Investigation of the Higher Order Zeeman Effect

By

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DEDICATION

I would like to dedicate this thesis to . . .

AKNOWLEDGEMENTS

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2.6.1 The quadratic Zeeman effect

The non-relativistic Hamiltonian for an n electron system in atomic units can be expressed as follows

$$\hat{H} = \sum_{i=1}^{n} \frac{\left(\vec{p}_i - e\vec{A}_i\right)^2}{2m} + V \tag{1}$$

using the cannonical momentum instead of the classical momentum is essential to account for electromagnetic interactions. The vector potential $\vec{A}$ can be described in terms of the magnetic field

$$\vec{A} = \frac{B}{2}\left(y\hat{x} - x\hat{y}\right) \tag{2}$$

which when expanded gives us the operator corresponding to the quadratic Zeeman effect

$$\hat{H}_Z^{(2)} = \frac{B^2 e^2}{8m} \sum_{i=1}^{n} (x_i^2 + y_i^2) \tag{3}$$

This operator can be expressed in spherical coordinates, written in terms of Legendre Polynomials

$$\hat{H}_Z^{(2)} = \frac{B^2 e^2}{12m} \sum_{i=1}^{n} r_i^2 \left(P_0(\cos\theta) - P_2(\cos\theta)\right) \tag{4}$$

$^3He^+$ is a system which contains only one electron, so we can drop the summation which accounts for all electrons to get our final quadratic Zeeman operator

$$\hat{H}_Z^{(2)}(^3\mathrm{He}^+) = \frac{B^2 e^2}{12m} r^2 \left(P_0(\cos\theta) - P_2(\cos\theta)\right) \tag{5}$$

2.6.2 The magnetic dipole moment operator

The magnetic dipole moment operator represents the interaction of a magnetic dipole moment with an external magnetic field. It is described via the following relation

$$Q_{M1} = \mu_B \left(1 - \frac{2p^2}{3m^2c^2} + \frac{Ze^2}{3mc^2 r}\right) \vec{\sigma} \cdot \vec{B} \tag{6}$$

Where μ_B is the Bohr magneton

$$\mu_B = \frac{e\hbar}{2mc} \tag{7}$$

The second term in the brackets of the magnetic dipole moment operator accounts for the relativistic correction to the kinetic energy of the electron, and the third term is the potential energy due to the Coulomb interaction between the electron and the nucleus. The first term corresponds to the ordinary Zeeman Effect, which does not contribute to the sum over states due to orthogonality.

The ordinary Zeeman effect contributes to Q_{M1} in $^3He^+$ because it has non-zero spin due to the missing electron. For systems such as ^3He, the ordinary Zeeman effect will not contribute.

2.6.3 The relativistic correction to ^{3}He$^+$

Combining the magnetic dipole moment with the quadratic Zeeman operator, we can write down the relativistic corrections for $^3He^+$. Written in terms of pseudostates the relativistic correction is

$$C_{\mathrm{rel}}^{(2)} = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{\langle\psi_0|H_Z^{(2)}|\psi_n\rangle\langle\psi_n|Q_{M1}|\psi_0\rangle}{E_0 - E_n} \tag{8}$$

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