Augmented-SVM: Automatic space partitioning for combining non-linear dynamics

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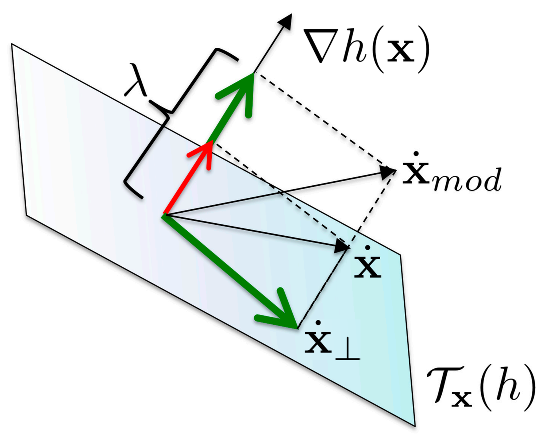
1 Introduction

- A new generalized support vector formulation, **Augmented-SVM**, is proposed.
- A-SVM framework combines the classifier value and derivative learning within one optimization problem.
- We present one application : Combining multiple non-linear dynamics into one dynamical system (DS) with multiple locally-stable attractors.
- For clarity, we have highlighted parts of the formulation that are similar to the standard SVM [1].

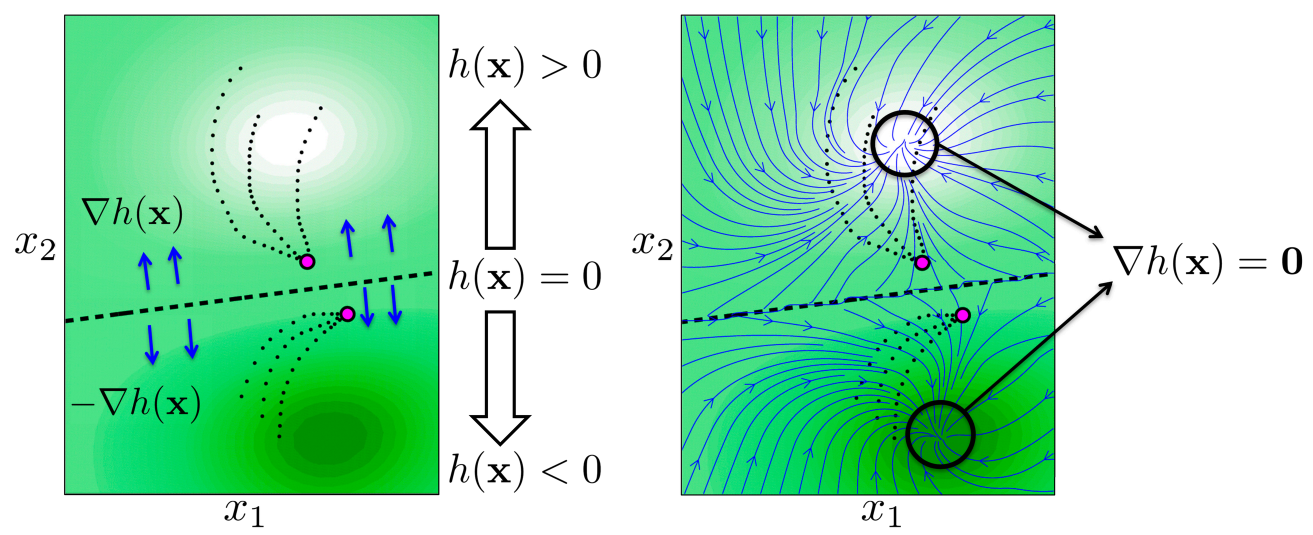
2 DS Modulation

$$\dot{\mathbf{x}}_{mod} = \lambda(\mathbf{x})\nabla h(\mathbf{x}) + \dot{\mathbf{x}}_{\perp}$$

$$\lambda(\mathbf{x}) = \begin{cases} \max\left(\epsilon, \nabla h(\mathbf{x})^T\dot{\mathbf{x}}\right) & \text{if } h(\mathbf{x}) > 0 \\ \min\left(-\epsilon, \nabla h(\mathbf{x})^T\dot{\mathbf{x}}\right) & \text{if } h(\mathbf{x}) < 0 \end{cases} \; ; \; \epsilon > 0$$



- **SVM → Misplaced attractors** and **inaccurate dynamics**.
- Need a classifier augmented with ***dynamic compatibility*** and ***stability.***



