Technical Appendices

A Diagrams

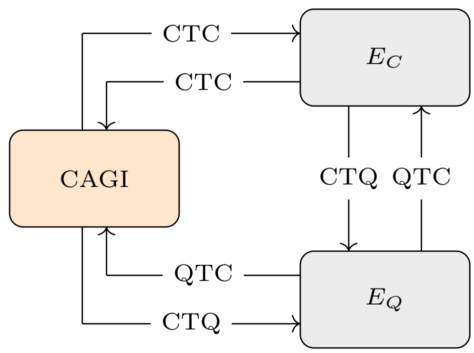

Fig. 1. Classical agent (CAGI) interacting via CTC, CTQ or QTC maps with classical E_C or quantum E_Q environments

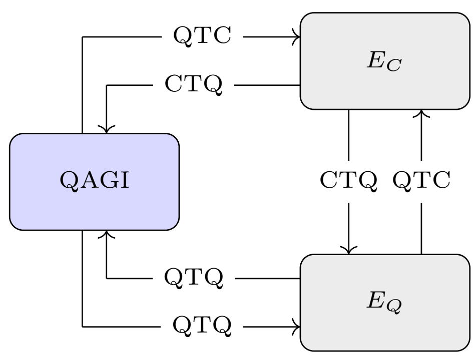

Fig. 2. Quantum agent (QAGI) interacting via QTC, CTQ or QTQ maps.

B Hamiltonian Dynamics

Classical Evolution Evolution is described by Hamilton's equations:

$$\dot{q}_i = \frac{\partial H_C}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H_C}{\partial q_i}. \tag{11}$$

This can be expressed more abstractly using the Poisson bracket. For two observables f, g, their Poisson bracket is:

$$\{f, g\}_{PB} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right). \tag{12}$$

The time evolution of any observable f is then given by $\dot{f} = \{f, H_C\}_{PB}$. When $\{f, g\}_{\mathrm{PB}} = 0$ the Poisson bracket vanishes, so the observables f and g *commute* and their values can, in principle, be fixed simultaneously with arbitrary accuracy. Classical logic and computation implicitly assume this independence: the truth of one proposition (or the content of one register) leaves another untouched unless an explicit coupling Hamiltonian H_C is present. Consequently, for a classical AGI we write the control Hamiltonian as a direct sum

$$H_C \;=\; \sum_k H_{C,k},$$

where each term $H_{C,k}$ drives a distinct functional block—learning (e.g. gradient-descent updates [33]), reasoning (e.g. a Hopfield-network energy or constraint-satisfaction term), or sensorimotor exchange. The mutual commutativity of these blocks, and of the variables they address, underpins the semantics of classical computation.

In information theoretic terms, classical mechanical dynamics of CAGI can be expressed as follows. Let $\{\mathcal{R}_i\}_{i=1}^n$ be the classical registers of the agent, each described by a *commutative* von Neumann algebra $\mathcal{V}_i = L^\infty(\Omega_i, \mu_i)$. A microstate of the whole agent–environment system is therefore a point $(\mathbf{q}, \mathbf{p}) \in \mathcal{M} = T^*\mathcal{C}$ with

$$q_i := X_i(\omega_i), \qquad p_i := M_i\,\dot{X}_i(\omega_i),$$

where $X_i \in \mathcal{V}_i$ is the random variable realised by register $\mathcal{R}_i$ and M_i is an information-theoretic weight term (e.g. an inverse learning rate or buffer capacity). The classical Hamiltonian functional $H_C\colon \mathcal{M} \to \mathbb{R}$ involves coordinates:

$$\dot{q}_i \;=\; \frac{\partial H_C}{\partial p_i}, \qquad \dot{p}_i \;=\; -\frac{\partial H_C}{\partial q_i}, \tag{13}$$

but now (13) is understood to act on *probability densities* $f_t(\mathbf{q}, \mathbf{p})$ pushed forward by the CTC channel $\mathrm{CTC}_t : L^\infty(\Omega) \to L^\infty(\Omega)$. For any pair of observables $f, g \in \bigoplus_i \mathcal{V}_i$ we retain the Poisson bracket:

$$\{f, g\}_{\mathrm{PB}} \;=\; \sum_{i=1}^{n} \Big(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i} \Big),$$

so the time derivative of f is $\dot{f} = \{f, H_C\}_{\mathrm{PB}}$. In information terms $\{f, g\}_{\mathrm{PB}} = 0$ *iff* the corresponding *classical channels* commute.

