Time Series Clustering

Machine Learning for Behavioral Data

April 29, 2025

Today's Topic

Week	Lecture/Lab
9	Unsupervised Learning
10	Spring Break
11	**Unsupervised Learning**
12	Ethical Machine Learning
13	Ethical Machine Learning
14	Reserve
15	Poster Presentations

- **K-Means, Spectral Clustering**
- **Choosing the optimal K***
- **Clustering time-series data**

Getting ready for today’s lecture…

- **If not done yet**: clone the repository containing the Jupyter notebook and data for today’s lecture into your Noto workspace
- SpeakUp room for today’s lecture:

https://go.epfl.ch/speakup-mlbd2025

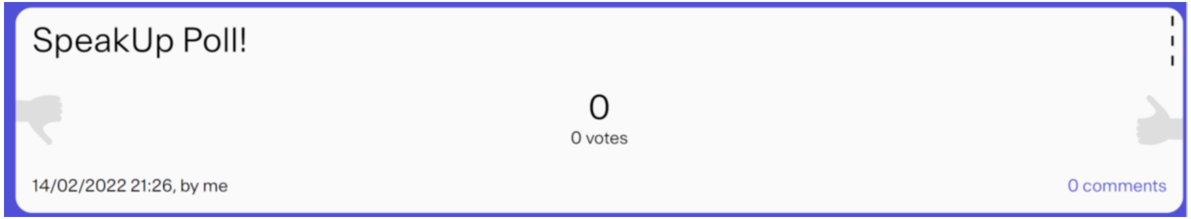
Short quiz about the past…

In K-Means, which of the following parameters affect the goodness of the solution?

a) Number of iterations

b) Initial positioning of cluster centers

c) Choice of k



SpeakUp Poll!

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0 votes

14/02/2022 21:26, by me

0 comments

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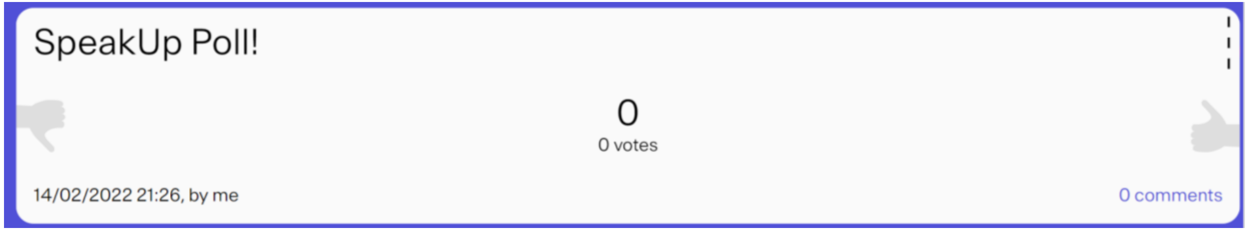
Short quiz about the past…

K-Means is useful when dealing with non-convex clusters:

a) True

b) False



SpeakUp Poll!

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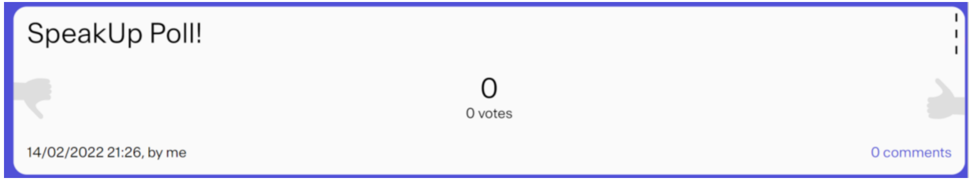
Short quiz about the past…

In a binary classification problem, it is appropriate to use the following activation function for the output layer:

a) Linear

b) Tanh

c) Sigmoid



SpeakUp Poll!

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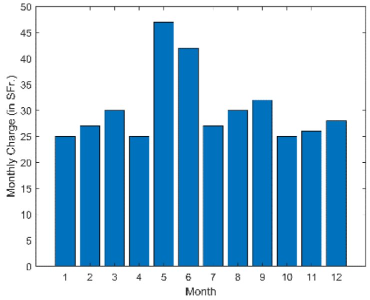
Today – Clustering Time Series Data

1. Aggregating features over time
2. Defining fixed time intervals (weeks, levels in a game, etc.)
3. Dynamic Time Warping



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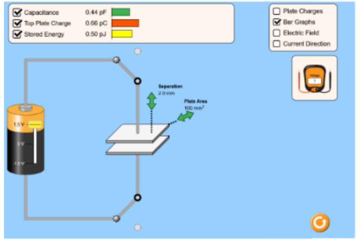
4. String Metrics
5. Markov Models



☑ Capacitance 0.44 pF
☑ Top Plate Charge 0.66 pC
☑ Stored Energy 0.50 pJ

☐ Plate Charges
☑ Bar Graphs
☐ Electric Field
☐ Current Direction

Action Sequences

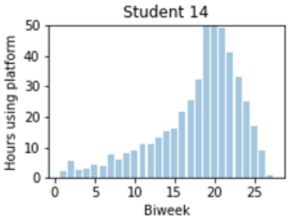
Learning Objectives

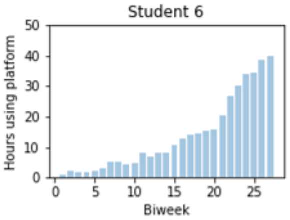
You should be able to:

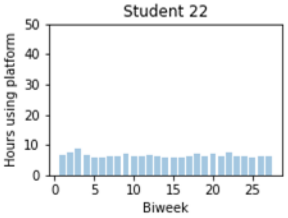
- Explain the different approaches to time series clustering
- Describe their advantages and disadvantages and when it is appropriate to use them
- Implement these approaches (lecture/lab session)
- Apply them to real-world data (lab session)

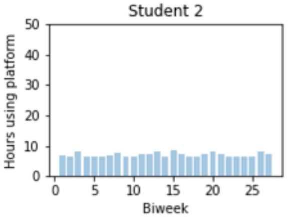
Today’s Use Case

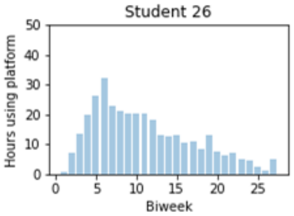
- Synthetic data of 30 high school students
- Time spent on an e-learning platform over one year (computed per biweek)
- Three clusters: 1) precrastinators, 2) regular, 3) procrastinators

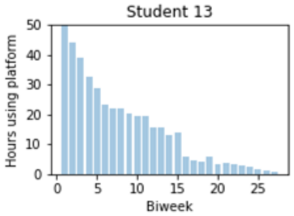












Agenda

- **Aggregating features over time**
- Defining fixed time intervals (weeks, levels in a game, etc.)
- Dynamic Time Warping
- String Metrics
- Markov Models

Aggregating features over time

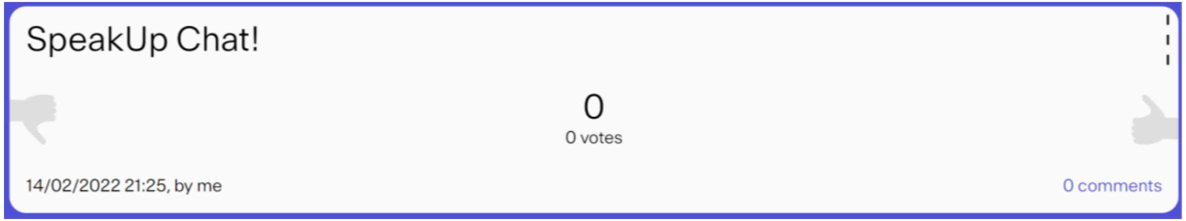
- We compute the value of the feature over the whole time series (average, maximum, range, standard deviation)
- We do not explicitly represent changes in features over time

➡ We can use standard distance/similarity measures

Your Turn – Aggregated Data

Run spectral clustering on the average number of hours:

- Can we interpret the different clusters?
- Are we able to retrieve the procrastination patterns? If not, why not?