3 Binary A-SVM Formulation

Primal

$$h(\mathbf{x}) = \mathbf{w}^{\mathrm{T}}\boldsymbol{\phi}(\mathbf{x}) + b\,, \quad \nabla h(\mathbf{x}) = \mathrm{J}(\mathbf{x})\mathbf{w} \text{ where } \boldsymbol{\phi} \in \mathbb{R}^F, \mathrm{J} \in \mathbb{R}^{F\times N}$$

$$\min_{\mathbf{w},\xi_i} \tfrac{1}{2}\|\mathbf{w}\|^2 + c\sum_i \xi_i \text{ subject to } \begin{array}{rll} y_i\left(\mathbf{w}^T\boldsymbol{\phi}(\mathbf{x}_i) + b\right) & \geq 1 & \forall i = 1\cdots M \\ y_i \mathbf{w}^T \mathrm{J}(\mathbf{x}_i)\hat{\dot{\mathbf{x}}}_i + \xi_i & > 0 & \forall i = 1\cdots M \\ \xi_i & > 0 & \forall i = 1\cdots M \\ \mathbf{w}^T\mathrm{J}(\mathbf{x}^*)\mathbf{e}_i & = 0 & \forall i = 1\cdots N \end{array}$$

→ ***Classification***
→ ***DS Compatibility*** (second and third constraints)
→ ***Stability*** (fourth constraint)

Lagrangian, KKT

Dual

$$\min_{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma}} \frac{1}{2}\left[\boldsymbol{\alpha}^T\boldsymbol{\beta}^T\boldsymbol{\gamma}^T\right]\begin{bmatrix} \mathrm{K} & \mathrm{G} & -\mathrm{G}_* \\ \mathrm{G}^{\mathrm{T}} & \mathrm{H} & -\mathrm{H}_* \\ -\mathrm{G}_*^{\mathrm{T}} & -\mathrm{H}_*^{\mathrm{T}} & \mathrm{H}_{**}\end{bmatrix}\begin{bmatrix}\boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma}\end{bmatrix} - \boldsymbol{\alpha}^T\bar{\mathbf{1}} \text{ subject to } \begin{array}{ll} 0 \leq \alpha_i & \forall i = 1\cdots M \\ 0 \leq \beta_i \leq c & \forall i : y_i = +1 \\ \sum_{i=1}^M \alpha_i y_i = 0 & \end{array}$$

Closed form, Globally convergent SMO-like updates

α updates same as SVM-SMO

$$\beta_i^{new} = \beta_i^{old} - \frac{\hat{\dot{\mathbf{x}}}_i^T\nabla_i h(\mathbf{x}_i)}{(\mathrm{H})_{ii}}$$

$$\gamma_i^{new} = \gamma_i^{old} + \frac{\mathbf{e}_i^T\nabla_* h(\mathbf{x}^*)}{(\mathrm{H}_{**})_{ii}}$$

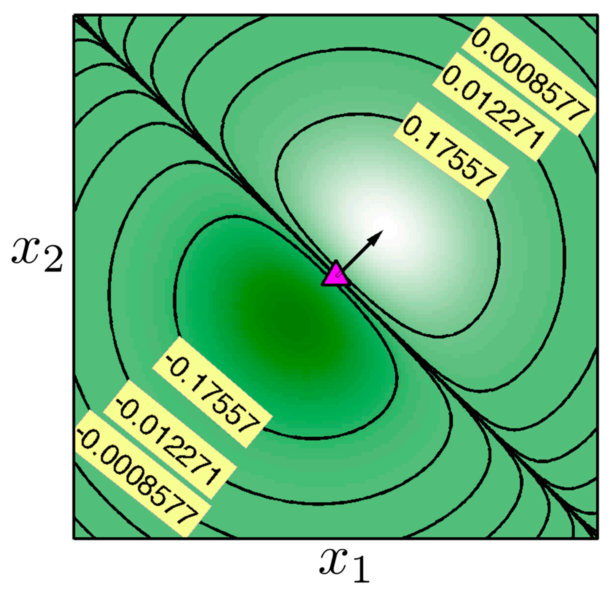
Classifier

$$h(\mathbf{x}) = \sum_{i=1}^{M}\alpha_i y_i k(\mathbf{x},\mathbf{x}_i) + \underbrace{\sum_{i=1}^{M}\beta_i\hat{\dot{\mathbf{x}}}_i^{\mathrm{T}}\frac{\partial k(\mathbf{x},\mathbf{x}_i)}{\partial \mathbf{x}_i}}_{\text{New } \beta\text{- SVs}} - \underbrace{\sum_{i=1}^{N}\gamma_i\mathbf{e}^{\mathrm{T}}\frac{\partial k(\mathbf{x},\mathbf{x}^*)}{\partial \mathbf{x}^*}}_{\text{Non-linear bias}} + b$$

