

# Recurrent Neural Networks

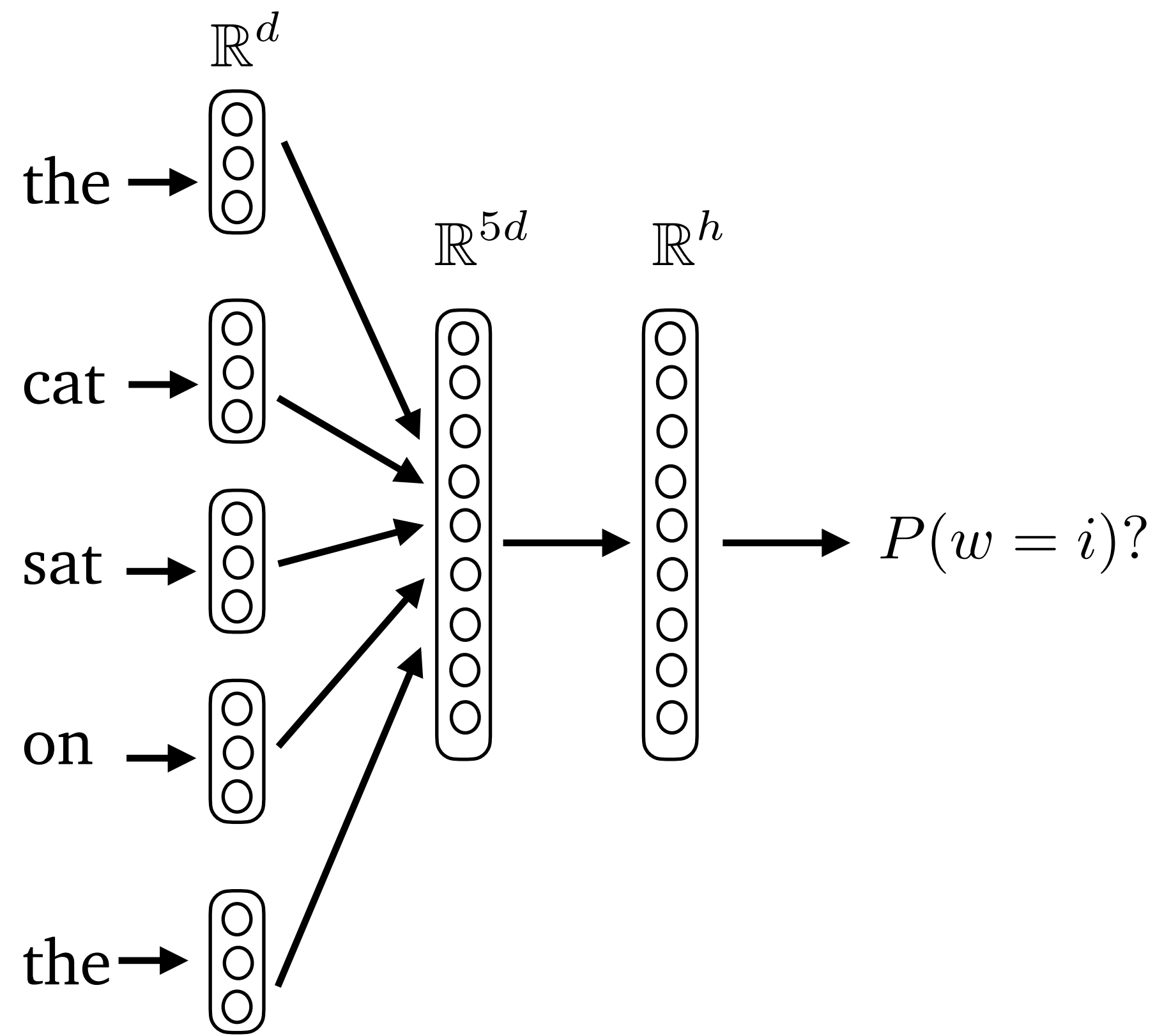
Antoine Bosselut

# Section Outline

- **Fixing the context bottleneck:** recurrent neural networks
- **Training recurrent neural networks:** backpropagation through time
- **Training Challenges:** Vanishing Gradients
- **Mitigations:** LSTMs, GRUs

# Fixed-context Neural Language Models

- $P(\text{mat} \mid \text{the cat sat on the}) = ?$



- Input layer ( $n = 5$ ):

$$\mathbf{x} = [\mathbf{e}_{\text{the}}; \mathbf{e}_{\text{cat}}; \mathbf{e}_{\text{sat}}; \mathbf{e}_{\text{on}}; \mathbf{e}_{\text{the}}] \in \mathbb{R}^{dn}$$

- Hidden layer

$$\mathbf{h} = \tanh(\mathbf{W}\mathbf{x} + \mathbf{b}) \in \mathbb{R}^h$$

- Output layer (softmax)

$$\mathbf{z} = \mathbf{U}\mathbf{h} \in \mathbb{R}^{|V|}$$

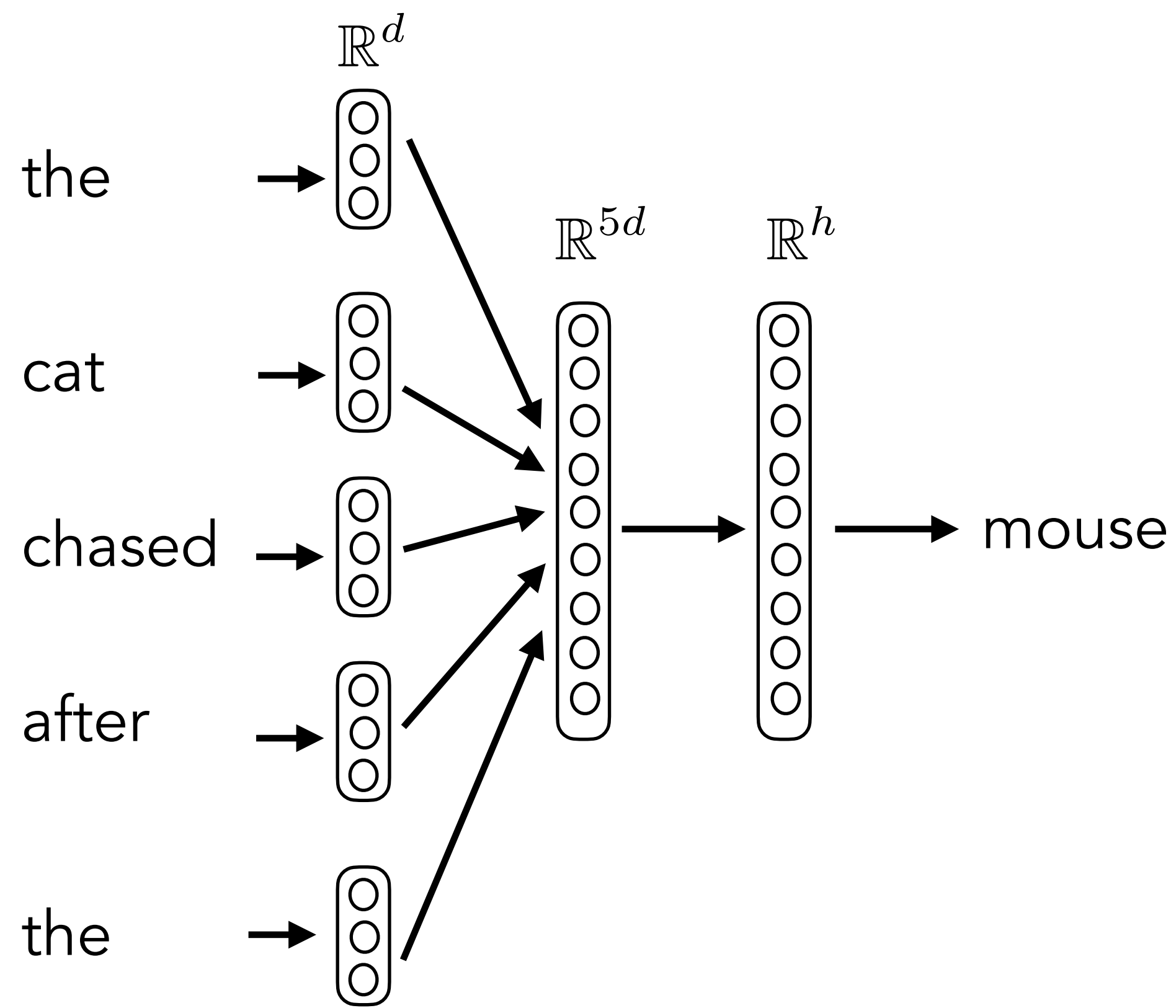
$$P(w = i \mid \text{the cat sat on the})$$

$$= \text{softmax}_i(\mathbf{z}) = \frac{e^{z_i}}{\sum_k e^{z_k}}$$

# Advantages vs. Disadvantages

- No more sparsity problem
  - All sequences can be estimated with non-zero probability ! **Why?**
- Model size is much smaller!
  - Depends on number of weights in model, not number of sequences!
- Fixed windows are still too small to encode **long-range dependencies**

# Fixed-context Neural Language Models



$P(\text{mouse} \mid \text{the cat chased the})$  ✓

$P(\text{mouse} \mid \text{the starving cat chased the})$  ✓

$P(\text{mouse} \mid \text{starving cat chased after the})$  ✓

$P(\text{mouse} \mid \text{cat fanatically chased after the})$  ✓

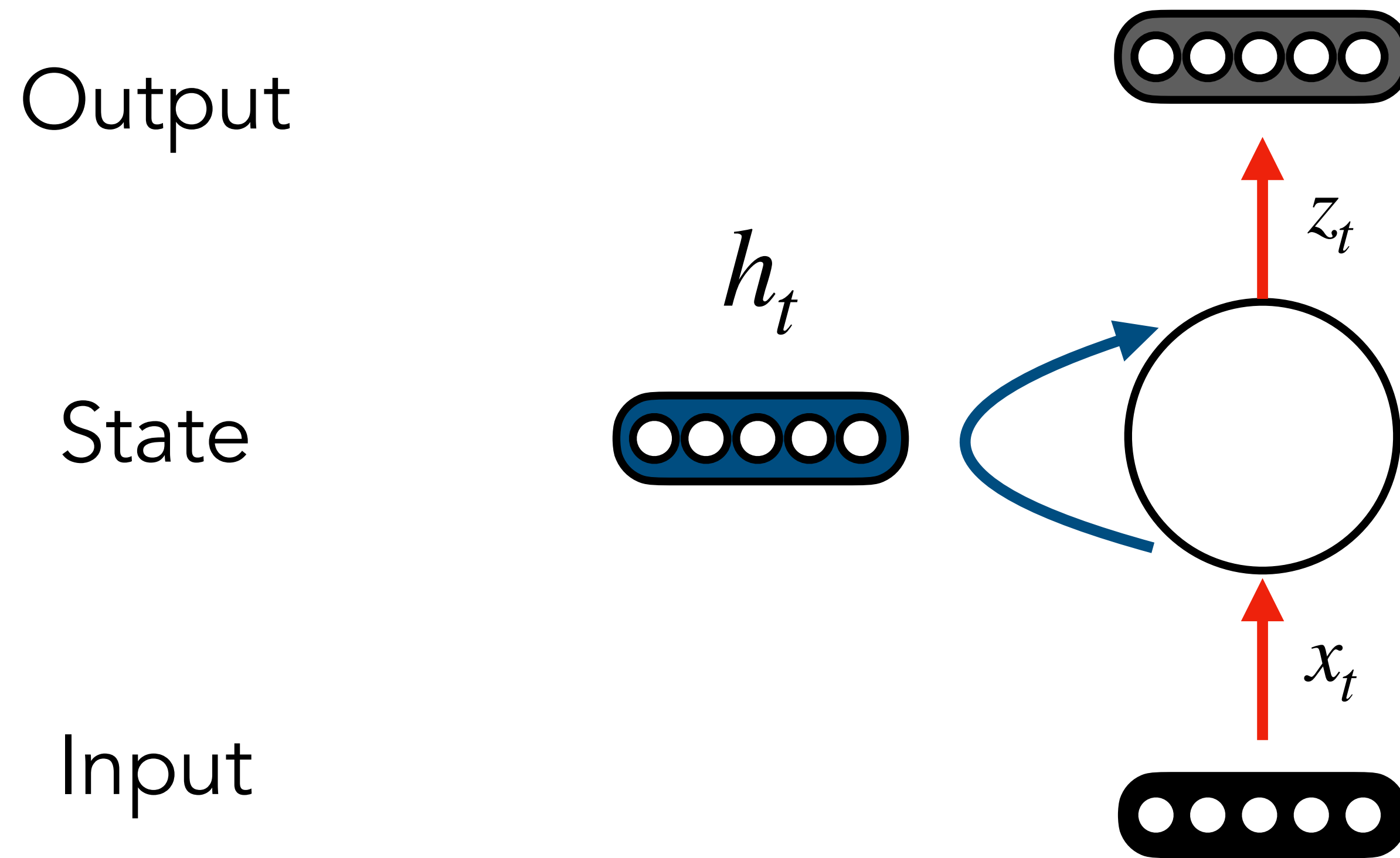
$P(\text{mouse} \mid \text{fanatically chased after the elusive})$  ✗

# Advantages vs. Disadvantages

- No more sparsity problem
  - All sequences can be estimated with non-zero probability ! **Why?**
- Model size is much smaller!
  - Depends on number of weights in model, not number of sequences!
- Fixed windows are still too small to encode **long-range dependencies**
- Weights in **W** aren't shared across embeddings in the window (computationally inefficient!)
- Enlarging the window size makes the weight matrix **W** larger (more computationally expensive!)

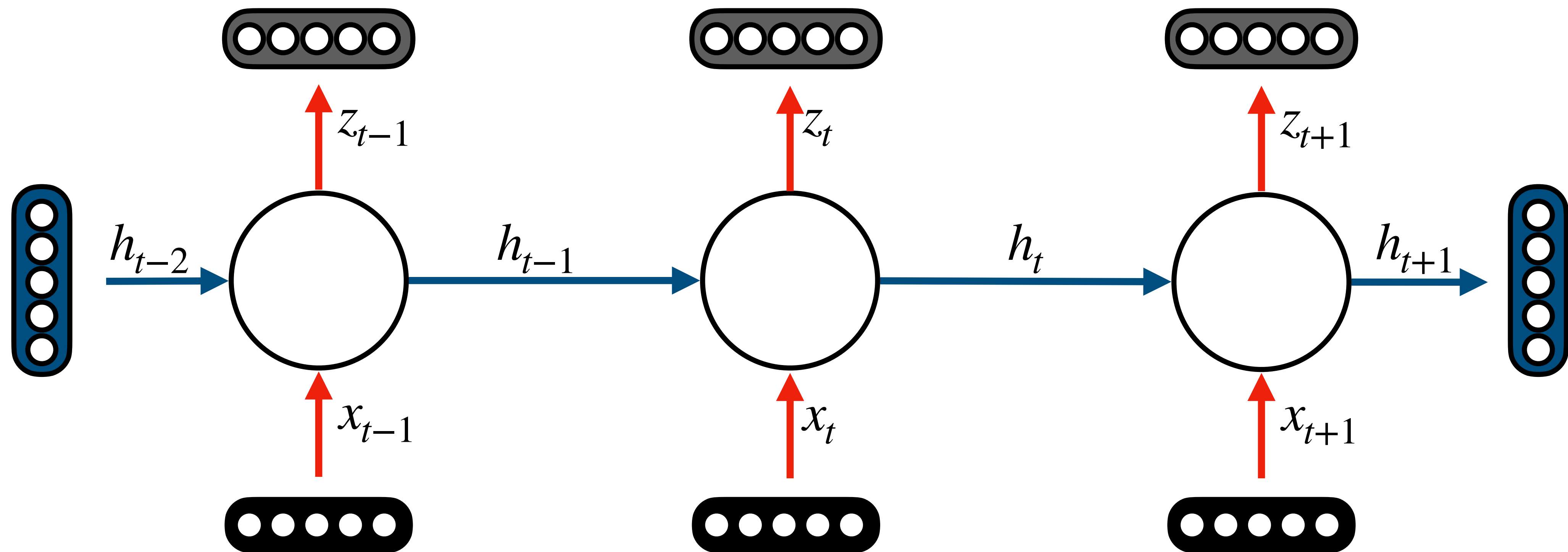
# Recurrent Neural Networks

- **Solution:** Recurrent neural networks — NNs with feedback loops



# Recurrent Neural Networks

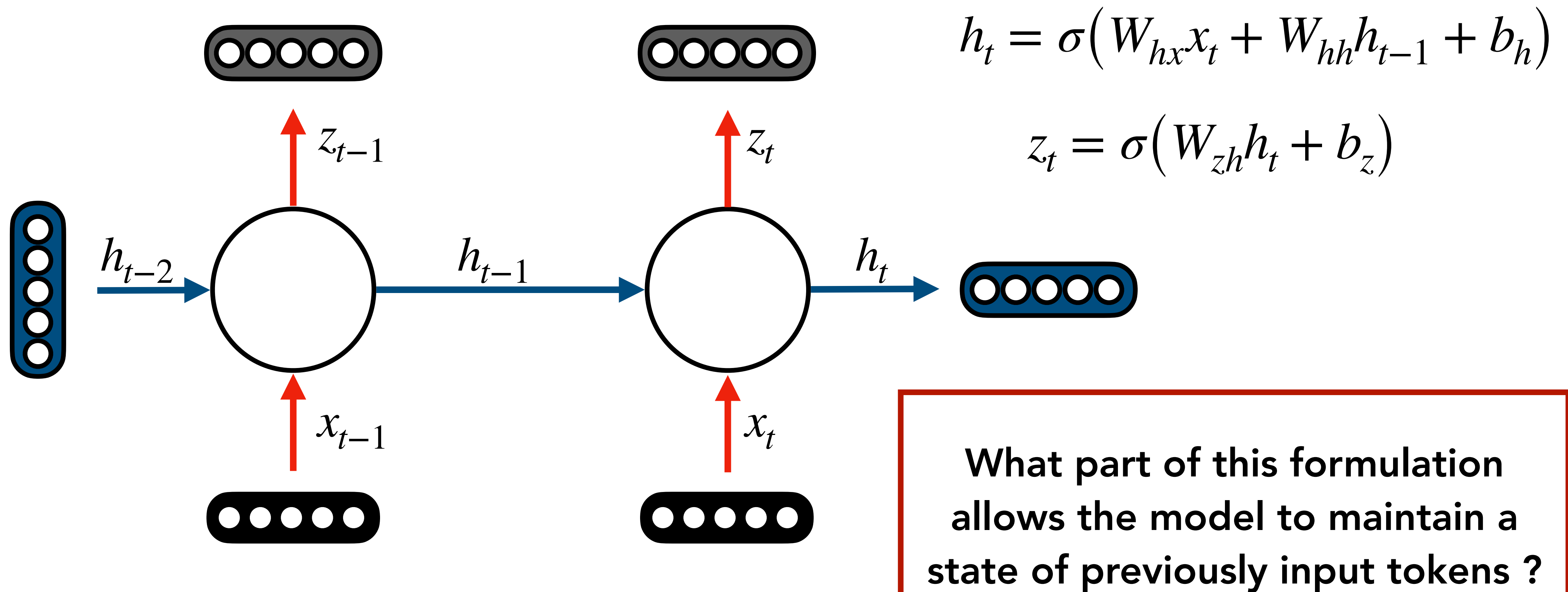
Unrolling the RNN across all time steps gives full computation graph



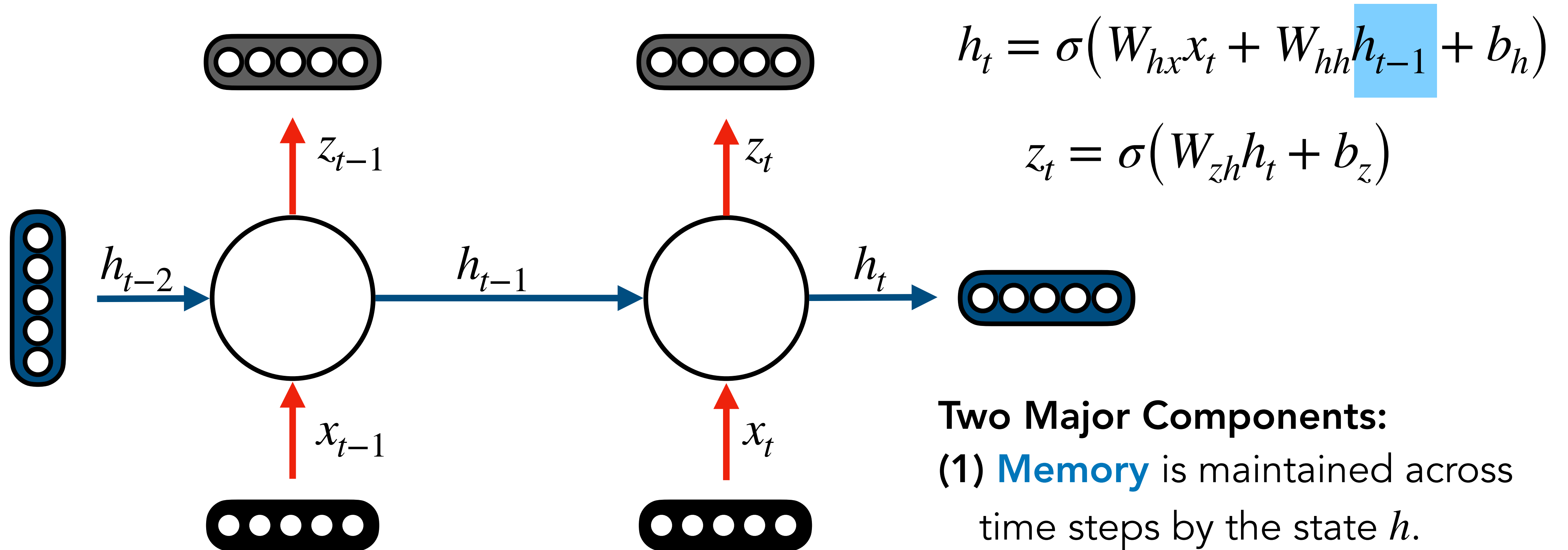
Allows for learning from entire sequence history, regardless of length