SpeakUp Chat!

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Agenda

- Aggregating features over time
- **Defining fixed time intervals (weeks, levels in a game, etc.)**
- Dynamic Time Warping
- String Metrics
- Markov Models
- Additional Practice

Using fixed time intervals

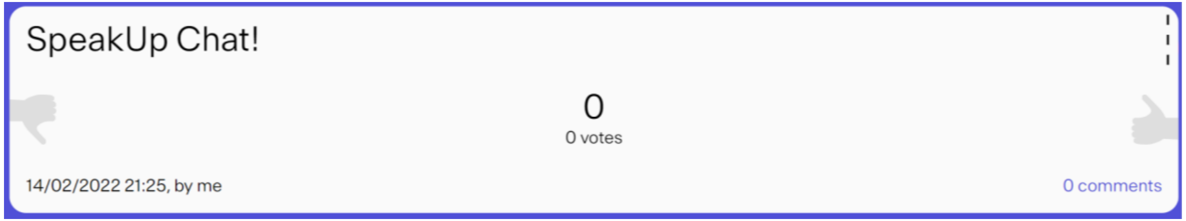
- Compute the feature value at fixed points in time (e.g., weeks, level in a game)
- We obtain feature vectors with the same length for every student

➡ We can use standard distance measures

Your Turn – Fixed Time Intervals

Run spectral clustering on the vectors of biweeks (dimension = 27) using Euclidean distance:

- What is the optimal number of clusters?
- How do the results differ from the aggregated feature results?



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Agenda

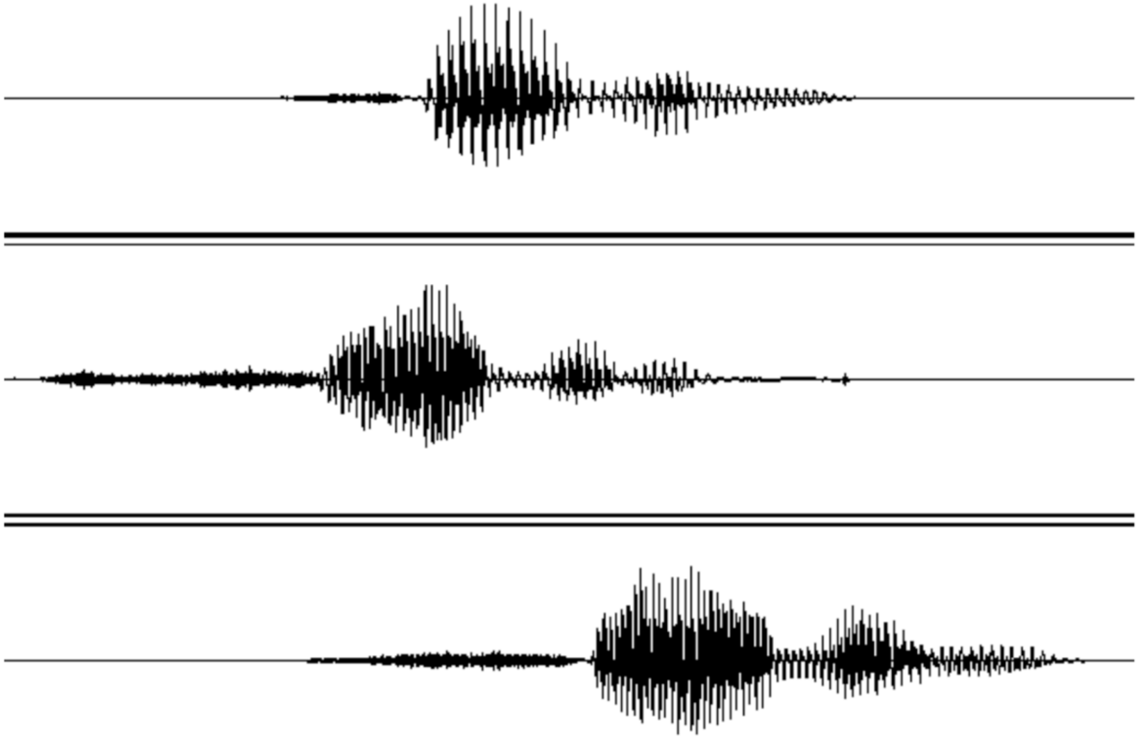
- Aggregating features over time
- Defining fixed time intervals (weeks, levels in a game, etc.)
- **Dynamic Time Warping**
- String Metrics
- Markov Models
- Additional Practice (if time permits)

Dynamic Time Warping

- Compute distance between two time series, which may vary in speed
- Time series can have different lengths

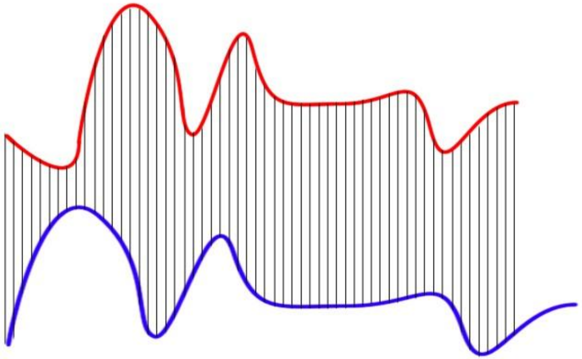
➡ Develop a **one-to-many** match, i.e. find an *optimal alignment* between two time series

Example: Spoken Digits

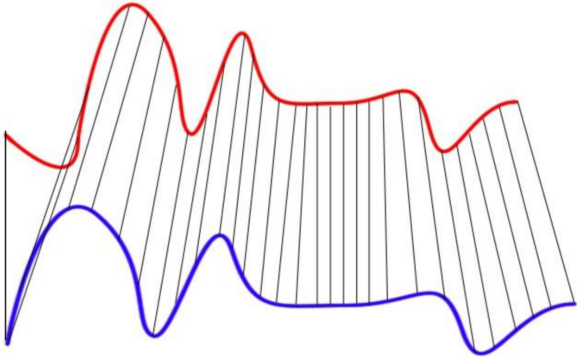


[*Image Credit: CS 498, Signals AI, University of Illinois*]