B.1 Quantum Hamiltonian dynamics

Upon shifting to a *quantum* substrate, the AGI's state lives as a vector $|\psi\rangle$ in a Hilbert space $\mathcal{H}$ (or, more generally, as a density operator ρ on $\mathcal{H}$). Observables correspond to self-adjoint operators A acting on that space, and evolution follows the Schrödinger–von Neumann equation

$$i\hbar\,\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = [H_Q, \rho(t)],$$

where $[A, B] \equiv AB - BA$ is the *commutator*—the quantum analogue of the Poisson bracket—and H_Q is the total quantum Hamiltonian. The critical algebraic shift from the classical picture is that operators need not commute: when $[A, B] \neq 0$, simultaneous precise values are forbidden, giving rise to distinctively quantum effects discussed below. For a quantum AGI we likewise decompose

$$H_Q \;=\; \sum_k H_{Q,k},$$

each $H_{Q,k}$ generating a functional capability—learning, reasoning, perception, actuation, and so forth. Now, however, the *commutation relations* among these generators, and with other key observables, govern behaviour: if the learning term $H_{Q,\mathrm{learn}}$ fails to commute with the sensing term $H_{Q,\mathrm{sens}}$, then observation can disturb learning (and vice versa) in a way with no classical counterpart. Such non-commutative structure underlies quantum phenomena like entanglement and contextuality and therefore reshapes the semantics of computation in a quantum-enabled AGI. This non-commutativity is fundamental and has profound consequences which may be a constraint or benefit.

For a quantum AGI we again write the control Hamiltonian as a sum of functional generators,

$$H_Q \;=\; \sum_k H_{Q,k}.$$

Each $H_{Q,k}$ is now an *operator*, so the commutators among these terms—and with other observables—govern the agent's evolution. If, say, the learning generator $H_{Q,\mathrm{learn}}$ fails to commute with the sensing generator $H_{Q,\mathrm{sens}}$, environmental measurement can disturb learning (and vice versa) in a manner with no classical analogue. This non-commutative architecture underlies quantum hallmarks such as entanglement and contextuality, which may represent either valuable resources or formidable challenges for a QAGI.

B.2 Quantum information formulation

In quantum information terms, transitioning to quantum involves replacing every classical register $\mathcal{V}_i = L^\infty(\Omega_i)$ by a *non-commutative* von Neumann algebra $\mathcal{V}_i = B(\mathcal{H}_i)$ acting on a Hilbert space $\mathcal{H}_i$. The full agent–environment is reflected by the tensor algebra $\mathcal{V} = \bigotimes_i \mathcal{V}_i \subseteq \mathcal{B}(\mathcal{H})$, with $\mathcal{H} = \bigotimes_i \mathcal{H}_i$. States are represented by density operators $\rho \in \mathcal{D}(\mathcal{H}) = \{\rho \geq 0,\ \mathrm{Tr}\,\rho = 1\}$, and a observable is an element $A \in \mathcal{V}$. When the evolution is closed and reversible the channel on $\mathcal{V}$ is the adjoint action of a unitary U_t:

$$\Phi_t^{(\mathrm{u})}(A) = U_t^\dagger A U_t, \qquad U_t = \exp\!\big(-\tfrac{i}{\hbar} H_Q t\big),$$

where the *Hamiltonian operator* $H_Q \in \mathcal{V}$ is the quantum analogue of H_C. In Schrödinger form this yields the familiar

$$i\hbar\dot{\rho}(t) = \big[H_Q, \rho(t)\big], \tag{14}$$

which is the generator $\mathcal{L}_{H_Q} = -\frac{i}{\hbar}[H_Q, \cdot]$ of a one-parameter group of QTC channels. Realistic AGI modules interact with—and are monitored by—their environment, so the fundamental dynamical object is a quantum channel $\Phi_t = \exp(t\mathcal{L})$, with Lindblad superoperator:

$$\mathcal{L}(\rho) = -\frac{i}{\hbar}[H_Q, \rho] + \sum_\alpha \Big(L_\alpha \rho L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \Big) \tag{15}$$

where the L_α's represent QTC measurement-and-feedback registers).