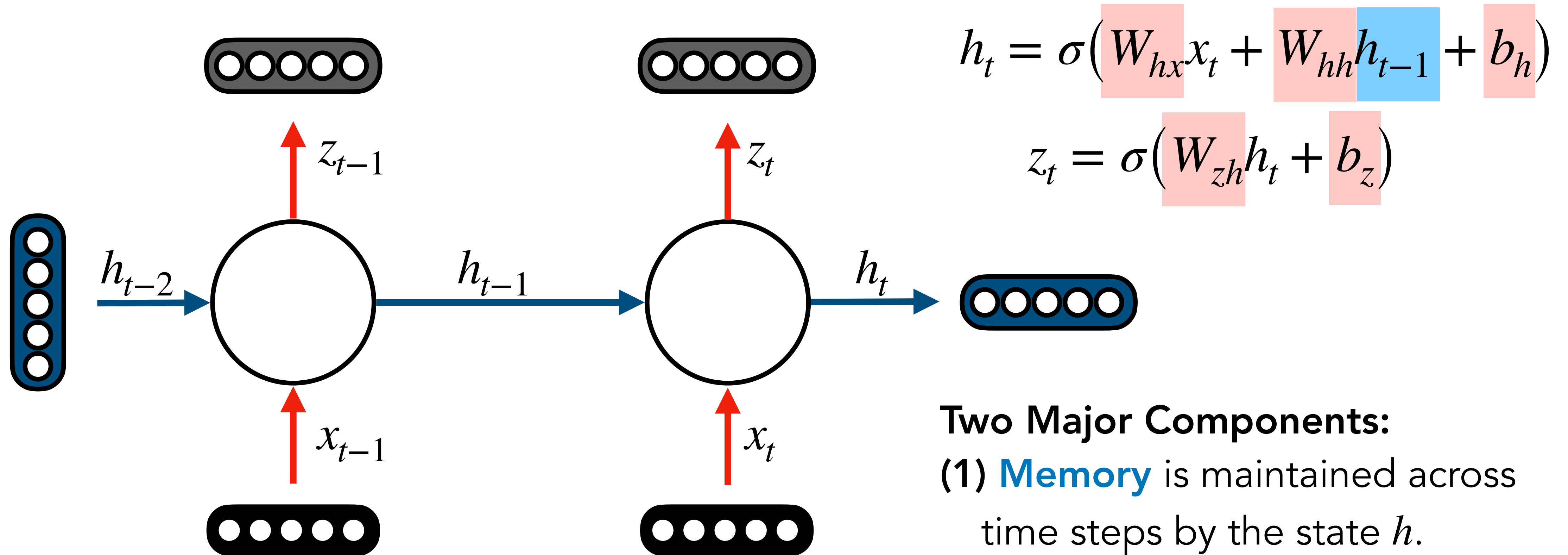
# Classical RNN: Elman Network



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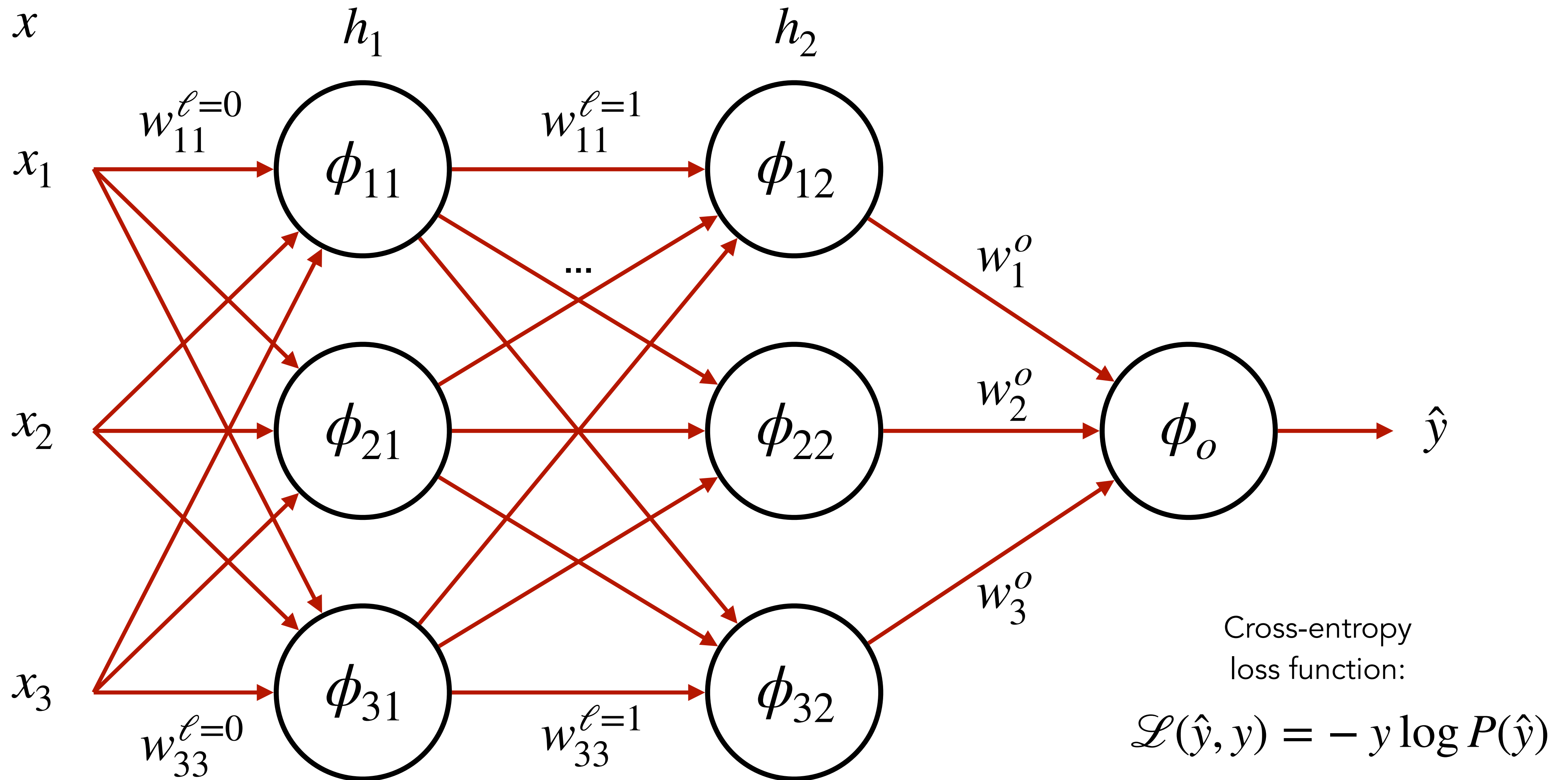
**Two Major Components:**

- (1) **Memory** is maintained across time steps by the state  $h$ .
- (2) **Parameter matrices** are shared across all time steps.

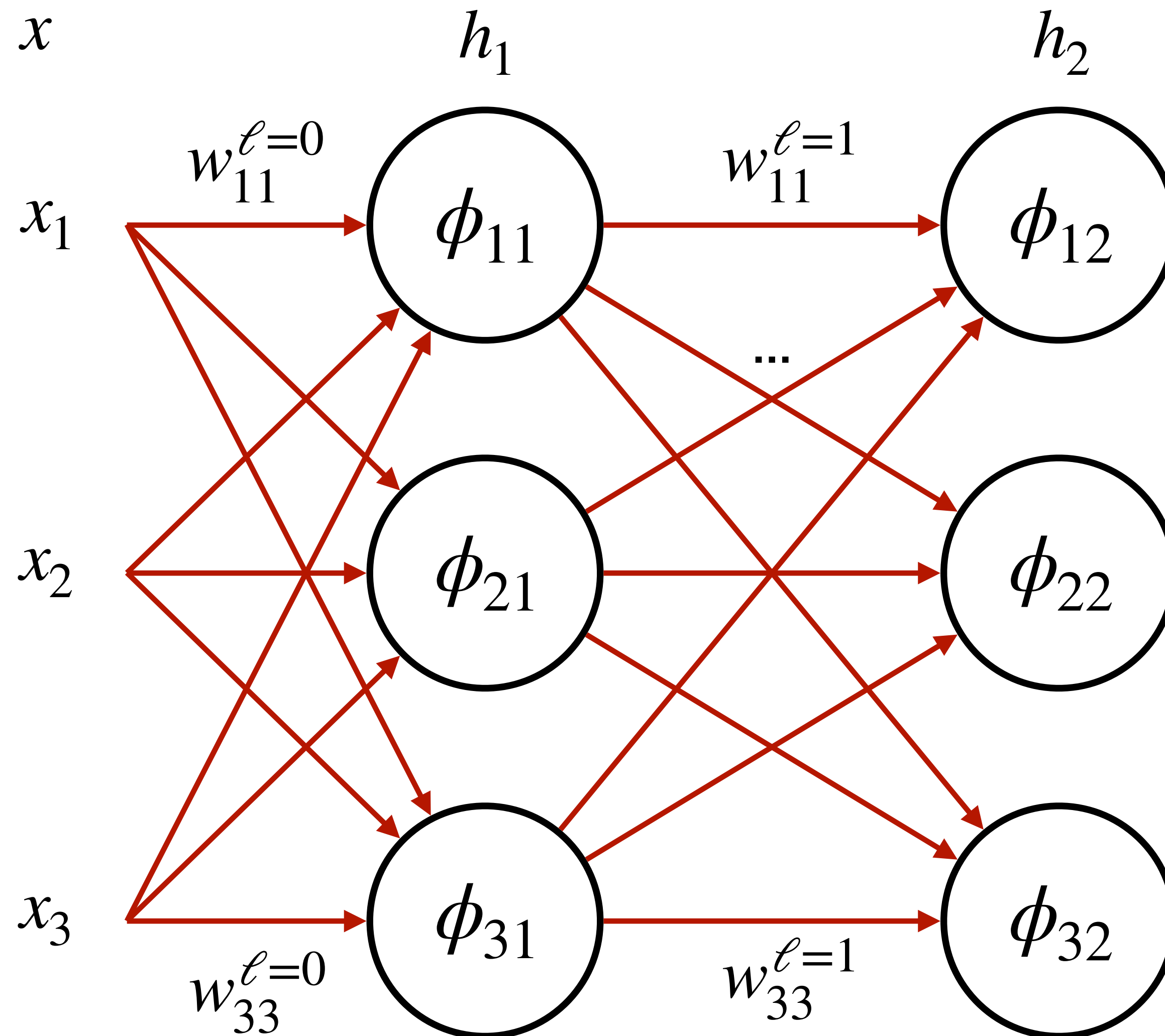
# Objective

**Learn parameter matrices  $W_{hh}$ ,  $W_{hx}$ ,  $W_{zh}$  in the Elman network such that the RNN can represent textual sequences**

# Backpropagation Review: FFNs



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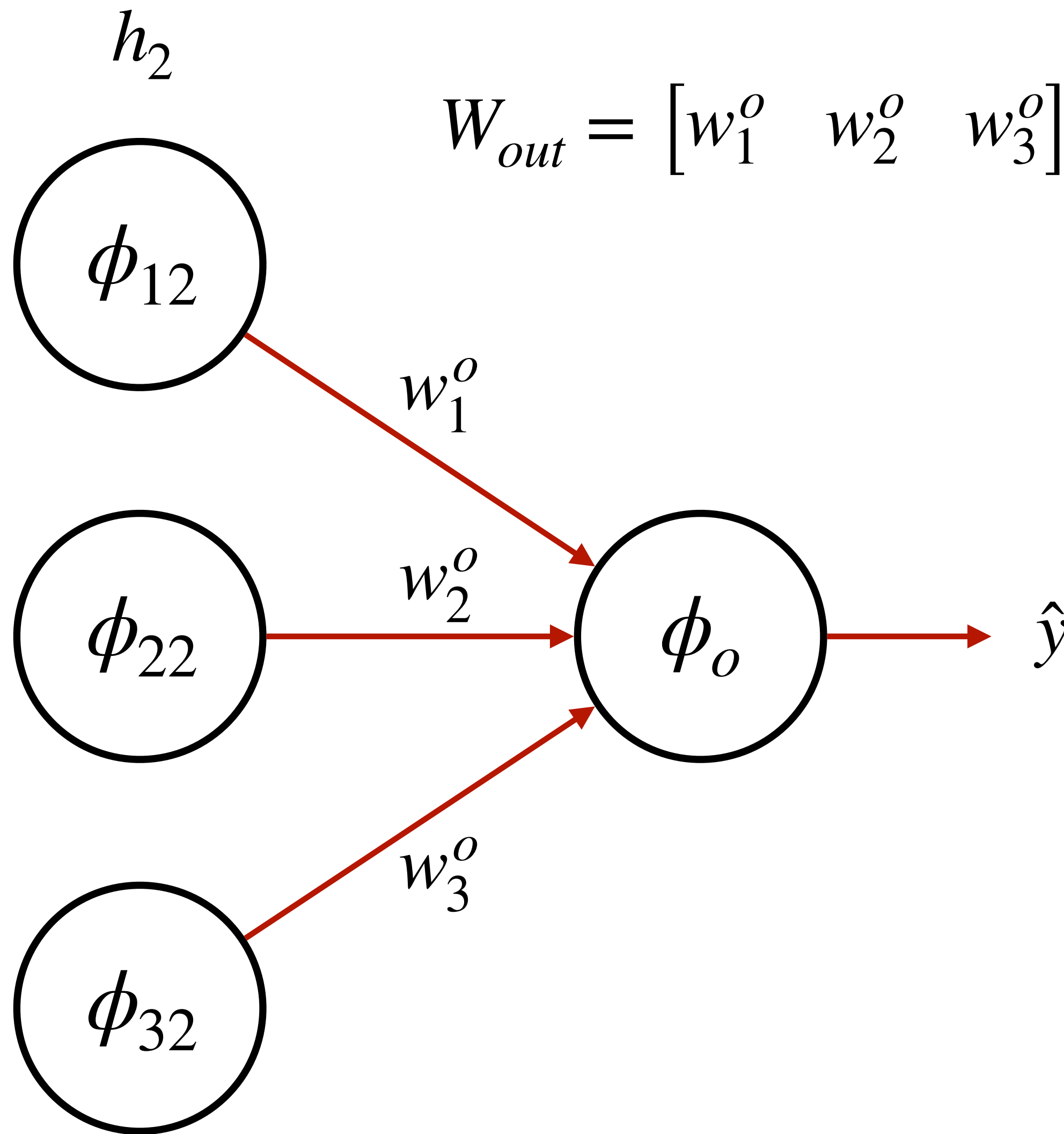
$$W_0 = \begin{bmatrix} w_{11}^{\ell=0} & w_{21}^{\ell=0} & w_{31}^{\ell=0} \\ w_{12}^{\ell=0} & w_{22}^{\ell=0} & w_{32}^{\ell=0} \\ w_{13}^{\ell=0} & w_{23}^{\ell=0} & w_{33}^{\ell=0} \end{bmatrix}$$

$$W_1 = \begin{bmatrix} w_{11}^{\ell=1} & w_{21}^{\ell=1} & w_{31}^{\ell=1} \\ w_{12}^{\ell=1} & w_{22}^{\ell=1} & w_{32}^{\ell=1} \\ w_{13}^{\ell=1} & w_{23}^{\ell=1} & w_{33}^{\ell=1} \end{bmatrix}$$

$$h_1 = \phi_1(W_0 x)$$

$$h_2 = \phi_2(W_1 h_1)$$

# Backpropagation Review: FFNs



$$W_{out} = \begin{bmatrix} w_1^o & w_2^o & w_3^o \end{bmatrix}$$

$$\mathcal{L}(\hat{y}, y) = -y \log P(\hat{y})$$

$$P(\hat{y}) = \phi_o(u)$$

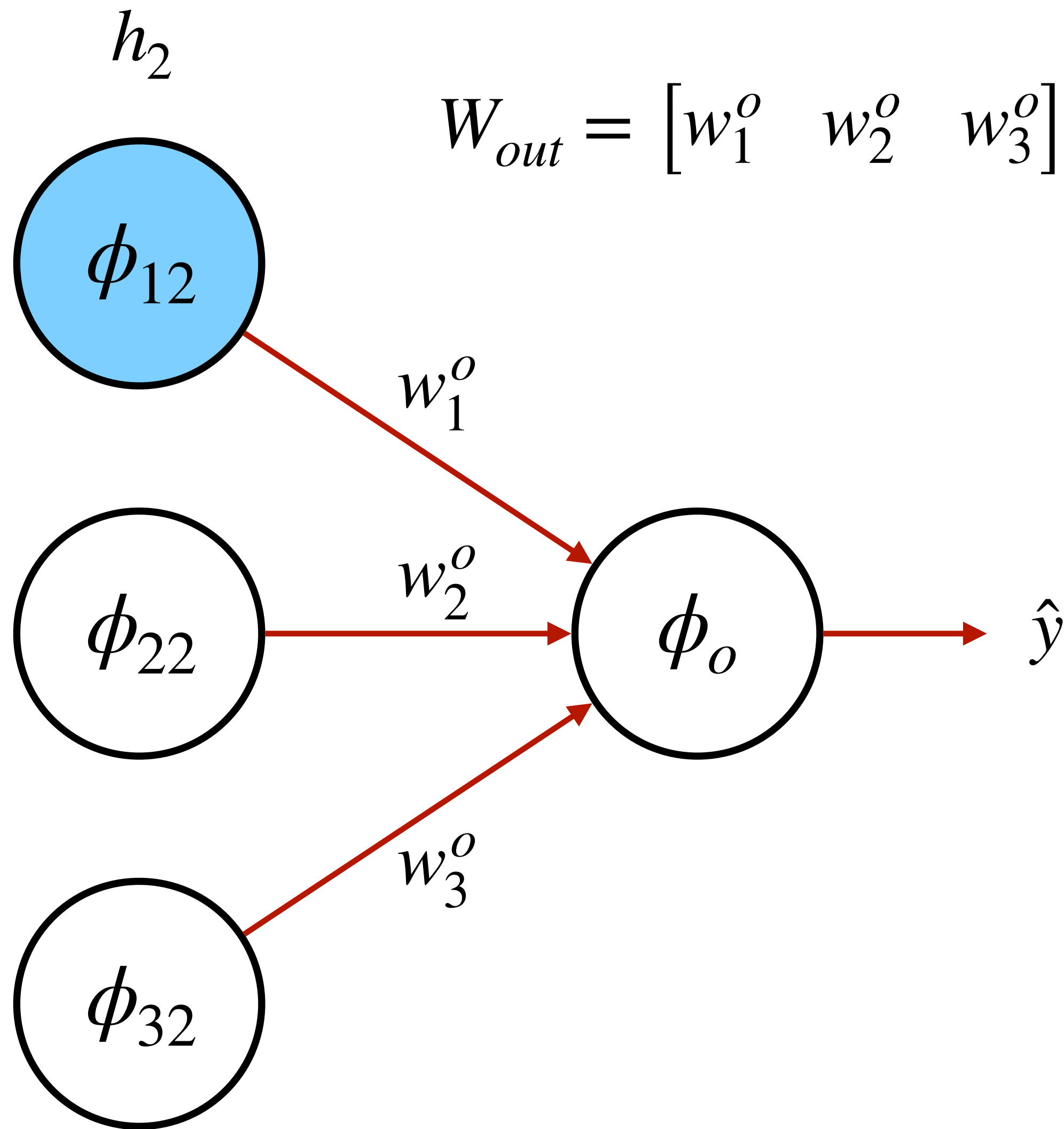
$$u = W_{out}h_2 = w_1^o \times \phi_{12}(\cdot) + w_2^o \times \phi_{22}(\cdot) + w_3^o \times \phi_{32}(\cdot)$$

$$\frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \phi_{12}(\cdot)} = \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial u} \frac{\partial u}{\partial \phi_{12}(\cdot)}$$

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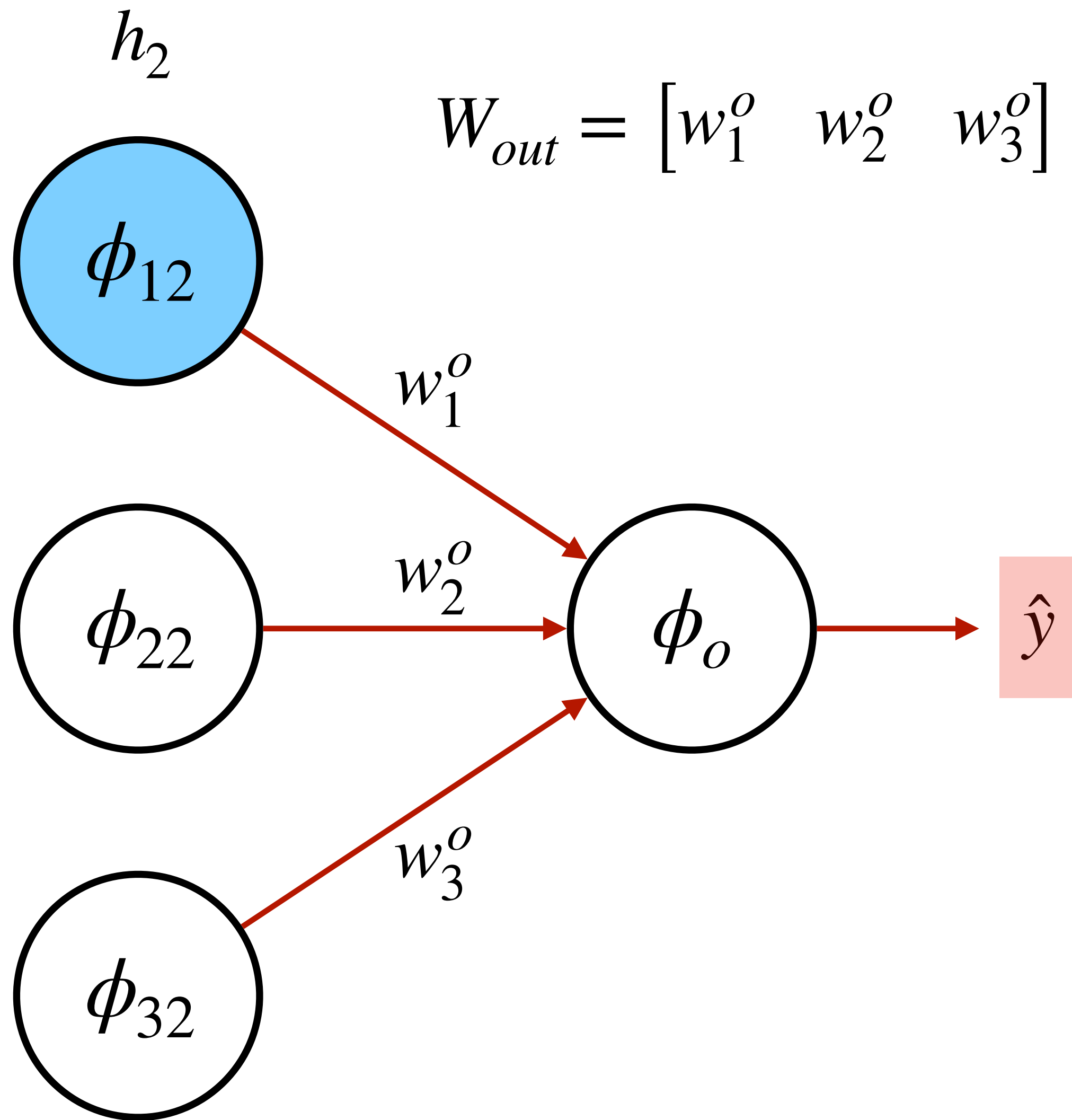
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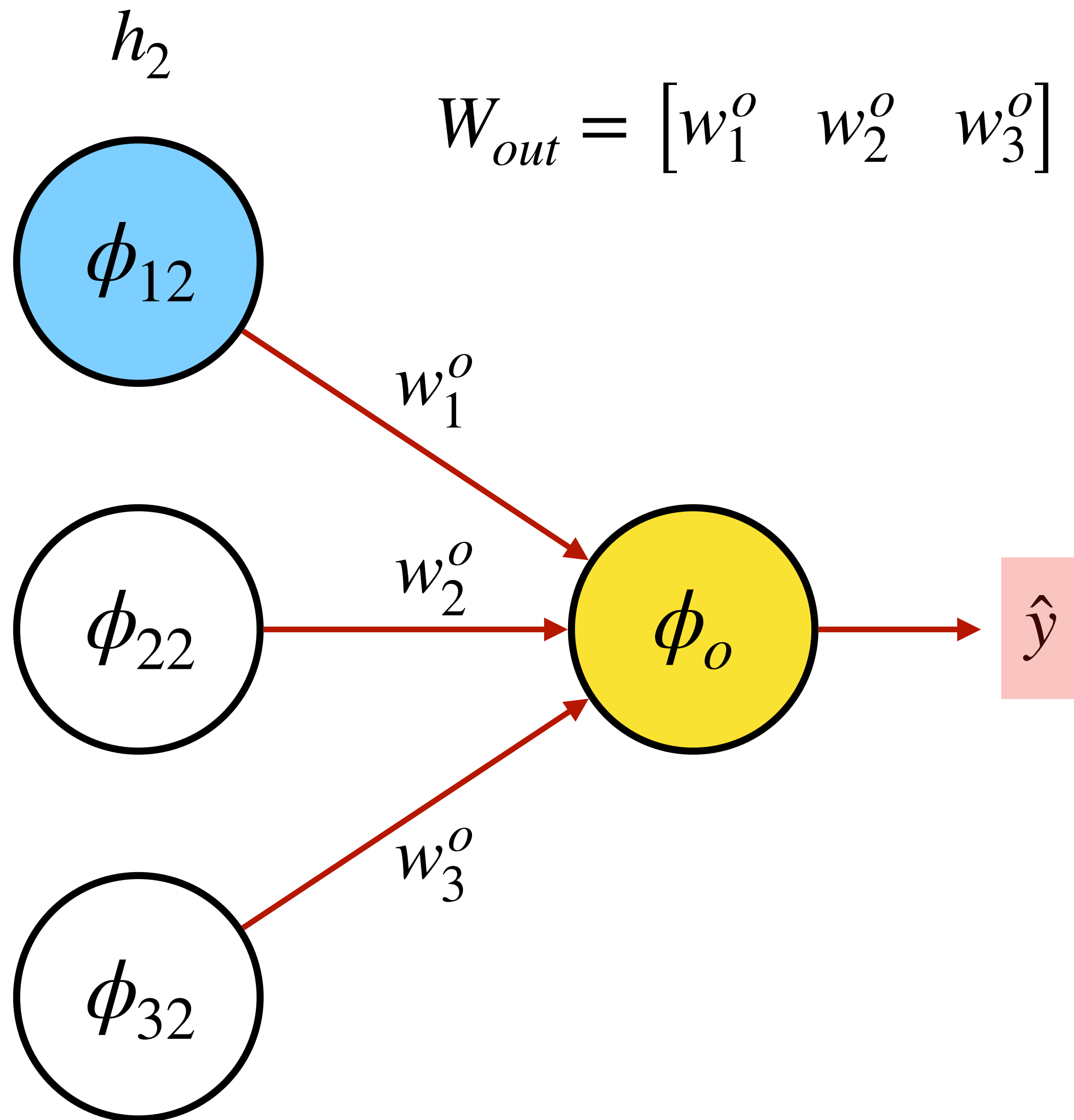
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Depends on label  $y$

