Augmented-SVM: Automatic space partitioning for combining multiple non-linear dynamics

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Dynamical Systems

- Learning autonomous dynamical systems (DS) from data

$$\{\mathbf{x}_i, \dot{\mathbf{x}}_i\}_{i=1\cdots M}\,; \mathbf{x}_i \in \mathbb{R}^N \quad \Rightarrow \quad \dot{\mathbf{x}} = f(\mathbf{x}); f : \mathbb{R}^N \mapsto \mathbb{R}^N$$

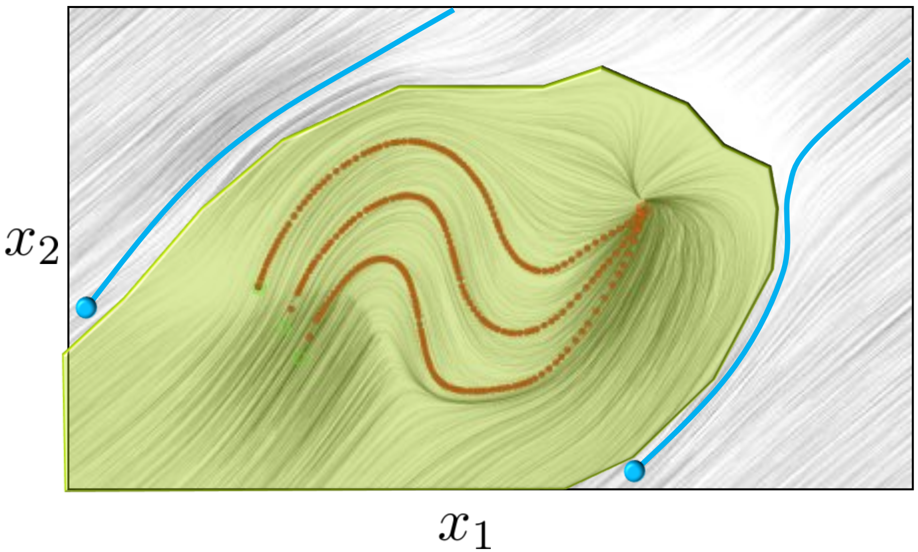
- Regression (GPR, SVR, GMR, LWR, LWPR) , Latent space models (GPDM)
- Stability!
  - Important for motion synthesis
  - Progress toward a goal state

Single attractor dynamics

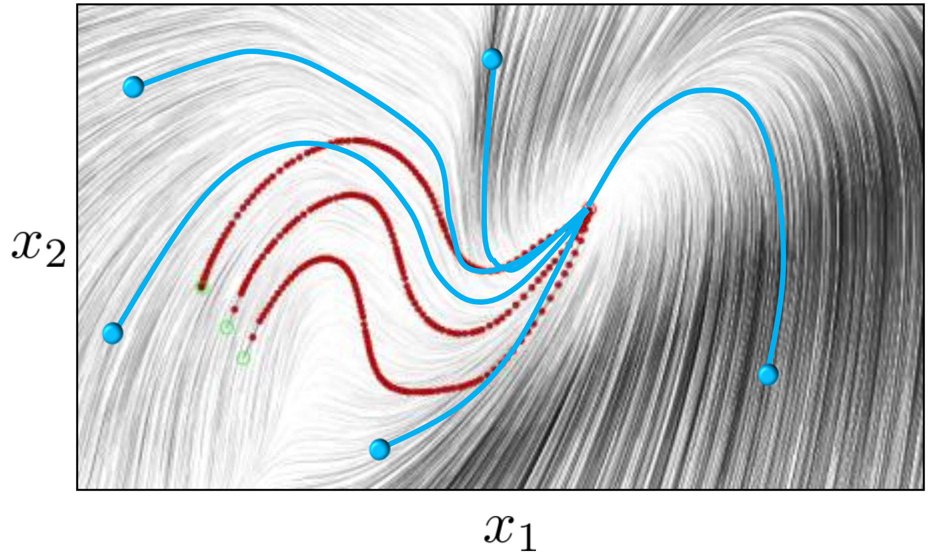
- For a DS $\dot{\mathbf{x}} = f(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{x}^*$ is an *attractor* iff $\exists$ some open neighborbood $\mathcal{B}$ of $\mathbf{x}^*$ such that $\mathbf{x}(t) \rightarrow \mathbf{x}^*$ as $t \rightarrow \infty \quad \forall \mathbf{x}(0) \in \mathcal{B}$

