

$$1) W(n) = W(n-1) + 2$$

root: 2 → balanced  
level 1: 2

max cost of level: 2  
# levels: n → O(n)

$$W(n) = W(n-1) + n^c, c \geq 1$$

root:  $n^c$  → since  $c \geq 1$  → balanced  
level 1:  $(n-1)^c$

max cost of level:  $n^c$   
# levels: n → O(n \cdot n^c) = O(n^{c+1})

$$W(n) = W(\sqrt{n}) + 1$$

root: 1 → balanced  
level 1: 1

max cost of level: 1  
# levels:  $n^{\frac{1}{2}}$  =  $\log_2 n$  → O(log n)

$$2) \text{Algorithm A: } W(n) = 5W(n/2) + n$$

root: n  
level 1:  $\frac{n}{2}$  → leaf dominated

# nodes > 5:  
# levels:  $\log_2 n$  → O( $n^{\log_2 5}$ )

$$\text{Algorithm B: } W(n) = 2W(n-1) + c$$

root: c → leaf dominated  
level 1:  $2^c$

# nodes:  $2^c$   
# levels: n → O(2^n)

$$\text{Algorithm C: } W(n) = 9W(n/3) + n^2$$

root:  $n^2$   
level 1:  $9\left(\frac{n}{3}\right)^2 = n^2$  → balanced

max cost of level:  $n^2$   
# levels:  $\log_3 n$  → O( $n^2 \log_3 n$ )

Choose Algorithm A because  $O(n^{\log_2 5}) < O(n^2 \log_3 n) \leq n^2$