# Knowledge Tracing

Machine Learning for Behavioral Data

March 27, 2023

# Today's Topic

| Week | Lecture/Lab |
|---|---|
| 1 | Introduction |
| 2 | Data Exploration |
| 3 | Regression |
| 4 | Classification |
| 5 | Model Evaluation |
| **6** | **Time Series Prediction** |
| 7 | Time Series Prediction |
| 8 | Time Series Prediction |

**Supervised learning on time series:**

- **Probabilistic graphical models**
- **GLMMs**
- **Neural networks: LSTM, GRU, etc.**

# Getting ready for today’s lecture…

- **If not done yet**: clone the repository containing the Jupyter notebook and data for today’s lecture into your Noto workspace
- SpeakUp room for today’s lecture:

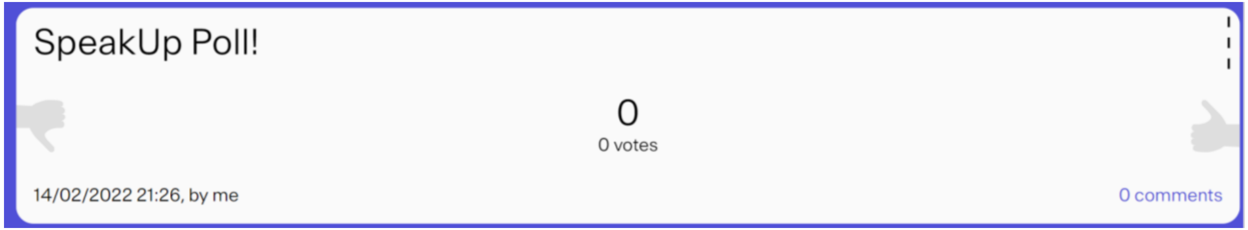
**https://go.epfl.ch/speakup-mlbd2025**

# Short quiz about the past…

Given the data set D = {1,2,3,4} , one possible bootstrap data set of D is {1,1,1,1}:

a) True

b) False




https://go.epfl.ch/speakup-mlbd2025

# Short quiz about the past…

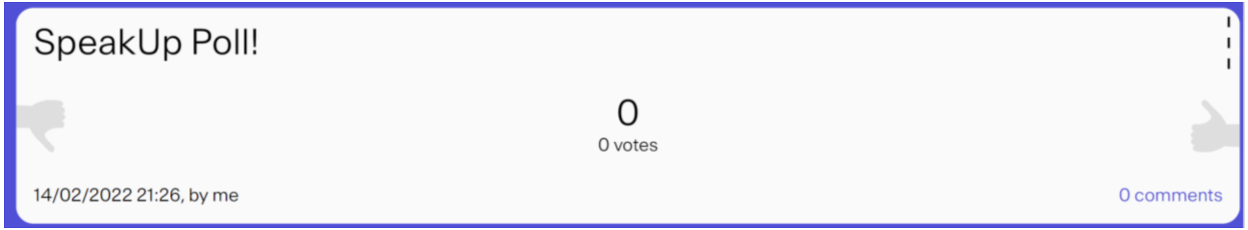
Which of the following statements about k-fold cross validation are wrong? $N$ denotes the number of samples in the data set, $k$ the number of folds:

a) $k$ must always be smaller than $N$.

b) The smaller $k$ is, the more expensive it is to compute the error.

c) Cross validation can be used to tune model hyperparameters.

d) Cross validation is not a valid method for computing the generalization error of a model.

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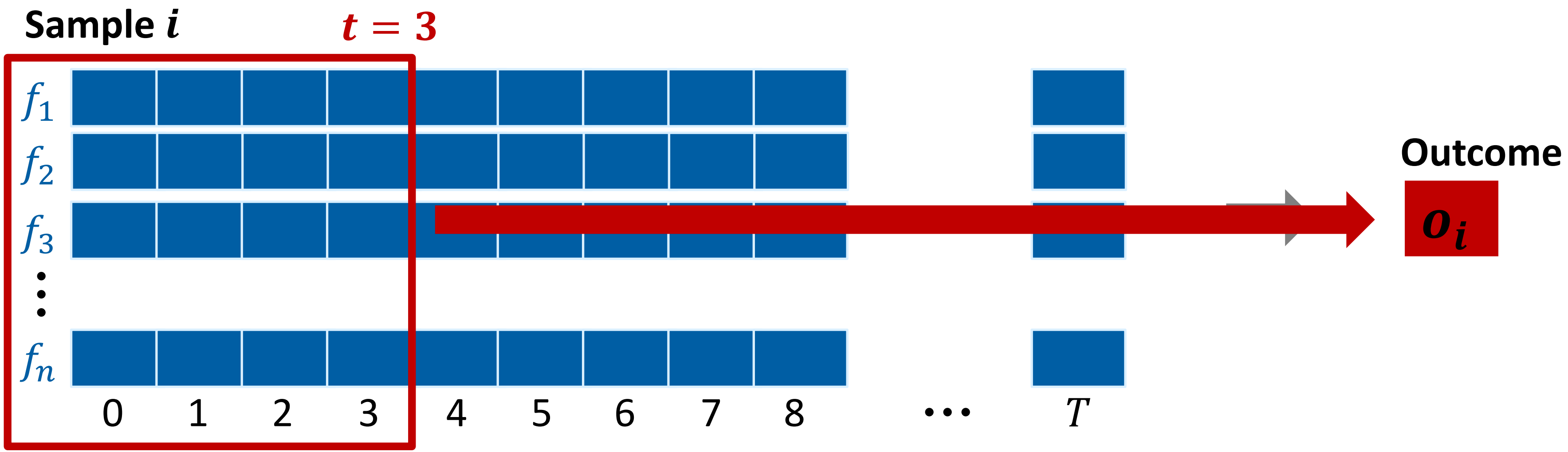
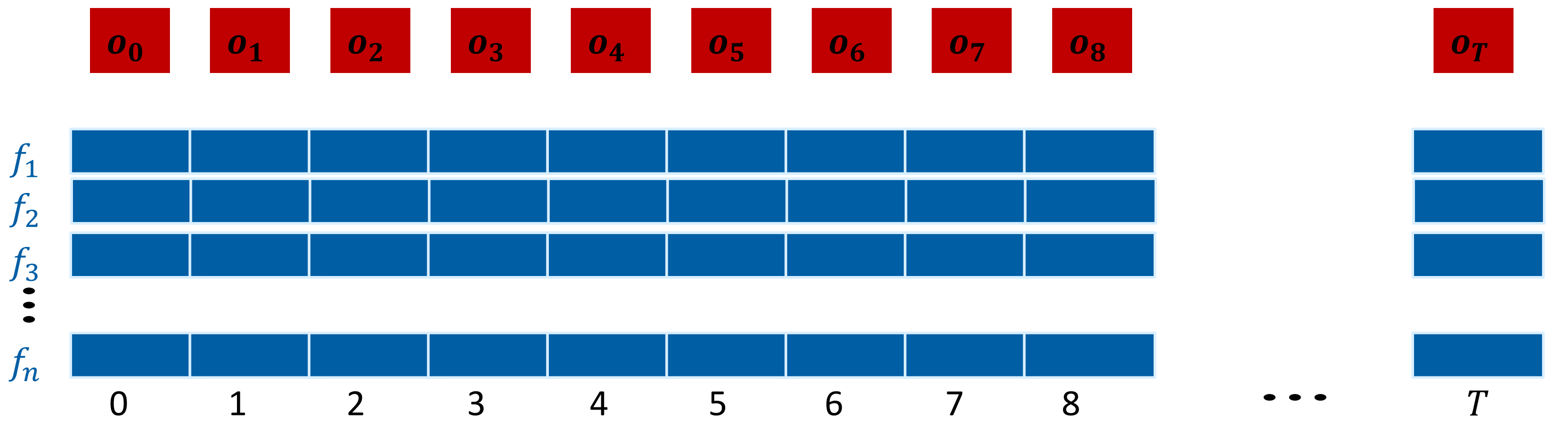
# Time Series – Prediction Task

- Prediction of a target variable after $t < T$ time steps, where $T$ is the total number of time steps



# Time Series – Tracing Task

- Prediction of a target variable after $t < T$ time steps, where $T$ is the total number of time steps
- Prediction of a variable in time step $t+1$, based on time steps $0, \dots, t$



# Today: Tracing Student Knowledge

- Is the student learning?
  - Measure what the student *knows* at a specific time *t*
  - More specifically: knowledge of the student about relevant knowledge components (skills)

| | | | |
|---|---|---|---|
| Task: | $50 - 23 = ?$ | $75 - 12 = ?$ | $38 - 14 = ?$ |
| Answer: | $27$ | $61$ | $24$ |

# Tracing Knowledge – why is it useful?

- Is the student learning?
  - Measure what the student *knows* at a specific time *t*
  - More specifically: knowledge of the student about relevant knowledge components (skills)

➡ Choose the next appropriate activity

➡ Know which activities support learning

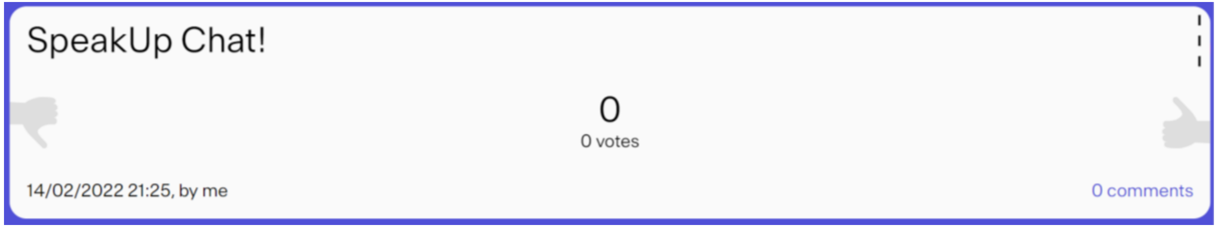
# Today’s Use Case

- ASSISTments is a free tool for assigning and assessing math problems and homework
- All math problems (tasks/items) are associated to a specific skill/knowledge component
- 4,217 middle-school students
- 525,534 observations

# Today: Tracing Student Knowledge

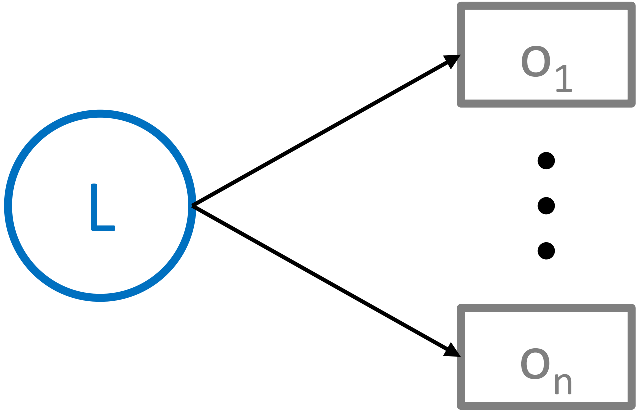
- **Bayesian Knowledge Tracing (BKT)**
  - Latent variables
  - BKT – Inference
  - Practical Example

# What is a latent variable?



# What is a latent variable?

- A latent variable $L$ is a variable which is not directly observable/cannot be measured
- It is assumed to affect the outcome of other variables $\boldsymbol{o}$, which can be observed (directly measured)

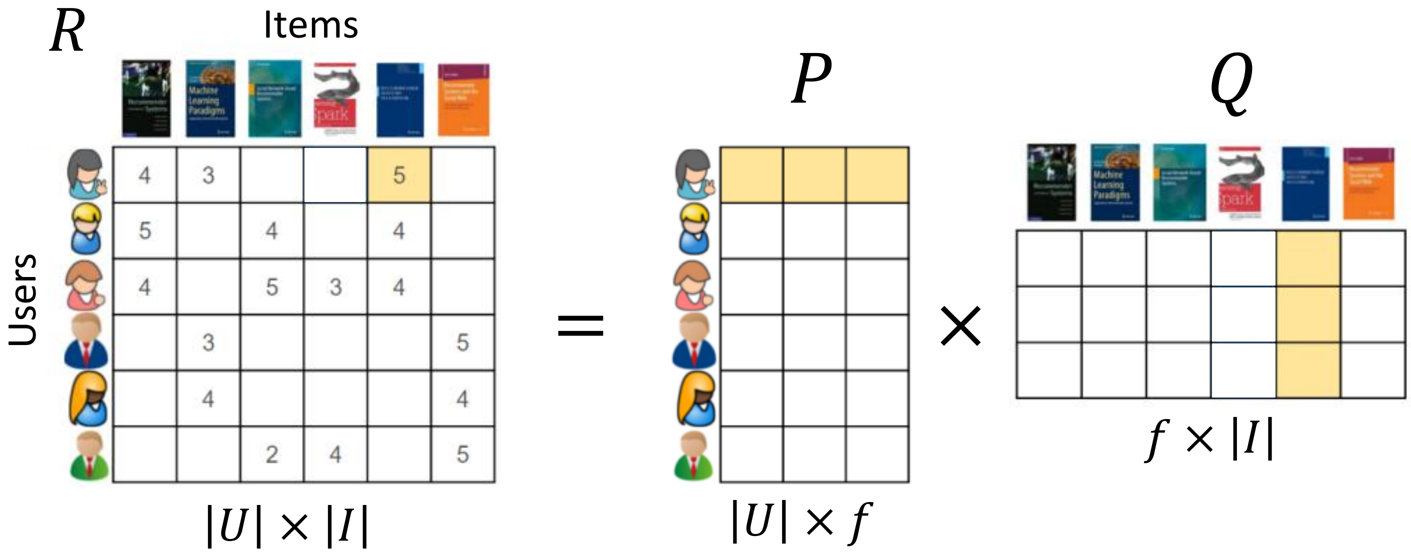


# Why should we use latent variables?

- In many scientific fields, we are interested in concepts/factors that cannot directly be measured/observed:
  - Political sciences: leadership, political competence, etc.
  - Psychology: stress,  self-worth, personality characteristics, talent, etc.
  - Education: memory, spatial ability, cognitive abilities, etc.
- We represent underlying concepts/factors by latent variables and infer them from the observed variables

# Example 1: Recommender Systems

- Given: ratings of users $u$ for items $i$ (e.g., books)




[Image source: https://heartbeat.fritz.ai/recommender-systems-with-python-part-iii-collaborative-filtering-singular-value-decomposition-5b5dcb3f242b]