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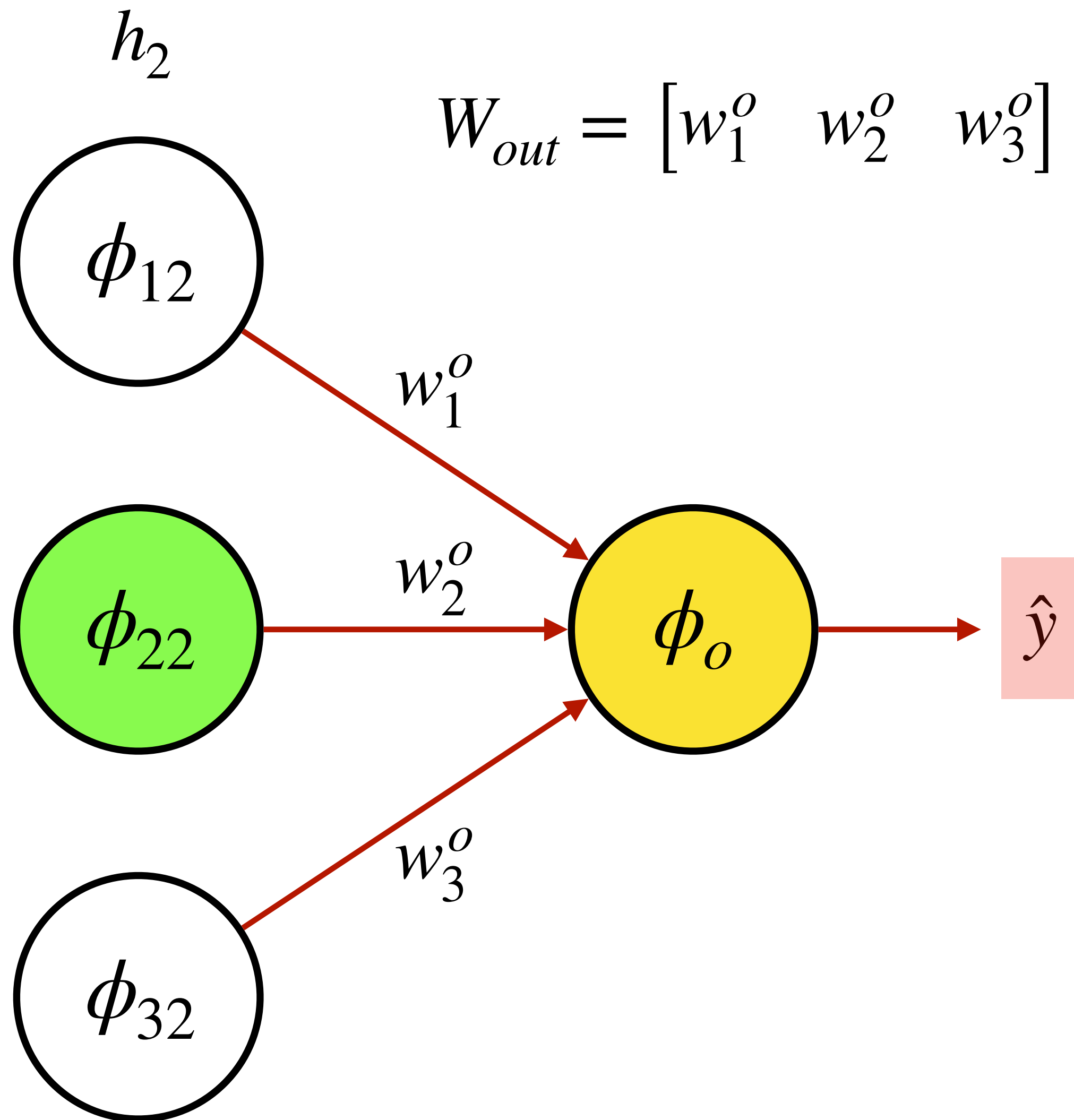
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Depends on label  $y$

Depends on  $\phi_o$

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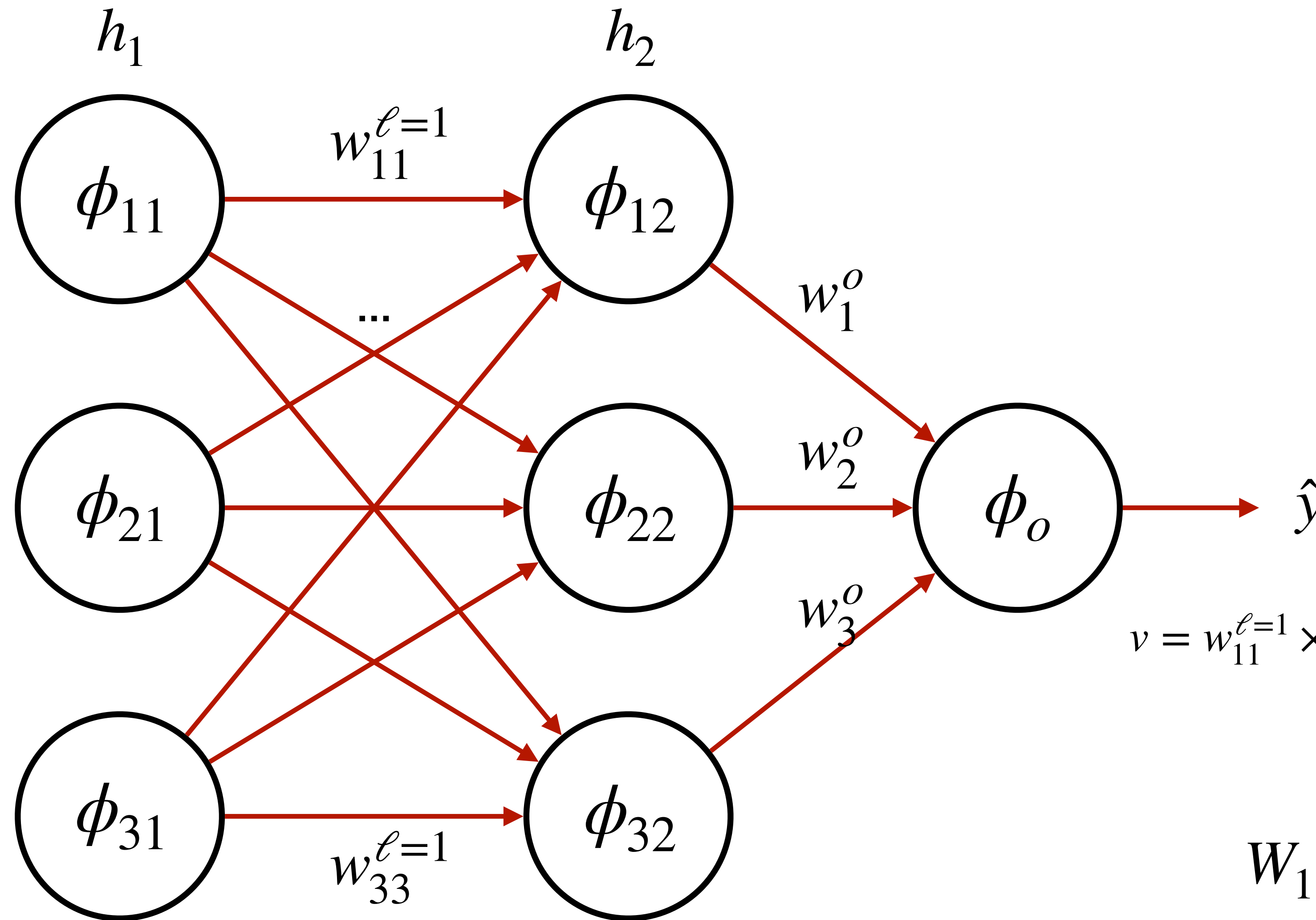
$$\frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \phi_{22}(\cdot)} = \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial u} \frac{\partial u}{\partial \phi_{22}(\cdot)}$$

$$= \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \hat{y}} \frac{\partial \phi_o(u)}{\partial u} w_2^o$$

Depends on label  $y$

Depends on  $\phi_o$

# Backpropagation Review: FFNs



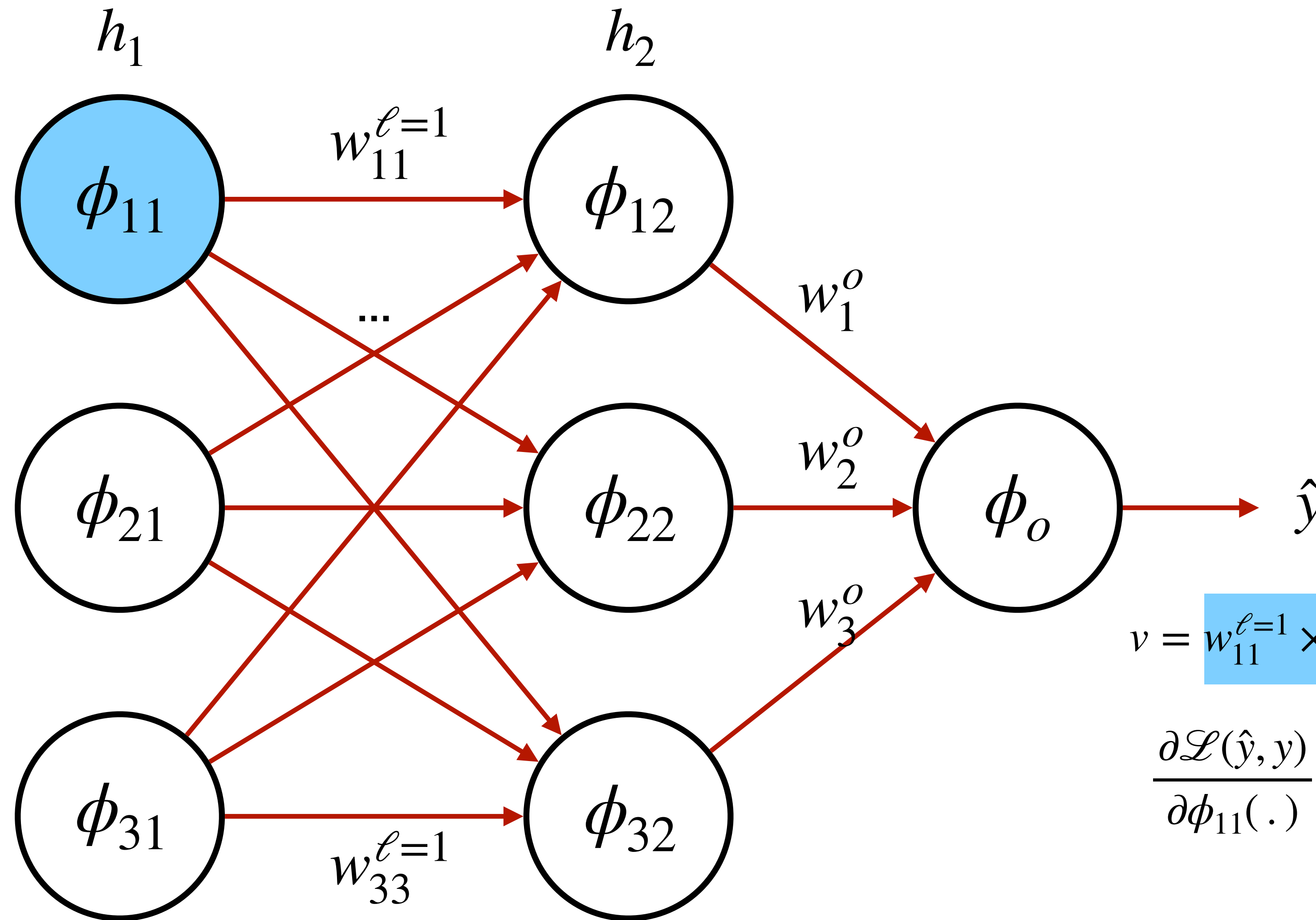
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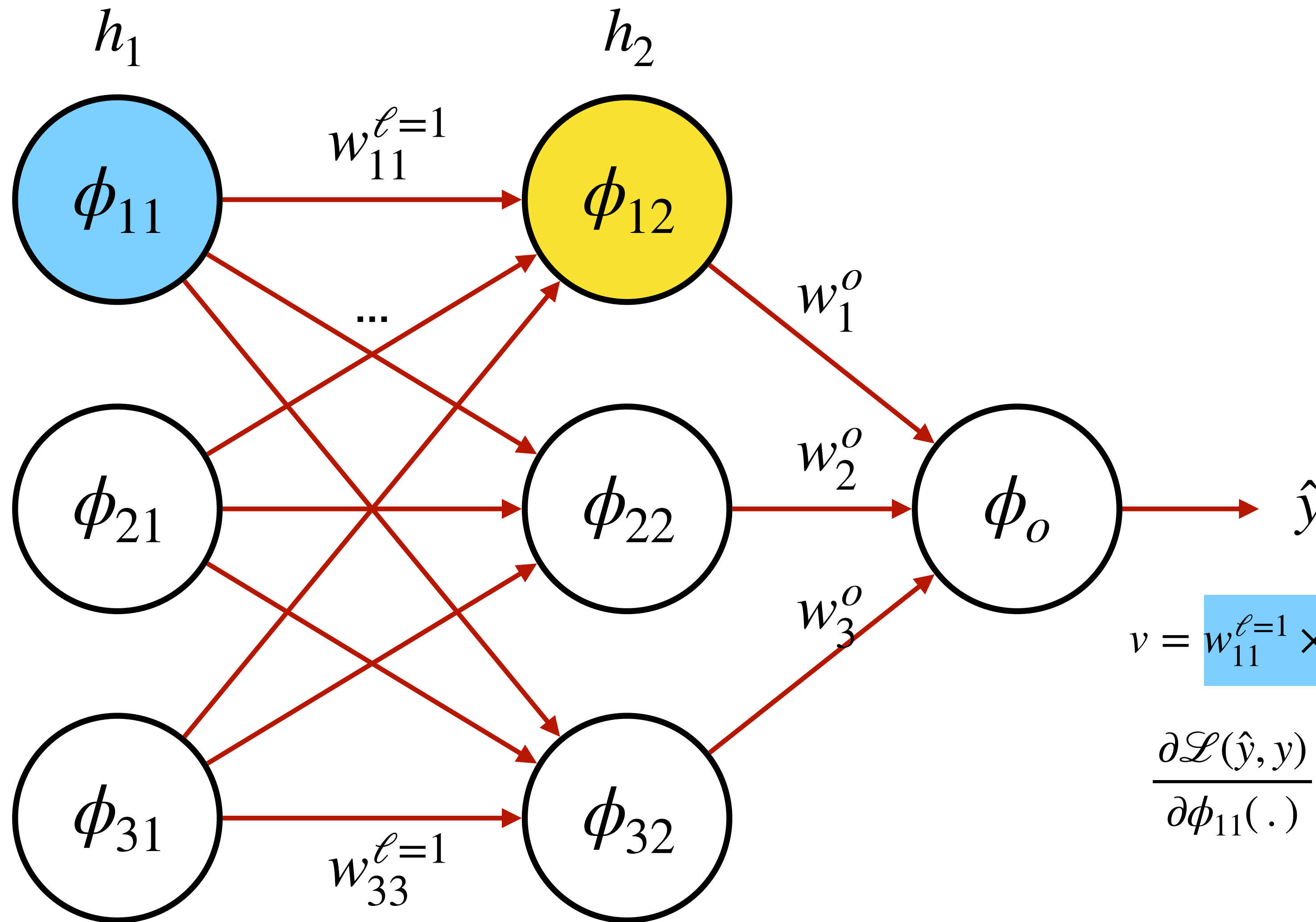
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Depends on  $\phi_{12}$

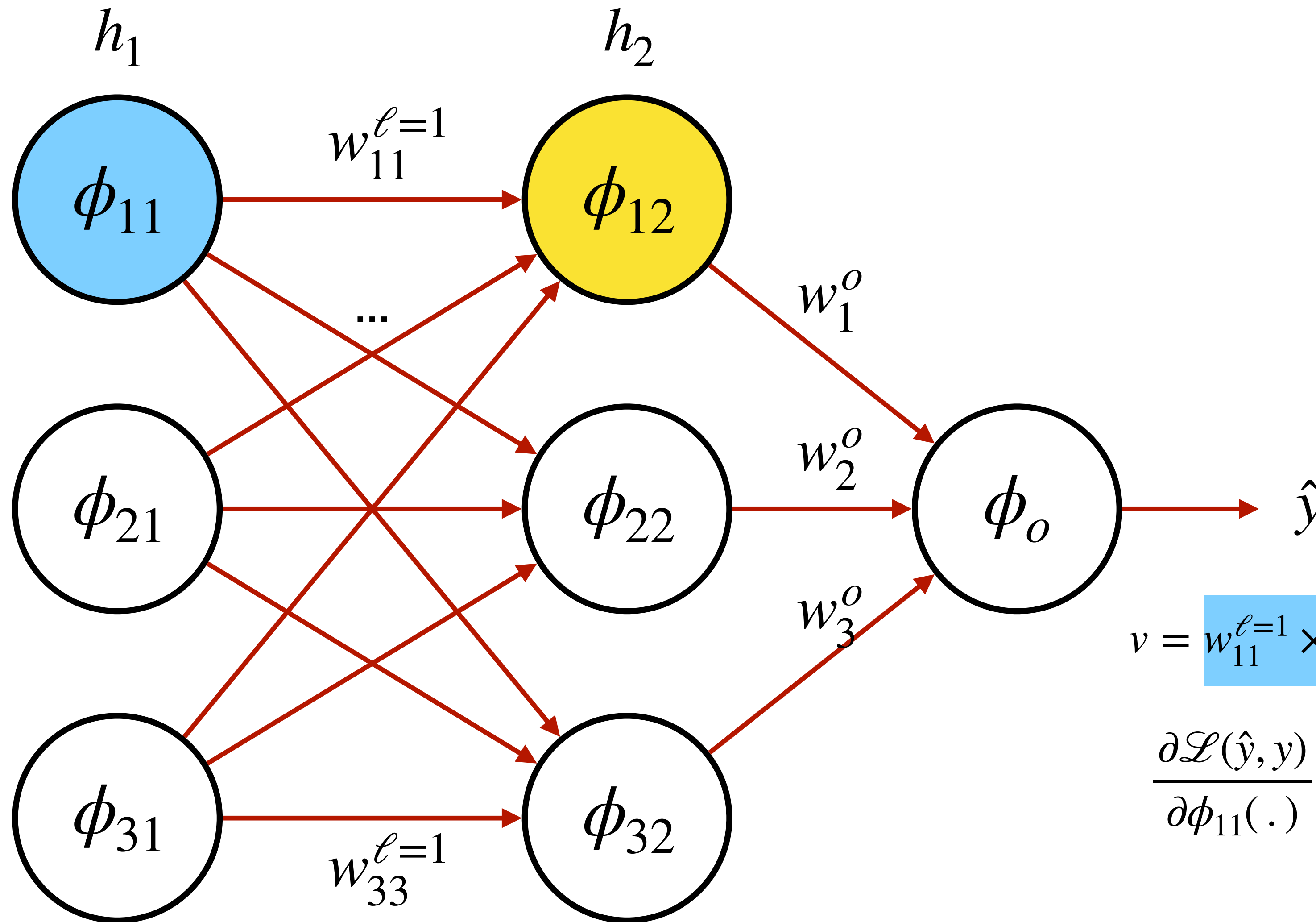
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$$= \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \hat{y}} \frac{\partial \phi_o(u)}{\partial u} w_1^o \frac{\partial \phi_{12}(v)}{\partial v} w_{11}^{l=1}$$



# Backpropagation Review: FFNs



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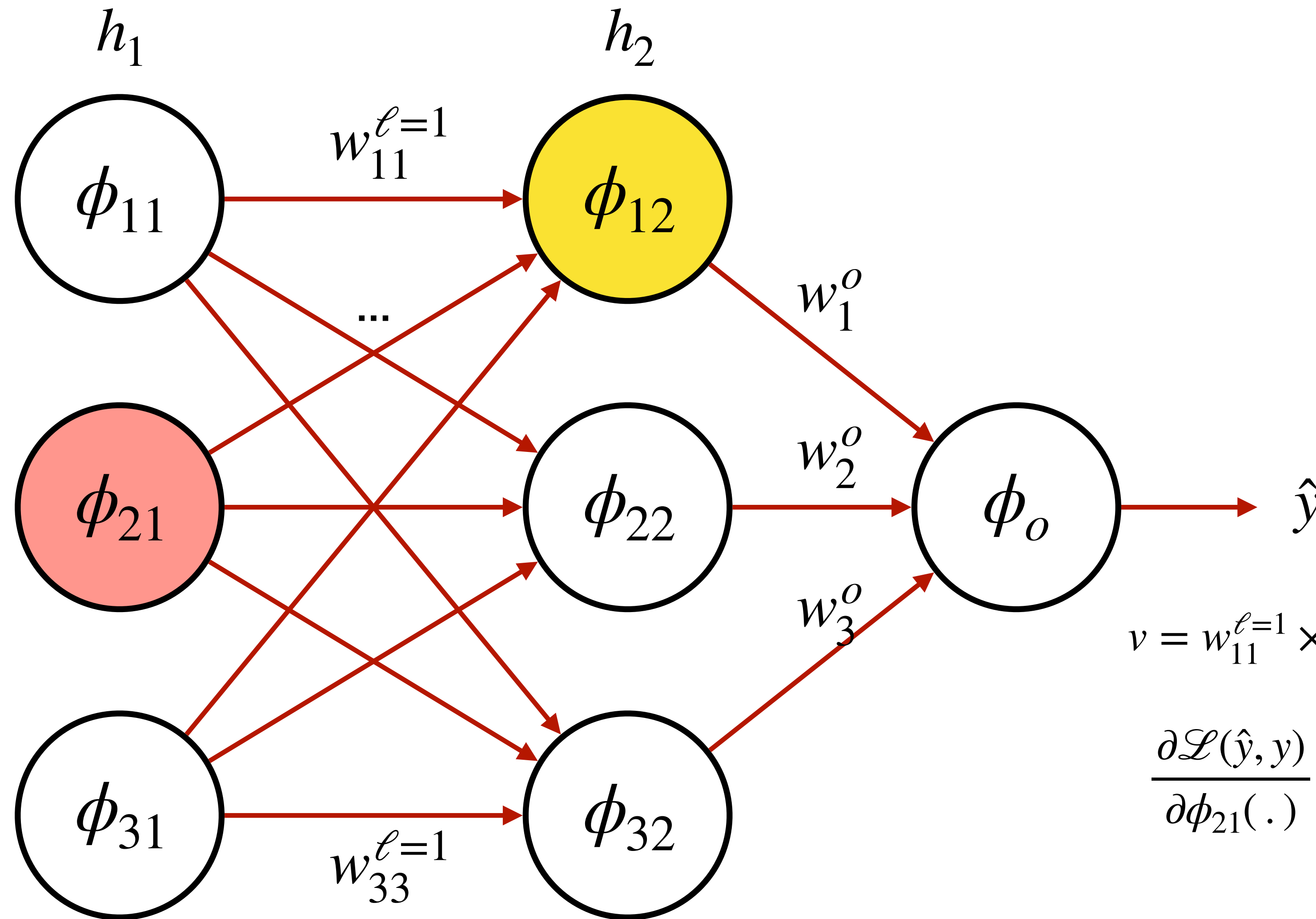
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# Backpropagation Review: FFNs



$$\frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \phi_{12}(\cdot)} = \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial u} \frac{\partial u}{\partial \phi_{12}(\cdot)}$$