Dynamic Time Warping vs. Euclidean Distance



Euclidean Distance



Dynamic Time Warping

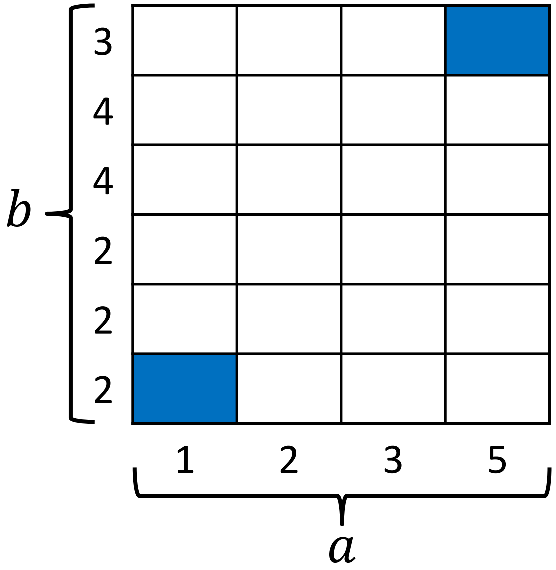
[*Image Credit: https://towardsdatascience.com/dynamic-time-warping-3933f25fcdd*]

Dynamic Time Warping: Rules

- **Goal**: minimize $D(a, b) = \min_{\emptyset} \sum_k d(a_{\emptyset(k)}, b_{\emptyset(k)})$
- **Rules** (given two sequences $\boldsymbol{a}$ and $\boldsymbol{b}$):
  - Every index of $\boldsymbol{a}$ must be matched with one or more indices from $\boldsymbol{b}$, and vice versa
  - The first index from $\boldsymbol{a}$ must be matched with the first index from $\boldsymbol{b}$ (but it does not have to be its only match)
  - The last index from $\boldsymbol{a}$ must be matched with the last index from $\boldsymbol{b}$ (but it does not have to be its only match)
  - The mapping of the indices from $\boldsymbol{a}$ to indices from $\boldsymbol{b}$ must be monotonically increasing, and vice versa, i.e. if $j > i$ are indices from $\boldsymbol{a}$, then there must not be two indices $m > n$ in $\boldsymbol{b}$, such that index i is matched with index m and index j is matched with index n, and vice versa

Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$



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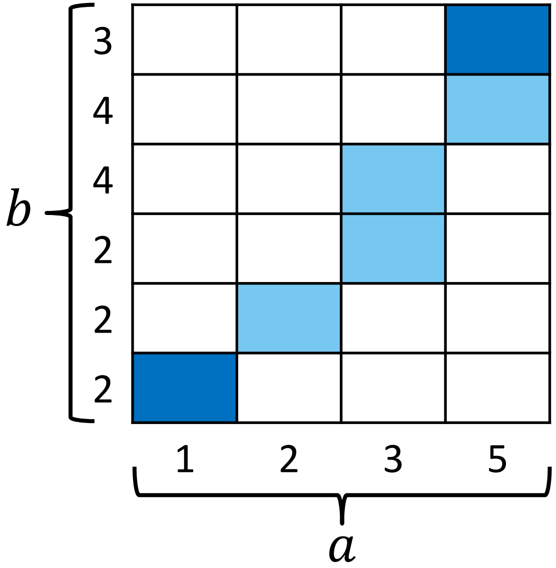

Dynamic Time Warping: Example

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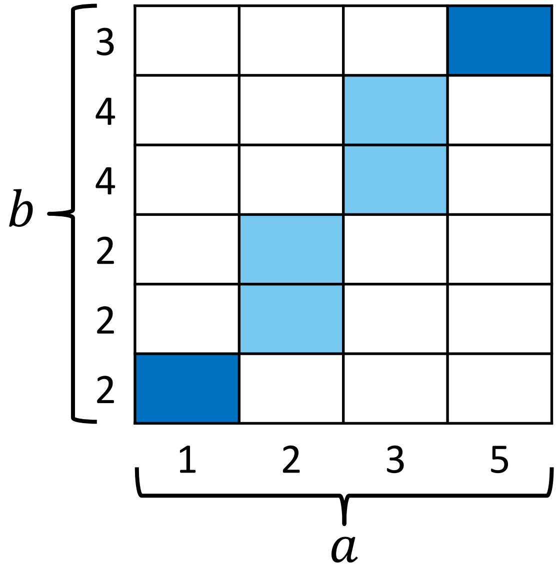
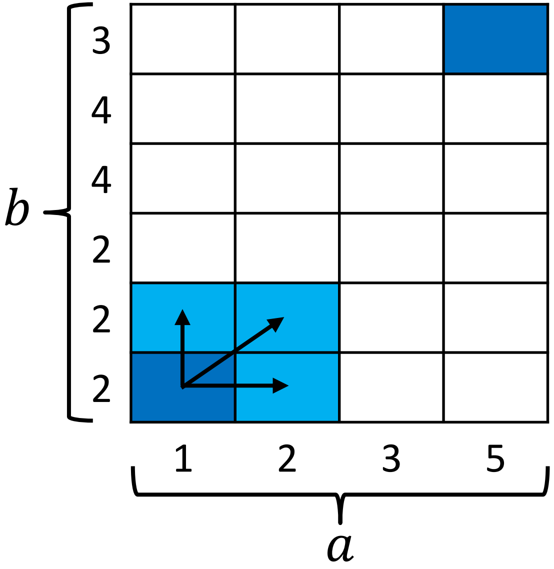


Dynamic Time Warping: Possible Paths

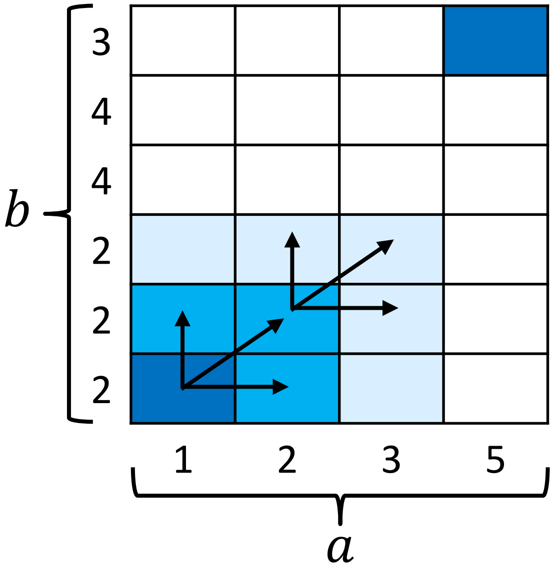
$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$



- Three possible paths from each square

Dynamic Time Warping: Possible Paths

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

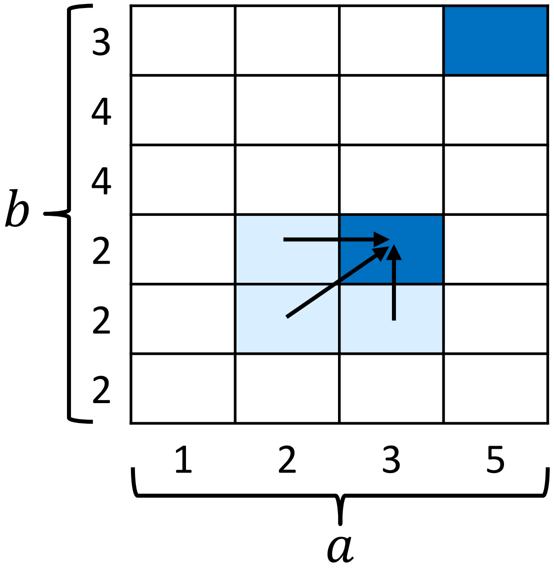


- Three possible paths from each square
- Every choice leads to three more possible paths