# Example 2: Education

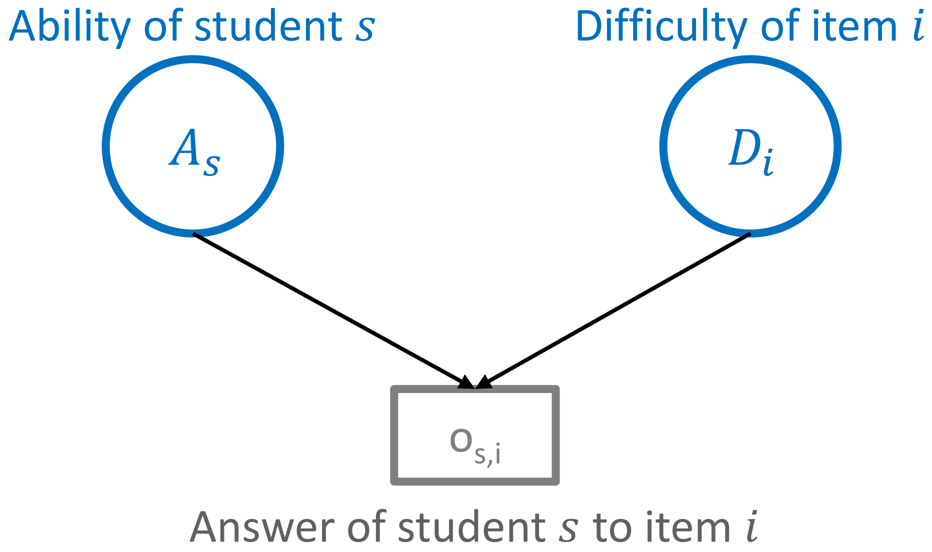
- Observations: binary answers (correct/wrong) of students to items (tasks)

$\boxed{o_{s,i}}$

Answer of student $s$ to item $i$

# Example 2: Education

- Observations: binary answers (correct/wrong) of students to items (tasks)



# Is the student learning?

| | | | |
|---|---|---|---|
| Task: | $50 - 23 = ?$ | $75 - 12 = ?$ | $38 - 14 = ?$ |
| Answer: | $27$ | $61$ | $24$ |

# What are we measuring?

| | | | |
|---|---|---|---|
| Task: | $50 - 23 = ?$ | $75 - 12 = ?$ | $38 - 14 = ?$ |
| Answer: | 27 | 61 | 24 |
| | 1 | 0 | 1 |

# Binary observations of student answers

**Subtraction 0-100**

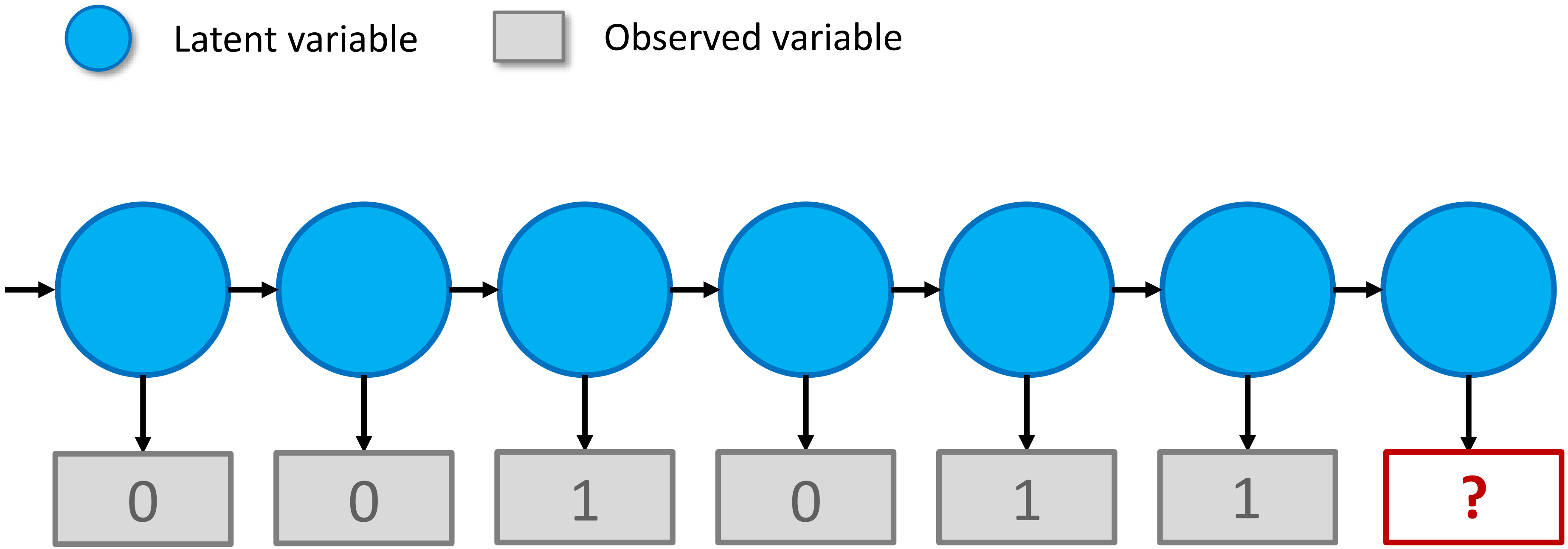
| 1 | 2 | | ••• | | n |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 1 |

# Predicting future performance

**Subtraction 0-100**

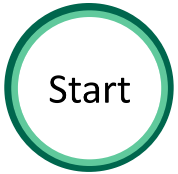
| 1 | 2 | | … | | n | n+1 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 1 | ? |

# Bayesian Knowledge Tracing (BKT)



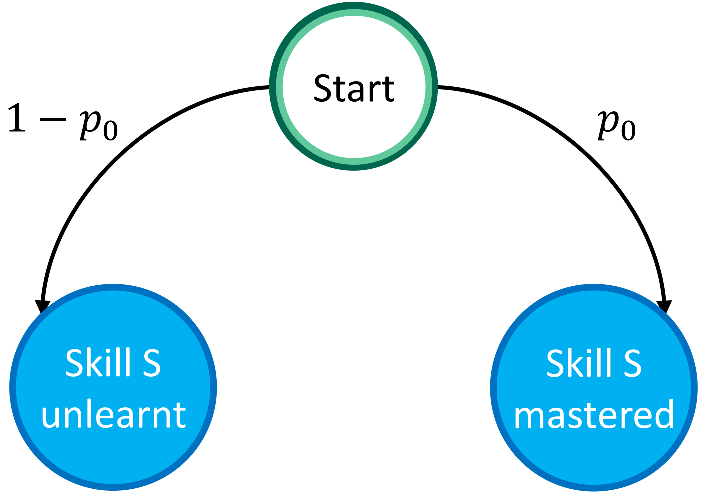

[Corbett & Anderson, UMUAI 1994]

# Bayesian Knowledge Tracing (BKT)



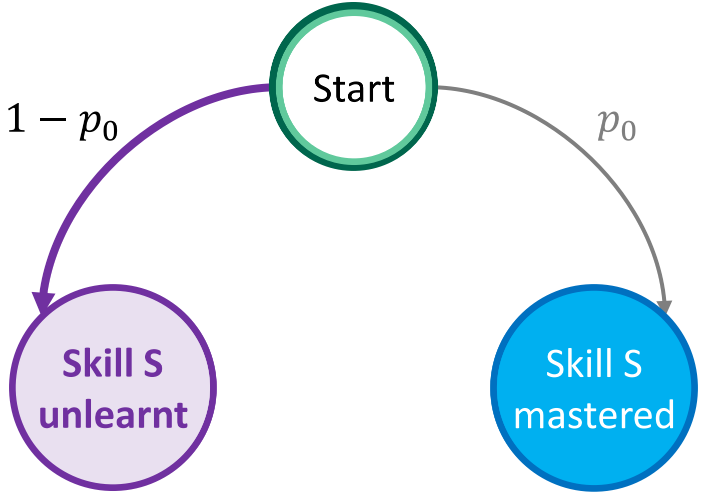

Observations for student $s$:

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

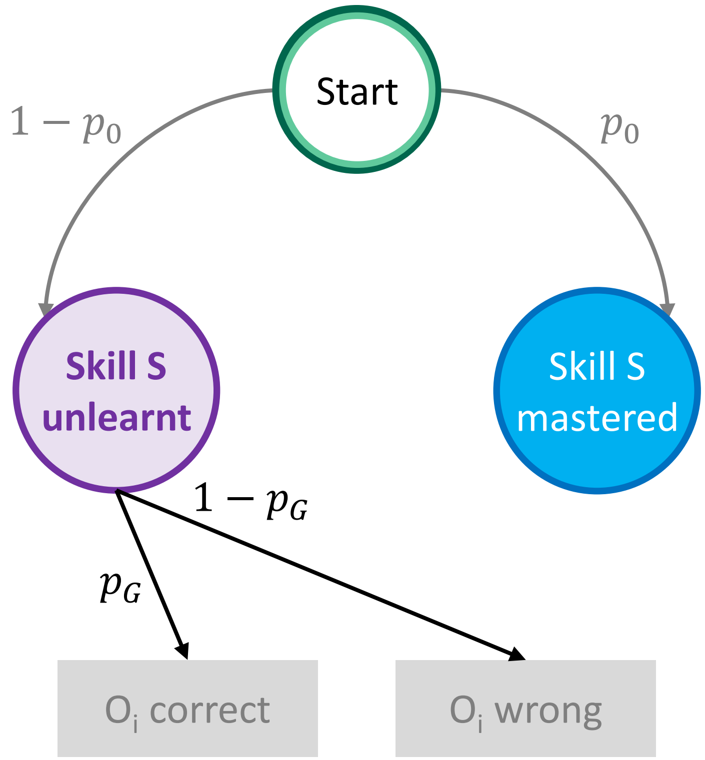
# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$:

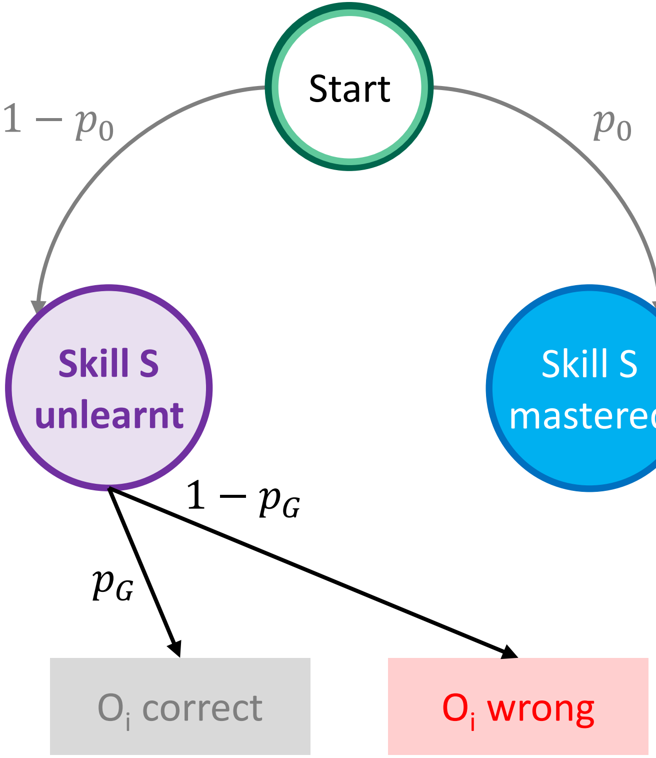
# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$:

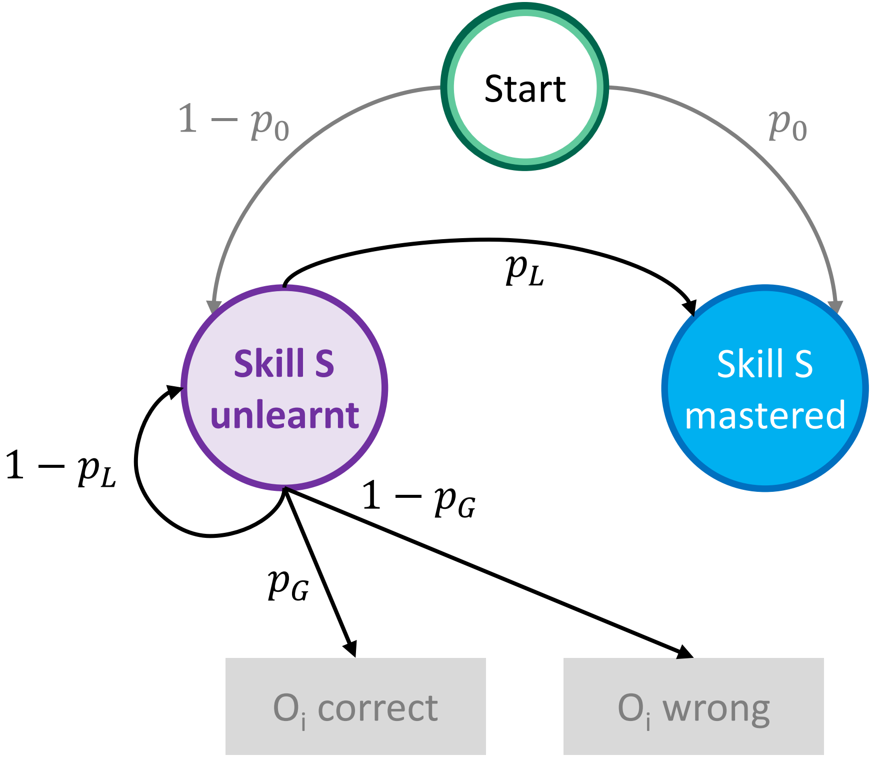
# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

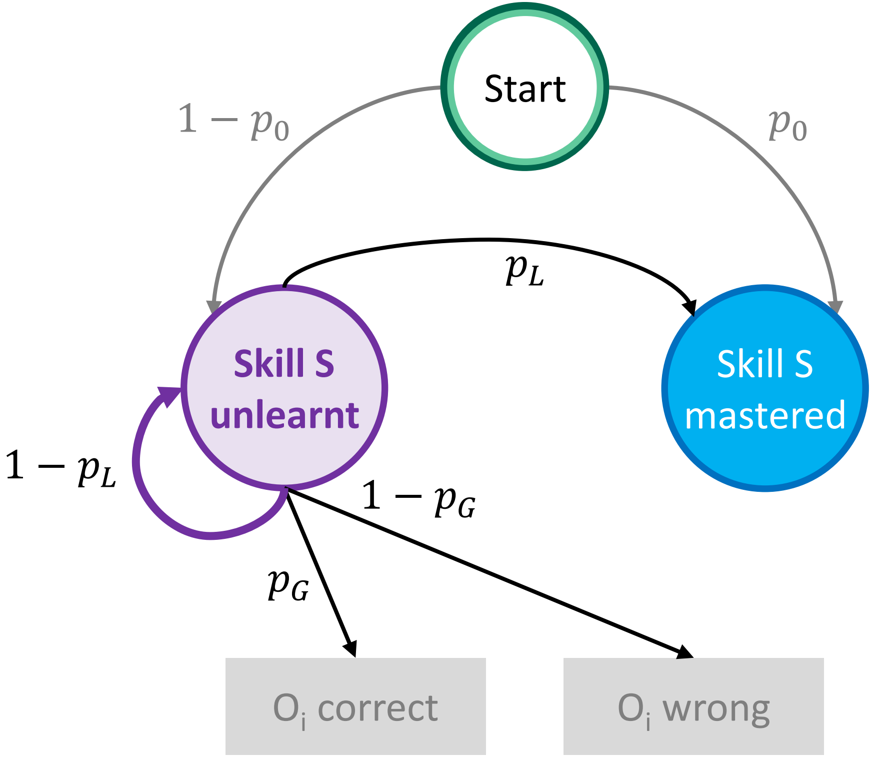
# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