- $\mathcal{B}$ → basin of attraction

Regression



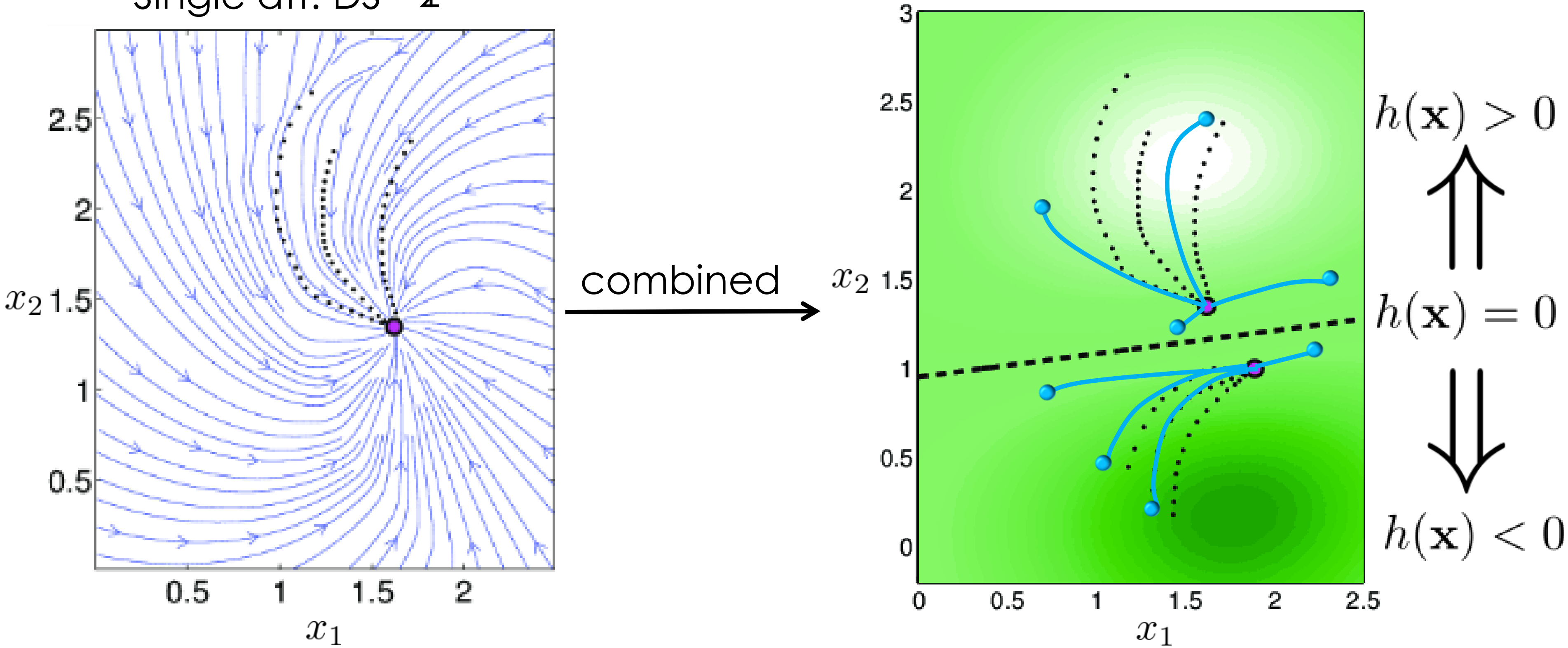
Regression + Stability



Multiple attractor dynamics

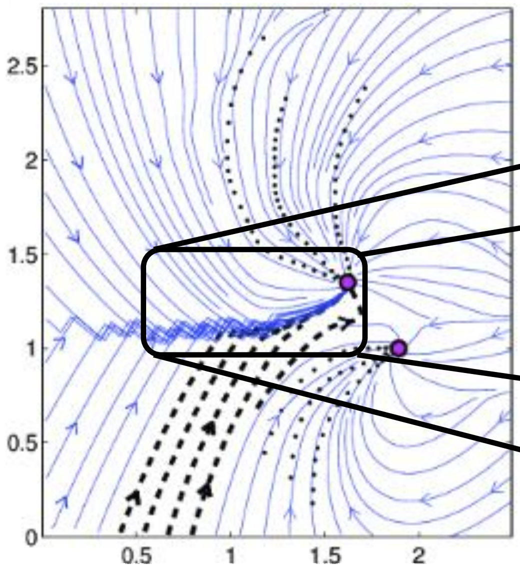
- All attractors must have finite basins of attraction → space partitions

Single att. DS - 2

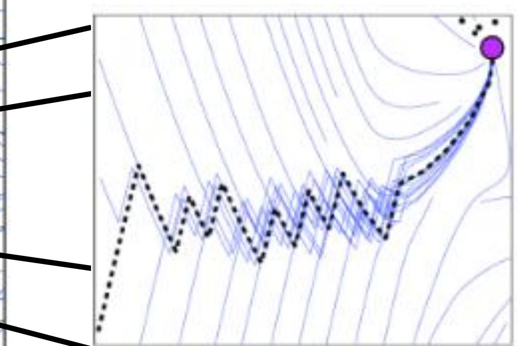


Combination of dynamics

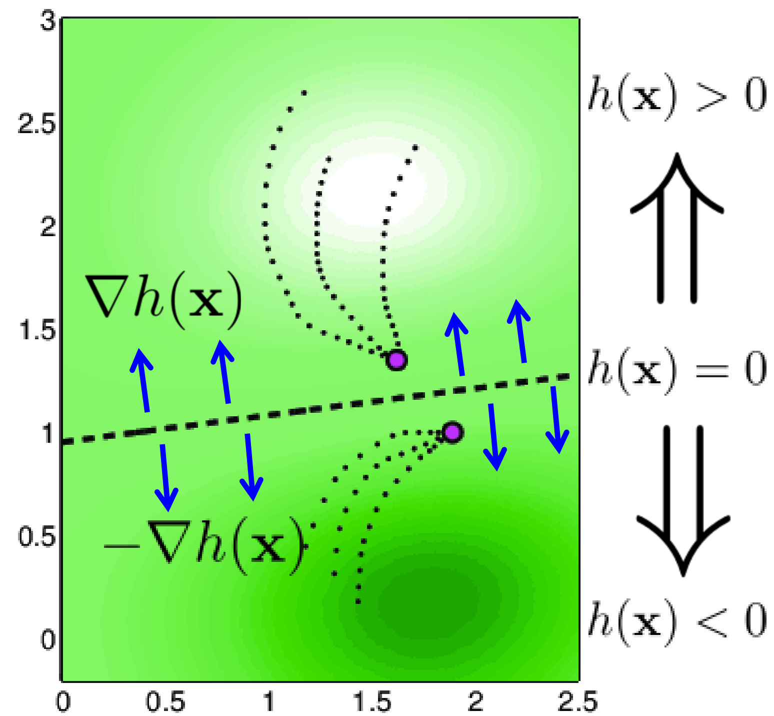
- The naïve approach : SVM classifier based switched dynamical system