➡ $\approx 3^{4 \cdot 6}$ options

Dynamic Time Warping: Minimum Path

$a = [1,2,3,5] \quad b = [2,2,2,4,4,3]$



- For any cell C (matching indices i, j): three possible precursor cells
- Minimum cost (distance) for getting to C

$$d(i,j) + \min(D(i-1,j), D(i-1,j-1), D(i,j-1))$$

Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

b

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

b

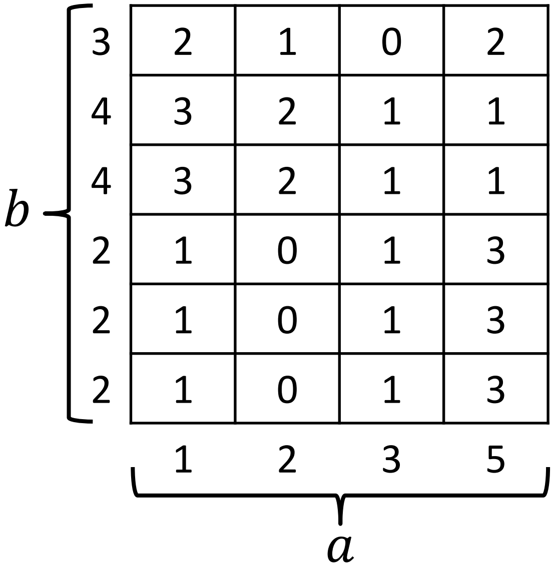
3				
4				
4				
2				
2				
2				
	1	2	3	5

a

2. Compute minimum distance path

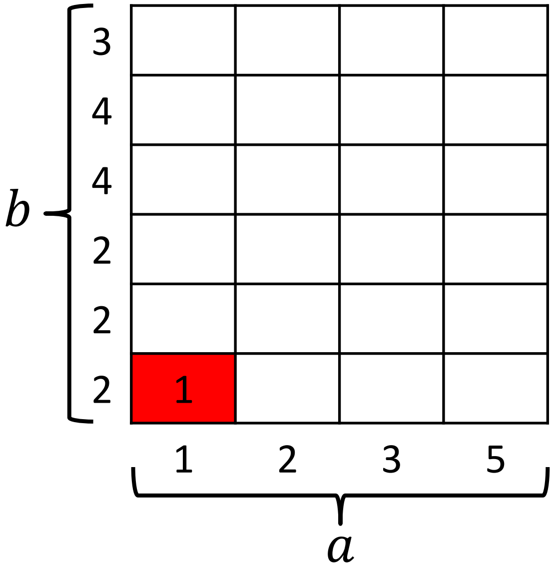
Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$



b	3	2	1	0	2
	4	3	2	1	1
	4	3	2	1	1
	2	1	0	1	3
	2	1	0	1	3
	2	1	0	1	3
		1	2	3	5
			a		

1. Compute pairwise distances



b	3				
	4				
	4				
	2				
	2				
	2	🟥 1			
		1	2	3	5
			a		

2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5]$ $b = [2,2,2,4,4,3]$

b

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

b

3	11			
4	9			
4	6			
2	3			
2	2			
2	1			
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

b

3	2	1	0	2
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4	3	2	1	1
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1. Compute pairwise distances

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3	11			
4	9			
4	6			
2	3			
2	2			
2	1	1	2	5
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: Example

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b

3	2	1	0	2
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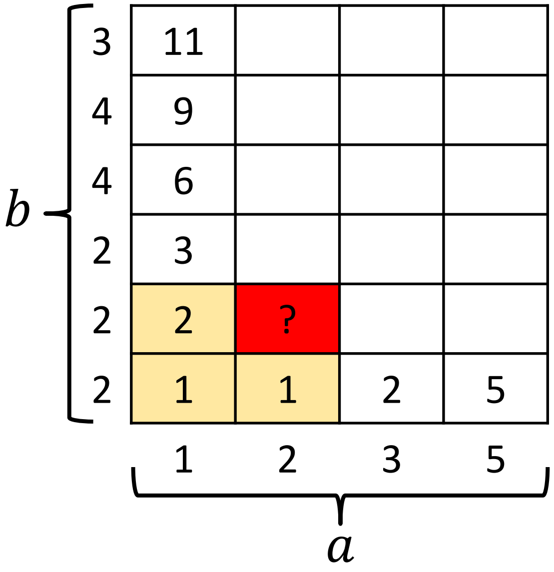
1. Compute pairwise distances



b

3	11			
4	9			
4	6			
2	3			
2	2	?		
2	1	1	2	5
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
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2	1	0	1	3
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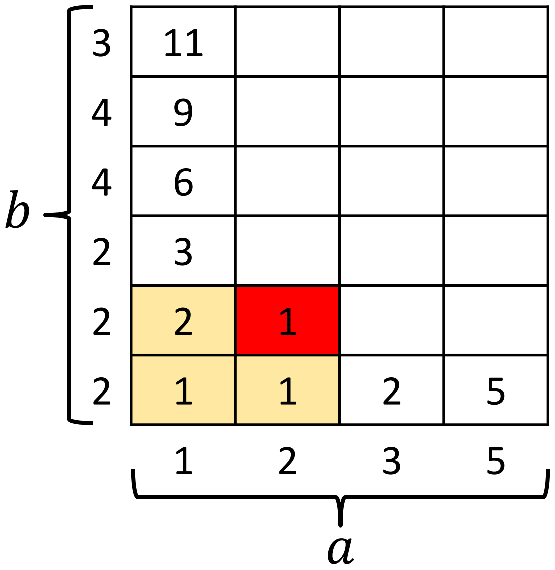
b a

1. Compute pairwise distances



3	11			
4	9			
4	6			
2	3			
2	2	1		
2	1	1	2	5
	1	2	3	5

b a

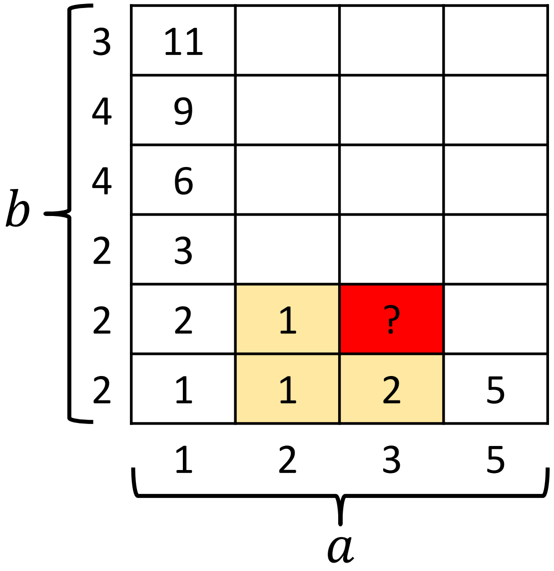
2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

b				
3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
a	1	2	3	5

1. Compute pairwise distances



b				
3	11			
4	9			
4	6			
2	3			
2	2	1	?	
2	1	1	2	5
a	1	2	3	5

2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5]$ $b = [2,2,2,4,4,3]$

b

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2	1	0	1	3
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2. Compute minimum distance path