# Bayesian Knowledge Tracing (BKT)

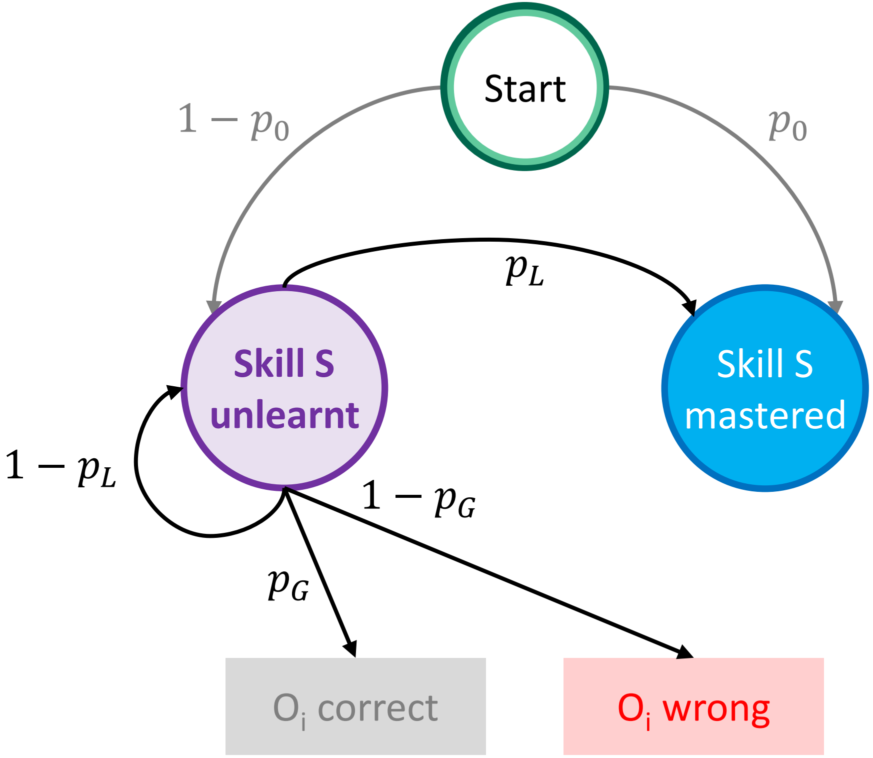



Observations for student $s$:

$t = 0$: 0

$t = 1$:

# Bayesian Knowledge Tracing (BKT)

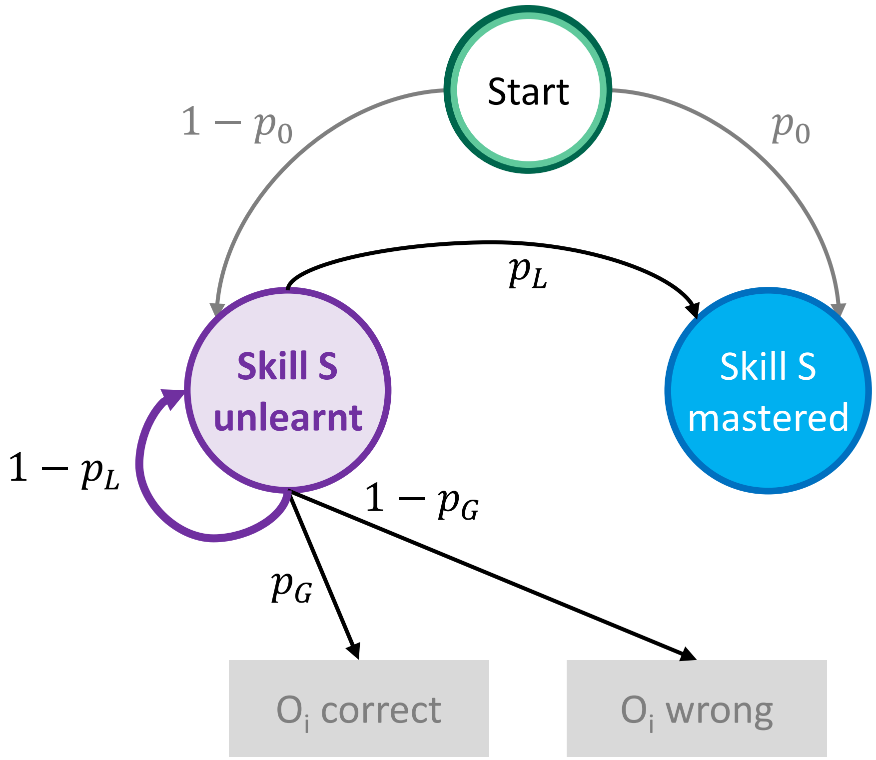



Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

# Bayesian Knowledge Tracing (BKT)



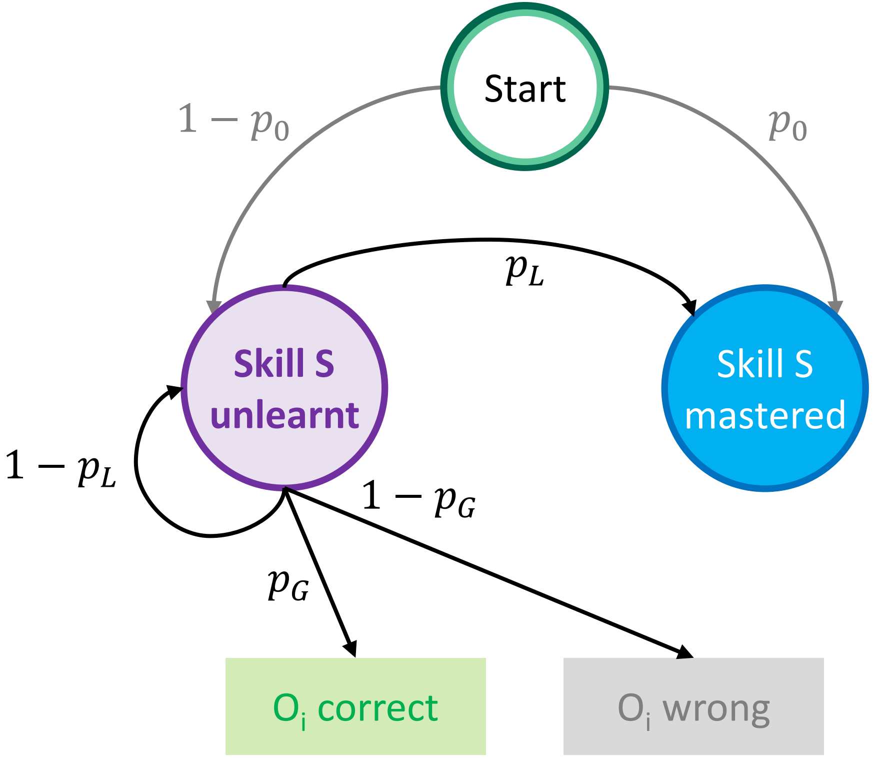

Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

$t = 2$:

# Bayesian Knowledge Tracing (BKT)



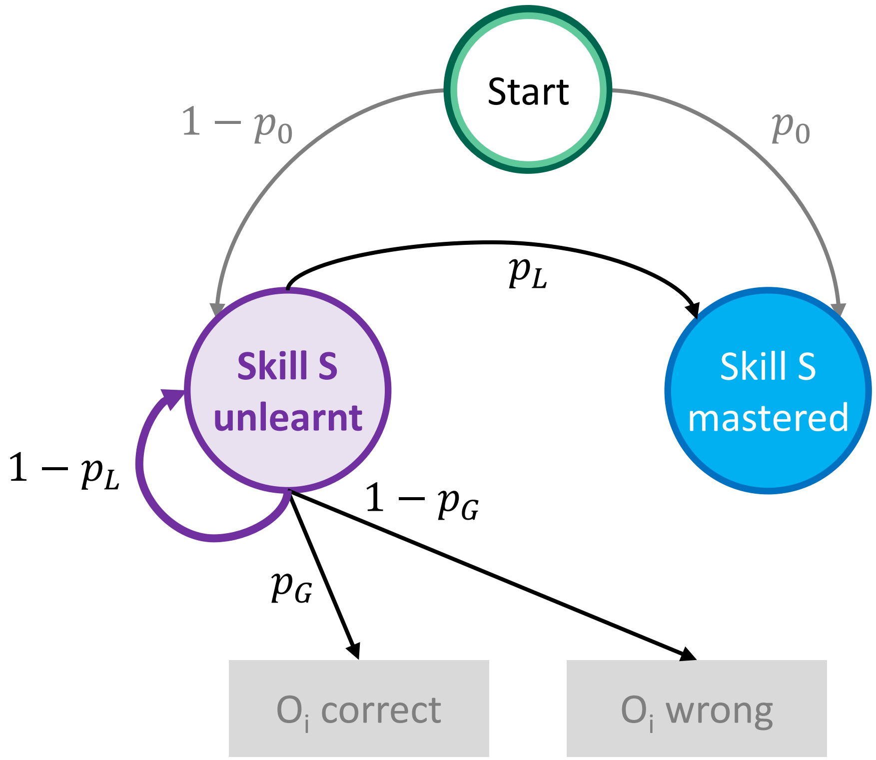

Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

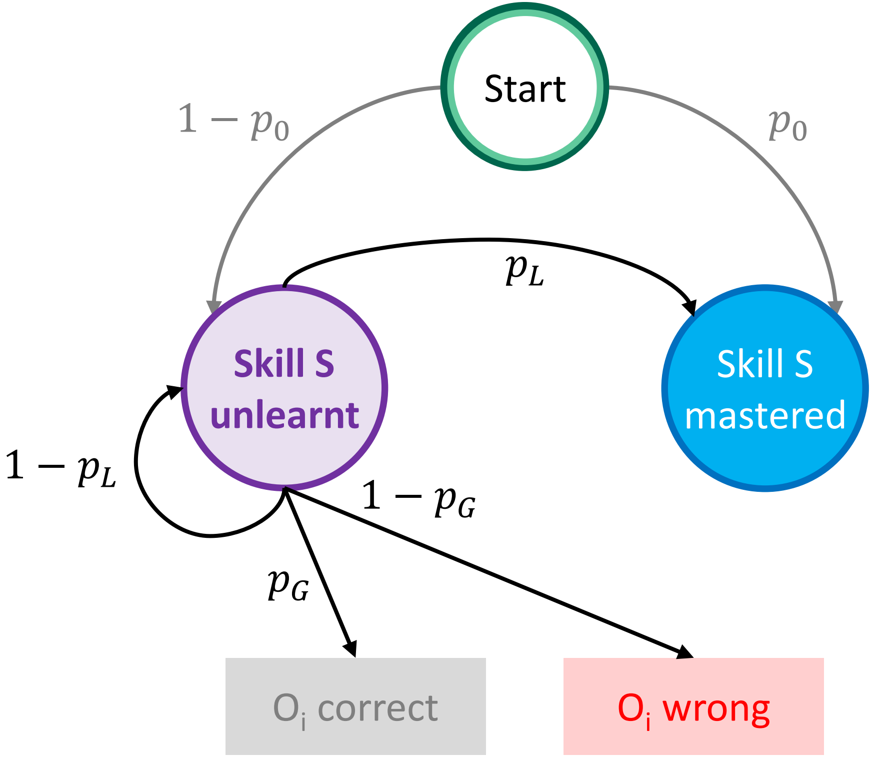
$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

$t = 3$:

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

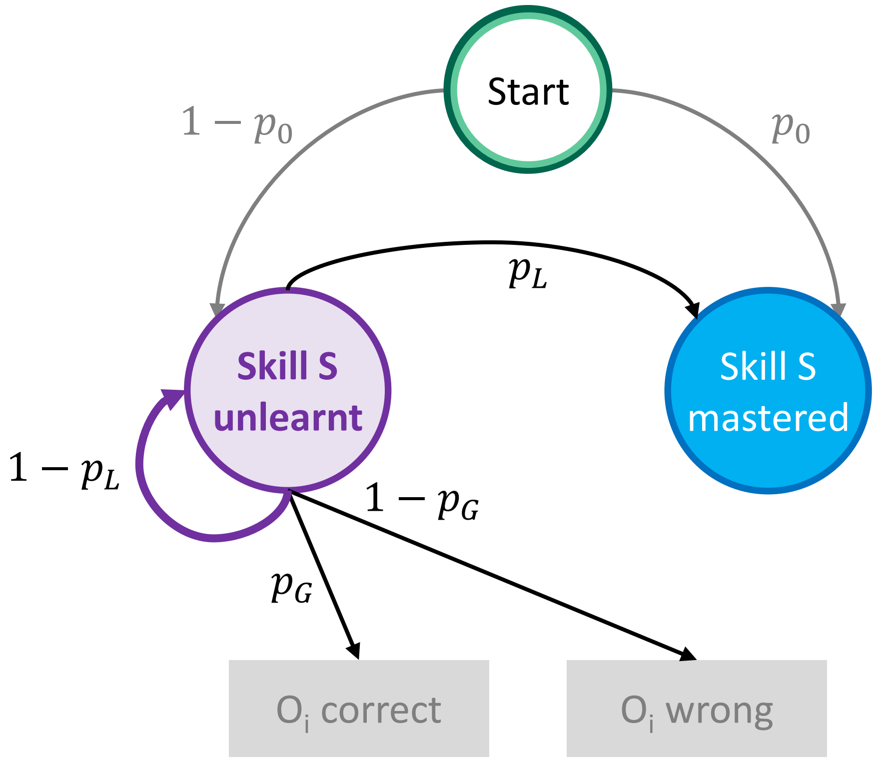
$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

$t = 3$: 0

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

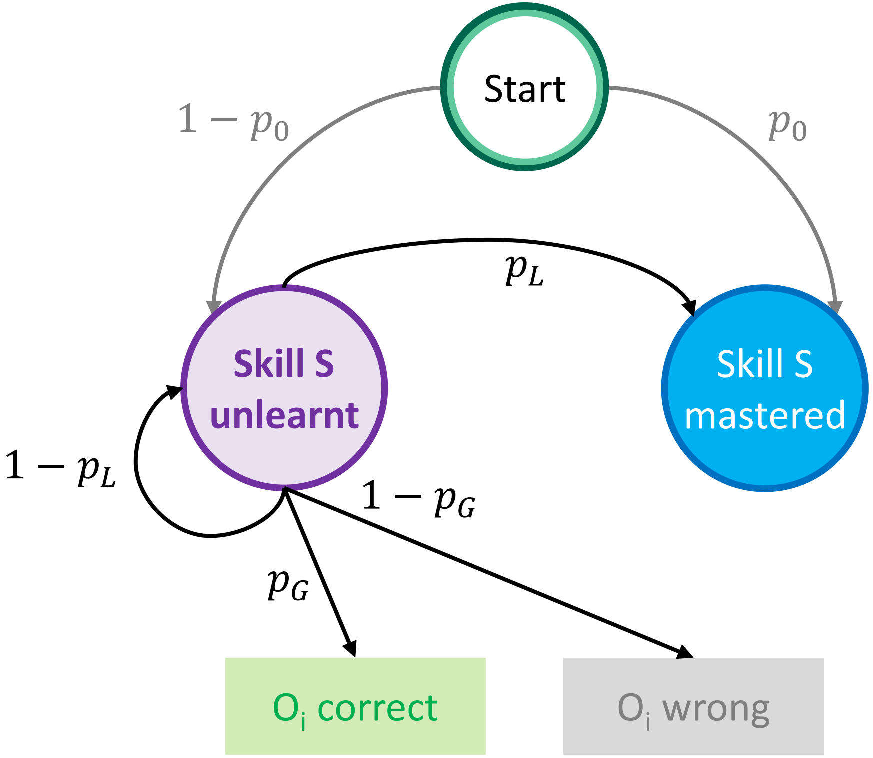
$t = 1$: 0

$t = 2$: 1

$t = 3$: 0

$t = 4$:

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

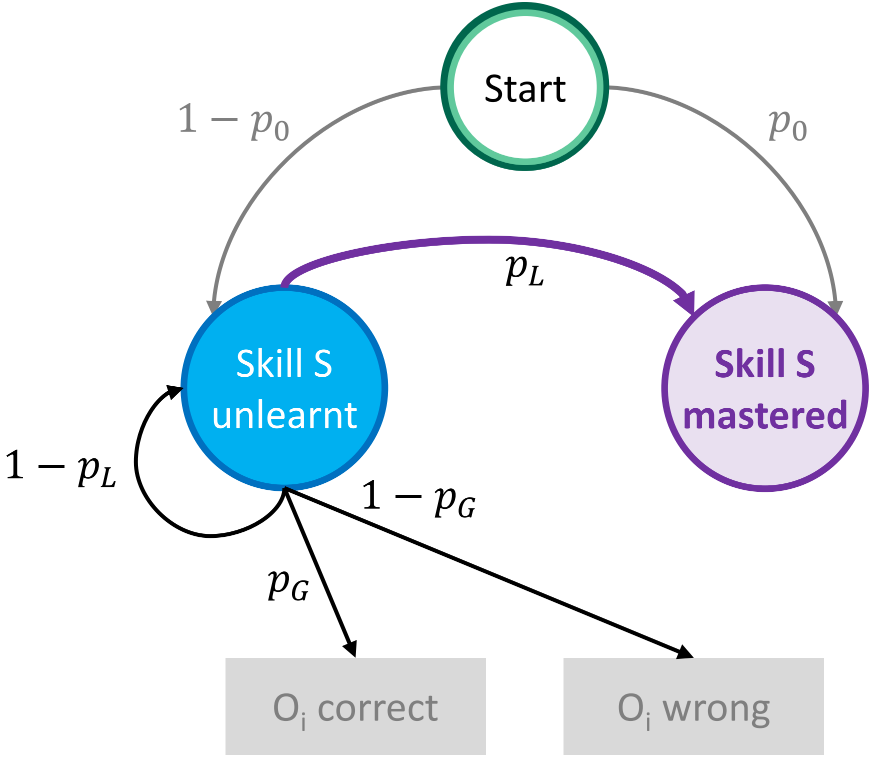
$t = 1$: 0

$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

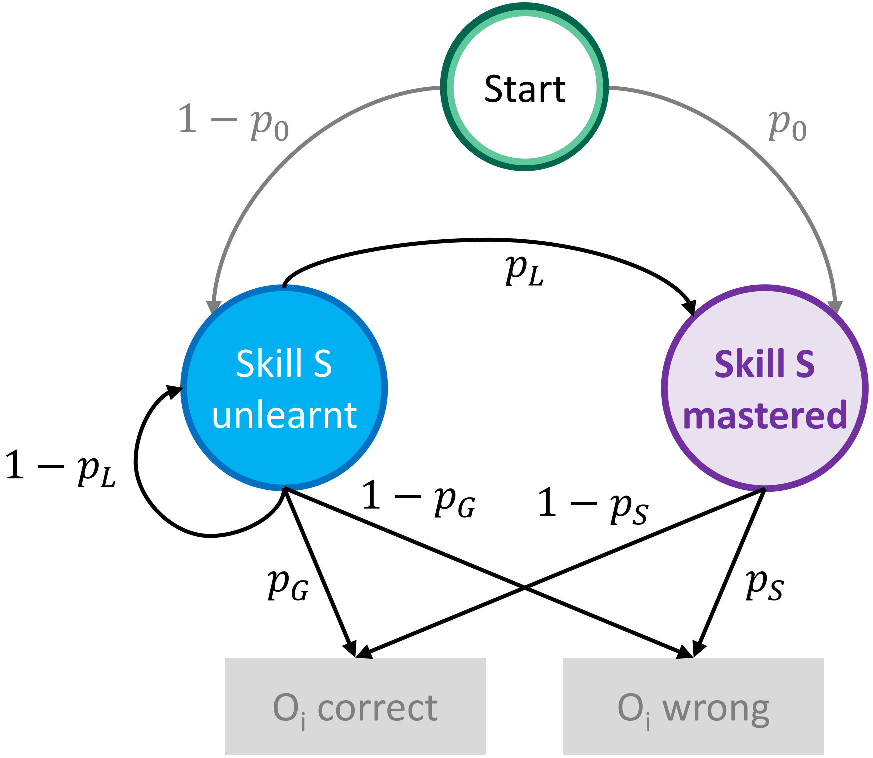
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$:

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

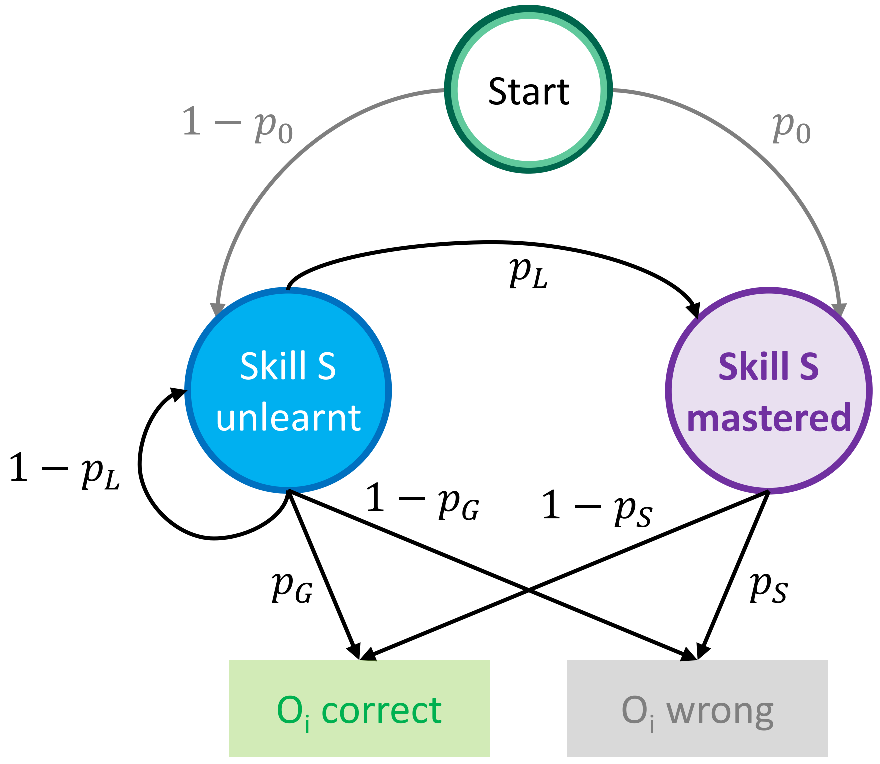
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$:

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

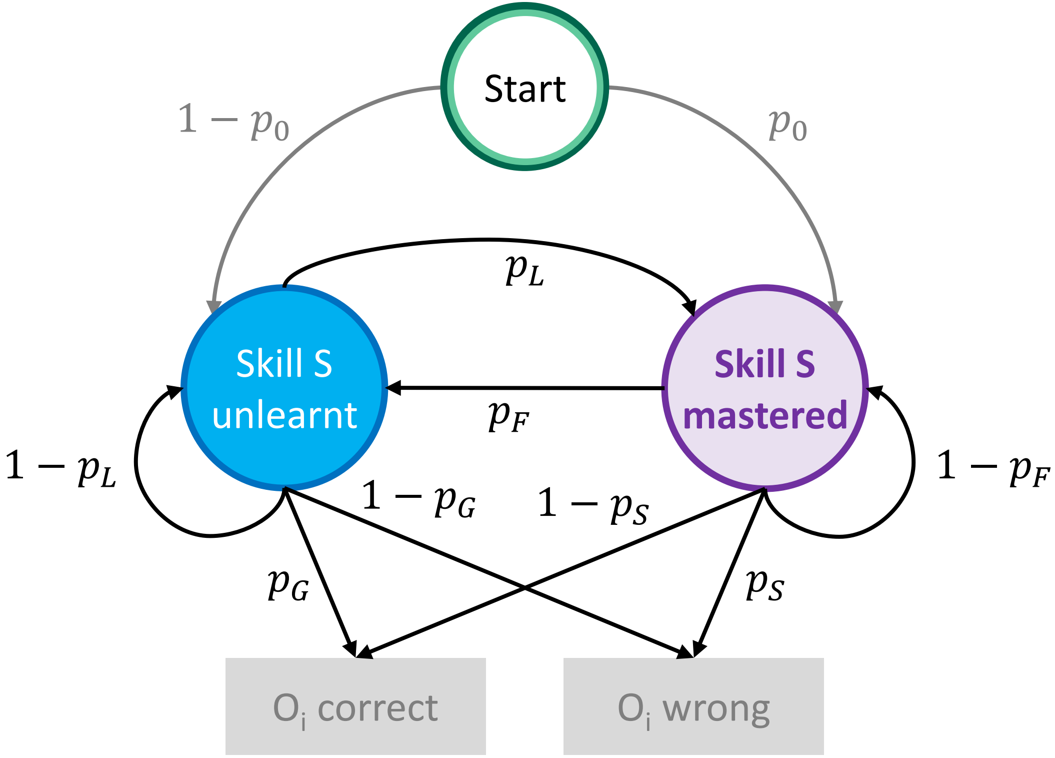
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

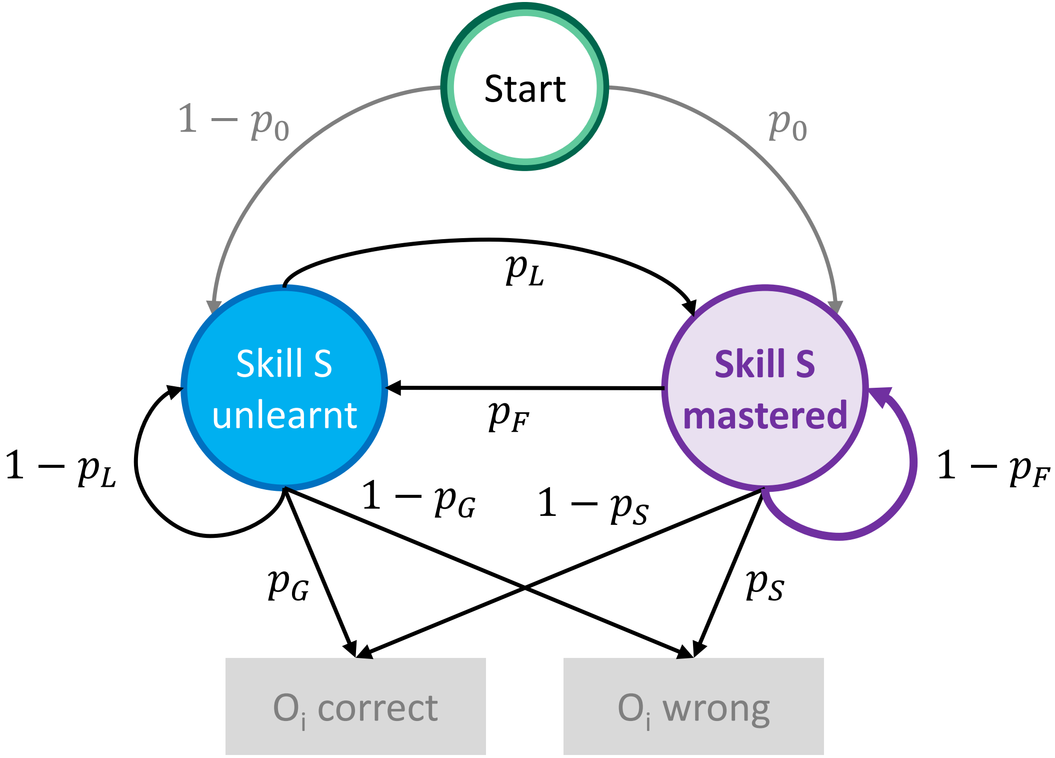
$t = 2$: 1

$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

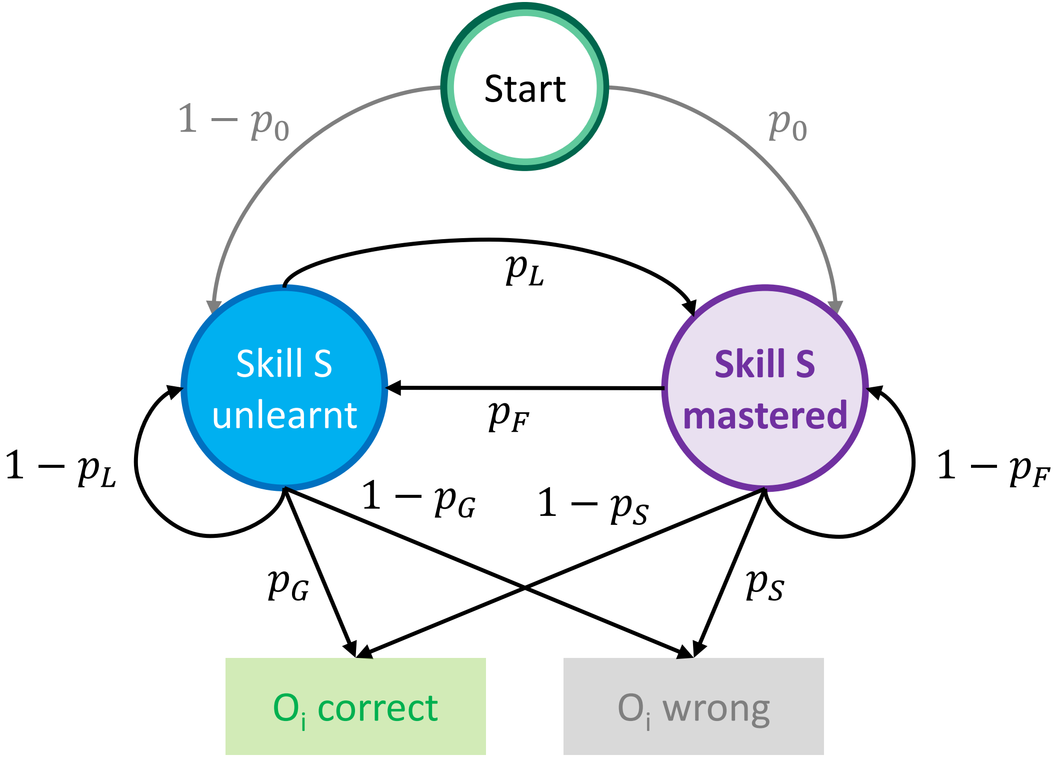
$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

$t = 6$:

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t = 0$: 0

$t = 1$: 0

$t = 2$: 1

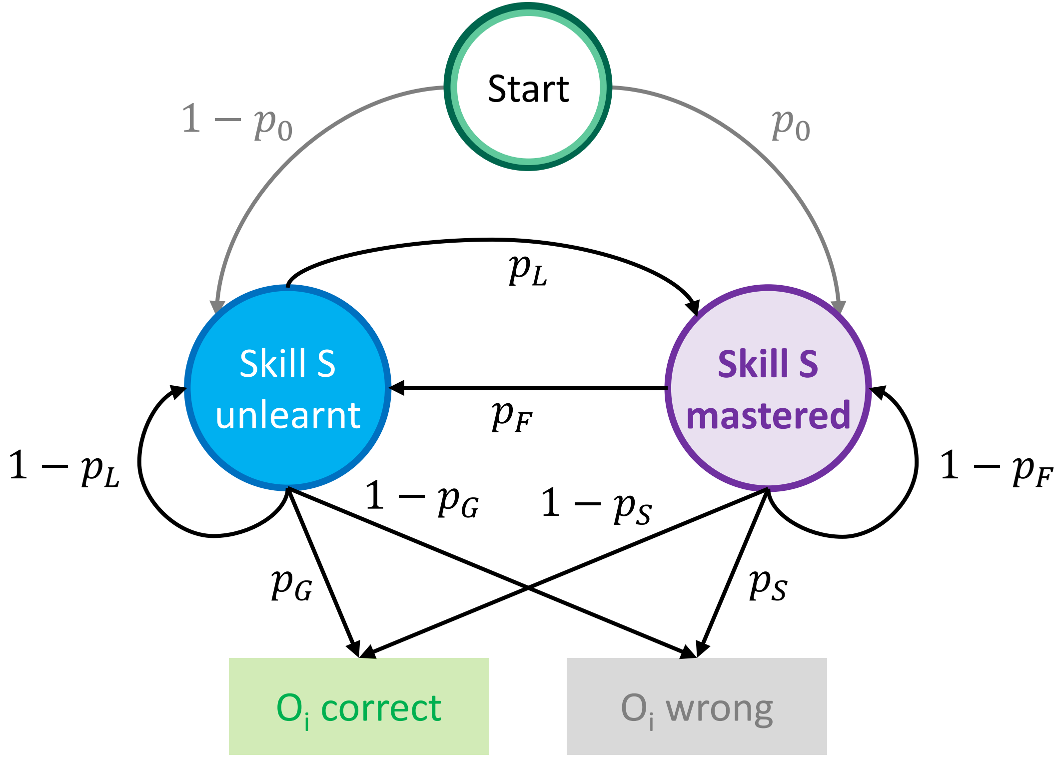
$t = 3$: 0

$t = 4$: 1

$t = 5$: 1

$t = 6$: 1

# Bayesian Knowledge Tracing (BKT)




Observations for student $s$:

$t=0$: 0

$t=1$: 0

$t=2$: 1

$t=3$: 0

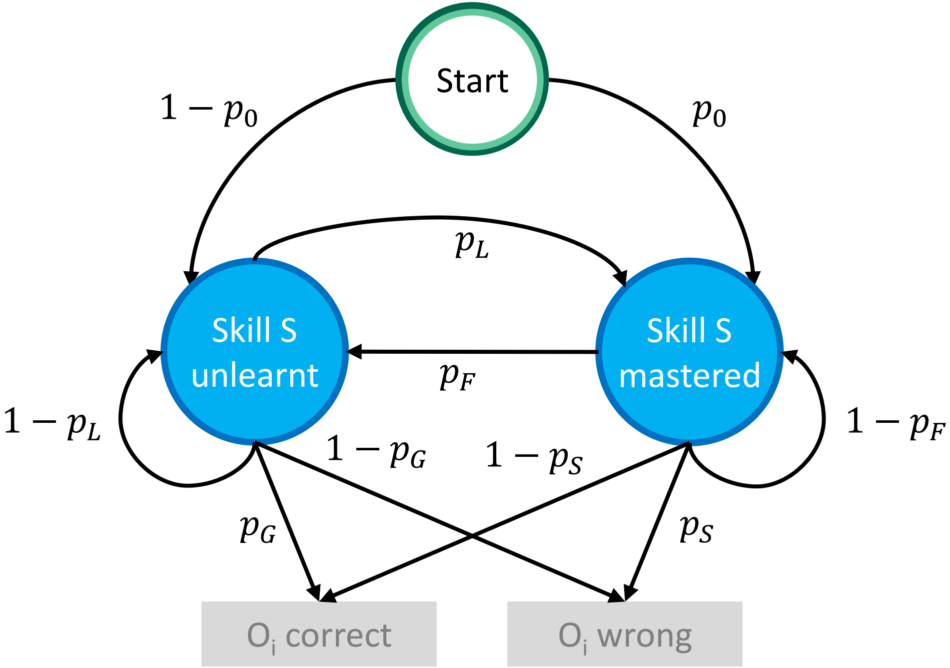
$t=4$: 1

$t=5$: 1

$t=6$: 1

$$\boxed{\boldsymbol{o_s} = [0,0,1,0,1,1,1]}$$

# BKT - Terminology



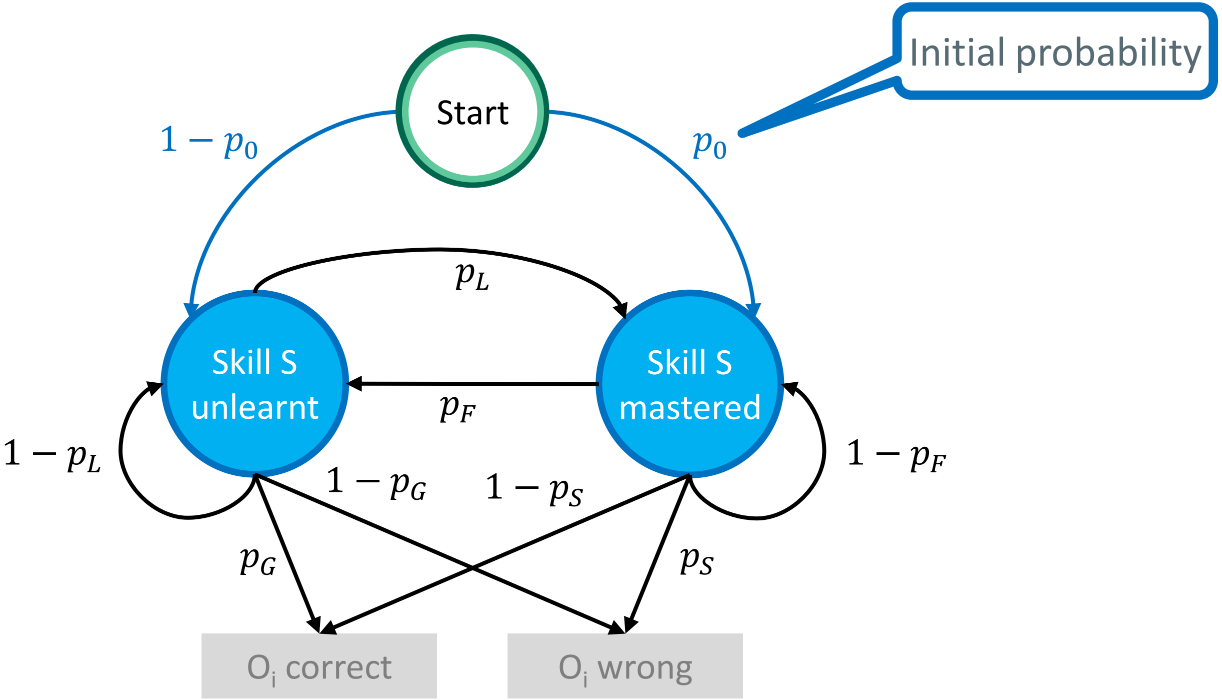

One latent variable ($S$) with two possible states

Observations (also binary)

Five parameters:

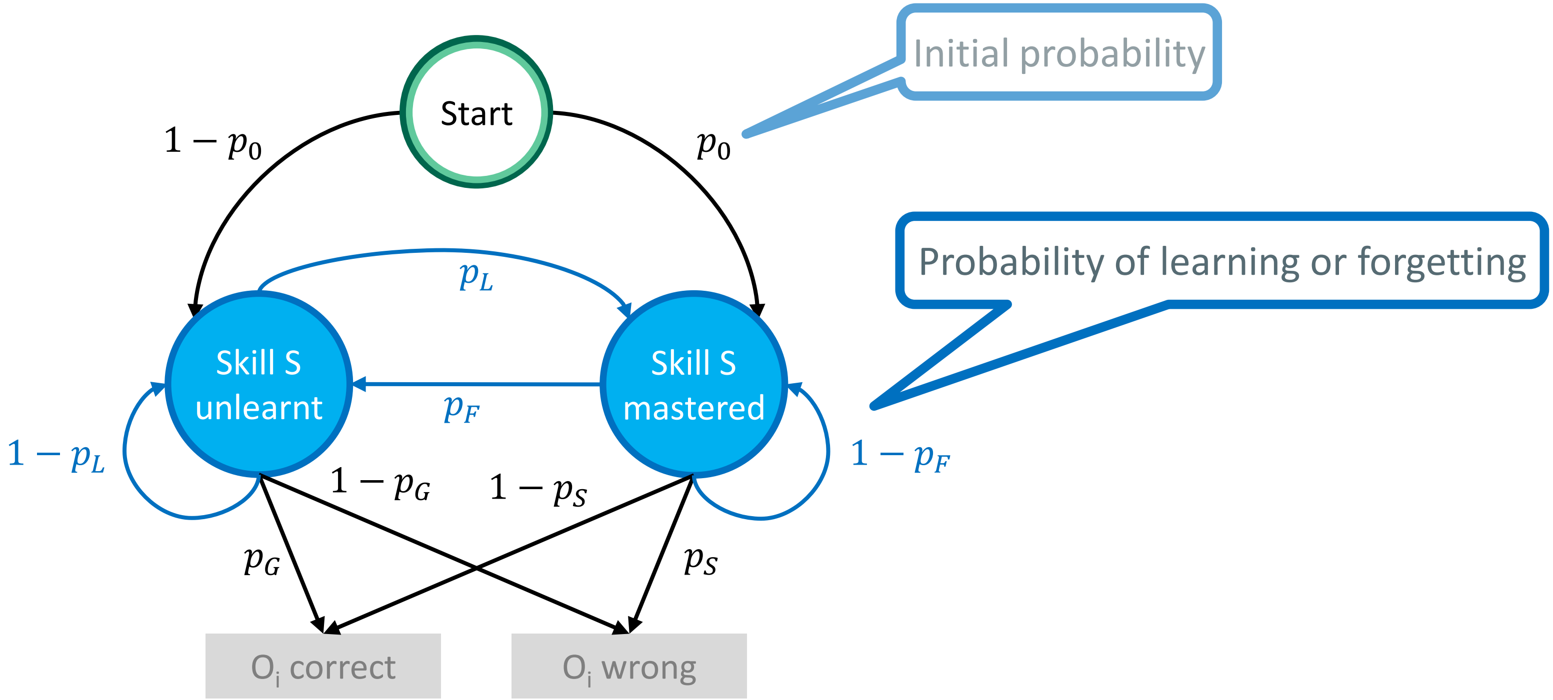
$$\theta = \{p_0, \underbrace{p_L, p_F}_{\text{Transition probabilities}}, \overbrace{p_S, p_G}^{\text{Emission probabilities}}\}$$

# BKT parameters are interpretable




[Corbett & Anderson, UMUAI 1994]

# BKT parameters are interpretable

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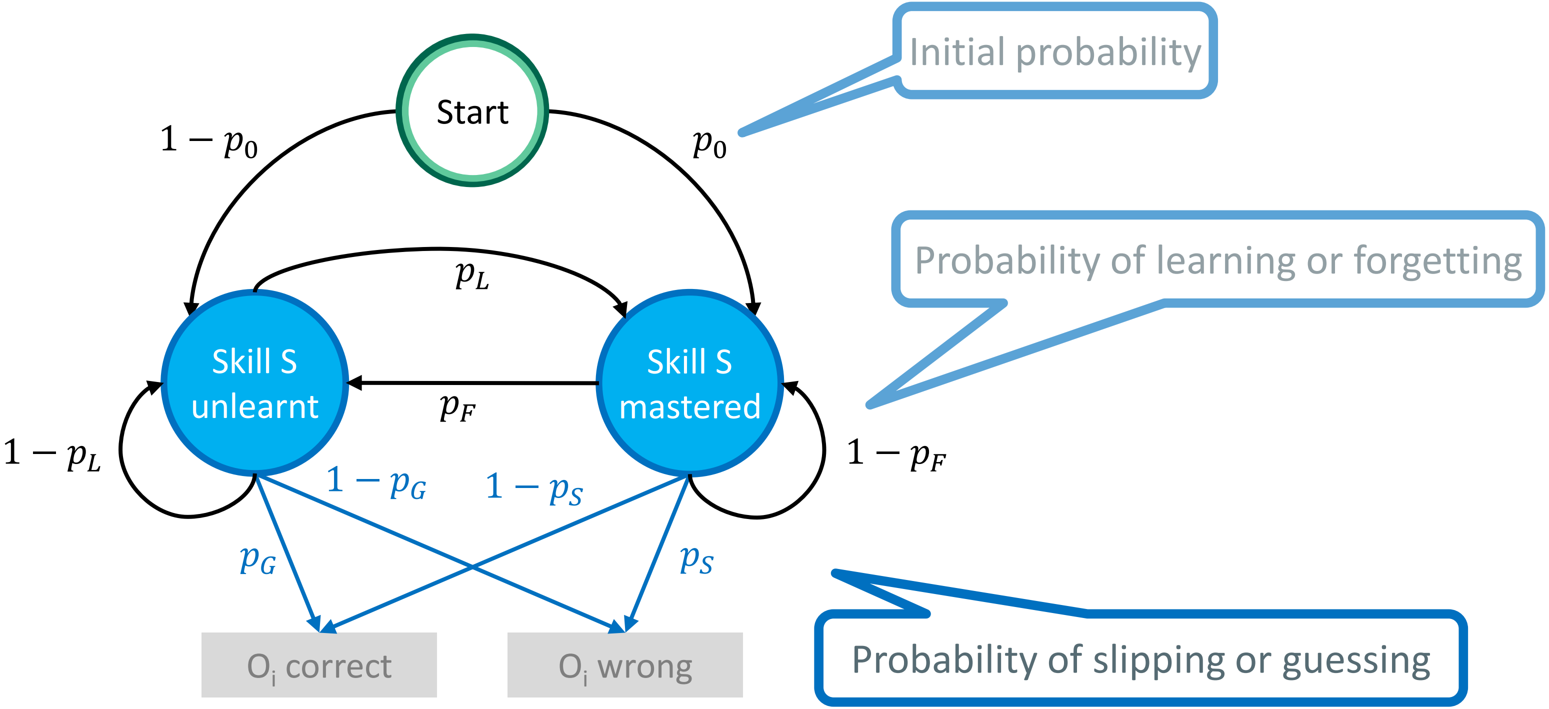
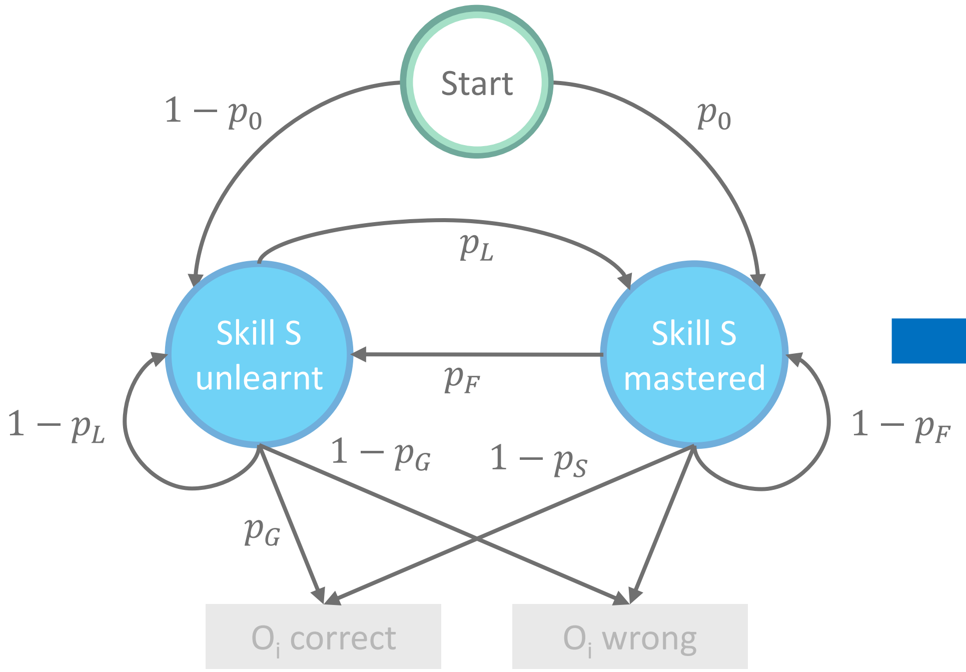
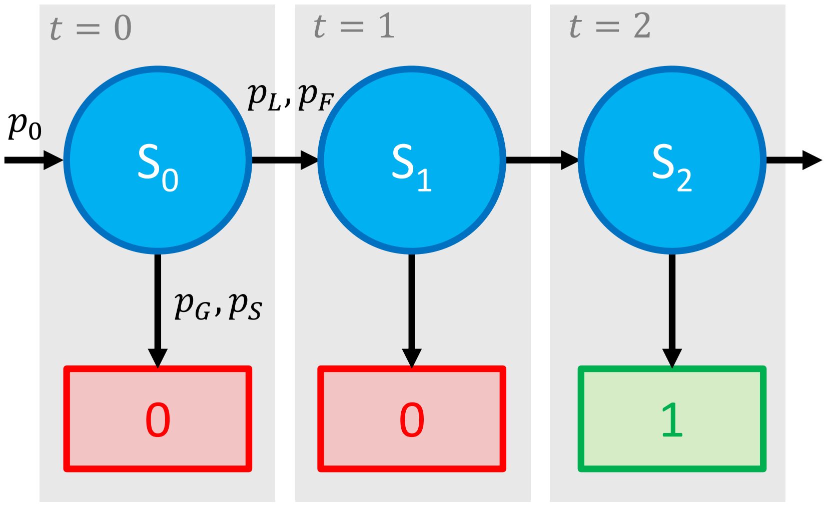
# BKT parameters are interpretable