Crossing over



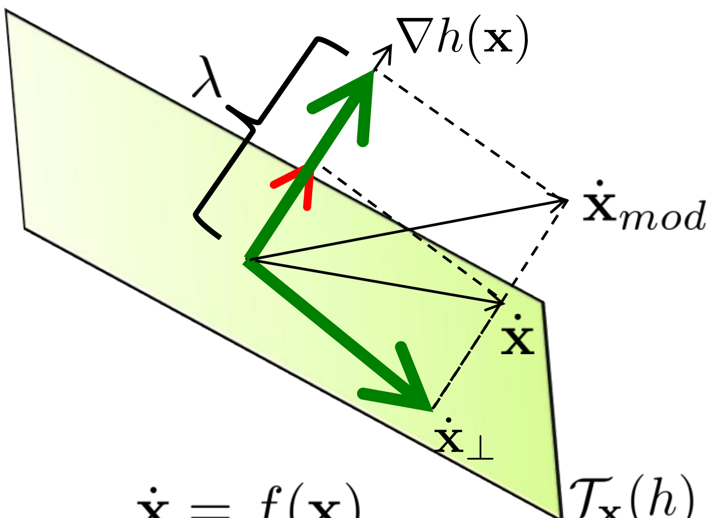
Jitter

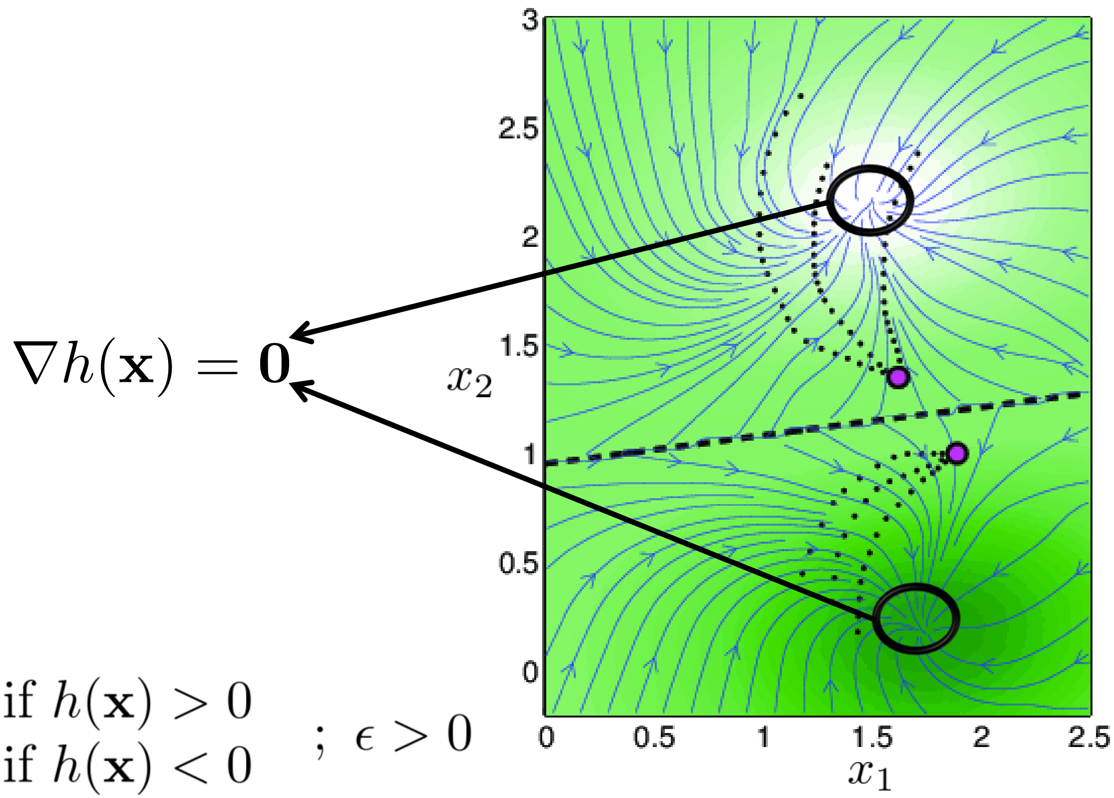


- ***Incompatibility*** between the classifier function and dynamics

Boundary avoiding

- **Dynamic gradient method** [1] : trajectories in the positive (negative) region move along ***increasing (decreasing)*** values of the classifier function.





$$\dot{\mathbf{x}} = f(\mathbf{x})$$

$$\dot{\mathbf{x}}_{mod} = \lambda(\mathbf{x})\nabla h(\mathbf{x}) + \dot{\mathbf{x}}_{\perp}$$

$$\lambda(\mathbf{x}) = \begin{cases} \max\left(\epsilon, \nabla h(\mathbf{x})^T \dot{\mathbf{x}}\right) & \text{if } h(\mathbf{x}) > 0 \\ \min\left(-\epsilon, \nabla h(\mathbf{x})^T \dot{\mathbf{x}}\right) & \text{if } h(\mathbf{x}) < 0 \end{cases} \quad ; \ \epsilon > 0$$

[1] **Lee, J**. *Dynamic gradient approaches to compute the closest unstable equilibrium point for stability region estimate and their computational limitations*. IEEE TRO Automatic Control, 2003.

Designing the classifier function: Augmented-SVM

- Data $\{\mathbf{x}_i, \dot{\mathbf{x}}_i, y_i\}_{i=1\cdots M}$; $\mathbf{x} \in \mathbb{R}^N, y_i = \pm 1$
- Feature space transformation and its Jacobian

$$\begin{aligned} h(\mathbf{x}) &= \mathbf{w}^{\mathrm{T}} \boldsymbol{\phi}(\mathbf{x}) + b \; ; \; \boldsymbol{\phi} \in \mathbb{R}^F \\ \nabla h(\mathbf{x}) &= \mathrm{J}(\mathbf{x})\mathbf{w} \qquad ; \; \mathrm{J} \in \mathbb{R}^{F\times N} \end{aligned}$$

- Constraints
  - Classification $\quad y_i h(\mathbf{x}_i) \geq 1$
  - DS compatibility $\quad y_i \nabla h(\mathbf{x}_i)^{\mathrm{T}} \hat{\dot{\mathbf{x}}}_i + \xi_i \geq 0$
  - Stability $\quad \nabla h(\mathbf{x}^*) = 0$
- Primal :

$$\min_{\mathbf{w},\xi_i} \frac{1}{2}\|\mathbf{w}\|^2 + c\sum_{i=1}^{M} \xi_i \quad \textbf{subject to}$$

$$\begin{aligned} y_i\left(\mathbf{w}^T \boldsymbol{\phi}(\mathbf{x}_i) + b\right) &\geq 1 && \forall i = 1\cdots M \\ y_i \mathbf{w}^T \mathrm{J}(\mathbf{x}_i)\hat{\dot{\mathbf{x}}}_i + \xi_i &> 0 && \forall i = 1\cdots M \\ \xi_i &> 0 && \forall i = 1\cdots M \\ \mathbf{w}^T \mathrm{J}(\mathbf{x}^*)\mathbf{e}_i &= 0 && \forall i = 1\cdots N \end{aligned}$$