Dynamic Time Warping: Example

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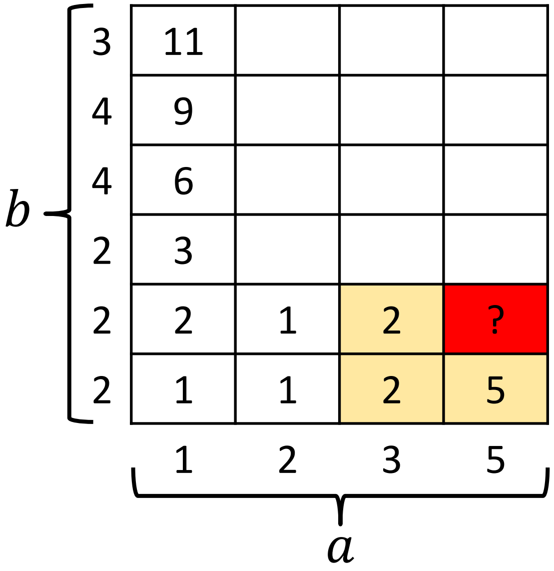
2	1	0	1	3
	1	2	3	5
		a		

1. Compute pairwise distances



b				
3	11			
4	9			
4	6			
2	3			
2	2	1	2	?
2	1	1	2	5
	1	2	3	5
		a		

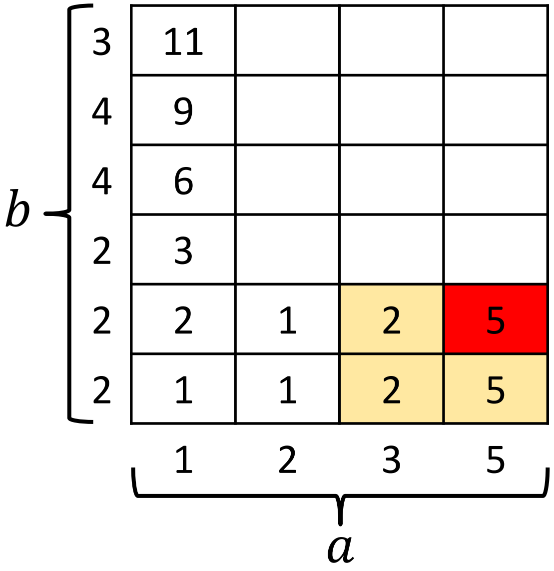
2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

	3	2	1	0	2
	4	3	2	1	1
b	4	3	2	1	1
	2	1	0	1	3
	2	1	0	1	3
	2	1	0	1	3
		1	2	3	5
			a		

1. Compute pairwise distances



	3	11			
	4	9			
b	4	6			
	2	3			
	2	2	1	2	5
	2	1	1	2	5
		1	2	3	5
			a		

2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5] \quad b = [2,2,2,4,4,3]$

b

3	2	1	0	2
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4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

Goal

b

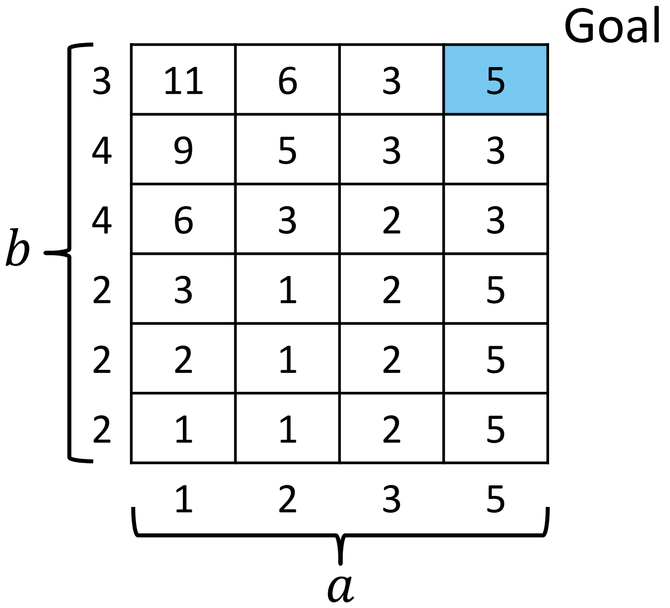
3	11	6	3	5
4	9	5	3	3
4	6	3	2	3
2	3	1	2	5
2	2	1	2	5
2	1	1	2	5
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5] \quad b = [2,2,2,4,4,3]$



3	11	6	3	5
4	9	5	3	3
4	6	3	2	3
2	3	1	2	5
2	2	1	2	5
2	1	1	2	5
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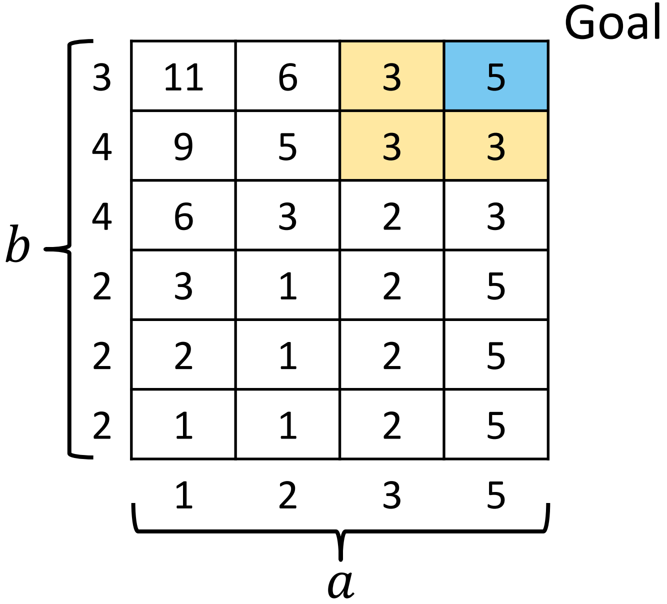
2. Compute minimum distance path

Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$



3	11	6	3	5
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2	3	1	2	5
2	2	1	2	5
2	1	1	2	5
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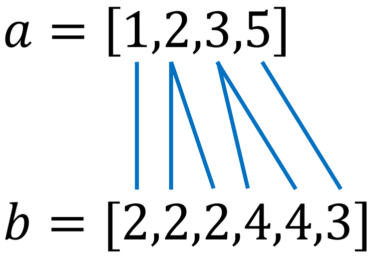
2. Compute minimum distance path

