# Learning Dynamical Systems with Bifurcations

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*Abstract*— **Trajectory planning through dynamical systems (DS) provenly provides robust control for robots in numerous applications, ranging from locomotion to object manipulation. Which DS should be used remains a task-dependent problem. Current learning approaches for multiple behaviors embed DS separately and switch at run time across these. Since this switching can be brittle and subject to instabilities, we discuss an approach to learn and embed multiple dynamics in a single DS. The proposed form of DS presents a set of parameters learned through a two-step optimization, and, exploiting Hopf bifurcations, allows to explicitly and smoothly transit across periodic and non-periodic phases, in addition to changing the equilibrium's location and the limit cycle's amplitude. The approach is validated with a real 7 DOF KUKA LWR 4+ manipulator to control wiping and with a humanoid robot in simulation.**

*Index Terms*— **Learning from Demonstration, Model Learning for Control, Motion Control, Optimization and Optimal Control.**

## I. Introduction

Research in robotic learning has seen rapid growth in the past two decades, currently allowing us to reproduce robustly a large range of tasks [1]. In more recent years, there has been a focus on the use of dynamical systems (DS) based frameworks to generate trajectories for robot motion, in place of traditional planning, in [2], [3]. DS offers robustness and immediate adaptation of the plan in the face of disturbances, as well as stability and convergence guarantees.

Transitions across dynamics can be exploited, e.g. in wiping tasks, when the robot should be able to rapidly transit from reaching the piece to wiping it, as pieces are conveyed through. An example of such transient transition across dynamics for wiping using time-dependent DS was offered in [4]; however, it required a separate DS to encode the switching. Other applications can be found in locomotion in [5], when transiting from a walking gait to a standing position, and where the modulation of velocity and amplitude permits the adaptation of the biped walking gait to the terrain and to change between gait types (walking or running) [6].

Most state of the art algorithms model differently dynamics involving periodic motions, such as polishing and wiping, and dynamics involving stable point attractors, such as pick-and-place, and then switch across these dynamics through on-line selection and identification of the required behavior [7], [8]. This, however, may lead to jittering and jerky motions when switching across dynamics with different equilibriums. In other words, the transient boundary over switching cannot be guaranteed to be small enough for a smooth motion [9].

Efforts to enable a single representation for periodic and non-periodic DS were offered in [10], a framework to iteratively learn an initial discrete motion followed by a periodic one using a time-dependent DS. We here offer an approach that encodes periodic and discrete dynamics in a single time-invariant DS by exploiting the concept of bifurcation, allowing for smooth transitions between the two. The proposed approach uses time-invariant DS which makes tracking more resilient to external disturbances [2]. We introduce a form of such DS in both 2D and 3D spaces. Furthermore, similarly to [4], [10], we propose a parameterization of the DS to offer an explicit way to control for its behavior, namely speed, location of the target and shape of the limit cycle.

Next, we introduce bifurcations and discuss previous research on the identification of their parameters. Besides, we explain how bifurcations can be employed to encapsulate two separate behaviors of dynamical systems.

### A. Dynamical systems with Hopf bifurcation

We consider a DS with one bifurcation parameter $\mu$ representing *local bifurcations*, i.e. systems where the change can be studied by considering the vector field in a neighborhood of an equilibrium point or a closed orbit.

$$f: \quad \mathbb{R}^n \to \mathbb{R}^n, \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mu), \quad \mathbf{x} \in \mathbb{R}^N, \quad \mu \in \mathbb{R}. \qquad (1)$$

The system in 1 presents a bifurcation at $\mu^* \in \mathbb{R}^{>}$ if, for small changes around $\mu^*$, the system incurs a topological change, leading to a change in the type of equilibria from a stable fixed point to limit cycle. Such bifurcations may occur in systems with at least 2 state dimensions and are characterized by the appearance of a limit cycle bifurcating from an equilibrium point, thus changing its stability. They have been extensively studied within the framework of Hopf [11] and Poincaré-Andronov-Hopf bifurcations [12].

We are interested in reproducing the *supercritical* Hopf bifurcation case, with a stable limit cycle appearing from a stable equilibrium when crossing the bifurcation value, so as to smoothly transit across periodic and discrete movement.

Systems presenting a Hopf bifurcation have been included in the formulation of time-dependent DS reviewed previously [4], [10]. In these approaches, however, the bifurcation parameter was restricted to the periodic dynamics only.

### B. Learning DS with bifurcations

Most of the work in the literature on identifying bifurcations has focused on reconstructing the bifurcation diagram of

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