$\{\mathbf{e}_i\} \rightarrow$ Cannonical basis of $\mathbb{R}^N$

A-SVM Dual

- The Lagrangian with multipliers $\alpha_i, \beta_i, \gamma_i$

$$\boxed{\mathcal{L}(\mathbf{w}, b, \alpha_i, \beta_i, \gamma_i) = \frac{1}{2}\|\mathbf{w}\|^2 + c\sum_{i=1}^{M}\xi_i - \sum_{i=1}^{M}\mu_i\xi_i - \sum_{i=1}^{M}\alpha_i\left(y_i(\mathbf{w}^T\boldsymbol{\phi}(\mathbf{x}_i)+b)-1\right)}$$

$$-\sum_{i=1}^{M}\beta_i\left(y_i\mathbf{w}^T\mathrm{J}(\mathbf{x}_i)\hat{\dot{\mathbf{x}}}_i+\xi_i\right)+\sum_{i=1}^{N}\gamma_i\mathbf{w}^T\mathrm{J}(\mathbf{x}^*)\mathbf{e}_i$$

- Dual : Constrained quadratic program

$$\min \ \frac{1}{2}\left[\boxed{\boldsymbol{\alpha}^T}\boldsymbol{\beta}^T\boldsymbol{\gamma}^T\right]\begin{bmatrix}\boxed{\mathrm{K}} & \mathrm{G} & -\mathrm{G}_* \\ \mathrm{G}^{\mathrm{T}} & \mathrm{H} & -\mathrm{H}_* \\ -\mathrm{G}_*^{\mathrm{T}} & -\mathrm{H}_*^{\mathrm{T}} & \mathrm{H}_{**}\end{bmatrix}\begin{bmatrix}\boxed{\boldsymbol{\alpha}} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma}\end{bmatrix}\boxed{-\boldsymbol{\alpha}^T\bar{\mathbf{1}} \quad \textbf{subject to}} \quad \begin{array}{c}\boxed{0 \le \alpha_i} \\ 0 \le \beta_i \le C \\ \boxed{\sum_{i=1}^{P}\alpha_i y_i = 0}\end{array}$$

$$\boxed{(\mathrm{K})_{ij} = y_i y_j k(\mathbf{x}_i, \mathbf{x}_j)} \qquad (\mathrm{H})_{ij} = \hat{\dot{\mathbf{x}}}_i^T \frac{\partial^2 k(\mathbf{x}_i, \mathbf{x}_j)}{\partial \mathbf{x}_i \partial \mathbf{x}_j} \hat{\dot{\mathbf{x}}}_j$$

$$(\mathrm{G})_{ij} = y_i \left(\frac{\partial k(\mathbf{x}_i, \mathbf{x}_j)}{\partial \mathbf{x}_j}\right)^T \hat{\dot{\mathbf{x}}}_j \qquad (\mathrm{H}_*)_{ij} = \hat{\dot{\mathbf{x}}}_i^T \frac{\partial^2 k(\mathbf{x}_i, \mathbf{x}^*)}{\partial \mathbf{x}_i \partial \mathbf{x}^*} \mathbf{e}_j$$

$$(\mathrm{G}_*)_{ij} = y_i \left(\frac{\partial k(\mathbf{x}_i, \mathbf{x}^*)}{\partial \mathbf{x}^*}\right)^T \mathbf{e}_j \qquad (\mathrm{H}_{**})_{ij} = \mathbf{e}_i^T \frac{\partial^2 k(\mathbf{x}^*, \mathbf{x}^*)}{\partial \mathbf{x}^* \partial \mathbf{x}^*} \mathbf{e}_j$$

