Ejercicios 1

7.7 Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \vee C$.
b. $\neg A \vee \neg B \vee \neg C \vee \neg D$.
c. $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

7.4 Which of the following are correct?

a. $False \models True$.
b. $True \models False$.
c. $(A \wedge B) \models (A \Leftrightarrow B)$.
d. $A \Leftrightarrow B \models A \vee B$.
e. $A \Leftrightarrow B \models \neg A \vee B$.
f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$.
g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.
h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.
i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.
j. $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable.
k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.
l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

7.18 Consider the following sentence:

$$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party] .$$

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.
b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).
c. Prove your answer to (a) using resolution.

Ejercicios 2

8.19 Assuming predicates $Parent(p, q)$ and $Female(p)$ and constants $Joan$ and $Kevin$, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation $\exists^1$ to mean "there exists exactly one.")

a. Joan has a daughter (possibly more than one, and possibly sons as well).
b. Joan has exactly one daughter (but may have sons as well).
c. Joan has exactly one child, a daughter.
d. Joan and Kevin have exactly one child together.
e. Joan has at least one child with Kevin, and no children with anyone else.

8.10 Consider a vocabulary with the following symbols:

$Occupation(p, o)$: Predicate. Person p has occupation o.
$Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
$Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
$Doctor$, $Surgeon$, $Lawyer$, $Actor$: Constants denoting occupations.
$Emily$, Joe: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

a. Emily is either a surgeon or a lawyer.
b. Joe is an actor, but he also holds another job.
c. All surgeons are doctors.
d. Joe does not have a lawyer (i.e., is not a customer of any lawyer).
e. Emily has a boss who is a lawyer.
f. There exists a lawyer all of whose customers are doctors.
g. Every surgeon has a lawyer.

9.6 Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens:

a. Horses, cows, and pigs are mammals.
b. An offspring of a horse is a horse.
c. Bluebeard is a horse.
d. Bluebeard is Charlie's parent.
e. Offspring and parent are inverse relations.
f. Every mammal has a parent.