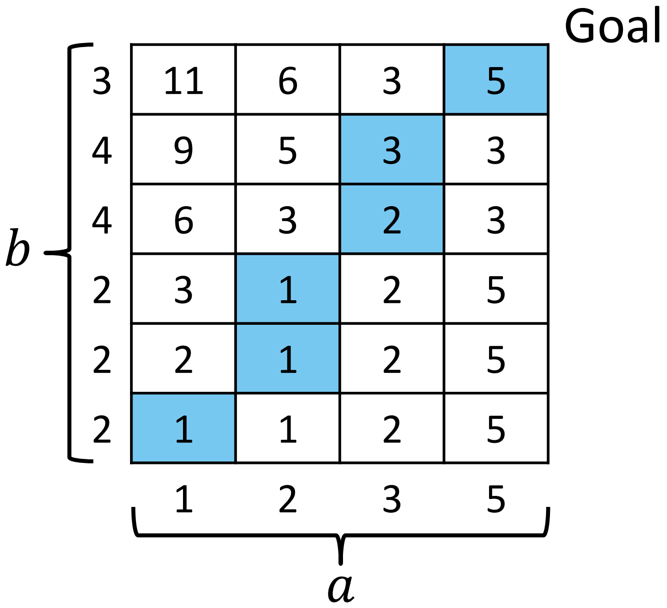
Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$



$a = [1,2,3,5]$

$b = [2,2,2,4,4,3]$



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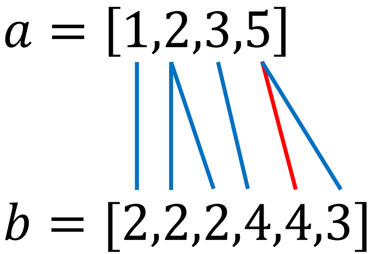
2	1	1	2	5
	1	2	3	5

2. Compute minimum distance path

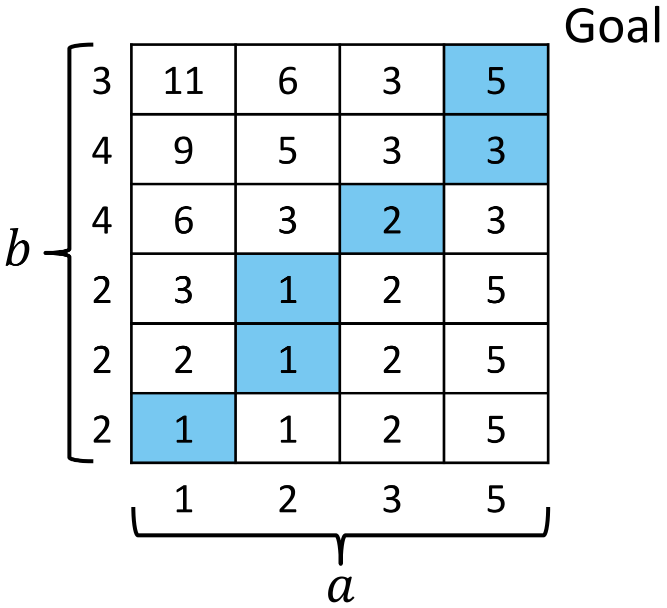
Dynamic Time Warping: Example

$$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$$



$$a = [1,2,3,5]$$

$$b = [2,2,2,4,4,3]$$

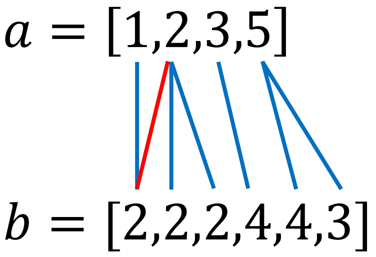


3	11	6	3	5
4	9	5	3	3
4	6	3	2	3
2	3	1	2	5
2	2	1	2	5
2	1	1	2	5
	1	2	3	5

2. Compute minimum distance path

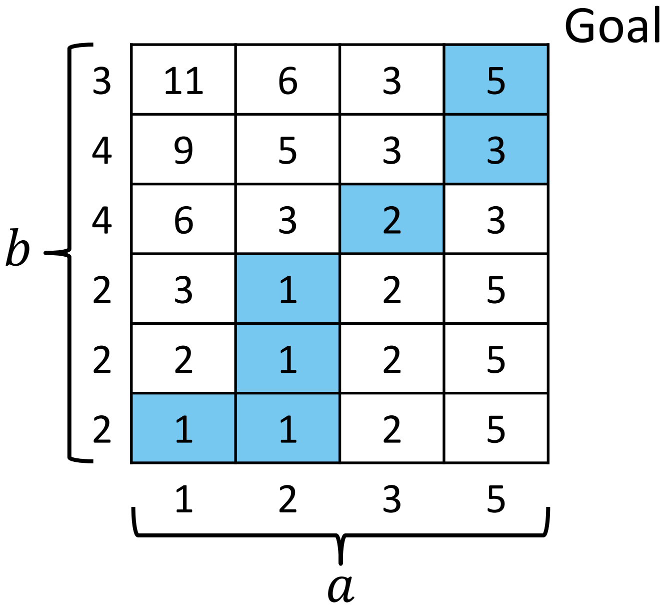
Dynamic Time Warping: Example

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$



$a = [1,2,3,5]$

$b = [2,2,2,4,4,3]$

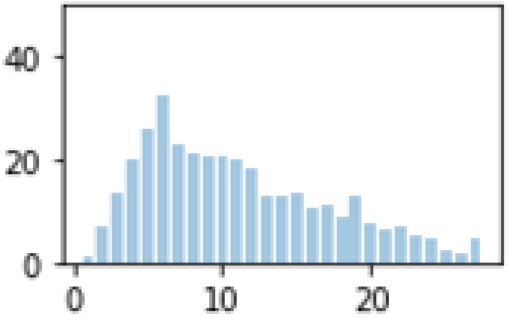


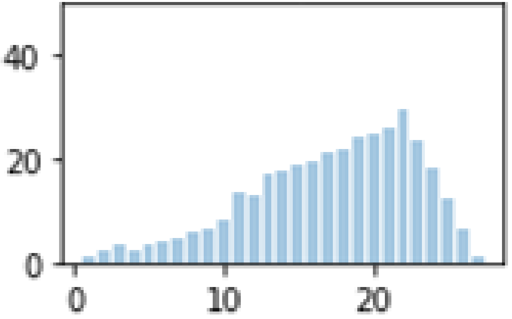
3	11	6	3	5
4	9	5	3	3
4	6	3	2	3
2	3	1	2	5
2	2	1	2	5
2	1	1	2	5
	1	2	3	5

2. Compute minimum distance path

Dynamic Time Warping: Window

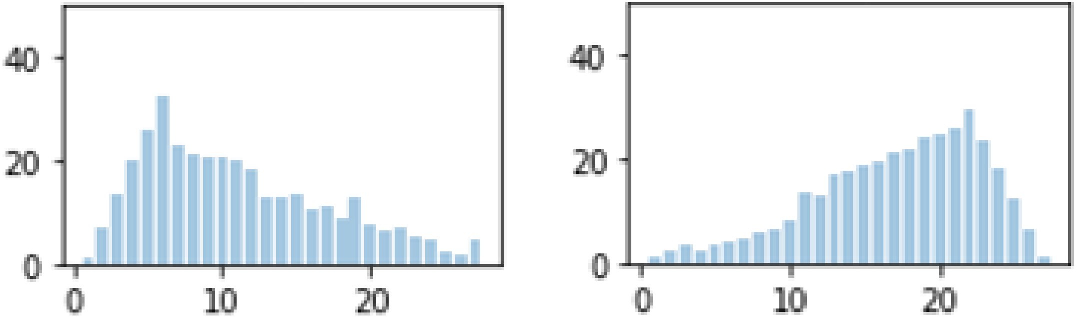
- Sometimes, we might want to constrain the mapping