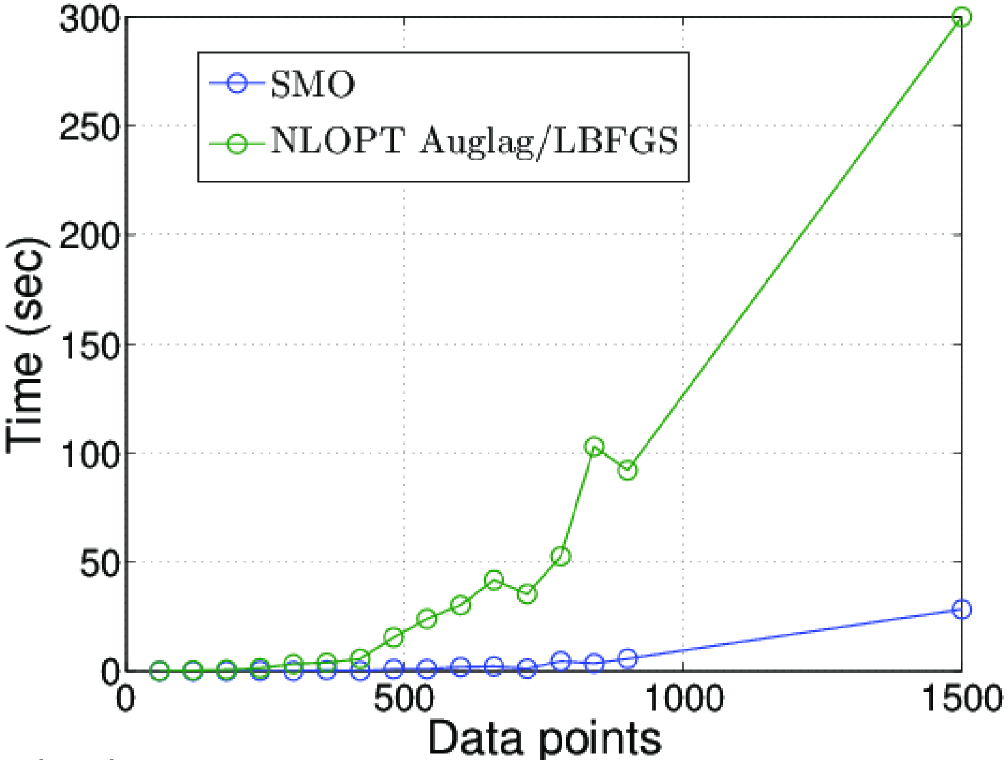
SMO & ν-variant for A-SVM

- **Closed form** & **Globally convergent** updates on Lagrange multipliers

α updates $\rightarrow$ same as SVM-SMO

$$\beta_i^{new} = \beta_i^{old} - \frac{\hat{\dot{\mathbf{x}}}_i^T \nabla_i h(\mathbf{x}_i)}{(\mathrm{H})_{ii}}$$

$$\gamma_i^{new} = \gamma_i^{old} + \frac{\mathbf{e}_i^T \nabla_* h(\mathbf{x}^*)}{(\mathrm{H}_{**})_{ii}}$$



- ν-ASVM variant can be derived where ν_1, ν_2 are the lower-bounds on the fraction of α, β -SV resp.

Classifier function

$$h(\mathbf{x}) = \underbrace{\boxed{\sum_{i=1}^{M} \alpha_i y_i k(\mathbf{x}, \mathbf{x}_i)}}_{\text{Standard SVM } \alpha \text{ - SVs}} + \underbrace{\sum_{i=1}^{M} \beta_i \hat{\dot{\mathbf{x}}}_i^{\mathrm{T}} \frac{\partial k(\mathbf{x}, \mathbf{x}_i)}{\partial \mathbf{x}_i}}_{\text{New } \beta \text{ - SVs}} - \underbrace{\sum_{i=1}^{N} \gamma_i \mathbf{e}_i^{\mathrm{T}} \frac{\partial k(\mathbf{x}, \mathbf{x}^*)}{\partial \mathbf{x}^*}}_{\text{Non-linear bias}} + \underbrace{\boxed{b}}_{\text{Const. bias}}$$