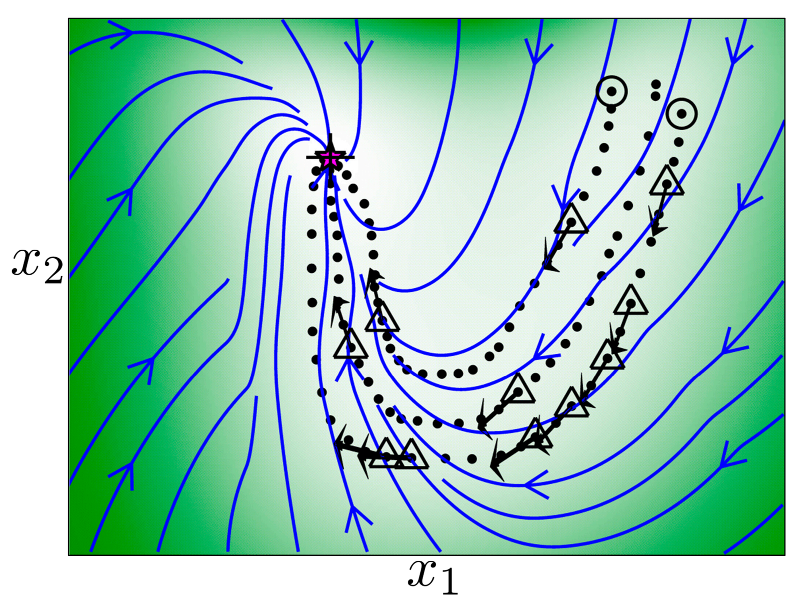
Classical SVM Formulation (highlighted)

4 New Support Vectors

$\blacktriangle \equiv \mathbf{x}_i$
$\rightarrow \equiv \hat{\dot{\mathbf{x}}}_i$
$\odot \equiv \alpha - \mathrm{SV}$
$\Delta \equiv \beta - \mathrm{SV}$

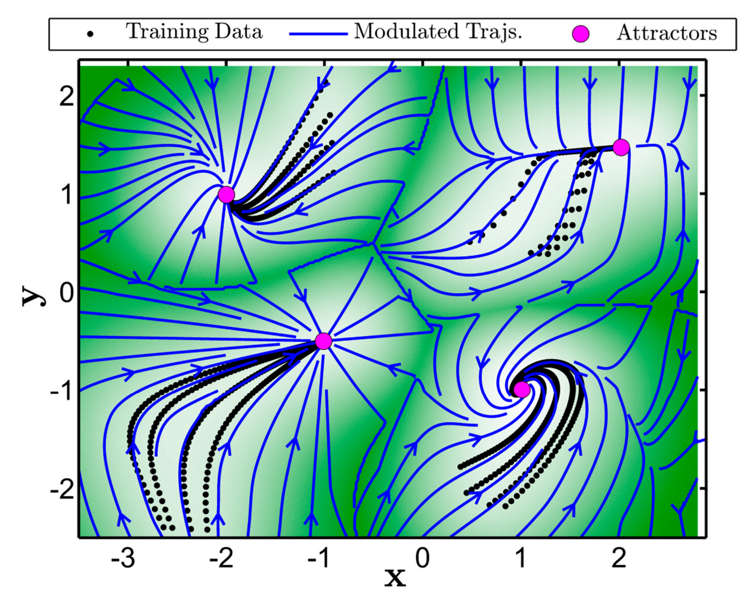


β-SV for rbf kernel
Creates a local positive slope directed along $\hat{\dot{\mathbf{x}}}_i$