Dynamic Time Warping: Window

- Sometimes, we might want to constrain the mapping



- We introduce an window size w: an element in sequence $\boldsymbol{a}$ at index i can only be mapped to elements at index $i - w, \dots, i + w$ in sequence $\boldsymbol{b}$

Dynamic Time Warping: $w = 2$

$a = [1,2,3,5]$ $\quad$ $b = [2,2,2,4,4,3]$

b

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

b

3				
4				
4				
2				
2				
2				
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: $w = 2$

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

b

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

b

3	-			
4	-			
4	-			
2	3			
2	2			
2	1			
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: $w = 2$

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

b

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

b

3	-	-		
4	-	-		
4	-	3		
2	-	1		
2	2	1		
2	1	1		
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: $w = 2$

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

b

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

b

3	-	-	-	
4	-	-	3	
4	-	-	2	
2	-	1	2	
2	2	1	2	
2	1	1	2	
	1	2	3	5

a

2. Compute minimum distance path

Dynamic Time Warping: $w = 2$

$a = [1,2,3,5] \qquad b = [2,2,2,4,4,3]$

b

3	2	1	0	2
4	3	2	1	1
4	3	2	1	1
2	1	0	1	3
2	1	0	1	3
2	1	0	1	3
	1	2	3	5

a

1. Compute pairwise distances

b

3	-	-	-	5
4	-	-	-	3
4	-	-	2	3
2	-	1	2	5
2	2	1	2	5
2	1	1	2	-
	1	2	3	5

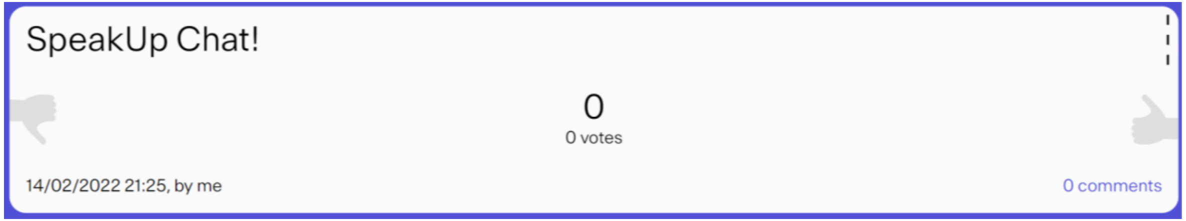
a

2. Compute minimum distance path

Your Turn – Dynamic Time Warping

Run spectral clustering using DTW with a window size of $w = 3$:

- How do the results differ from previous results?
- What happens if you set $w = 0$?
- And if you set $w = 27$?



SpeakUp Chat!

0

0 votes

14/02/2022 21:25, by me

0 comments

https://go.epfl.ch/speakup-mlbd2025

Agenda

- Aggregating features over time
- Defining fixed time intervals (weeks, levels in a game, etc.)
- Dynamic Time Warping
- **String Metrics**
- Markov Models

Example from Research: String Metrics

Using the pump repair manual

Use the manual to help you repair the pump. You should ONLY perform the checks and repairs that are necessary.

After you have repaired the pump, click Test Pump to make sure it is working.

Pump Repair Manual:
Common Problems with Hand Pumps

- Handle moves too easily and no water comes out.
- Water has a bad smell.
- Water is very cloudy.
- It is hard to move the handle or pump operation is noisy.
- Little or no water comes out.

What to Check For:
Handle has become disconnected from the piston rod.

How to Repair:
Reconnect handle to piston rod.

$C1 \rightarrow C2 \rightarrow C3 \rightarrow C4 \rightarrow R4 \rightarrow$
$P \rightarrow C5 \rightarrow R5 \rightarrow P$

$C4 \rightarrow R4 \rightarrow P \rightarrow C5 \rightarrow R5 \rightarrow P$

[*Visualization and Confirmatory Clustering of Sequence Data from a Simulation-Based Assessment Task*, Berger et al., EDM 2014]

Example from Research: String Metrics

- Levenstein distance: minimal number of single character edits (insertion, deletion, substitution) to change one string into the other
- Longest common subsequence (LCS): string similarity measure, find the longest common subsequence between two sequences

[*Visualization and Confirmatory Clustering of Sequence Data from a Simulation-Based Assessment Task*, Berger et al., EDM 2014]

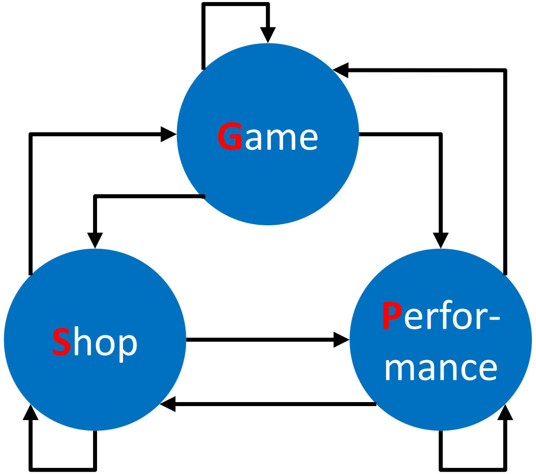
Agenda

- Aggregating features over time
- Defining fixed time intervals (weeks, levels in a game, etc.)
- Dynamic Time Warping
- String Metrics
- **Markov Models**

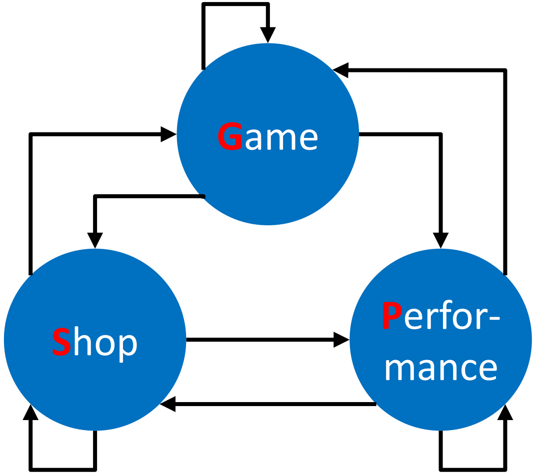
Markov Models

- Detailed action sequences provide rich temporal information
- Might contain a considerable amount of noise
- We might be interested not in the detailed sequence, but in patterns (which actions tend to follow each other)

Markov Models



Markov Models



$G \rightarrow S \rightarrow G \rightarrow S \rightarrow G \rightarrow P \rightarrow S \rightarrow G$

$G \rightarrow G \rightarrow G \rightarrow G \rightarrow P \rightarrow G \rightarrow G$