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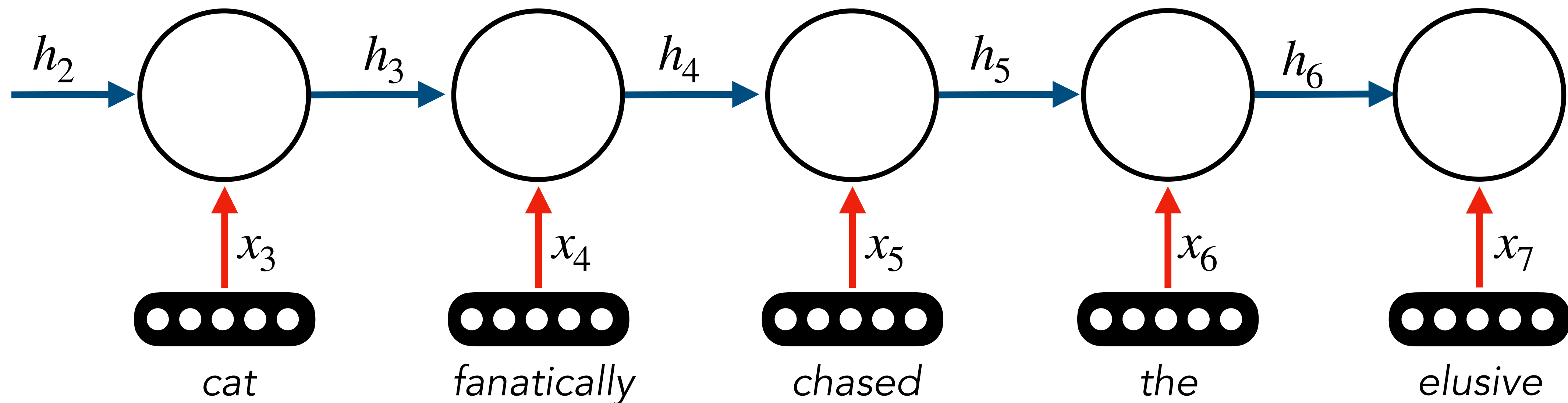


# Question

**How would we extend backpropagation  
to a recurrent neural network?**

# Recall

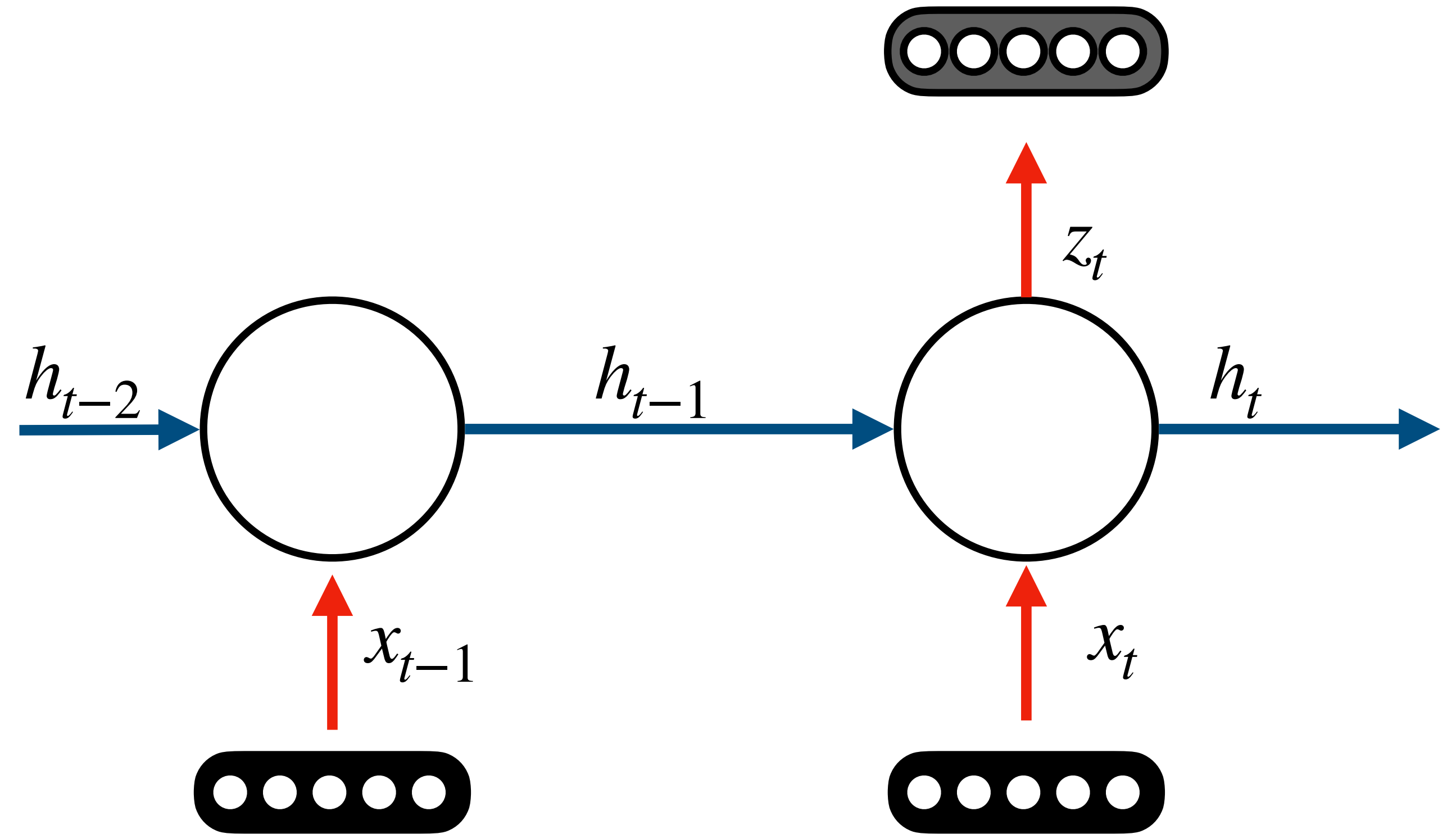
- RNN can be unrolled to a feedforward neural network
- Depth of feedforward neural network depends on length of the sequence



# Backpropagation through Time

$$z_t = \sigma(W_{zh}h_t + b_z)$$

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$



# Backpropagation through Time

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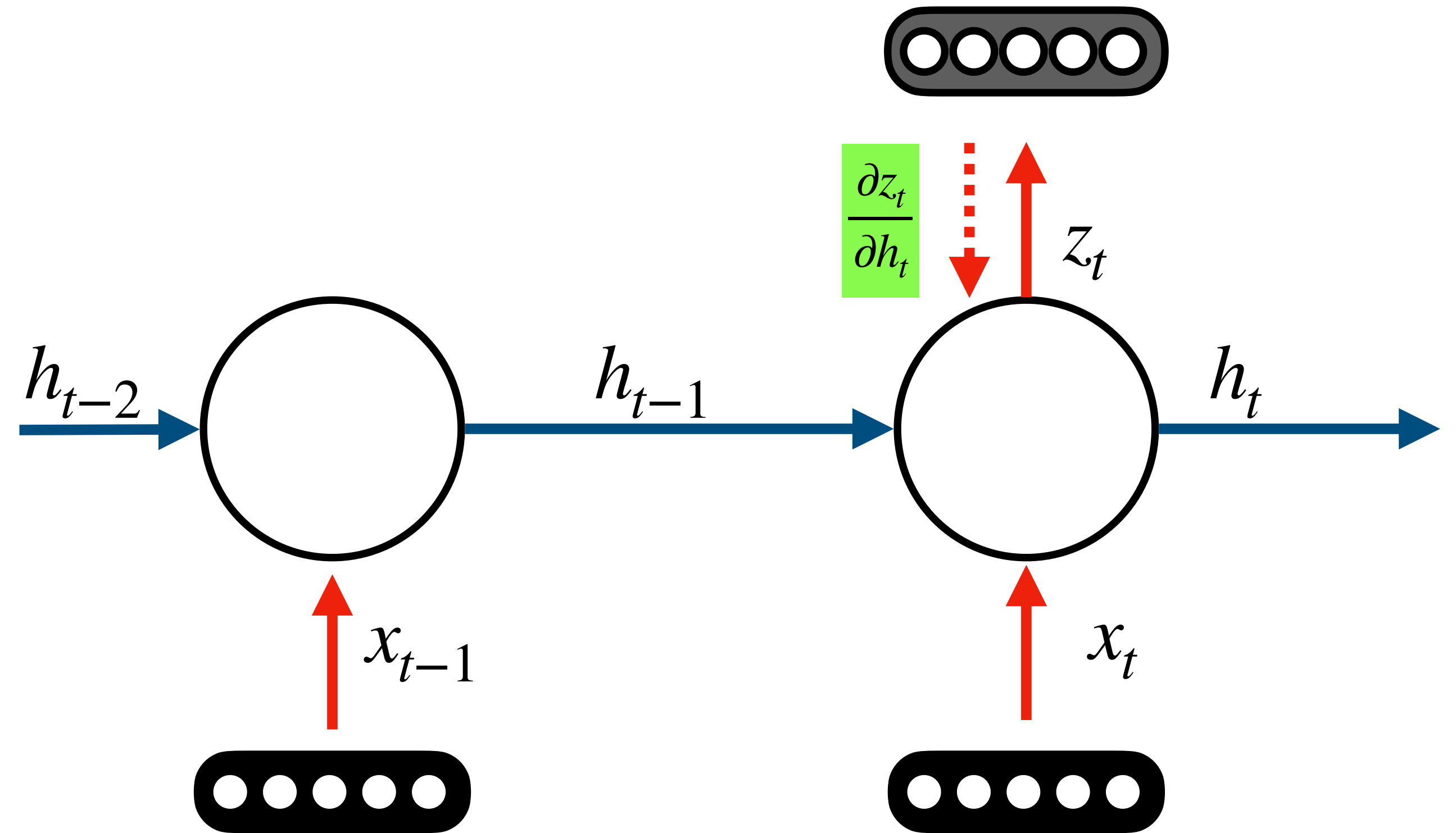
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$$v = W_{zh}h_t + b_z \quad z_t = \sigma(v)$$

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---

$$\frac{\partial z_t}{\partial h_t} = \frac{\partial \sigma(v)}{\partial v} \frac{\partial v}{\partial h_t} = \frac{\partial \sigma(v)}{\partial v} W_{zh}$$



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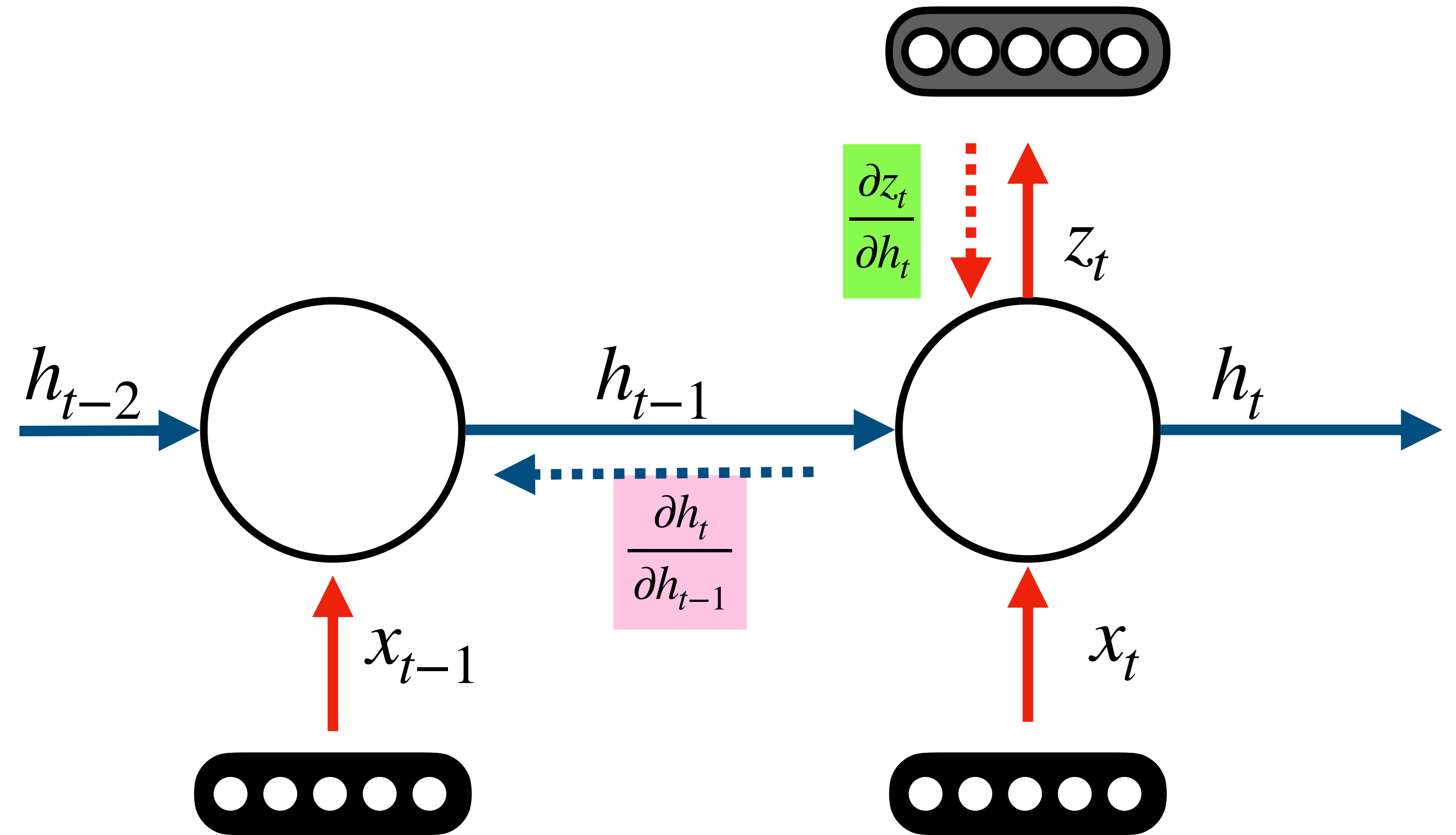

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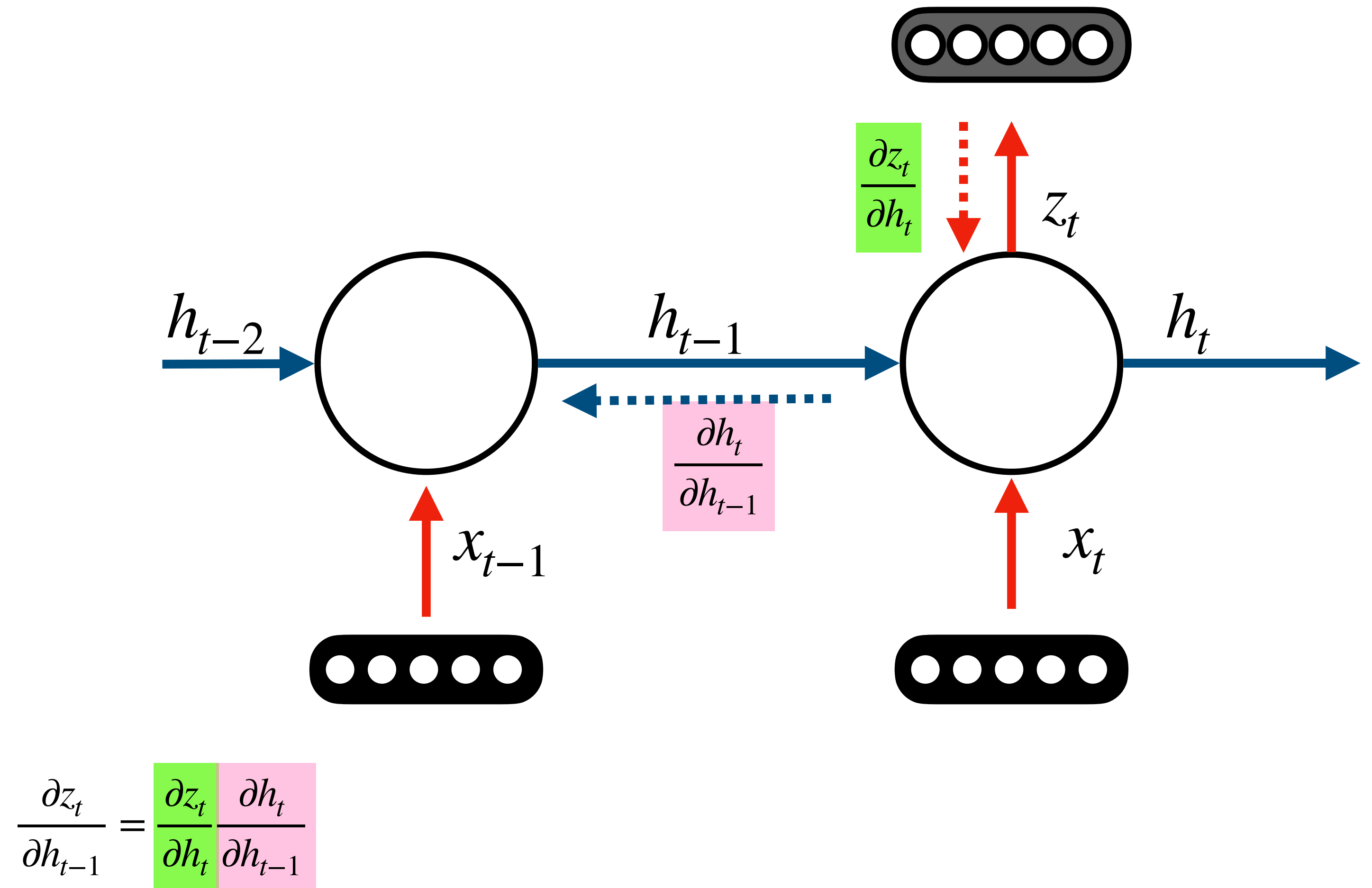

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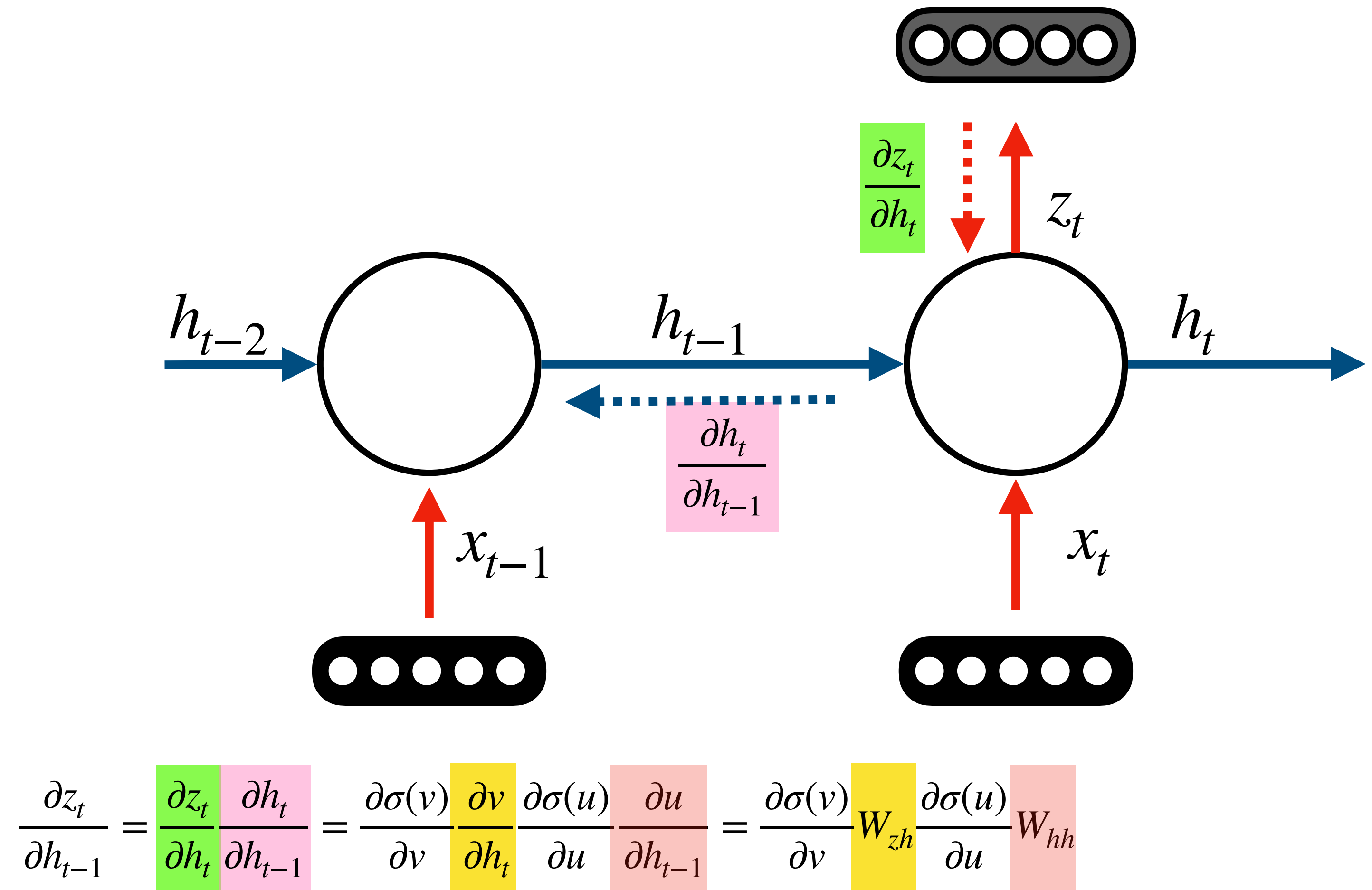

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$$v_t = W_{zh}h_t + b_z \quad z_t = \sigma(v_t)$$

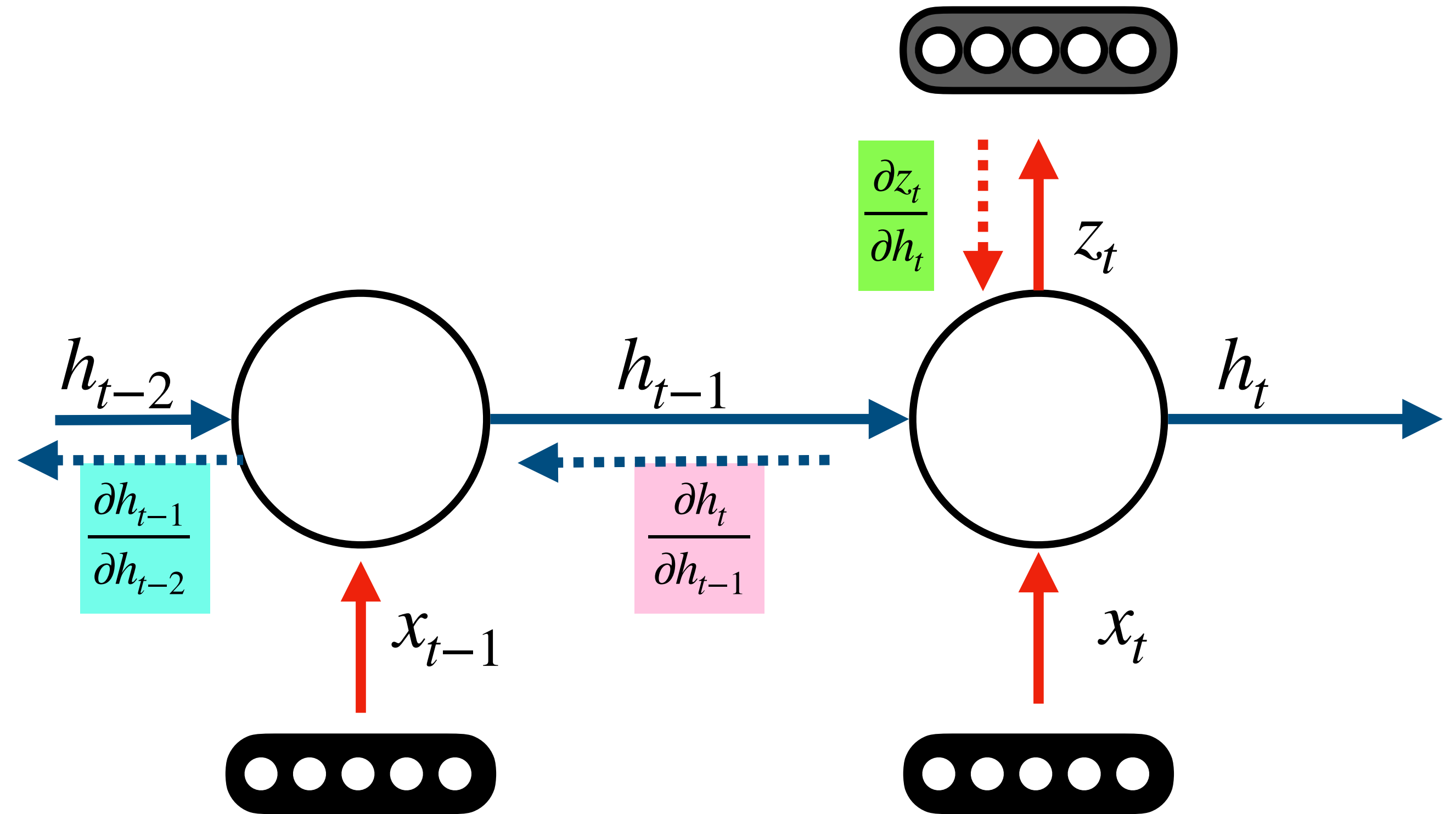
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$$\frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \frac{\partial u_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} W_{hh}$$



$$\frac{\partial z_t}{\partial h_{t-2}} = \frac{\partial z_t}{\partial h_t} \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(v_t)}{\partial v_t} W_{zh} \frac{\partial \sigma(u_t)}{\partial u_t} W_{hh} \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} W_{hh}$$