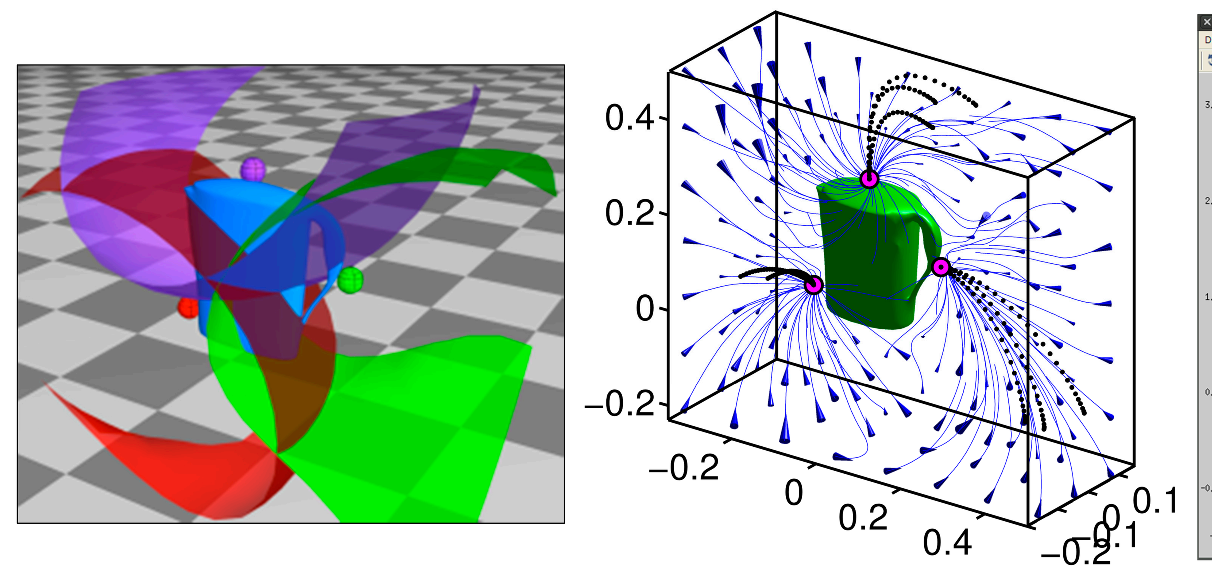


Several β-SVs molding the overall flow of the DS

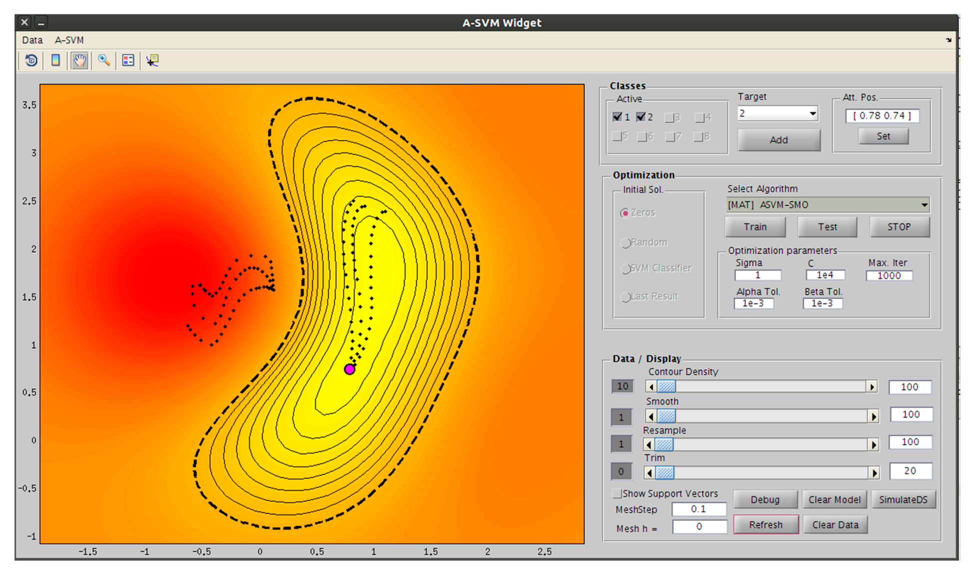
5 Multi-Class Implementation



4 attractor DS with clearly demarcated boundaries and attractor locations.



1-vs-all classifier surfaces **(left)** and resulting flow **(right)** for a 3-D pitcher object.



Source code + GUI available at

http://asvm.epfl.ch

6 References/Acknowledgements

[1] **Cortes, C. and Vapnik, V.** "Support Vector Networks." *Machine Learning*, 1995.

[2] **Lee, J.** "Dynamic gradient approaches to compute the closest unstable equilibrium point for stability region estimate and their computational limitations." *IEEE Transactions on Automatic Control*, 2003.

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