[Corbett & Anderson, UMUAI 1994]

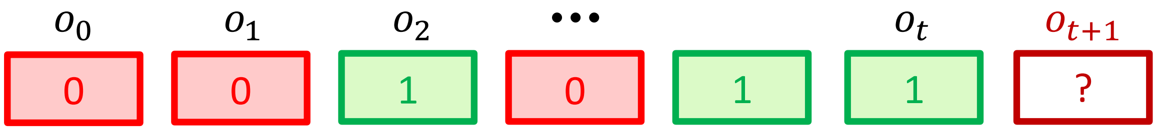
# BKT – unrolled over time



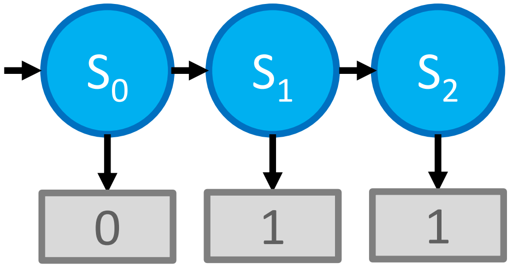
# Two tasks need to be solved in practice

- Given a model with parameters $\theta = \{p_0, p_L, p_F, p_S, p_G\}$ and a sequence of observations $\boldsymbol{o} = [o_0, \dots, o_t]$ from a student $s$, predict $o_{t+1}$



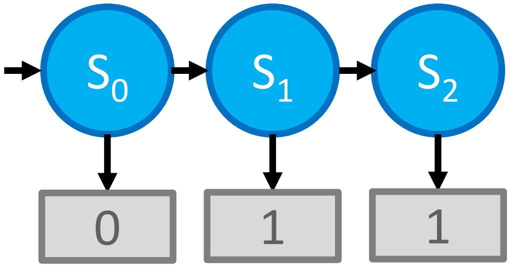

**Inference**

# Inference Example




$p_0 = 0.5$
$p_S = 0.2$
$p_G = 0.3$
$p_L = 0.4$
$p_F = 0.0$

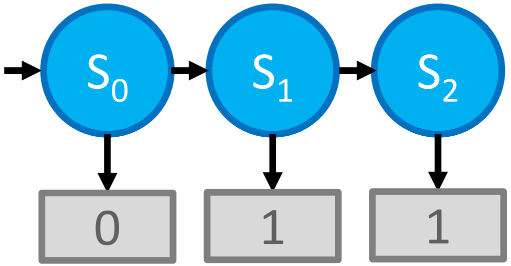
# Inference Example – Your Turn




$p_0 = 0.5$
$p_S = 0.2$
$p_G = 0.3$
$p_L = 0.4$
$p_F = 0.0$

- $p(s_0 = 1)$?
- $p(o_0 = 1)$?
- $p(s_1 = 1 | o_0 = 0)$?
- $p(o_1 = 1 | o_0 = 0)$?
- $p(s_2 = 1 | o_0 = 0, o_1 = 1)$?

# Inference Example – Your Turn




$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$

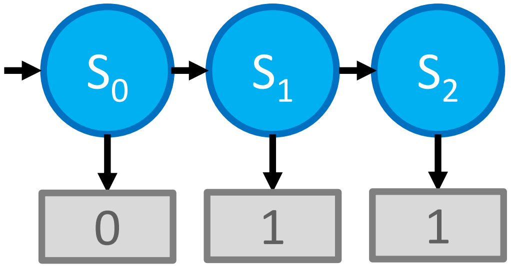
| $S_0$ | $p(S_0)$ |
|---|---|
| 1 | $p_0$ |
| 0 | $1\text{-}p_0$ |

| $S_t$ | $S_{t+1}$ | $p(S_{t+1}\|S_t)$ |
|---|---|---|
| 0 | 0 | $1 - p_L$ |
| 0 | 1 | $p_L$ |
| 1 | 0 | $p_F$ |
| 1 | 1 | $1 - p_F$ |

| $S_t$ | $O_t$ | $p(O_t\|S_t)$ |
|---|---|---|
| 0 | 0 | $1 - p_G$ |
| 0 | 1 | $p_G$ |
| 1 | 0 | $p_S$ |
| 1 | 1 | $1 - p_S$ |

- $p(s_0 = 1)$?
- $p(o_0 = 1)$?
- $p(s_1 = 1 | o_0 = 0)$?
- $p(o_1 = 1 | o_0 = 0)$?
- $p(s_2 = 1 | o_0 = 0, o_1 = 1)$?

# Inference Example – Your Turn




$p_0 = 0.5$
$p_S = 0.2$
$p_G = 0.3$
$p_L = 0.4$
$p_F = 0.0$

| $S_0$ | $p(S_0)$ |
|---|---|
| 1 | $p_0$ |
| 0 | 1-$p_0$ |

| $S_t$ | $S_{t+1}$ | $p(S_{t+1}\|S_t)$ |
|---|---|---|
| 0 | 0 | $1 - p_L$ |
| 0 | 1 | $p_L$ |
| 1 | 0 | $p_F$ |
| 1 | 1 | $1 - p_F$ |

| $S_t$ | $O_t$ | $p(O_t\|S_t)$ |
|---|---|---|
| 0 | 0 | $1 - p_G$ |
| 0 | 1 | $p_G$ |
| 1 | 0 | $p_S$ |
| 1 | 1 | $1 - p_S$ |

**Some useful rules:**

$$p(A, B) = p(A|B) \cdot p(B)$$

$$p(A|B) = \frac{p(B|A) \cdot p(A)}{p(B)}$$

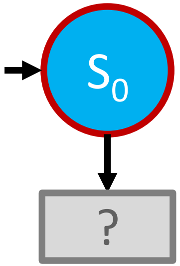
$$p(A = 1) = p(A = 1, B = 1) + p(A = 1, B = 0)$$

- $p(s_0 = 1)$?
- $p(o_0 = 1)$?
- $p(s_1 = 1|o_0 = 0)$?
- $p(o_1 = 1|o_0 = 0)$?
- $p(s_2 = 1|o_0 = 0, o_1 = 1)$?

# Inference in BKT models

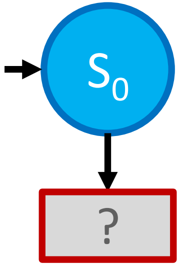
## Equations for time step 0:

$$p(s_0 = 1) = p_0$$
$$p(s_0 = 0) = 1 - p_0$$



# Inference in BKT models

## Equations for time step 0:




$$p(s_0 = 1) = p_0$$
$$p(s_0 = 0) = 1 - p_0$$

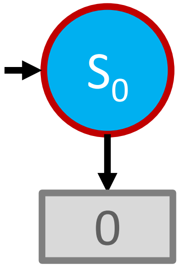
$$\begin{aligned} p(o_0 = 1) &= p(o_0 = 1, s_0 = 1) + p(o_0 = 1, s_0 = 0) \\ &= (1 - p_s) \cdot p_0 + p_G \cdot (1 - p_0) \end{aligned}$$

$$p(o_0 = 0) = p_S \cdot p_0 + (1 - p_G) \cdot (1 - p_0)$$

# Inference in BKT models

$$\underbrace{p(s_0 = 1|o_0 = 0)}_{p_{s_0|0}} = \frac{p(o_0 = 0|s_0 = 1) \cdot p(s_0 = 1)}{p(o_0 = 0)}$$

$$= \frac{p_S \cdot p_0}{p_s \cdot p_0 + (1 - p_G) \cdot (1 - p_0)}$$

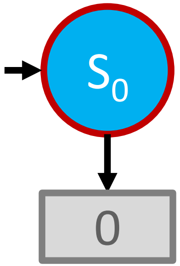



$$p(s_0 = 0|o_0 = 0) = 1 - p_{s_0|0}$$

# Inference in BKT models

$$\underbrace{p(s_0 = 1|o_0 = 1)}_{p_{s_0|1}} = \frac{p(o_0 = 1|s_0 = 1) \cdot p(s_0 = 1)}{p(o_0 = 1)}$$

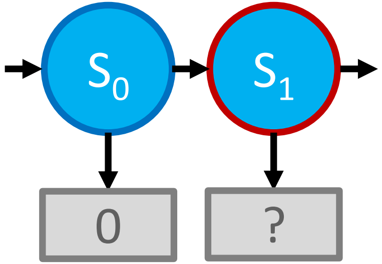
$$= \frac{(1 - p_S) \cdot p_0}{(1 - p_s) \cdot p_0 + p_G \cdot (1 - p_0)}$$




$$p(s_0 = 0|o_0 = 1) = 1 - p_{s_0|1}$$

# Inference in BKT models

## Equations for time step 1:




$$p(s_1 = 1 | o_0 = 0) = \frac{p(s_1 = 1, o_0 = 0)}{p(o_0 = 0)}$$

$$= \frac{p(s_1 = 1, s_0 = 1, o_0 = 0)}{p(o_0 = 0)} + \frac{p(s_1 = 1, s_0 = 0, o_0 = 0)}{p(o_0 = 0)}$$

$$= \frac{p(s_1 = 1 | s_0 = 1) \cdot p(o_0 = 0 | s_0 = 1) \cdot p(s_0 = 1)}{p(o_0 = 0)}$$

$$+ \frac{p(s_1 = 1 | s_0 = 0) \cdot p(o_0 = 0 | s_0 = 0) \cdot p(s_0 = 0)}{p(o_0 = 0)}$$

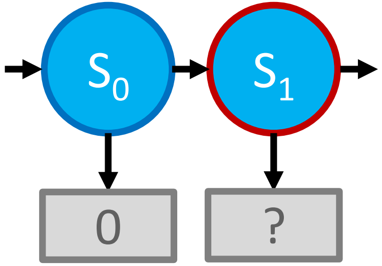
$$p(s_1 = 1 | o_0 = 0) = (1 - p_F) \cdot p_{s_0|0} + p_L \cdot (1 - p_{s_0|0})$$

# Inference in BKT models

$$p(s_1 = 1|o_0 = 1) = (1 - p_F) \cdot p_{s_0|1} + p_L \cdot (1 - p_{s_0|1})$$

$$p(s_1 = 1|o_0 = 0) = (1 - p_F) \cdot p_{s_0|0} + p_L \cdot (1 - p_{s_0|0})$$

$$p_{s_1|o_0} = (1 - p_F) \cdot p_{s_0|o_0} + p_L \cdot (1 - p_{s_0|o_0})$$

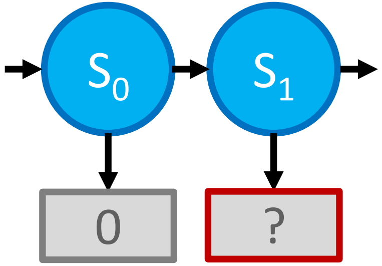


# Inference in BKT models

$$p(o_1 = 1|o_0 = 0) = \frac{p(o_1 = 1, o_0 = 0)}{p(o_0 = 0)}$$

$$= \frac{p(o_1 = 1, s_1 = 1, s_0 = 1, o_0 = 0)}{p(o_0 = 0)} + \frac{p(o_1 = 1, s_1 = 1, s_0 = 0, o_0 = 0)}{p(o_0 = 0)} + \frac{p(o_1 = 1, s_1 = 0, s_0 = 1, o_0 = 0)}{p(o_0 = 0)} + \frac{p(o_1 = 1, s_1 = 0, s_0 = 0, o_0 = 0)}{p(o_0 = 0)}$$

$$= p(o_1 = 1|s_1 = 1) \cdot (p(s_1 = 1|s_0 = 1) \cdot p(o_0 = 0|s_0 = 1) \cdot p(s_0 = 1)/p(o_0 = 0))$$
$$+ p(o_1 = 1|s_1 = 1) \cdot (p(s_1 = 1|s_0 = 0) \cdot p(o_0 = 0|s_0 = 0) \cdot p(s_0 = 0)/p(o_0 = 0))$$
$$+ p(o_1 = 1|s_1 = 0) \cdot (p(s_1 = 0|s_0 = 1) \cdot p(o_0 = 0|s_0 = 1) \cdot p(s_0 = 1)/p(o_0 = 0))$$
$$+ p(o_1 = 1|s_1 = 0) \cdot (p(s_1 = 0|s_0 = 0) \cdot p(o_0 = 0|s_0 = 0) \cdot p(s_0 = 0)/p(o_0 = 0))$$




$$p(o_1 = 1|o_0 = 0) = (1 - p_S) \cdot p_{s_1|o_0=0} + p_G \cdot (1 - p_{s_1|o_0=0})$$

