CS 320

Computer Language Processing Exercises: Weeks 1 and 2

February 28, 2025

1 Languages and Automata

Exercise 1 Consider the following languages defined by regular expressions:

- 1. $\{a, ab\}^*$
- 2. $\{aa\}^* \cup \{aaa\}^*$
- 3. a^+b^+

and the following languages defined in set-builder notation:

- A. $\{w \mid \forall i.0 \le i \le |w| \land w_{(i)} = b \implies (i > 0 \land w_{(i-1)} = a)\}$
- B. $\{w \mid \forall i.0 \le i < |w| 1 \implies w_{(i)} = b \implies w_{(i+1)} = a\}$
- C. $\{w \mid \exists i.0 < i < |w| \land w_{(i)} = b \land w_{(i-1)} = a\}$
- D. $\{w \mid (|w| = 0 \mod 2 \lor |w| = 0 \mod 3) \land \forall i.0 \le i < |w| \implies w_{(i)} = a\}$
- E. $\{w \mid \forall i.0 \le i < |w| 1 \land w_{(i)} = a \implies w_{(i+1)} = b\}$
- F. $\{w \mid \exists i.0 < i < |w| 1 \land (\forall y.0 \le y \le i \implies w_{(y)} = a) \land (\forall y.i < y < |w| \implies w_{(y)} = b)\}$

For each pair (e.g. 1-A), check whether the two languages are equal, providing a proof if they are, and a counterexample word that is in one but not the other if unequal.

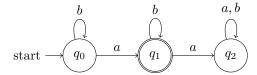
Exercise 2 For each the following languages, construct an NFA \mathcal{A} that recognizes them, i.e. $L(\mathcal{A}) = L_i$:

- 1. L_1 : binary strings divisible by 3
- 2. L_2 : binary strings divisible by 4
- 3. L_3 : $\{(w_1 \oplus w_2) \mid w_1 \in L_1 \land w_2 \in L_2 \land |w_1| = |w_2|\}$

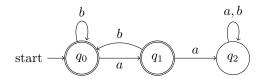
where \oplus is the bitwise-xor operation on binary strings.

Exercise 3 Give a verbal and a set-notational description of the language accepted by each of the following automata. You can assume that the alphabet is $\Sigma = \{a, b\}$.

1. \mathcal{A}_1



2. \mathcal{A}_2



2 Lexing

Consider a simple arithmetic language that allows you to compute one arithmetic expression, construct conditionals, and let-bind expressions. An example program is:

```
let x = 3 in
let y = ite (x > 0) (x * x) 0 in
  (2 * x) + y
```

The lexer for this language must recognize the following tokens:

For simplicity, *letter* is a shorthand for the set of all English lowercase letters $\{a-z\}$ and digit is a shorthand for the set of all decimal digits $\{0-9\}$.

Exercise 4 For each of the tokens above, construct an NFA that recognizes strings matching its regular expression.

A lexer is constructed by combining the NFAs for each of the tokens in parallel, assuming maximum munch. The resulting token is the first NFA in the token order that accepts a prefix of the string. Thus, tokens listed first have higher priority. We then continue lexing the remaining string. You may assume that the lexer drops any skip tokens.

Exercise 5 For each of the following strings, write down the sequence of tokens that would be produced by the constructed lexer, if it succeeds.

- 1. let x = 5 in x + 3
- 2. let5x2
- 3. xin
- 4. ==>
- 5. <===><==

Exercise 6 Construct a string that would be lexed differently if we ran the NFAs in parallel and instead of using token priority, simply picked the longest match.