$\{\mathbf{e}_i\} \rightarrow$ Cannonical basis of $\mathbb{R}^N$

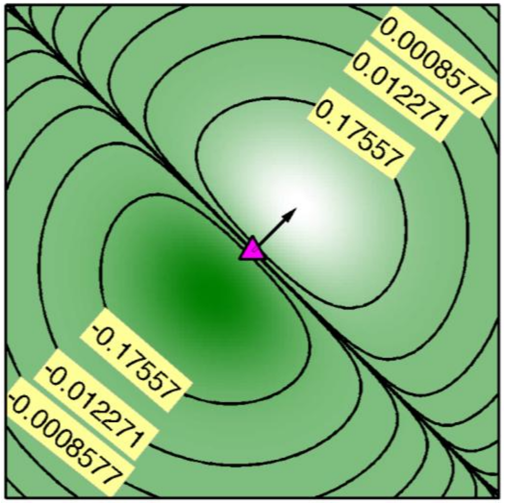
- β - SV for the rbf kernel

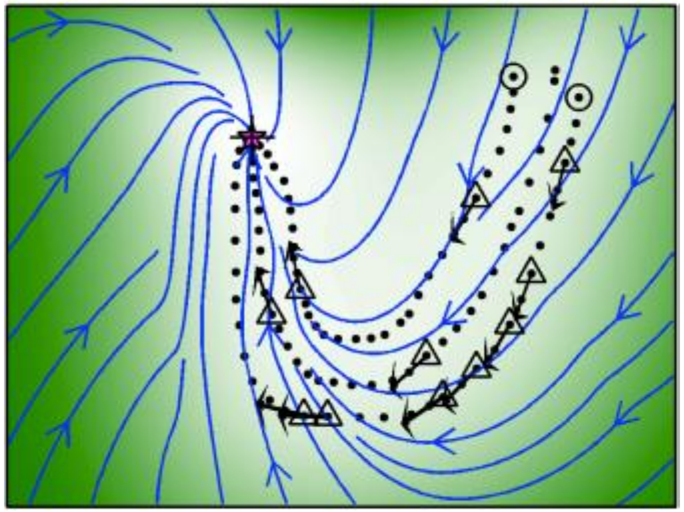
$\equiv \mathbf{x}_i$

$\equiv \hat{\dot{\mathbf{x}}}_i$

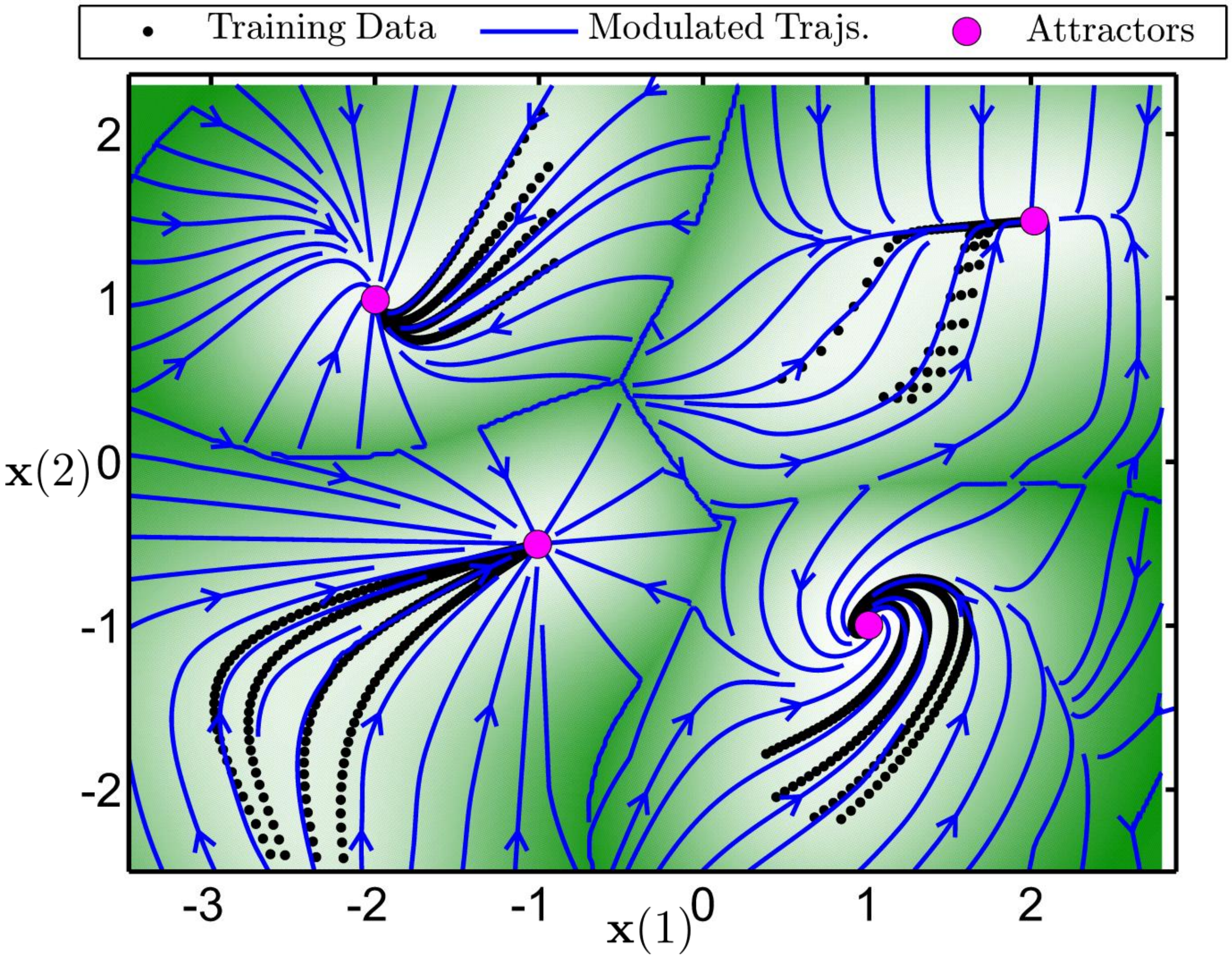