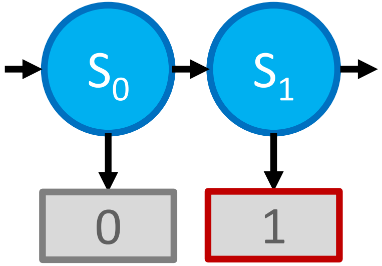
# Inference in BKT models

$$p(o_1 = 1|o_0 = 1) = (1 - p_S) \cdot p_{s_1|o_0=1} + p_G \cdot (1 - p_{s_1|o_0=1})$$

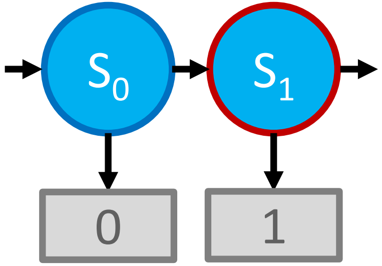
$$p(o_1 = 0|o_0 = 1) = p_S \cdot p_{s_1|o_0=1} + (1 - p_G) \cdot (1 - p_{s_1|o_0=1})$$

$$p(o_1 = 1|o_0) = (1 - p_S) \cdot p_{s_1|o_0} + p_G \cdot (1 - p_{s_1|o_0})$$

$$p(o_1 = 0|o_0) = p_S \cdot p_{s_1|o_0} + (1 - p_G) \cdot (1 - p_{s_1|o_0})$$



# Inference in BKT models




$$p(s_1 = 1 | o_1 = 1, o_0) = \frac{p(s_1 = 1, o_1 = 1, o_0)}{p(o_1 = 1, o_0)}$$

$$\begin{aligned} p(o_1 = 1, o_0) &= p(o_1 = 1 | o_0) \cdot p(o_0) \\ &= ((1 - p_S) \cdot p_{s_1|o_0} + p_G \cdot (1 - p_{s_1|o_0})) \cdot p(o_0) \end{aligned}$$

$$\begin{aligned} p(s_1 = 1, o_1 = 1, o_0) &= p(o_1 = 1 | s_1 = 1\,) \cdot p(s_1 = 1 | s_0 = 1\,) \cdot p(o_0 | s_0 = 1) \cdot p(s_0 = 1) \\ &\quad + p(o_1 = 1 | s_1 = 1\,) \cdot p(s_1 = 1 | s_0 = 0\,) \cdot p(o_0 | s_0 = 0) \cdot p(s_0 = 0) \\ &= (1 - p_s) \cdot p_{s_1|o_0} \cdot p(o_0) \end{aligned}$$

$$p(s_1 = 1 | o_1 = 1, o_0) = \frac{(1 - p_s) \cdot p_{s_1|o_0}}{((1 - p_S) \cdot p_{s_1|o_0} + p_G \cdot (1 - p_{s_1|o_0}))}$$

# Inference in BKT models

$\boldsymbol{o_{t-1}} = [o_0, \dots, o_{t-1}]$

**Equations for time $t = 0$:**

**Equations for time steps $t = 1, \dots, T$:**

**Belief about latent state before observation**

$$p(s_0 = 1) = p_0$$

$$p_{s_t|\boldsymbol{o_{t-1}}} = (1 - p_F) \cdot p_{s_{t-1}|\boldsymbol{o_{t-1}}} + p_L \cdot (1 - p_{s_{t-1}|\boldsymbol{o_{t-1}}})$$

**Predicted observation at time *t***

$$p(o_0 = 1) = (1 - p_s) \cdot p_0 + p_G \cdot (1 - p_0)$$
$$p(o_0 = 0) = p_S \cdot p_0 + (1 - p_G) \cdot (1 - p_0)$$

$$p(o_t = 1|\boldsymbol{o_{t-1}}) = (1 - p_S) \cdot p_{s_t|\boldsymbol{o_{t-1}}} + p_G \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})$$
$$p(o_t = 0|\boldsymbol{o_{t-1}}) = p_S \cdot p_{s_t|\boldsymbol{o_{t-1}}} + (1 - p_G) \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})$$

**Posterior: belief about latent state after observation**

$$\left.\begin{aligned} p_{s_0|1} &= \frac{(1 - p_S) \cdot p_0}{(1 - p_S) \cdot p_0 + p_G \cdot (1 - p_0)} \\ p_{s_0|0} &= \frac{p_S \cdot p_0}{p_S \cdot p_0 + (1 - p_G) \cdot (1 - p_0)} \end{aligned}\right\} p_{s_0|o_0}$$

$$\left.\begin{aligned} p_{s_t|1,\boldsymbol{o_{t-1}}} &= \frac{(1 - p_S) \cdot p_{s_t|\boldsymbol{o_{t-1}}}}{(1 - p_S) \cdot p_{s_t|\boldsymbol{o_{t-1}}} + p_G \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})} \\ p_{s_t|0,\boldsymbol{o_{t-1}}} &= \frac{p_S \cdot p_{s_t|\boldsymbol{o_{t-1}}}}{p_S \cdot p_{s_t|\boldsymbol{o_{t-1}}} + (1 - p_G) \cdot (1 - p_{s_t|\boldsymbol{o_{t-1}}})} \end{aligned}\right\} p_{s_t|\boldsymbol{o_t}}$$

[Corbett & Anderson, UMUAI 1994]

# Making predictions using a BKT model

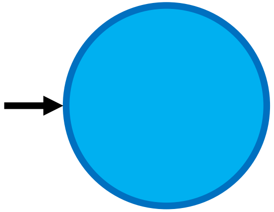
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



# Making predictions using a BKT model

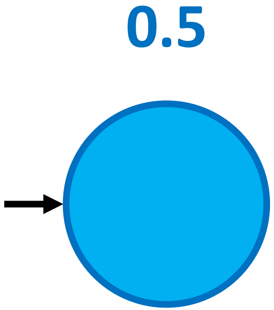
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

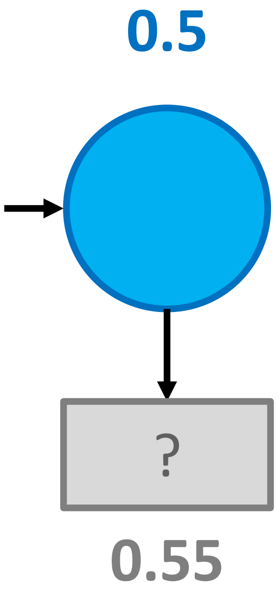
$p_F = 0.0$




$p(s_0 = 1) = p_0$

# Making predictions using a BKT model

$$p_0 = 0.5$$
$$p_S = 0.2$$
$$p_G = 0.3$$
$$p_L = 0.4$$
$$p_F = 0.0$$




$$p(s_0 = 1) = p_0$$

$$p(o_0 = 1) = (1 - p_s) \cdot p_0 + p_G \cdot (1 - p_0)$$

# Making predictions using a BKT model

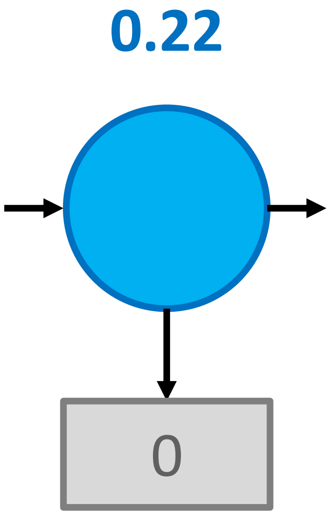
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$




$$p_{s_0|0} = \frac{p_S \cdot p_0}{1 - p(o_0 = 1)} = \frac{0.2 \cdot 0.5}{0.45}$$

# Making predictions using a BKT model

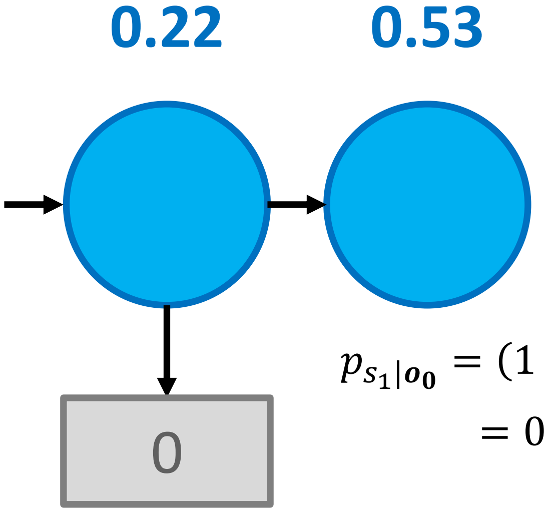
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$




$$p_{s_1|\boldsymbol{o_0}} = (1 - p_F) \cdot p_{s_0|\boldsymbol{o_0}} + \; p_L \cdot (1 - p_{s_0|\boldsymbol{o_0}})$$
$$= 0.22 + 0.4 \cdot 0.78$$

# Making predictions using a BKT model

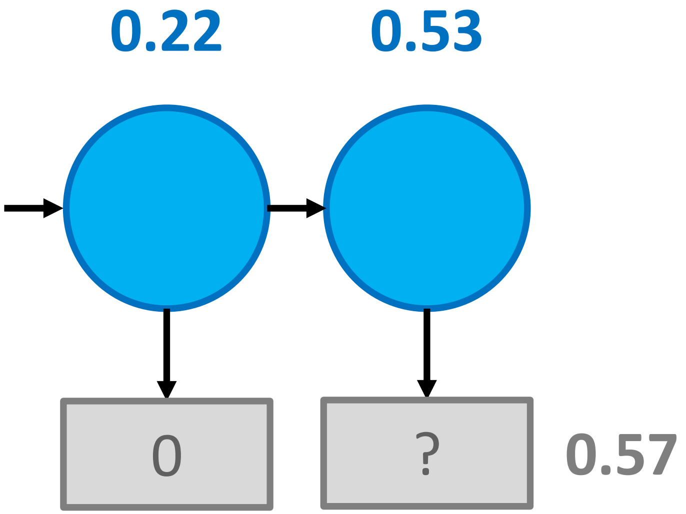
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$




$$p(o_1 = 1|\boldsymbol{o_0}) = (1 - p_S) \cdot p_{s_1|\boldsymbol{o_0}} + p_G \cdot (1 - p_{s_1|\boldsymbol{o_0}})$$
$$= 0.8 \cdot 0.53 + 0.3 \cdot 0.47$$

# Making predictions using a BKT model

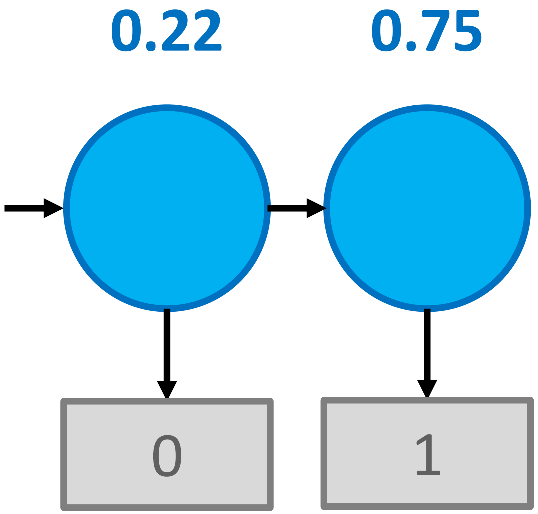
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$




$$p_{s_1|1,\boldsymbol{o_0}} = \frac{(1-p_s)\cdot p_{s_1|\boldsymbol{o_0}}}{(1-p_S)\cdot p_{s_1|\boldsymbol{o_0}} + p_G\cdot(1-p_{s_1|\boldsymbol{o_0}})}$$

$$= \frac{0.8\cdot 53}{0.57}$$

# Making predictions using a BKT model

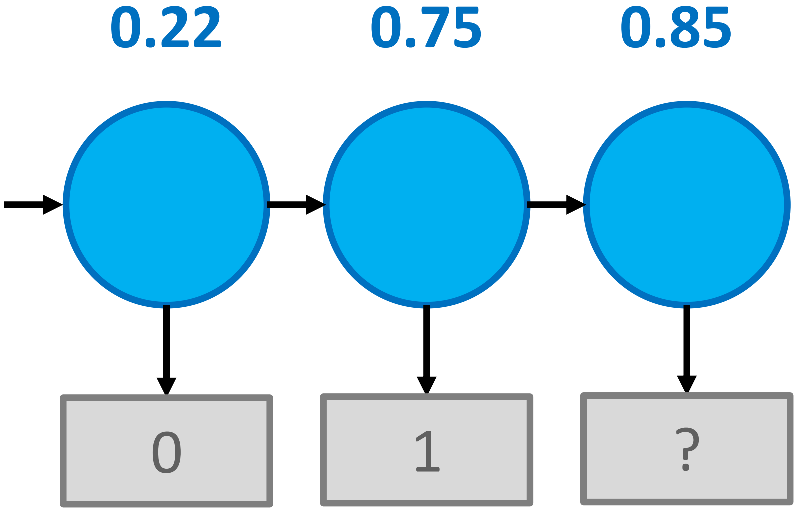
$$p_0 = 0.5$$

$$p_S = 0.2$$

$$p_G = 0.3$$

$$p_L = 0.4$$

$$p_F = 0.0$$



# Making predictions using a BKT model

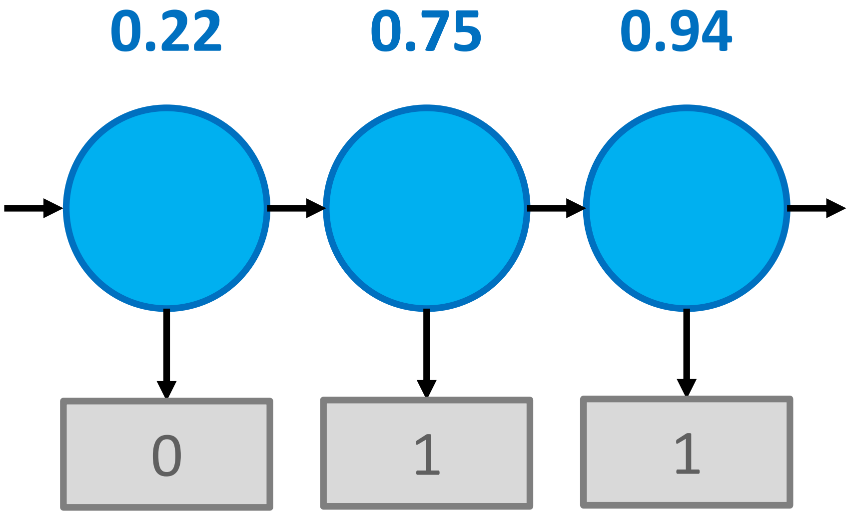
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