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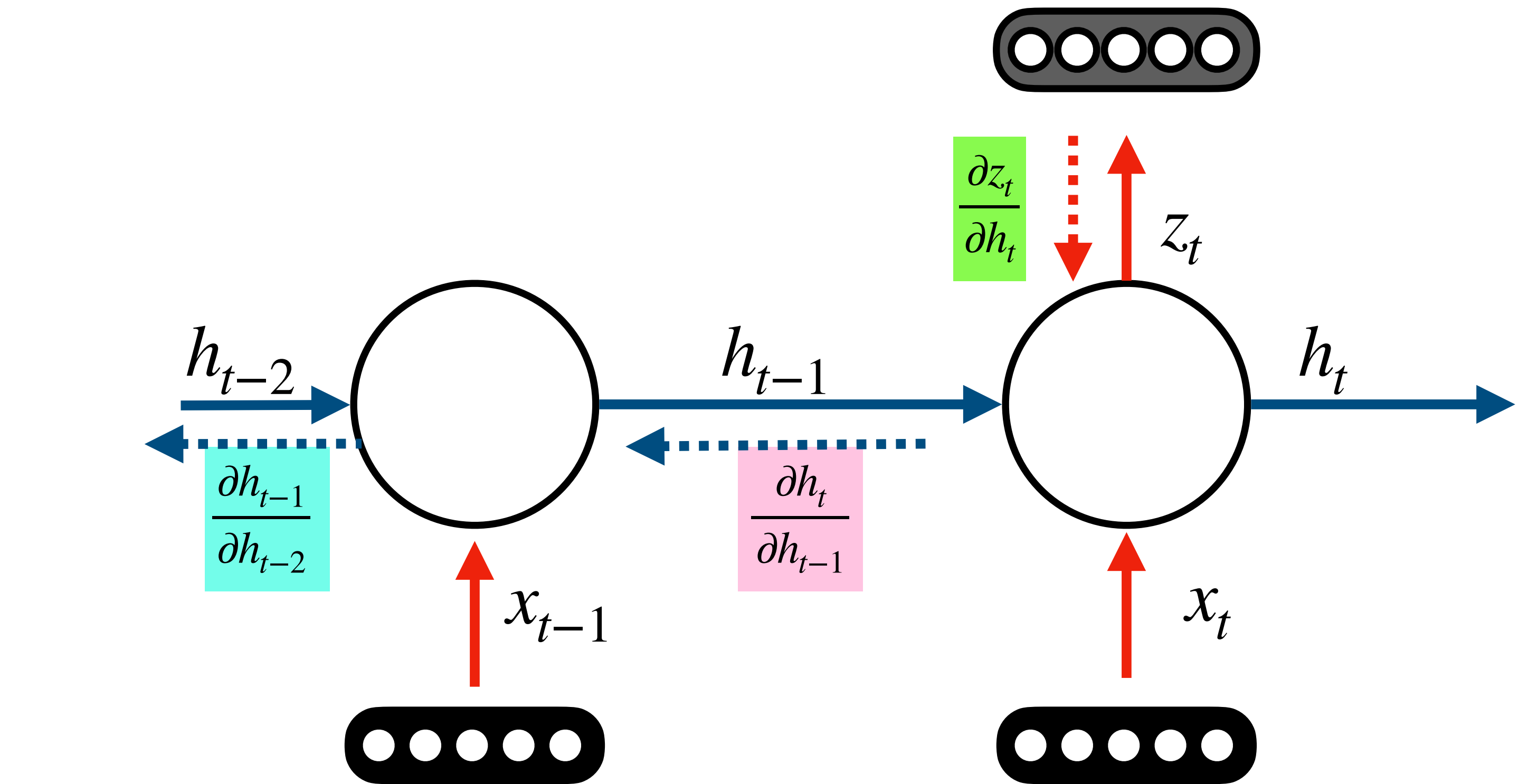
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---

$$\frac{\partial z_t}{\partial h_t} = \frac{\partial \sigma(v_t)}{\partial v_t} \frac{\partial v_t}{\partial h_t} = \frac{\partial \sigma(v_t)}{\partial v_t} W_{zh}$$

$$\frac{\partial h_t}{\partial h_{t-1}} = \frac{\partial \sigma(u_t)}{\partial u_t} \frac{\partial u_t}{\partial h_{t-1}} = \frac{\partial \sigma(u_t)}{\partial u_t} W_{hh}$$

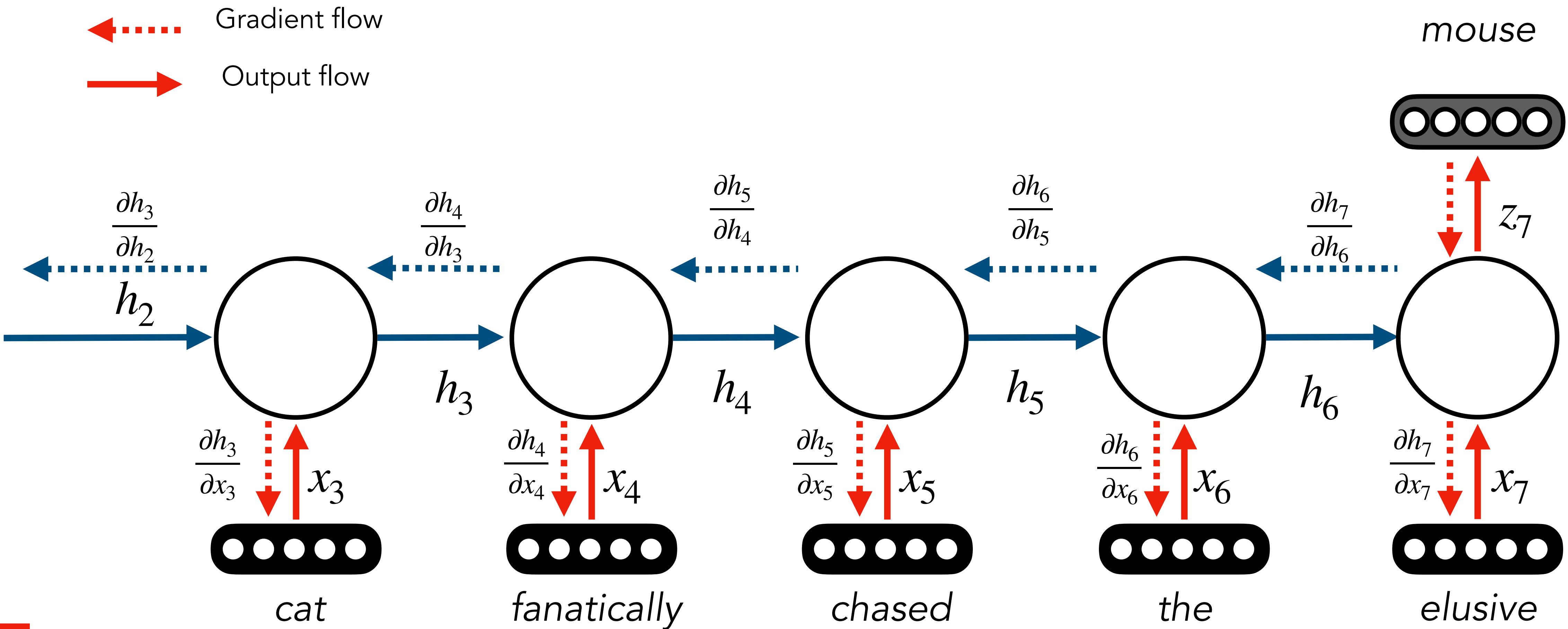
$$\frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \frac{\partial u_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} W_{hh}$$



$$\frac{\partial z_t}{\partial h_{t-2}} = \frac{\partial z_t}{\partial h_t} \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(v_t)}{\partial v_t} W_{zh} \frac{\partial \sigma(u_t)}{\partial u_t} W_{hh} \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} W_{hh}$$

Note that these are  
actually the same matrix

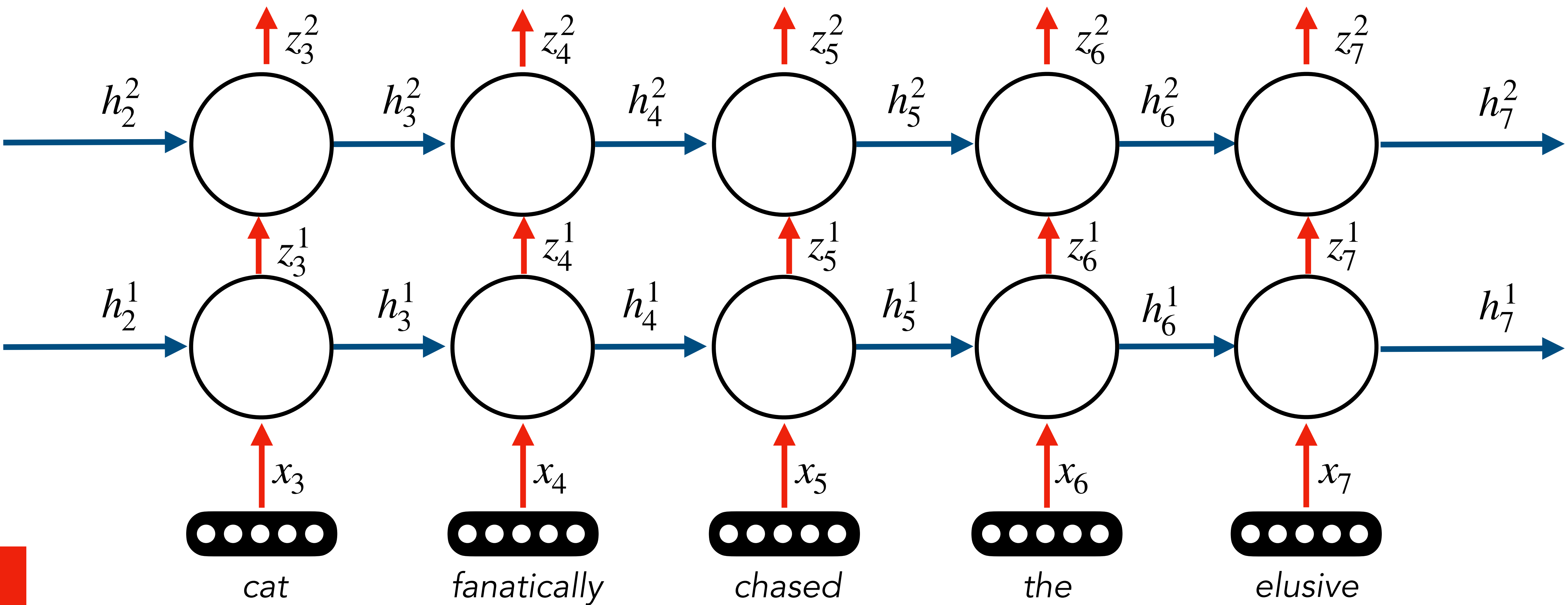
# Backpropagation through time

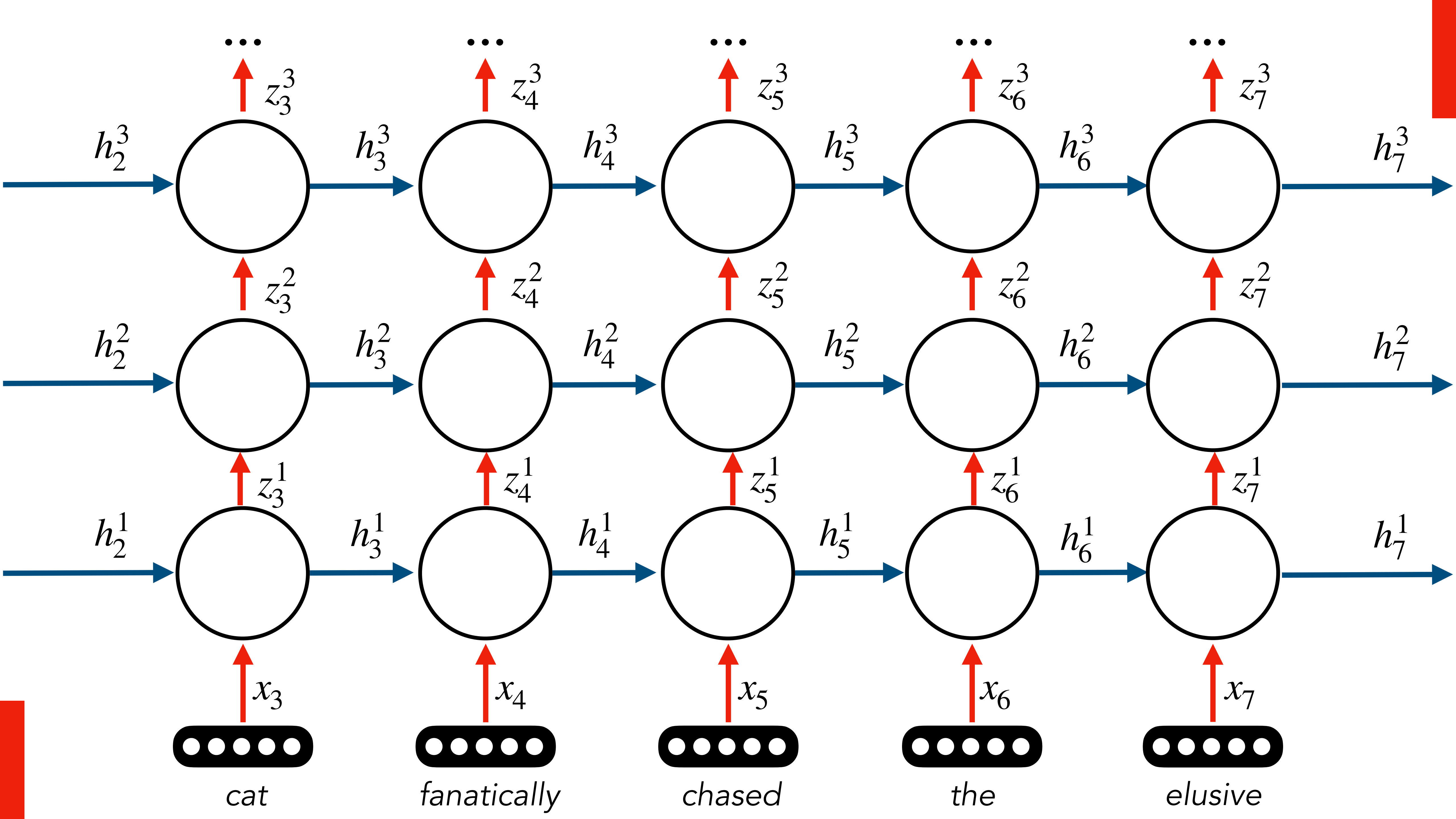


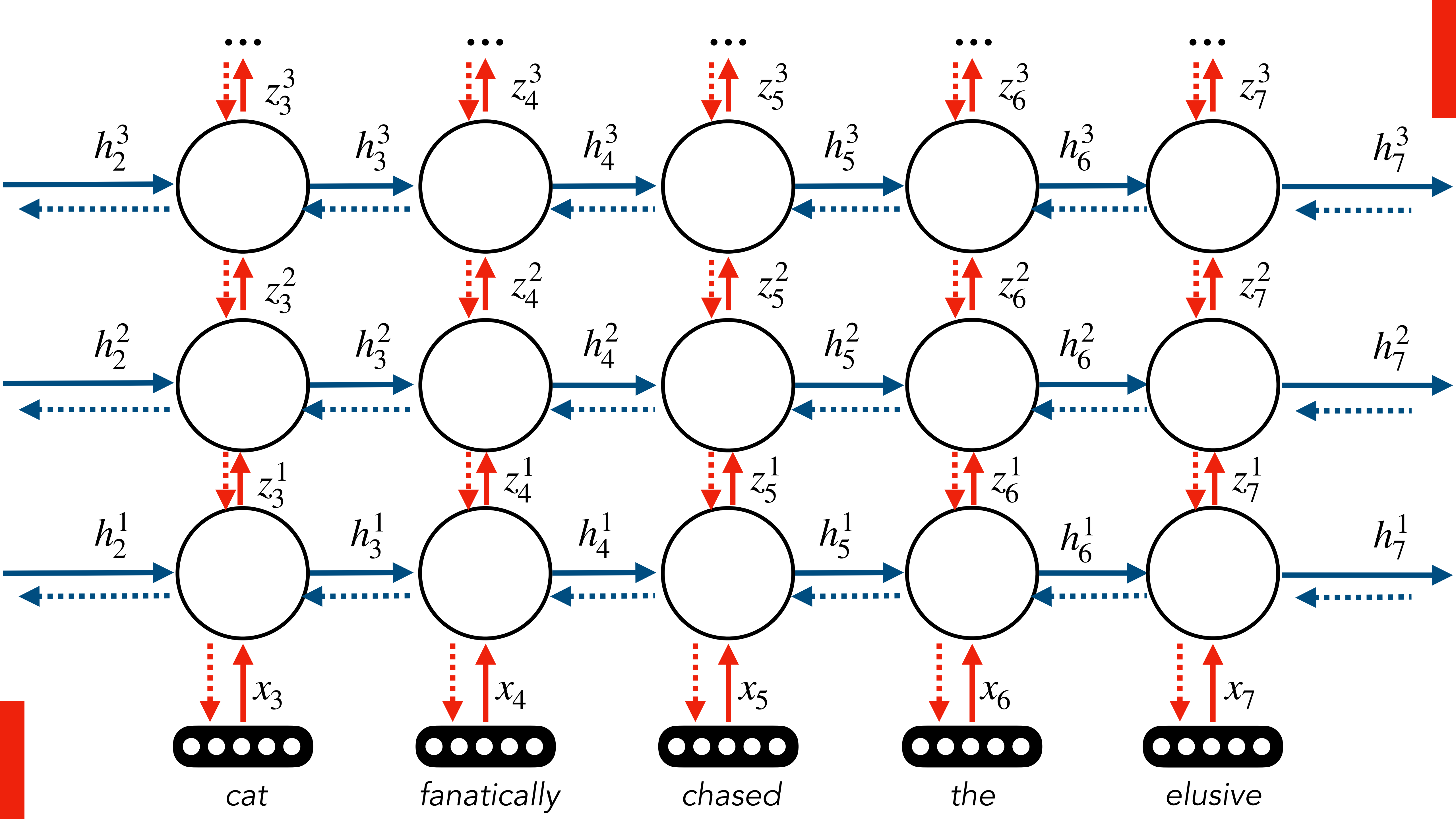
# RNNs In Practice

- Computing gradients by hand is hard!
  - Though potentially a good way to sanity check whether your network behaves the way you expect
- Most modern software packages for deep learning use automatic differentiation to compute gradients automatically from the forward pass
- Only need to define the forward pass of your model (much easier!)
- You'll use PyTorch in this class! You won't have to compute gradients by hand :)