$\odot \equiv \alpha - \mathrm{SV}$

$\Delta \equiv \beta - \mathrm{SV}$

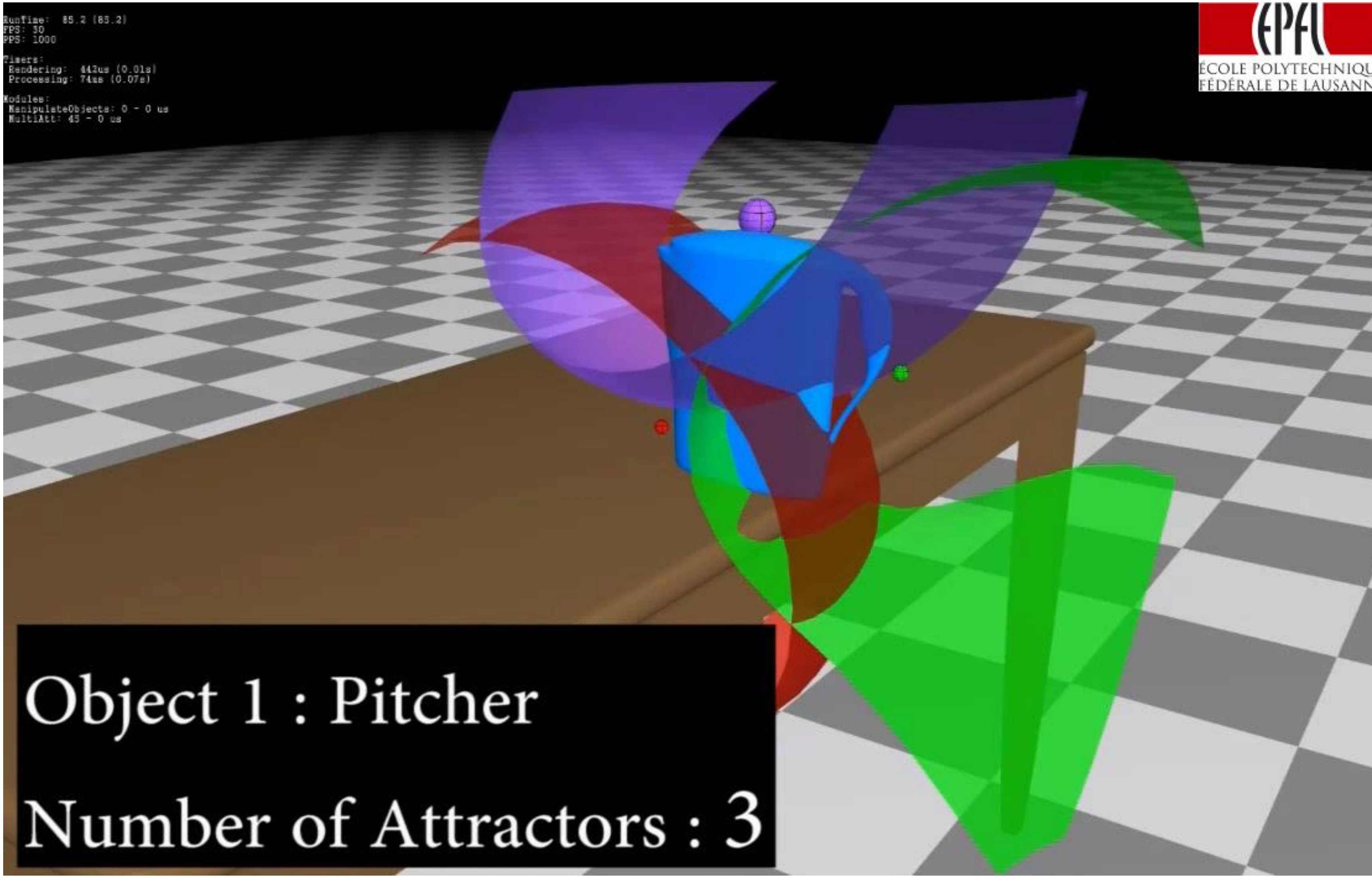




1-vs-rest multi-class implementation



Application : reach-to-grasp motions



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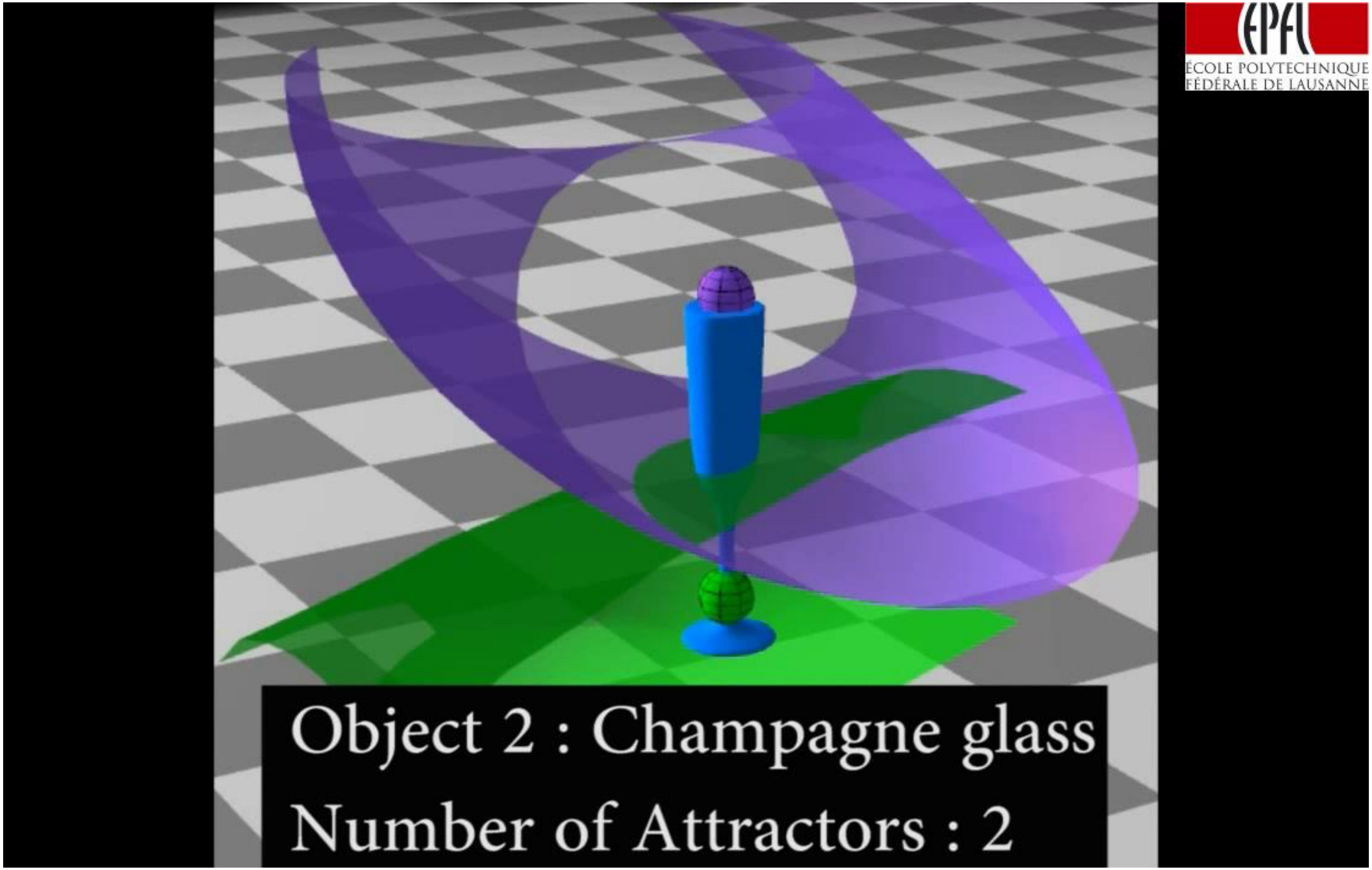

