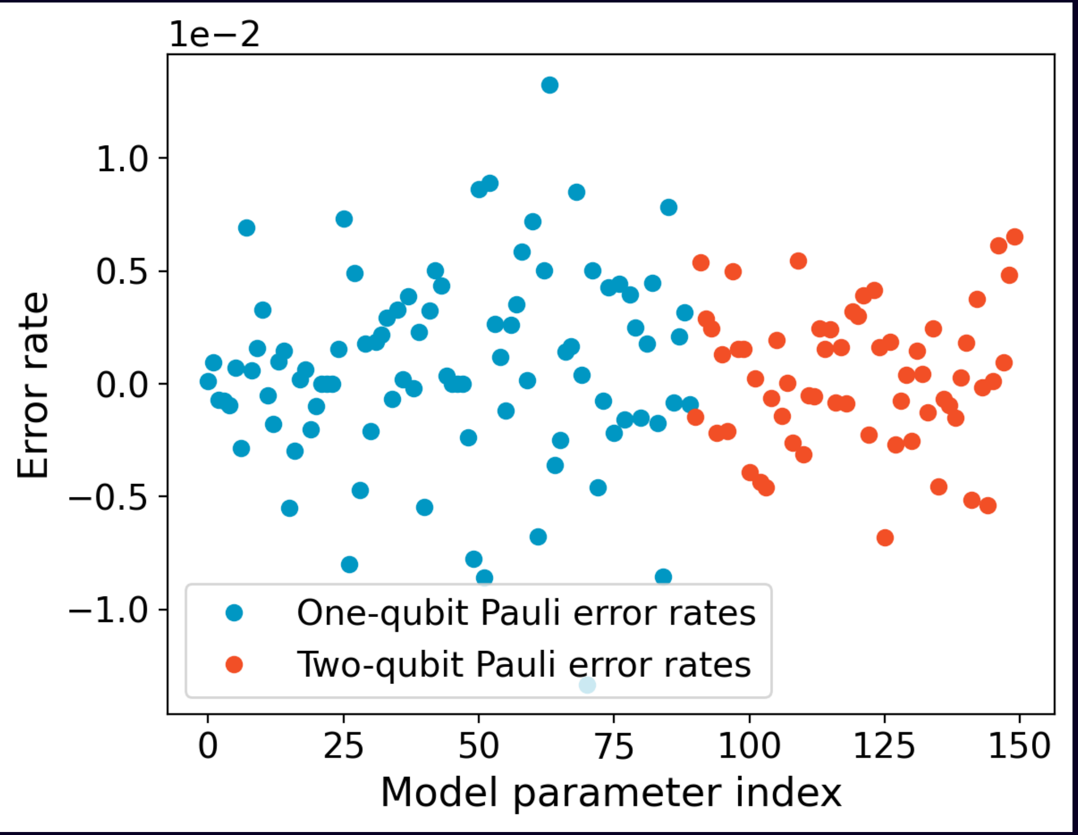




5-qubit IBM Osaka

4 mirror + 6 random, 5 circuits, no twirls





Results

Calibration data indicated a $6.2 \cdot 10^{-3}$ **CNOT error** between qubits 0 and 1 (Algiers). Reconstructing the estimated noise channels, we get an infidelity of $7.6 \cdot 10^{-3}$ for the **CZ gate** between the same qubits.

Looking at the **CZ gate** between qubits 0 and 1 (Osaka), we estimate an infidelity of $3.6 \cdot 10^{-2}$, while the reported **single-qubit ECR error** for qubit 2 is $3.8 \cdot 10^{-3}$.

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