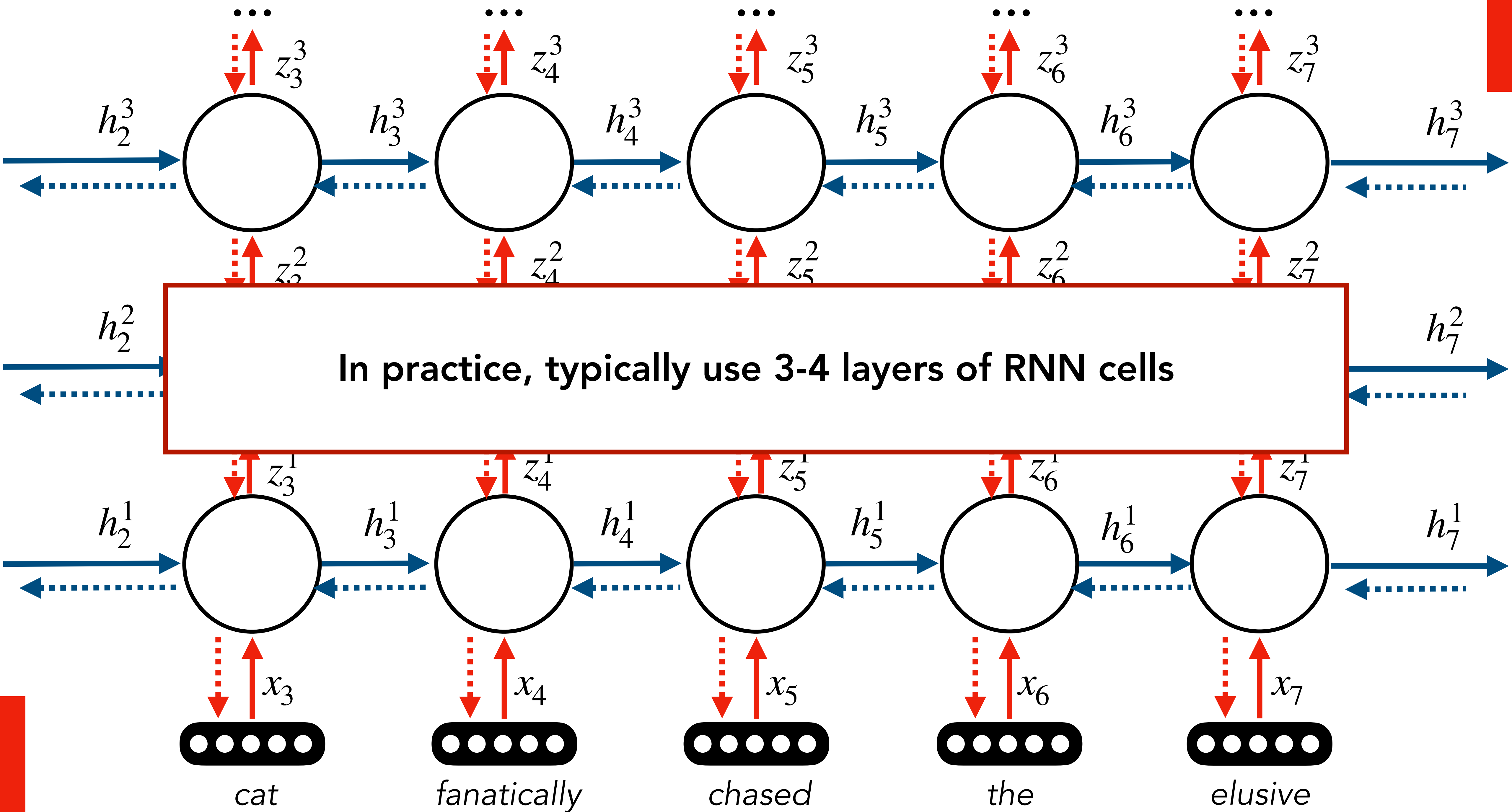
# Multiple Layers



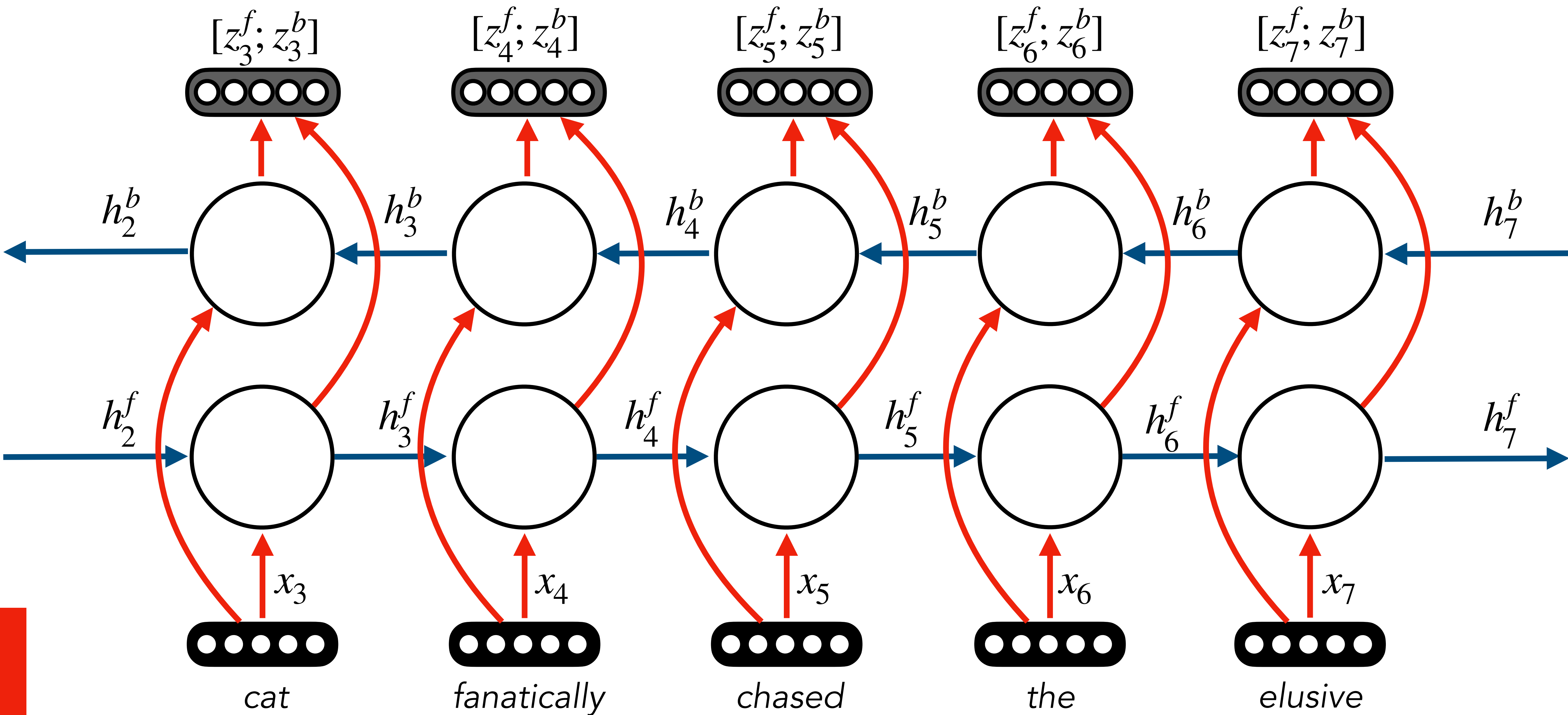






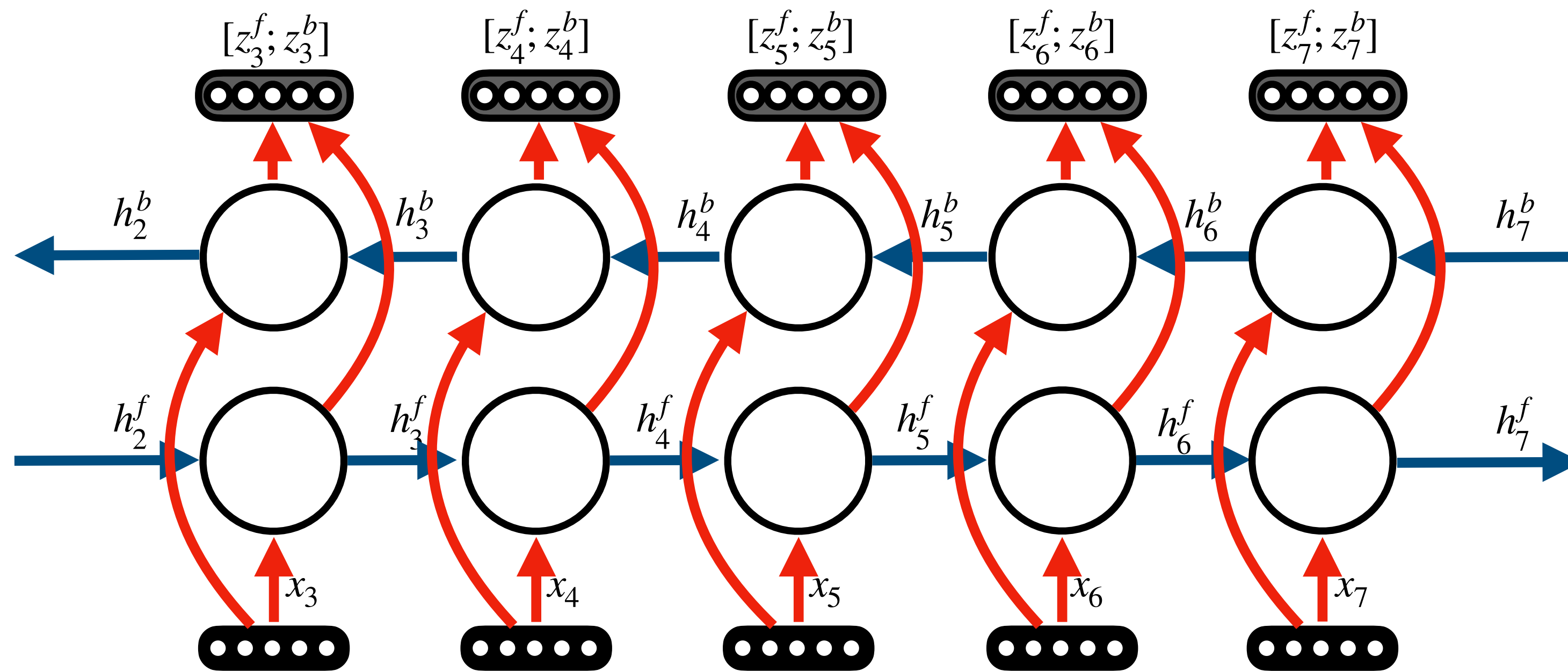


# Bidirectionality





# Bidirectionality



- **Concatenate** the output states for final representation at each step
  - Can also do mean / max
- Separate parameters for forward and backward RNNs
- If you can use the future text for the task, then you should use **bidirectionality**

# Question

**For which of the following task types  
can we use a bidirectional RNN?**

- (a) Classification**
- (b) Sequence labelling**
- (c) Text Generation**

# Recap

- Neural language models allow us to *share information* among similar sequences by learning neural representations that similarly represent them
- **Problem:** Fixed context language models can only process a limited window of the word history at a time
- **Solution:** recurrent neural networks can **theoretically** learn to model an **unbounded context length**

# Issue with Recurrent Models

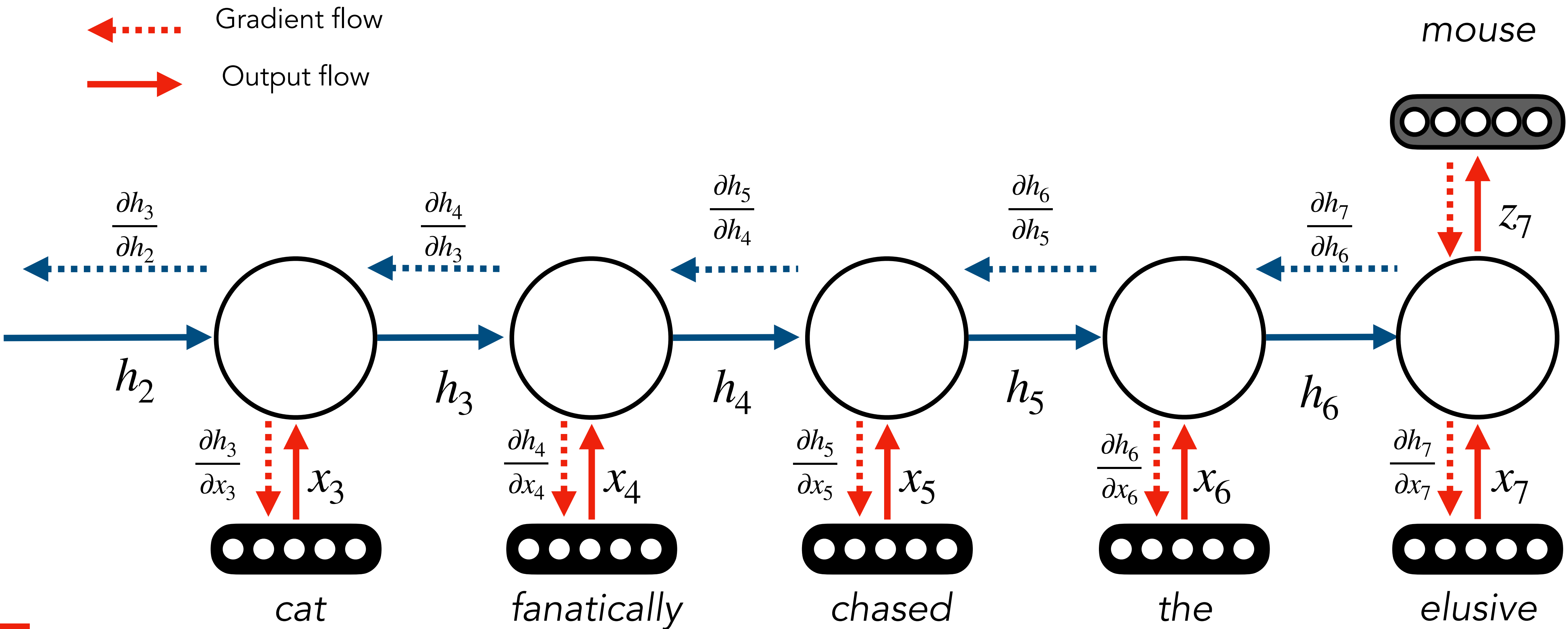
- Multiple steps of state overwriting makes it challenging to learn long-range dependencies.

*They tuned, discussed for a moment, then struck up a lively **jig**. Everyone joined in, turning the courtyard into an even more chaotic scene, people now **dancing** in circles, **swinging** and **spinning** in circles, everyone making up their own **dance steps**. I felt my feet tapping, my body wanting to move. Aside from writing, I 've always loved dancing .*

- Nearby words should affect each other more than farther ones, but RNNs make it challenging to learn any long-range interactions

# Backpropagation through time

← ... Gradient flow  
→ Output flow



# Vanishing Gradients

$$z_t = \sigma(W_{zh}h_t + b_z)$$

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$

$$\frac{\partial z_t}{\partial h_{t-2}} = \frac{\partial z_t}{\partial h_t} \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(v_t)}{\partial v_t} \textcolor{yellow}{W_{zh}} \frac{\partial \sigma(u_t)}{\partial u_t} \textcolor{pink}{W_{hh}} \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \textcolor{blue}{W_{hh}}$$

---


$$v_t = \textcolor{yellow}{W_{zh}}h_t + b_z \quad z_t = \sigma(v_t)$$

$$u_t = W_{hx}x_t + \textcolor{pink}{W_{hh}}h_{t-1} + b_h \quad h_t = \sigma(u_t)$$


---

$$\frac{\partial z_t}{\partial h_t} = \frac{\partial \sigma(v_t)}{\partial v_t} \textcolor{yellow}{\frac{\partial v_t}{\partial h_t}} = \frac{\partial \sigma(v_t)}{\partial v_t} \textcolor{yellow}{W_{zh}}$$

$$\frac{\partial h_t}{\partial h_{t-1}} = \frac{\partial \sigma(u_t)}{\partial u_t} \textcolor{pink}{\frac{\partial u_t}{\partial h_{t-1}}} = \frac{\partial \sigma(u_t)}{\partial u_t} \textcolor{pink}{W_{hh}}$$

$$\frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \textcolor{blue}{\frac{\partial u_{t-1}}{\partial h_{t-2}}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \textcolor{blue}{W_{hh}}$$

Generalising this:

$$\frac{\partial h_t}{\partial h_{t-T}} = \prod_{i=t-T}^{i=t} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{i=t-T}^{i=t} \frac{\partial \sigma(u_i)}{\partial u_i} \textcolor{blue}{W_{hh}}$$

# Vanishing Gradients

$$z_t = \sigma(W_{zh}h_t + b_z)$$

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$

$$\frac{\partial z_t}{\partial h_{t-2}} = \frac{\partial z_t}{\partial h_t} \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(v_t)}{\partial v_t} \boxed{W_{zh}} \frac{\partial \sigma(u_t)}{\partial u_t} \boxed{W_{hh}} \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \boxed{W_{hh}}$$

---


$$v_t = \boxed{W_{zh}}h_t + b_z \quad z_t = \sigma(v_t)$$

$$u_t = W_{hx}x_t + \boxed{W_{hh}}h_{t-1} + b_h \quad h_t = \sigma(u_t)$$


---

$$\frac{\partial z_t}{\partial h_t} = \frac{\partial \sigma(v_t)}{\partial v_t} \boxed{\frac{\partial v_t}{\partial h_t}} = \frac{\partial \sigma(v_t)}{\partial v_t} \boxed{W_{zh}}$$

$$\frac{\partial h_t}{\partial h_{t-1}} = \frac{\partial \sigma(u_t)}{\partial u_t} \boxed{\frac{\partial u_t}{\partial h_{t-1}}} = \frac{\partial \sigma(u_t)}{\partial u_t} \boxed{W_{hh}}$$

$$\frac{\partial h_{t-1}}{\partial h_{t-2}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \boxed{\frac{\partial u_{t-1}}{\partial h_{t-2}}} = \frac{\partial \sigma(u_{t-1})}{\partial u_{t-1}} \boxed{W_{hh}}$$

Generalising this:

$$\frac{\partial h_t}{\partial h_{t-T}} = \prod_{i=t-T}^{i=t} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{i=t-T}^{i=t} \frac{\partial \sigma(u_i)}{\partial u_i} \boxed{W_{hh}}$$

< 1 for many  
activation fns

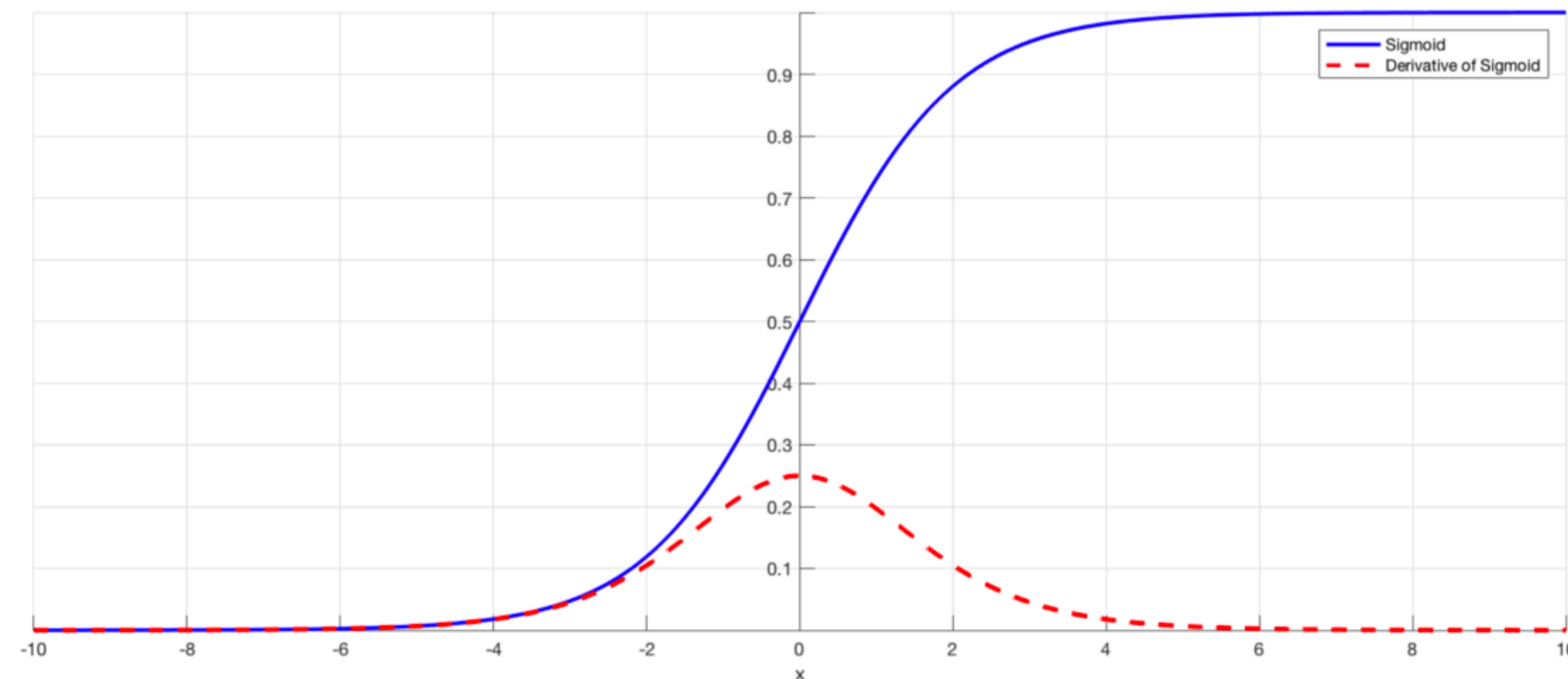
Typically small  
(Regularisation)



# Vanishing Gradients

- **Learning Problem:** Long unrolled networks will crush gradients that backpropagate to earlier time steps

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$
$$u = W_{hx}x_t + W_{hh}h_{t-1} + b_h$$
$$\frac{\partial h_t}{\partial h_{t-1}} = \frac{\partial \sigma(u)}{\partial u} \frac{\partial u}{\partial h_{t-1}} = W_{hh} \frac{\partial \sigma(u)}{\partial u}$$

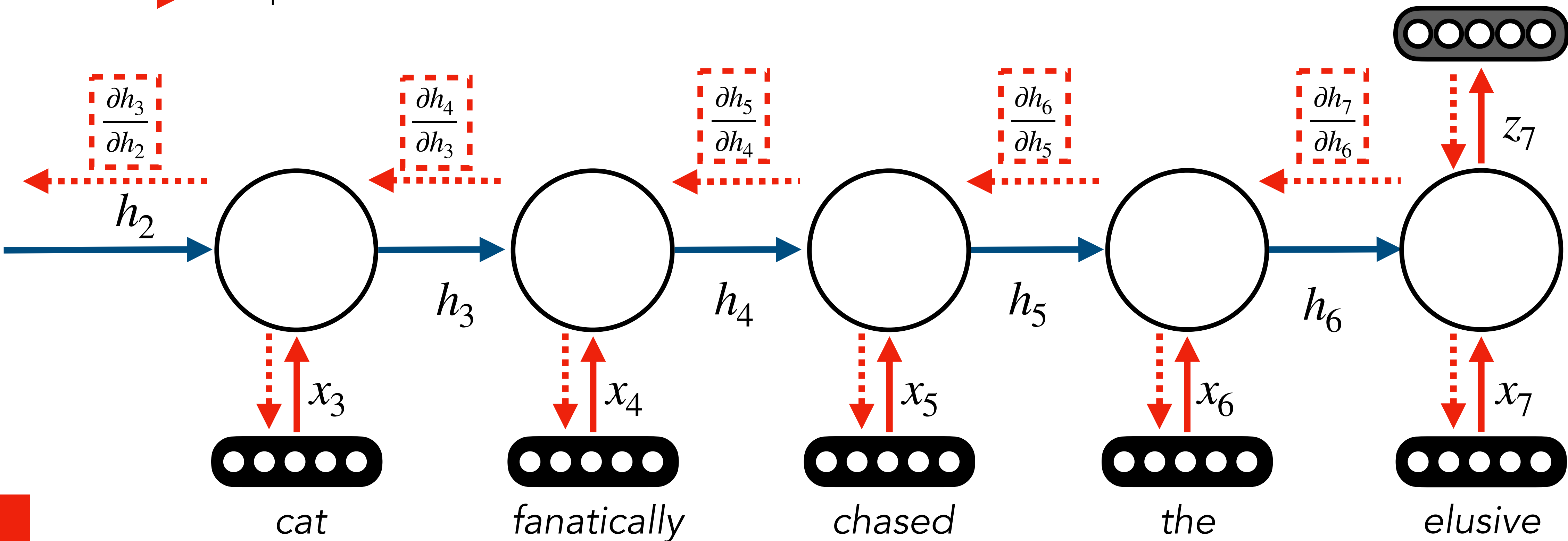
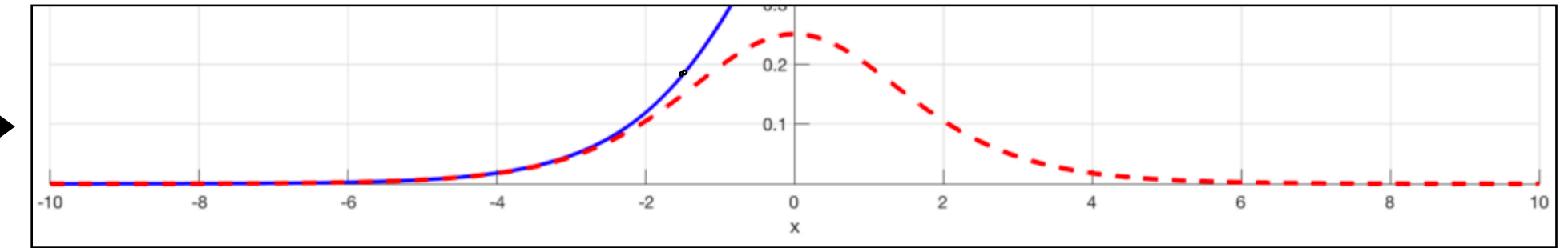




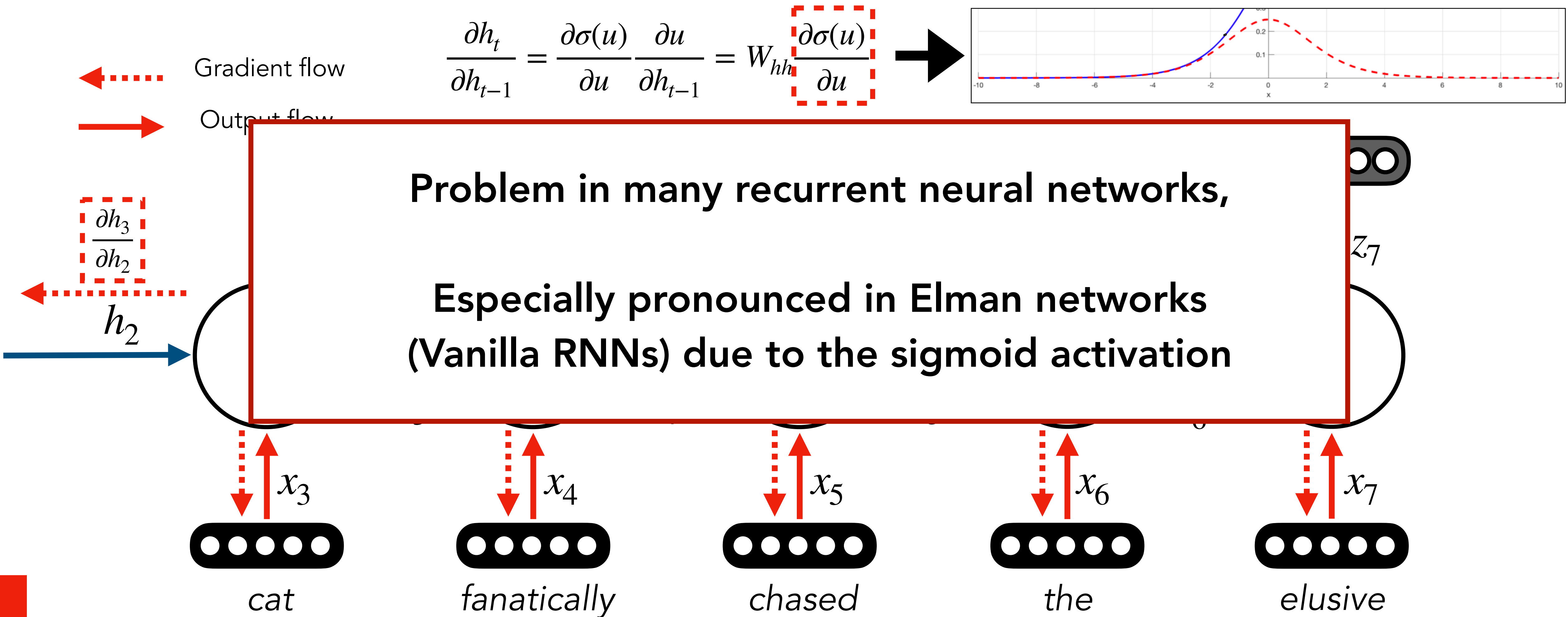
# Vanishing Gradients

← Gradient flow  
→ Output flow

$$\frac{\partial h_t}{\partial h_{t-1}} = \frac{\partial \sigma(u)}{\partial u} \frac{\partial u}{\partial h_{t-1}} = W_{hh} \frac{\partial \sigma(u)}{\partial u}$$