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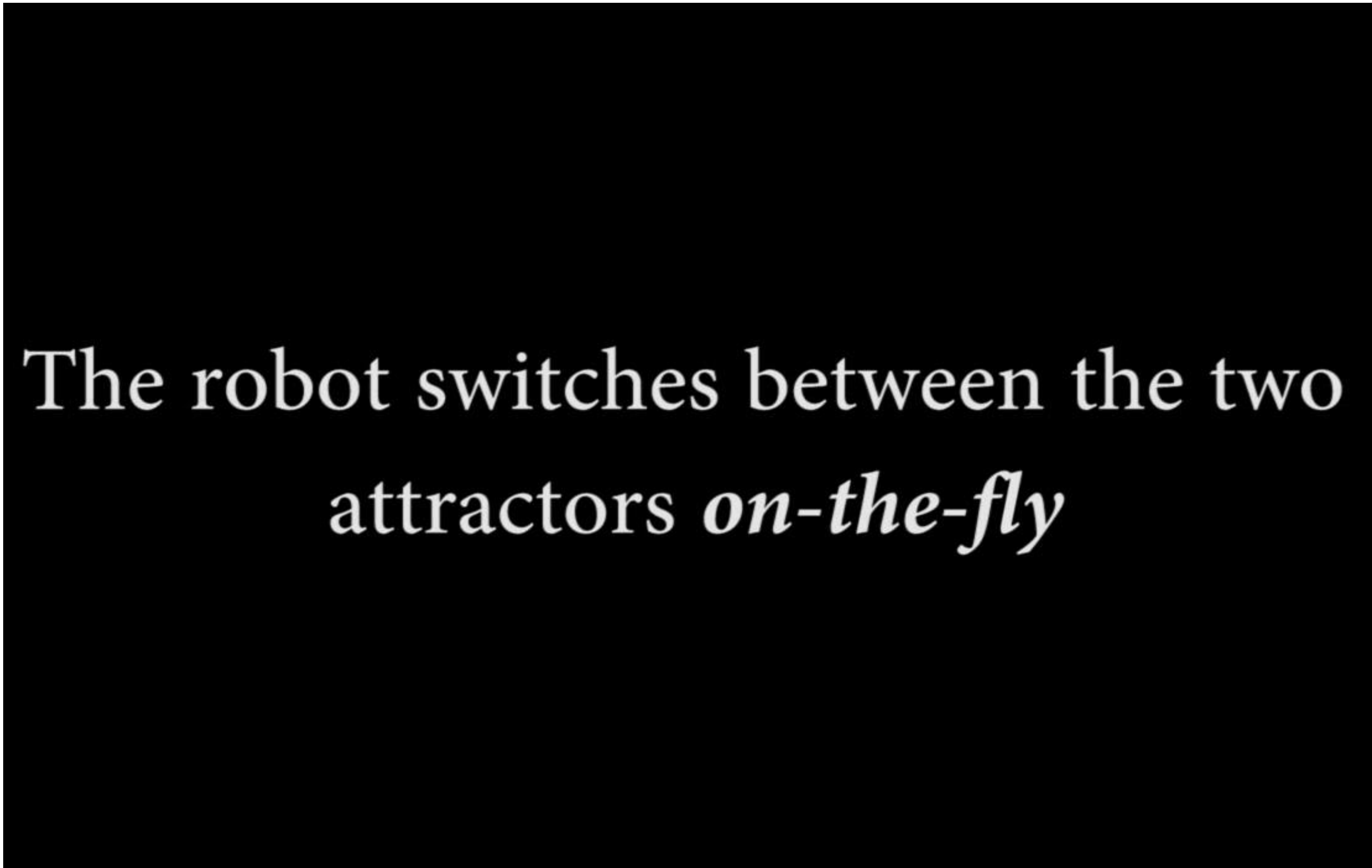
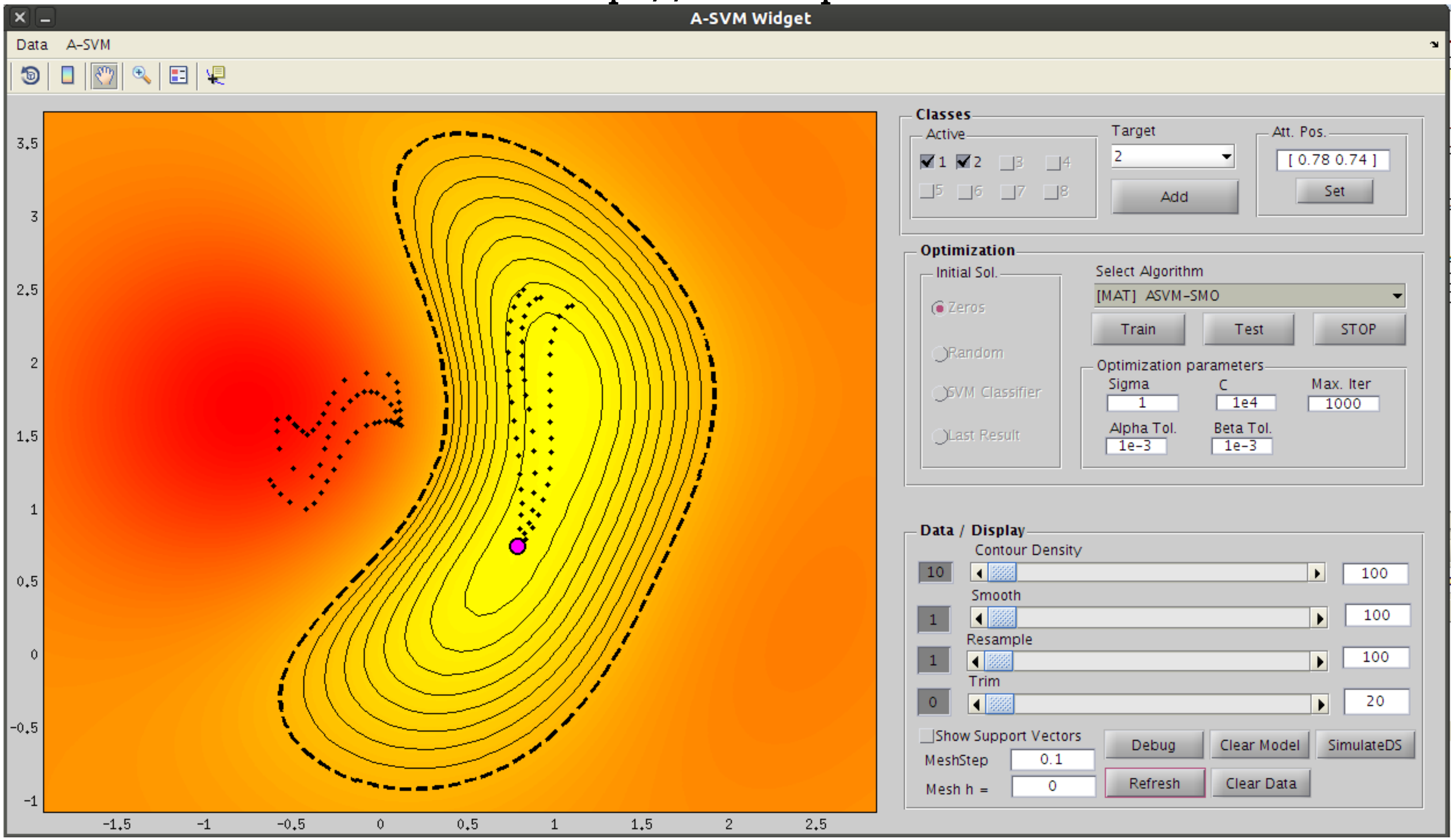


Fast decision-making + trajectory generation

W53

http://asvm.epfl.ch



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