# Making predictions using a BKT model

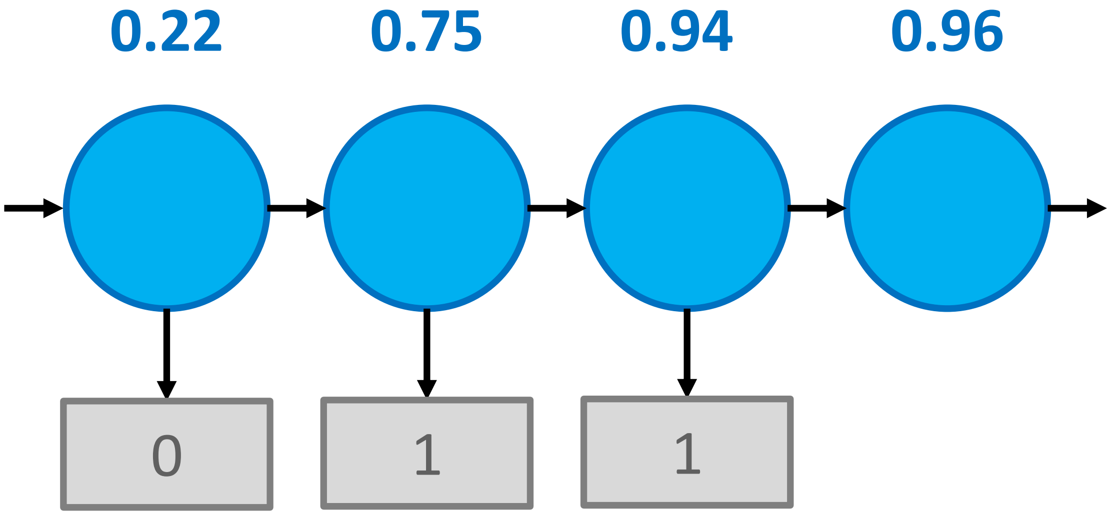
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



# Making predictions using a BKT model

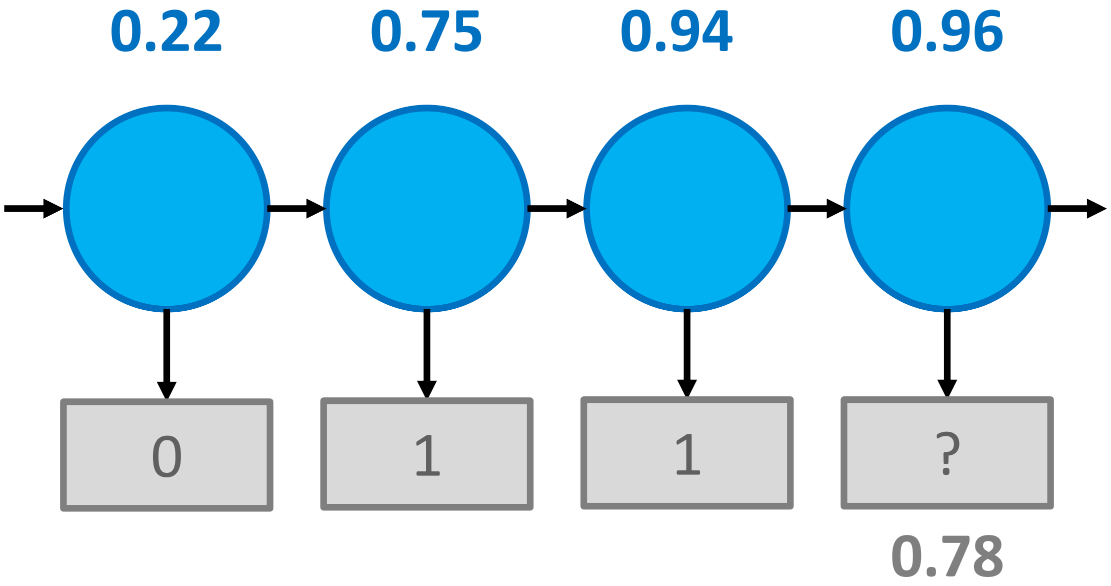
$p_0 = 0.5$

$p_S = 0.2$

$p_G = 0.3$

$p_L = 0.4$

$p_F = 0.0$



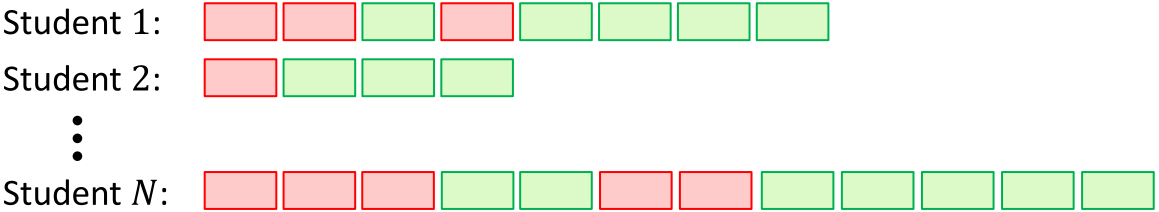
# Two tasks need to be solved in practice

- Given a model with parameters $\theta = \{p_0, p_L, p_F, p_S, p_G\}$ and a sequence of observations $\boldsymbol{o} = [o_0, \ldots, o_t]$ from a student $s$, predict $o_{t+1}$



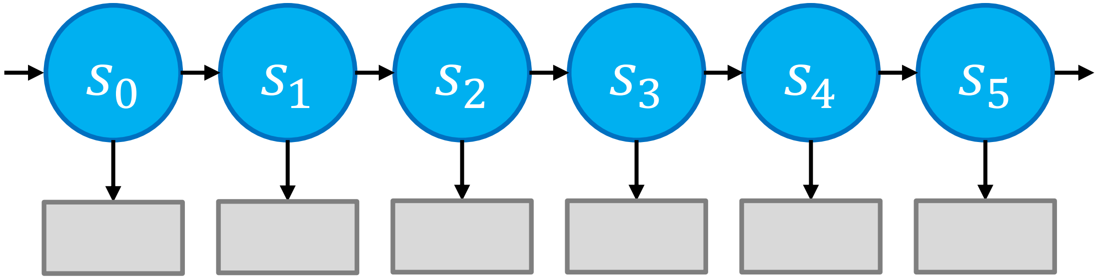

**Inference**

- Given sequences of observations $\boldsymbol{o} = [o_0, \ldots, o_T]$ of $N$ students, learn the parameters $\theta = \{p_0, p_L, p_F, p_S, p_G\}$ that maximize the likelihood of the observed data




**Parameter Learning**

# Training a BKT model




$$\boxed{\theta = \{p_0, p_L, p_F, p_S, p_G\}}$$

Student $l_0$: $\mathbf{o}_{l_0} = [0,1,1]$

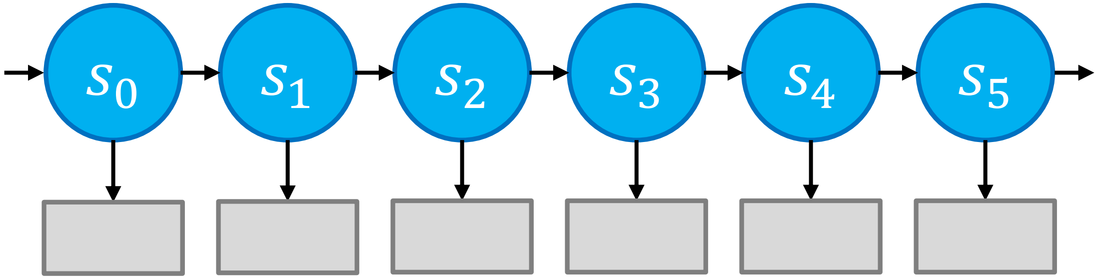
⋮

Student $l_{N-1}$: $\mathbf{o}_{l_{N-1}} = [1,0,1,1,1,0,0,1,1,1]$

Student $l_N$: $\mathbf{o}_{l_N} = [0,1,0,1]$

$$\boxed{\max_{\theta}\ p\left(\mathbf{o}_{l_0}, \ldots, \mathbf{o}_{l_{N-1}}, \mathbf{o}_{l_N}\right)}$$

# Training a BKT model




$$\theta = \{p_0, p_L, p_F, p_S, p_G\}$$

Student $l_0$: $\mathbf{o}_{l_0} = [0,1,1]$

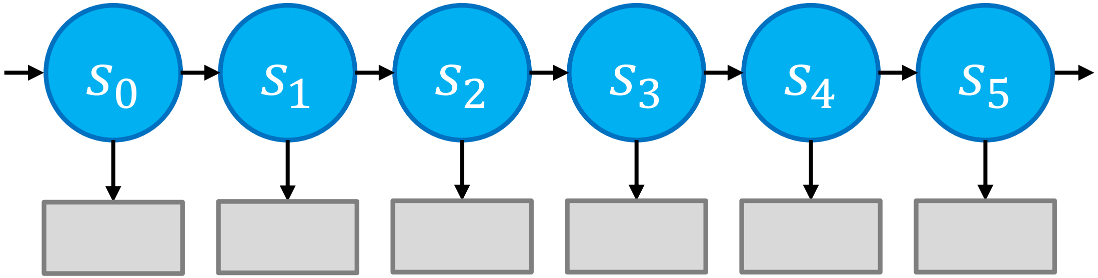
⋮

Student $l_{N-1}$: $\mathbf{o}_{l_{N-1}} = [1,0,1,1,1,0,0,1,1,1]$

Student $l_N$: $\mathbf{o}_{l_N} = [0,1,0,1]$

$$\max_{\theta} \prod_{i=1}^{N} p(\boldsymbol{o}_{l_i})$$

# Training a BKT model




$$\theta = \{p_0, p_L, p_F, p_S, p_G\}$$

Student $l_0$: $p(\mathbf{o}_{l_0}) = \sum_{\boldsymbol{s}} p(\mathbf{o}_{l_0}, \boldsymbol{s})$

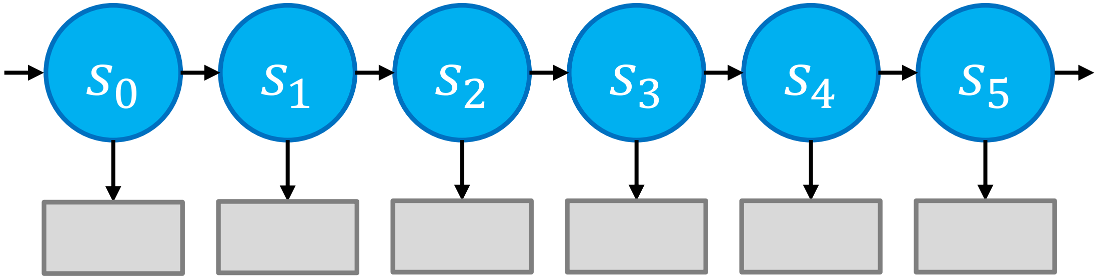
⋮

Student $l_{N-1}$: $p(\mathbf{o}_{l_0}) = \sum_{\boldsymbol{s}} p(\mathbf{o}_{l_0}, \boldsymbol{s})$

Student $l_N$: $p(\mathbf{o}_{l_0}) = \sum_{\boldsymbol{s}} p(\mathbf{o}_{l_0}, \boldsymbol{s})$

$$\max_{\theta} \prod_{i=1}^{N} \sum_{\boldsymbol{s}_{l_i}} p(\mathbf{o}_{l_i}, \boldsymbol{s}_{l_i})$$

# Training a BKT model




$$\theta = \{p_0, p_L, p_F, p_S, p_G\}$$

$$\max_{\theta} \prod_{i=1}^{N} \sum_{\mathbf{s}_{l_i}} p(\mathbf{o}_{l_i}, \mathbf{s}_{l_i}) \Rightarrow \min_{\theta} - \sum_{i=1}^{N} \log\left( \sum_{\mathbf{s}_{l_i}} p(\mathbf{o}_{l_i}, \mathbf{s}_{l_i}) \right)$$

- Brute-Force Grid Search
- Expectation Maximization
- Gradient Descent
- Nelder-Mead Optimization

# Your Turn – Evaluating a BKT model

- In the student notebook, you have:
  - A trained BKT model for six selected skills
  - A data frame containing the predictions of the BKT model for each observation in the test set
- Your task:
  - Compute the RMSE or AUC separately for each skill
  - Provide a visualization of the mean RMSE (or AUC) + standard deviation over all skills as well as the per skill RMSE (or AUC)

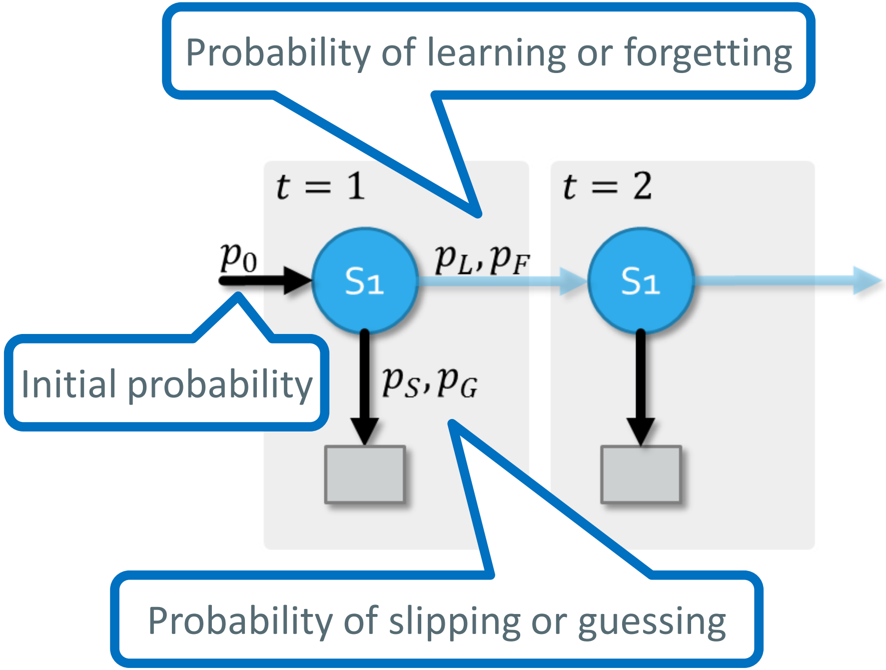
# Summary – Why tracing knowledge?

- Is the student learning?
  - Measure what the student *knows* at a specific time *t*
  - More specifically: knowledge of the student about relevant knowledge components (skills)

➡ Choose the next appropriate activity

➡ Know which activities support learning

# Summary - BKT




- Predict $p(o_{i_{s1},t} | o_0, \dots, o_{t-1})$, the probability that the student will solve task $i_{s1}$ correctly at time step $t$
- Predict $p(s_{1,t} | o_0, \dots, o_{t-1})$, the probability that the student has mastered skill $s_1$ at time step $t$

# Summary - Assumptions behind BKT

- Knowledge can be divided into different skills
- Definition of skills is accurate/detailed enough
- Each task corresponds to a single skill (original)
- There is **no** connection between the skills
- Mastery can be achieved through practice
- There is no forgetting: $p_F = 0$ (original)