# Vanishing Gradients



# Question

How could we fix this vanishing gradient problem?

$$\frac{\partial h_t}{\partial h_{t-T}} = \prod_{i=t-T}^{i=t} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{i=t-T}^{i=t} \frac{\partial \sigma(u_i)}{\partial u_i} W_{hh}$$

Typically less than one

Typically small (Regularisation)

# Recap

- Early neural language models (and n-gram models) suffer from **fixed context windows**
- Recurrent neural networks can **theoretically** learn to model an **unbounded context length** using back propagation through time (BPTT)
- Practically, **vanishing gradients** can still stop many RNNs from learning **long-range dependencies**

# Gated Recurrent Neural Networks

- Use gates to avoid dampening gradient signal every time step

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$

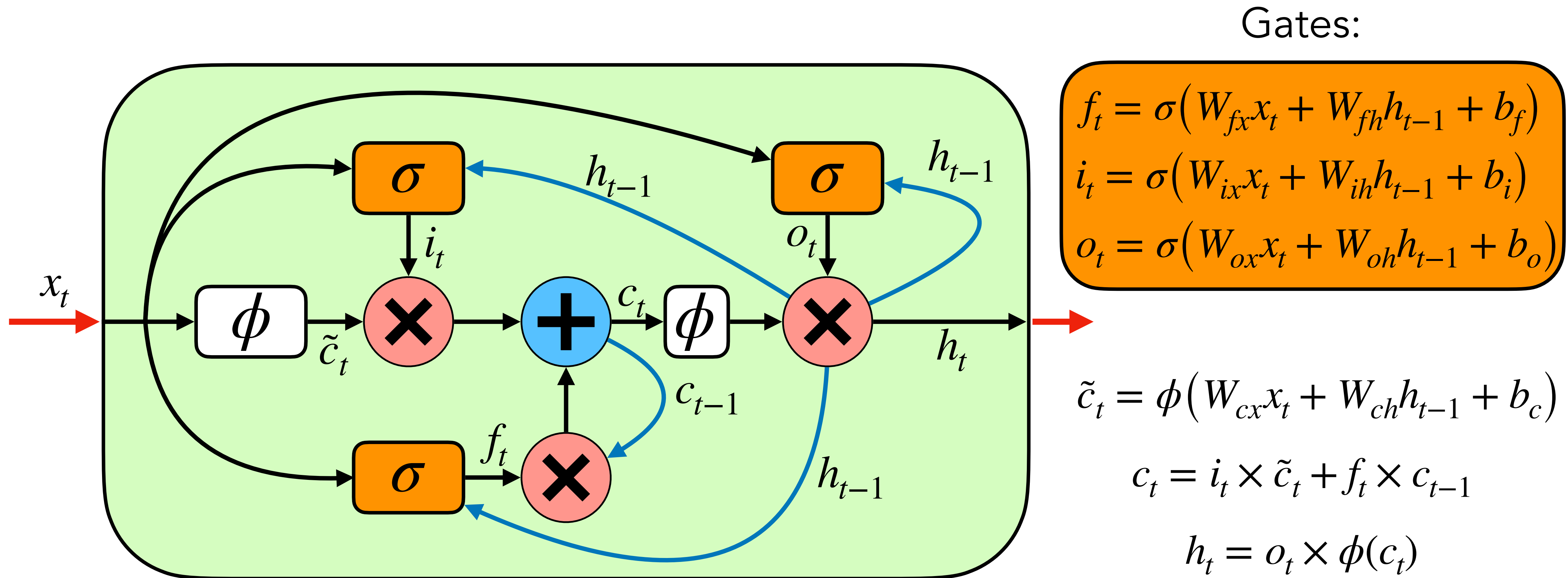
**Elman Network**

$$h_t = h_{t-1} \odot \mathbf{f} + \mathbf{func}(x_t)$$

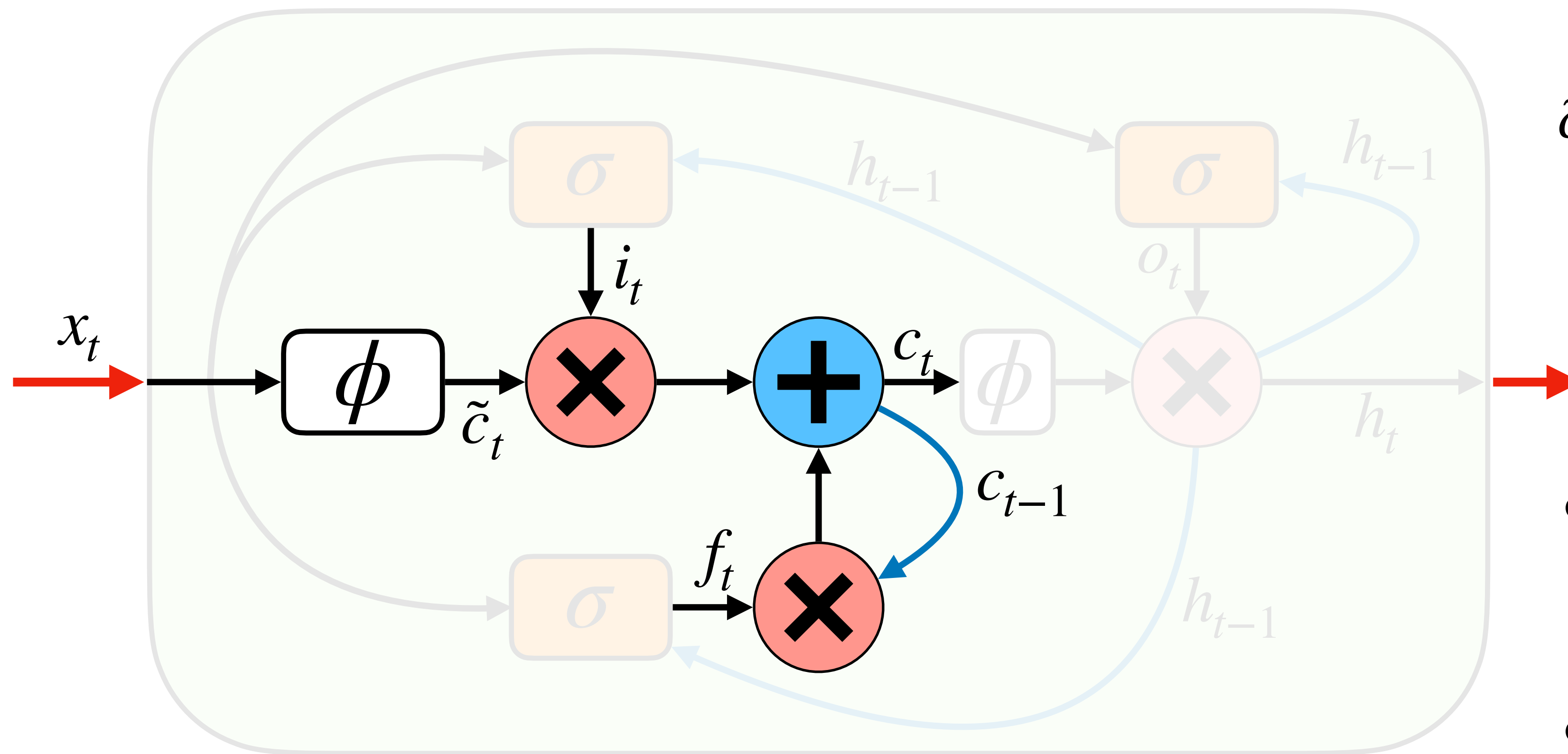
**Gated Network Abstraction**

- Gate value  $\mathbf{f}$  computes how much information from previous hidden state moves to the next time step  $\rightarrow 0 < \mathbf{f} < 1$
- Because  $h_{t-1}$  is no longer inside the activation function, it is not automatically constrained, reducing vanishing gradients!

# Long Short Term Memory (LSTM)



# Cell State



$$\tilde{c}_t = \phi(W_{cx}x_t + W_{ch}h_{t-1} + b_c)$$

$$c_t = i_t \times \tilde{c}_t + f_t \times c_{t-1}$$

- Hidden state  $h_{t-1}$  is now short-term memory
- Cell state  $c_t$  tracks longer-term dependencies



# What does the cell state track?

- Can visualise the activations of cell state (i.e., dimensions of **c**) and find semantic behaviour!
- Stack Overflow example:

```
#ifdef CONFIG_AUDITSYSCALL
static inline int audit_match_class_bits(int class, u32 *mask)
{
    int i;
    if (classes[class]) {
        for (i = 0; i < AUDIT_BITMASK_SIZE; i++)
            if (mask[i] & classes[class][i])
                return 0;
    }
    return 1;
}
```

# What does the cell state track?

- Can visualise the activations of cell state (i.e., dimensions of **c**) and find semantic behaviour!
- Stack Overflow example: **track indentation**

```
#ifdef CONFIG_AUDIT_SYSCALL
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            if (mask[i] & classes[class][i])
                return 0;
    }
    return 1;
}
```

- War and Peace:

"You mean to imply that I have nothing to eat out of.... On the contrary, I can supply you with everything even if you want to give dinner parties," warmly replied Chichagov, who tried by every word he spoke to prove his own rectitude and therefore imagined Kutuzov to be animated by the same desire.

Kutuzov, shrugging his shoulders, replied with his subtle penetrating smile: "I meant merely to say what I said."



# What does the cell state track?

- Can visualise the activations of cell state (i.e., dimensions of **c**) and find semantic behaviour!
- Stack Overflow example: track indentation