$G \rightarrow P \rightarrow S \rightarrow G \rightarrow P \rightarrow S$

Parameters: Maximum Likelihood Estimation

$$G \rightarrow S \rightarrow G \rightarrow S \rightarrow G \rightarrow P \rightarrow S \rightarrow G \rightarrow S \rightarrow S \rightarrow G \rightarrow G \rightarrow S \rightarrow G \rightarrow P \rightarrow G \rightarrow$$
$$P \rightarrow S \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow S \rightarrow S \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow S \rightarrow S \rightarrow G \rightarrow S$$

$$p(S|G) = \frac{10}{15} = 0.67$$

$$p(G|G) = \frac{2}{15} = 0.13$$

$$p(P|G) = \frac{3}{15} = 0.20$$

$$\begin{array}{cc} & \begin{array}{ccc} \boldsymbol{G} & \boldsymbol{S} & \boldsymbol{P} \end{array} \\ \begin{array}{c} \boldsymbol{G} \\ \boldsymbol{S} \\ \boldsymbol{P} \end{array} & \begin{pmatrix} 0.13 & 0.67 & 0.20 \\ 0.79 & 0.11 & 0 \\ 0.33 & 0.67 & 0 \end{pmatrix} \end{array}$$

Stationary Distribution

$$\boxed{\begin{array}{l} G \rightarrow S \rightarrow G \rightarrow S \rightarrow G \rightarrow P \rightarrow S \rightarrow G \rightarrow S \rightarrow S \rightarrow G \rightarrow G \rightarrow S \rightarrow G \rightarrow P \rightarrow G \rightarrow \\ P \rightarrow S \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow S \rightarrow S \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow S \rightarrow S \rightarrow G \rightarrow S \end{array}}$$

$$\begin{array}{cc} & \begin{array}{ccc} \boldsymbol{G} & \boldsymbol{S} & \boldsymbol{P} \end{array} \\ \begin{array}{c} \boldsymbol{G} \\ \boldsymbol{S} \\ \boldsymbol{P} \end{array} & \begin{pmatrix} 0.13 & 0.67 & 0.20 \\ 0.89 & 0.11 & 0 \\ 0.33 & 0.67 & 0 \end{pmatrix} \end{array}$$

$$\boxed{\pi\mathrm{T} = \pi}$$

Stationary Distribution

$$\boxed{\begin{array}{l} G \rightarrow S \rightarrow G \rightarrow S \rightarrow G \rightarrow P \rightarrow S \rightarrow G \rightarrow S \rightarrow S \rightarrow G \rightarrow G \rightarrow S \rightarrow G \rightarrow P \rightarrow G \rightarrow \\ P \rightarrow S \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow S \rightarrow S \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow G \rightarrow S \rightarrow \mathrm{G} \rightarrow S \rightarrow S \rightarrow G \rightarrow S \end{array}}$$

$$\begin{array}{cc} & \begin{array}{ccc} \boldsymbol{G} & \boldsymbol{S} & \boldsymbol{P} \end{array} \\ \begin{array}{c} \boldsymbol{G} \\ \boldsymbol{S} \\ \boldsymbol{P} \end{array} & \begin{pmatrix} 0.13 & 0.67 & 0.20 \\ 0.89 & 0.11 & 0 \\ 0.33 & 0.67 & 0 \end{pmatrix} \end{array}$$

$$\boxed{\pi \mathrm{T} = \pi}$$

$$\pi = [0.48 \quad 0.43 \quad 0.09]$$

Expected Frequencies

- When sequences get very long (n gets large), how often do we expect to observe the transitions?

$$\begin{array}{c c} & \begin{array}{ccc} \boldsymbol{G} & \boldsymbol{S} & \boldsymbol{P} \end{array} \\ \begin{array}{c} \boldsymbol{G} \\ \boldsymbol{S} \\ \boldsymbol{P} \end{array} & \begin{pmatrix} 0.06 & 0.32 & 0.10 \\ 0.38 & 0.05 & 0 \\ 0.03 & 0.06 & 0 \end{pmatrix} \end{array}$$

Distance Metrics

- Based on Frobenius Norm: equivalent to Euclidean distance over vectors

$$D_2(A, B) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(a_{ij} - b_{ij}\right)^2}$$

Distance Metrics

- Kullback-Leibler Divergence: measures difference between two probability distributions

$$D_{KL}(P||Q) = \sum_{x \in \mathrm{X}} P(x) \cdot \log(\frac{P(x)}{Q(x)})$$

- Jensen-Shannon Divergence: measures difference between two probability distributions

$$D_{JS}(P||Q) = \frac{1}{2} D_{KL}(P||M) + \frac{1}{2} D_{KL}(M||Q) \qquad \mathrm{M} = \frac{1}{2}(P+Q)$$

Distance Metrics

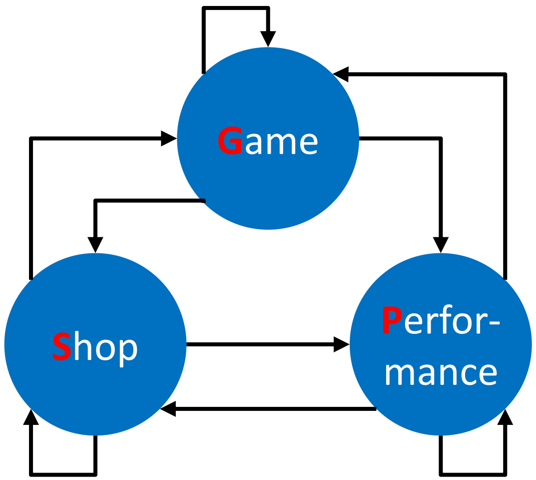
- Hellinger Distance: measures difference between two probability distributions

$$D_H(P||Q) = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{n} (\sqrt{p_i} - \sqrt{q_i})^2}$$

Distance between samples: Options

- Compute distance between stationary distributions: use Hellinger Distance (or Jensen-Shannon Divergence)
- Compute distance between transition matrices: use Frobenius Distance
- Compute distance between expected frequencies: use Hellinger Distance (or Jensen-Shannon Divergence)

Example from Research: Spelling Learning



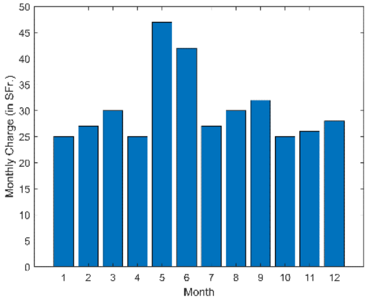
Three clusters:

- Focused on the task
- Children, who frequently check performance/shop in-between tasks
- Spend long amounts of time off-task

[*Temporally Coherent Clustering of Student Data*, Klingler et al., EDM 2016]

Summary - Handling Time Series Data

1. Aggregating features over time
2. Defining fixed time intervals (weeks, levels in a game, etc.)
3. Dynamic Time Warping



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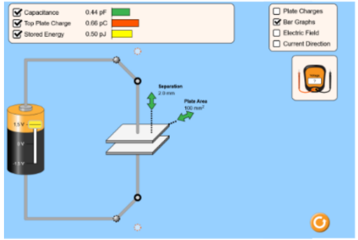
4. String Measures
5. Markov Models



☑ Capacitance 0.44 pF
☑ Top Plate Charge 0.66 pC
☑ Stored Energy 0.50 pJ

☐ Plate Charges
☑ Bar Graphs
☐ Electric Field
☐ Current Direction

Separation 2.0 mm
Plate Area 100 mm²

Action Sequences