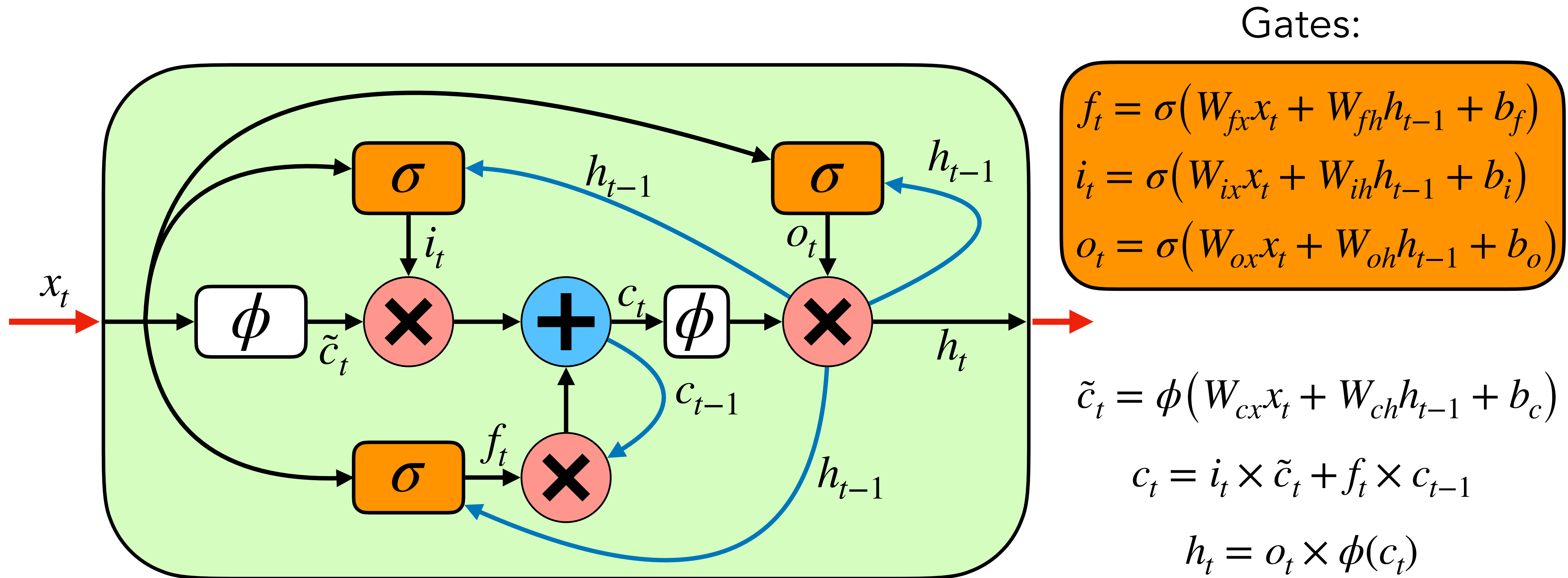
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        for (i = 0; i < AUDIT_BITMASK_SIZE; i++)
            if (mask[i] & classes[class][i])
                return 0;
    }
    return 1;
}
```

- War and Peace: **are we in a quote or not?**

"You mean to imply that I have nothing to eat out of.... On the contrary, I can supply you with everything even if you want to give dinner parties," warmly replied Chichagov, who tried by every word he spoke to prove his own rectitude and therefore imagined Kutuzov to be animated by the same desire.

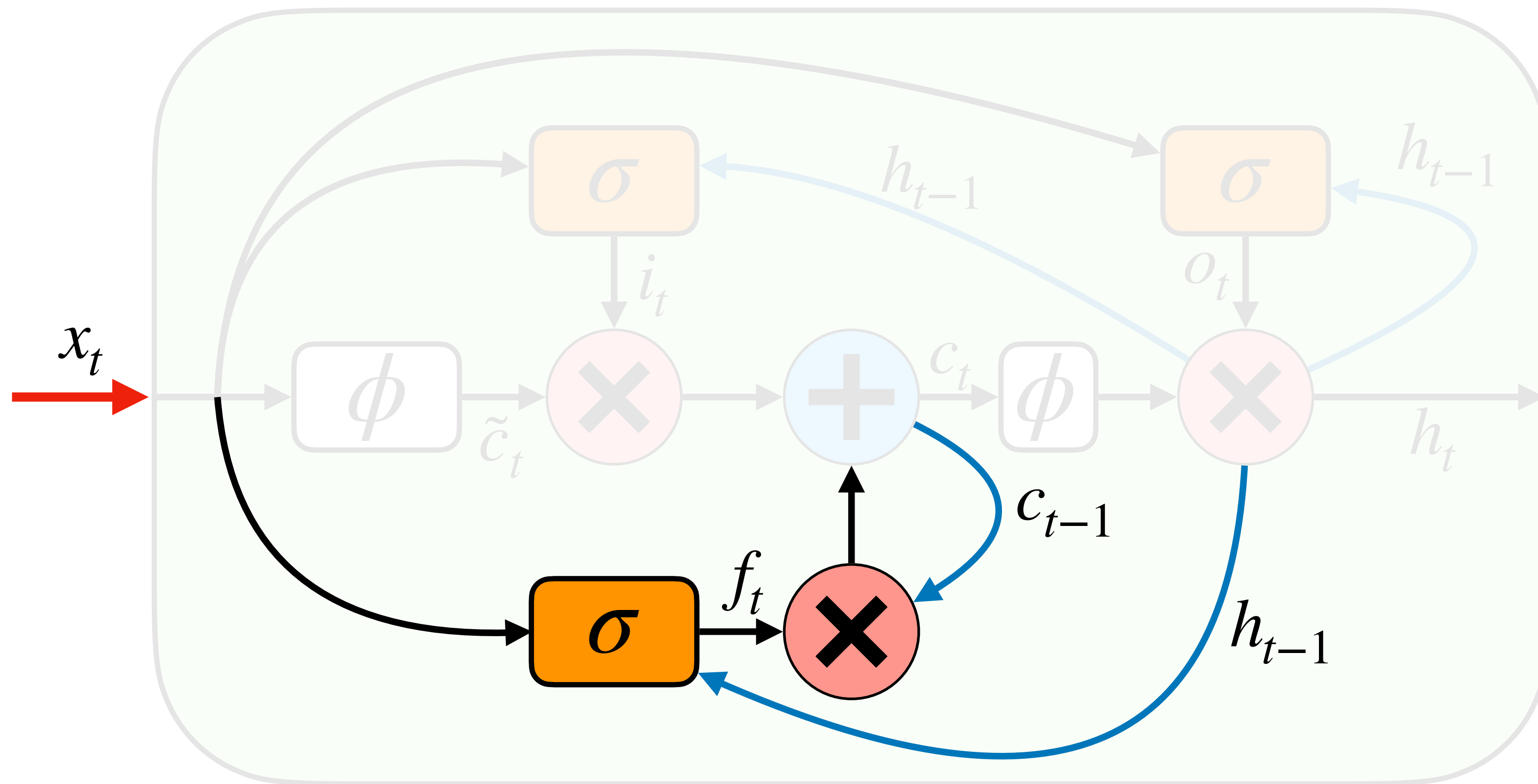
Kutuzov, shrugging his shoulders, replied with his subtle penetrating smile: "I meant merely to say what I said."

# Long Short Term Memory (LSTM)



# Forget Gate

I went to **the lecture**



$$f_t = \sigma(W_{fx}x_t + W_{fh}h_{t-1} + b_f)$$

$$\tilde{c}_t = \phi(W_{cx}x_t + W_{ch}h_{t-1} + b_c)$$

$$c_t = i_t \times \tilde{c}_t + f_t \times c_{t-1}$$

- Forget gate controls how much memory is forgotten
  - 1 -> remember the past
  - 0 -> forget everything up to now

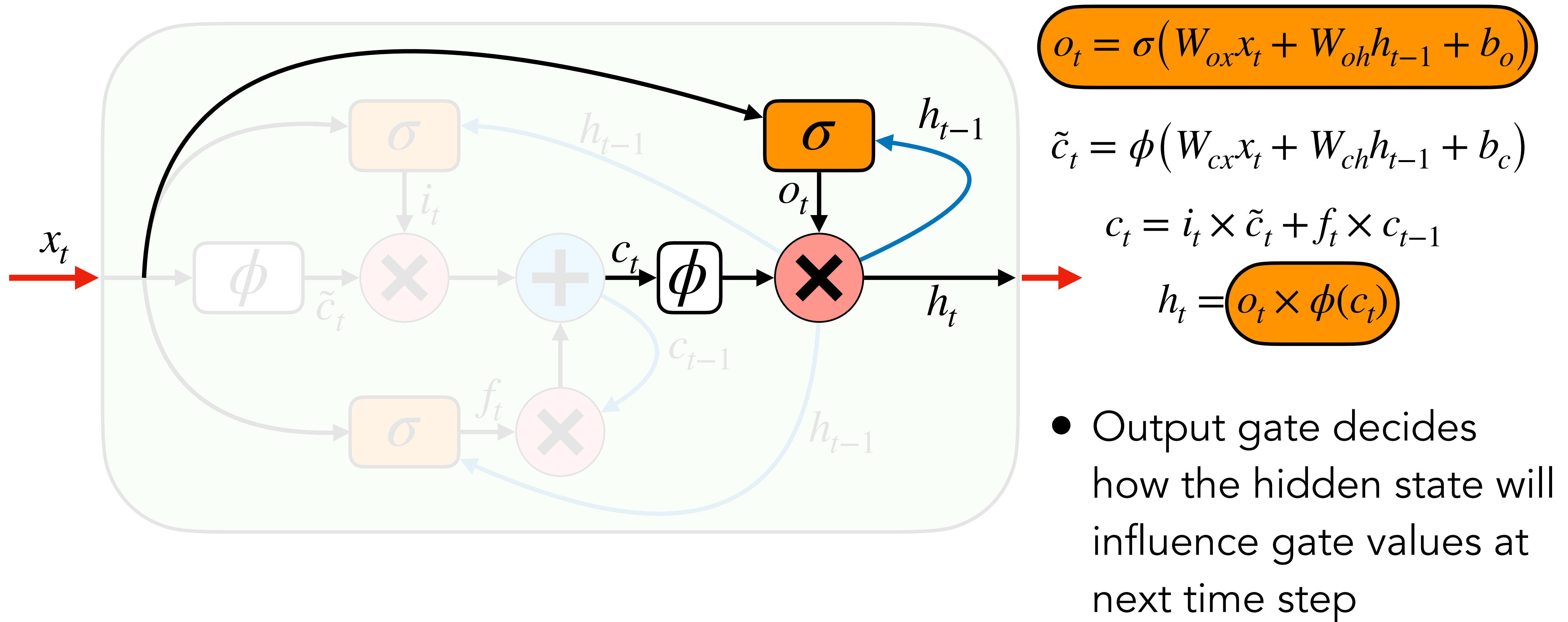
100

100

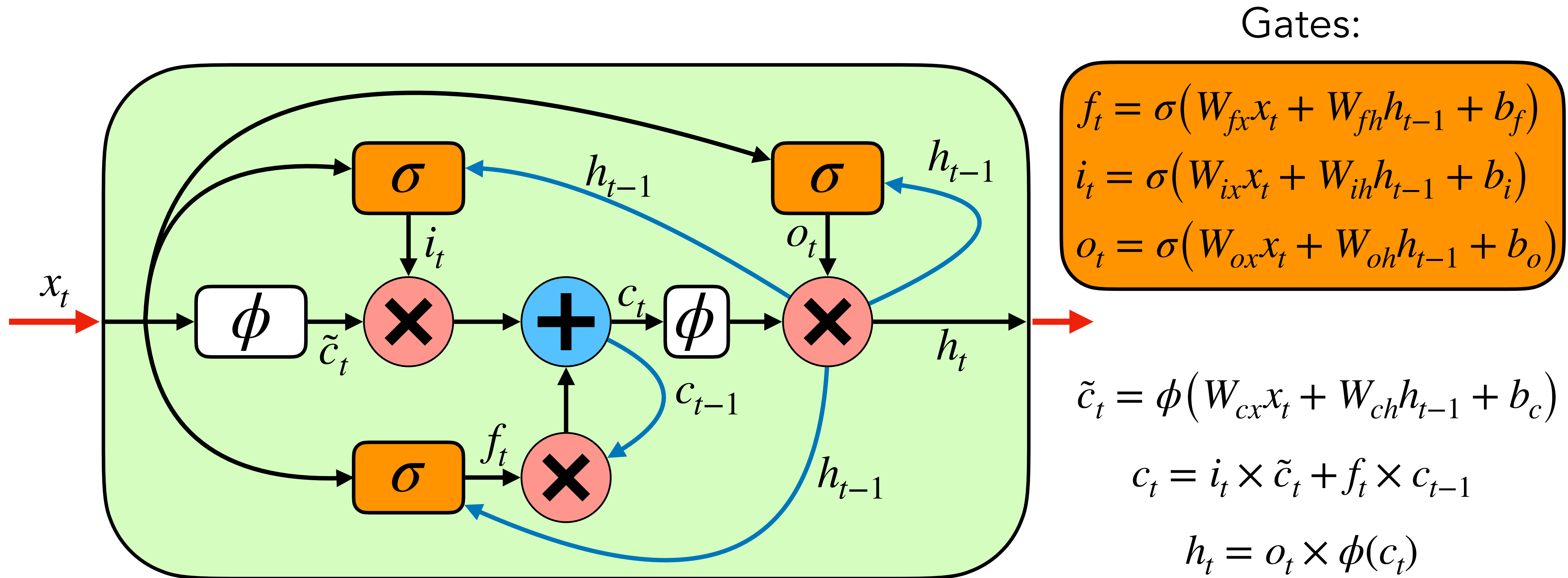
$$c_t = i_t \times \tilde{c}_t + f_t \times c_{t-1}$$



# Output Gate



# Long Short Term Memory (LSTM)



# Questions!

- For what type of input might the model learn to make the input gate be 0 ?
- What happens if the forget gate is 0?
- What happens if both the forget gate and input gate are 0 ?
- What happens if both the forget gate and input gate are 1 ?

Gates:

$$\begin{aligned}f_t &= \sigma(W_{fx}x_t + W_{fh}h_{t-1} + b_f) \\i_t &= \sigma(W_{ix}x_t + W_{ih}h_{t-1} + b_i) \\o_t &= \sigma(W_{ox}x_t + W_{oh}h_{t-1} + b_o)\end{aligned}$$

$$\tilde{c}_t = \phi(W_{cx}x_t + W_{ch}h_{t-1} + b_c)$$

$$c_t = i_t \times \tilde{c}_t + f_t \times c_{t-1}$$

$$h_t = o_t \times \phi(c_t)$$

# Gated Recurrent Unit (GRU)

- Also uses gates to avoid dampening gradient signal every time step

$$h_t = (1 - \mathbf{z}) \odot h_{t-1} + \mathbf{z} \odot \mathbf{func}(x_t, h_{t-1}) \quad h_t = h_{t-1} \odot \mathbf{f} + \mathbf{func}(x_t)$$

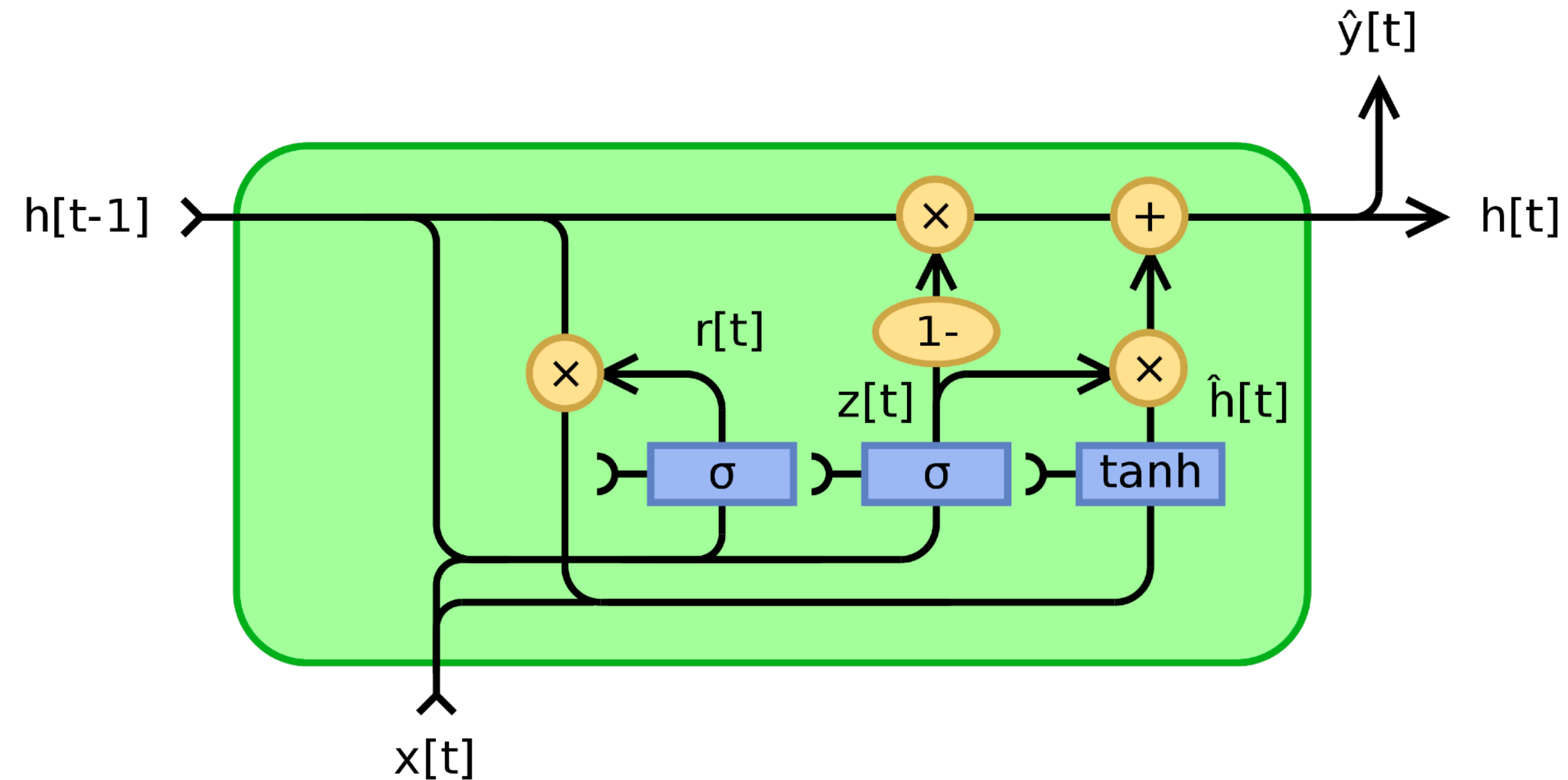
**GRU**

**LSTM**

- Works similarly to LSTM
  - Theoretically less powerful (find out why tomorrow!)
  - Typically faster to train and sometimes works better than LSTMs

# Gated Recurrent Unit (GRU)

- $\mathbf{z}$  is update gate (used to update hidden state),  $\mathbf{r}$  is reset gate (used to reset hidden state)
- The single hidden state and simpler update gate gives simpler mixing algorithm than in LSTMs



$$z_t = \sigma_g(W_z x_t + U_z h_{t-1} + b_z)$$

$$r_t = \sigma_g(W_r x_t + U_r h_{t-1} + b_r)$$

$$h_t = (1 - z_t) \circ h_{t-1} + z_t \circ \sigma_h(W_h x_t + U_h(r_t \circ h_{t-1}) + b_h)$$

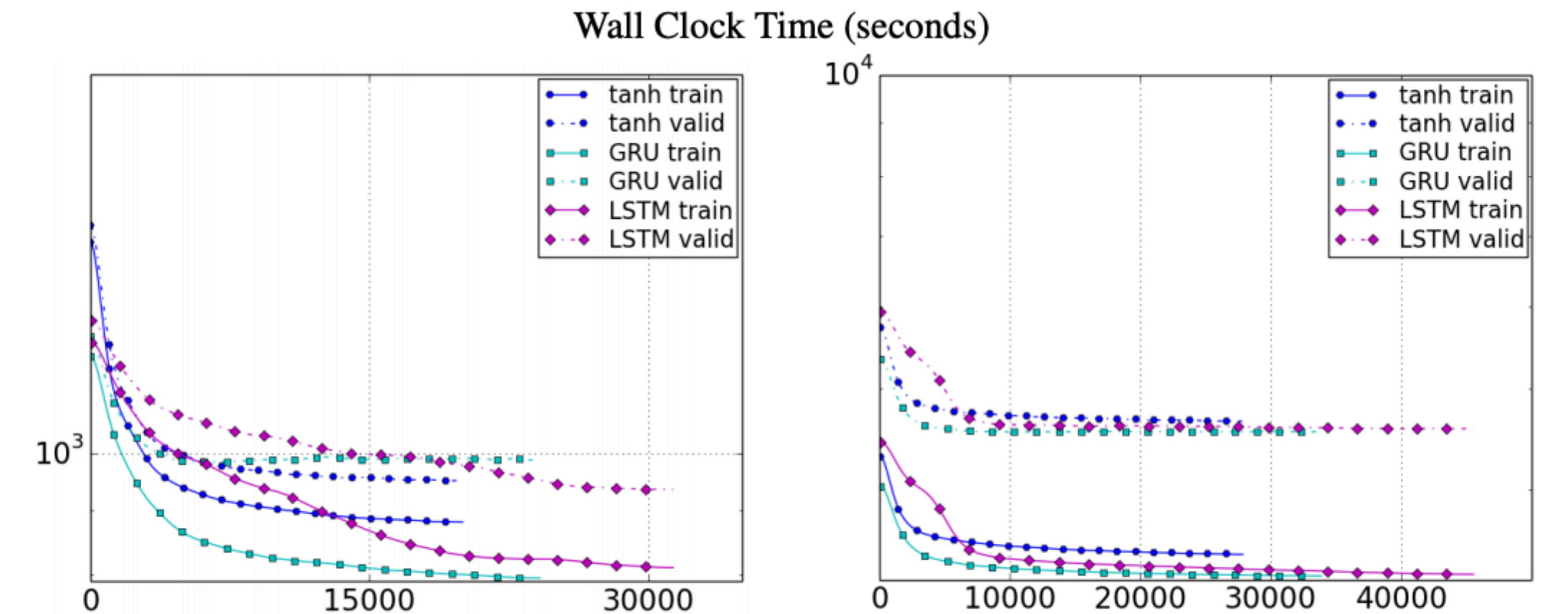
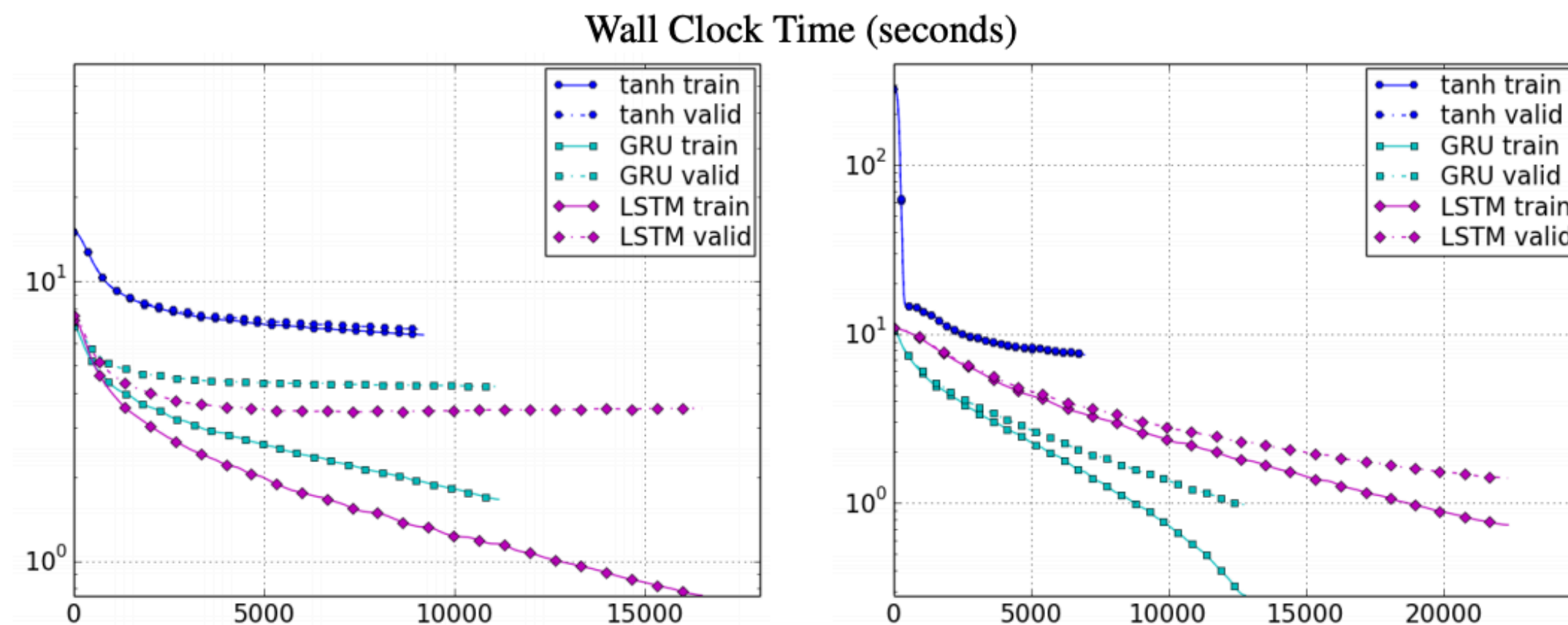
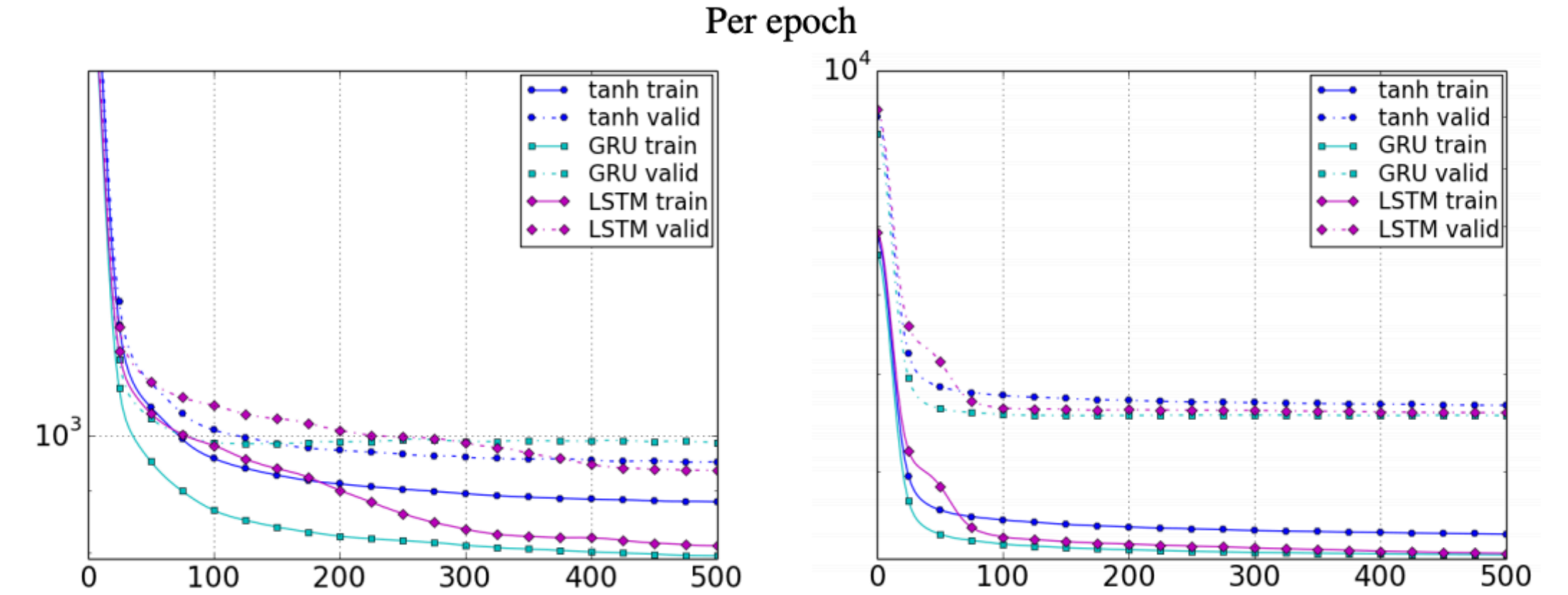
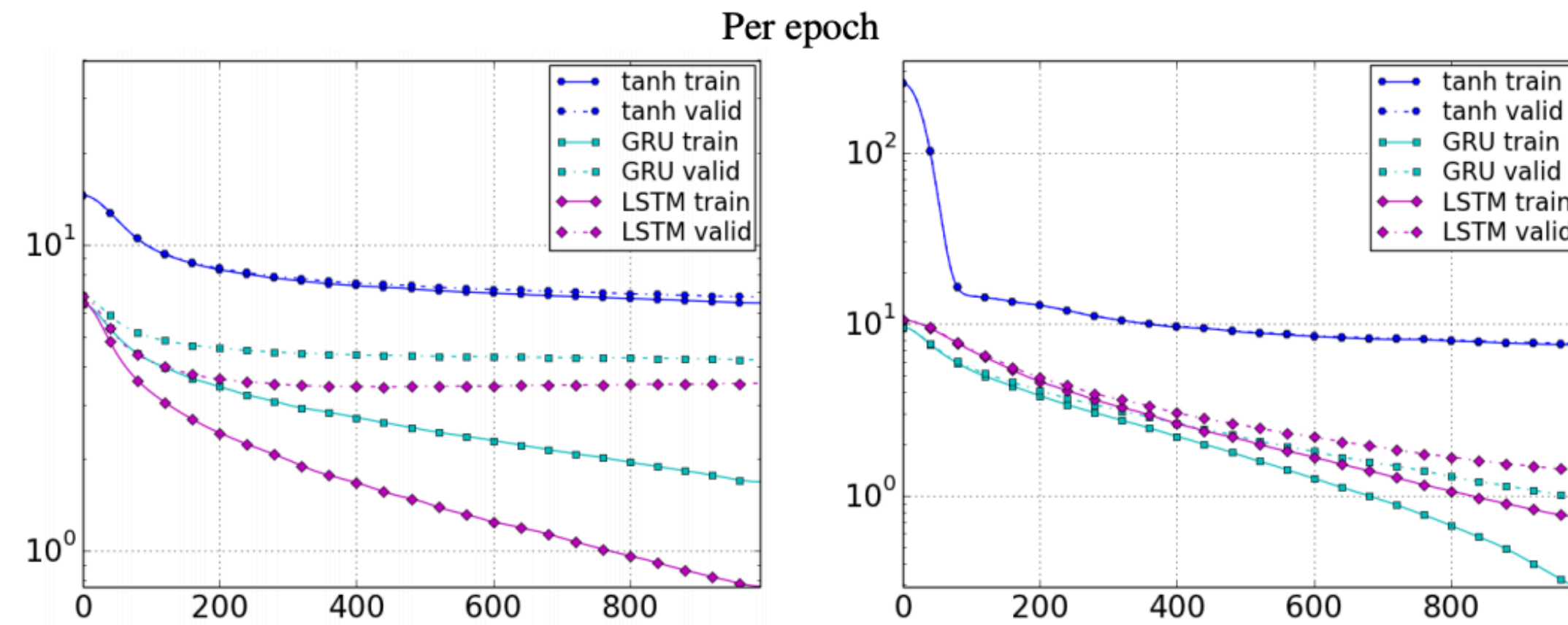


# Which is better?

Speech Signal Modeling

Music Modeling

Negative Loglikelihood



(a) UbiSoft Dataset A

(b) UbiSoft Dataset B

(a) Nottingham Dataset

(b) MuseData Dataset

# Question

**What are the advantages of using LSTMs and GRUs?**



# Vanishing Gradients?

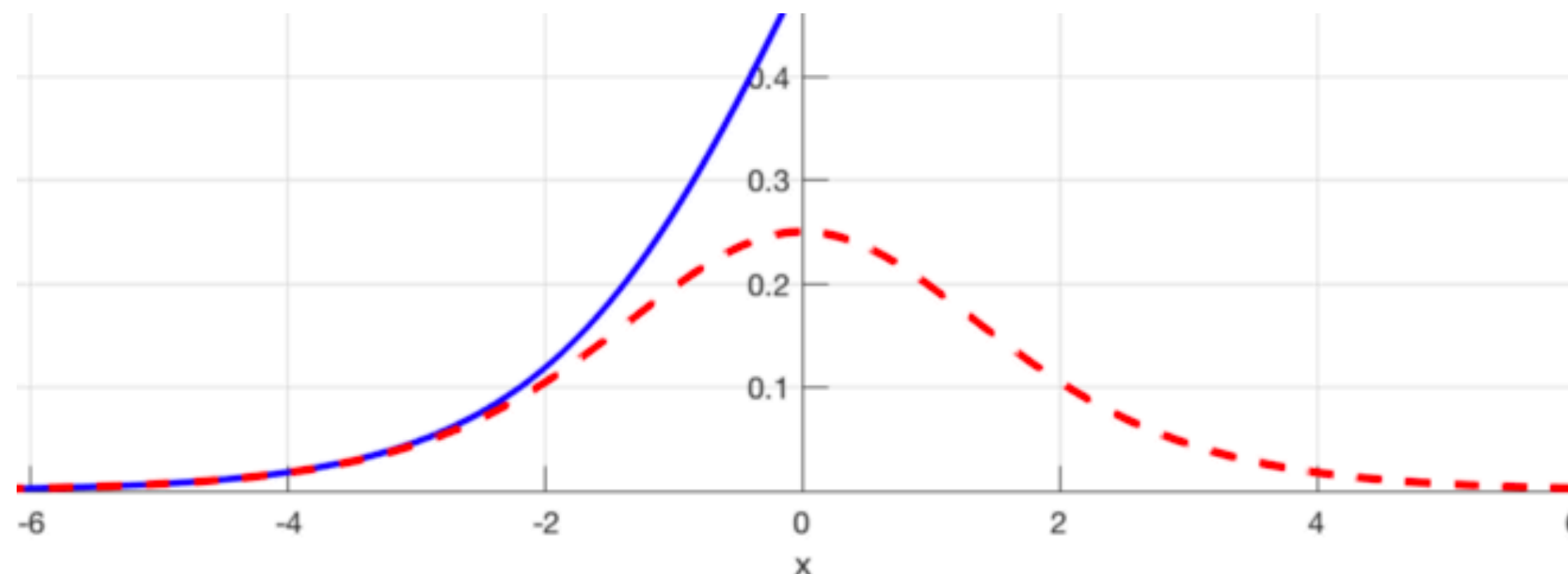
Recurrent Neural Networks

Long Short Term Memory

State maintained by hidden state feedback

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$

Gradient systemically squashed by sigmoid



State maintained by cell value

$$c_t = i_t \times \tilde{c}_t + f_t \times c_{t-1}$$

Gradient set by value of forget gate

$$\frac{\partial c_t}{\partial c_{t-1}} = f_t$$

Can still vanish, but only if forget gate closes!

# Question

**What's a disadvantage of using a LSTM or GRU?**

# Question

What's a disadvantage of using a LSTM or GRU?

More parameters!

$$f_t = \sigma(W_{fx}x_t + W_{fh}h_{t-1} + b_f)$$

$$i_t = \sigma(W_{ix}x_t + W_{ih}h_{t-1} + b_i)$$

$$o_t = \sigma(W_{ox}x_t + W_{oh}h_{t-1} + b_o)$$

$$\tilde{c}_t = \phi(W_{cx}x_t + W_{ch}h_{t-1} + b_c)$$

$$c_t = i_t \times \tilde{c}_t + f_t \times c_{t-1}$$

$$h_t = o_t \times \phi(c_t)$$

$$z_t = \sigma(W_{zh}h_t + b_z)$$

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h)$$

# Question

**Could there be better architectures than GRUs and LSTMs?**

# Optimal Architectures?

MUT1:

$$\begin{aligned} z &= \text{sigm}(W_{xz}x_t + b_z) \\ r &= \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r) \\ h_{t+1} &= \tanh(W_{hh}(r \odot h_t) + \tanh(x_t) + b_h) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

MUT2:

$$\begin{aligned} z &= \text{sigm}(W_{xz}x_t + W_{hz}h_t + b_z) \\ r &= \text{sigm}(x_t + W_{hr}h_t + b_r) \\ h_{t+1} &= \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

MUT3:

$$\begin{aligned} z &= \text{sigm}(W_{xz}x_t + W_{hz} \tanh(h_t) + b_z) \\ r &= \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r) \\ h_{t+1} &= \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

Arch.	Arith.	XML	PTB
Tanh	0.29493	0.32050	0.08782
LSTM	0.89228	0.42470	0.08912
LSTM-f	0.29292	0.23356	0.08808
LSTM-i	0.75109	0.41371	0.08662
LSTM-o	0.86747	0.42117	0.08933
LSTM-b	0.90163	0.44434	0.08952
GRU	0.89565	0.45963	0.09069
MUT1	<b>0.92135</b>	<b>0.47483</b>	0.08968
MUT2	0.89735	<b>0.47324</b>	0.09036
MUT3	0.90728	0.46478	<b>0.09161</b>

Arch.	5M-tst	10M-v	20M-v	20M-tst
Tanh	4.811	4.729	4.635	4.582 (97.7)
LSTM	4.699	4.511	4.437	4.399 (81.4)
LSTM-f	4.785	4.752	4.658	4.606 (100.8)
LSTM-i	4.755	4.558	4.480	4.444 (85.1)
LSTM-o	4.708	4.496	4.447	4.411 (82.3)
LSTM-b	4.698	4.437	4.423	<b>4.380 (79.83)</b>
GRU	4.684	4.554	4.559	4.519 (91.7)
MUT1	4.699	4.605	4.594	4.550 (94.6)
MUT2	4.707	4.539	4.538	4.503 (90.2)
MUT3	4.692	4.523	4.530	4.494 (89.47)

# Recap

- Recurrent neural networks can **theoretically** learn to model an **unbounded context length**
  - no increase in model size because weights are shared across time steps
- Practically, however, **vanishing gradients** stop vanilla RNNs from learning useful **long-range dependencies**
- LSTMs and GRUs are variants of recurrent networks that mitigate the vanishing gradient problem
  - used for for **many sequence-to-sequence tasks (up next!)**

# References

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