ECON 616: Machine Learning

Ed Herbst

Last Updated: March 13, 2025

Background

- Stuff written by economists: Varian (2014), Mullainathan and Spiess (2017), Athey (2018)

- Useful books: Hastie, Tibshirani, and Friedman (2009),

- Gentle introduction: Machine Learning on `http://coursera.org`; many other things on the internet (of varying quality).

- Computation: `scikit-learn` (python).

“Machine Learning” definition

- Hard to define; context dependent;

- Athey (2018):

  > *... a field that develops algorithms designed to applied to datasets with the main areas of focus being prediction (regression), classification, and clustering or grouping tasks.*

- Broadly speaking, two branches:
  - Supervised: dependent variables known (think predicting output growth)
  - Unsupervised: dependent variables unknown (think classifying recessions)

A Dictionary

	Econometrics	ML
$\underbrace{y}_{T\times 1} = \{y_{1:T}\}$	Endogenous	outcome
$\underbrace{X}_{T\times n} = \{x_{1:T}\}$	Exogenous	Feature
$1 : T$	“in sample”	“training”
$T : T + V$	??? not enough data!	“validation”
$T + V : T + V + O$	“out of sample”	“testing”

Today I’ll concentrate on prediction (regression) problems.

$$\hat{y} = f(X; \theta)$$

Economists would call this modeling the `conditional expectation`, MLers the `hypothesis function`.

It all starts with a loss funciton

Generally speaking, we can write (any) estimation problem as essentially a loss minizimation problem.

Let L

$$L(\hat{y}, y) = L(f(X; \theta), y)$$

Be a `loss function` (sometimes called a "cost" function).

Estimation in ML: pick θ to minimize loss.

ML more concerned with minimizing my loss than inference on θ per se.

Forget standard errors. . .

Gradient Descent

- In practice, it is often not possible to minimize the loss function analytically.

- In fact, most machine learning models correspond to functions $f(\cdot;\theta)$ that are highly nonlinear in θ.

- In addition to an explosition of datasets, a large part of the success of machine learning algorithms is the development of robust minimization routines.

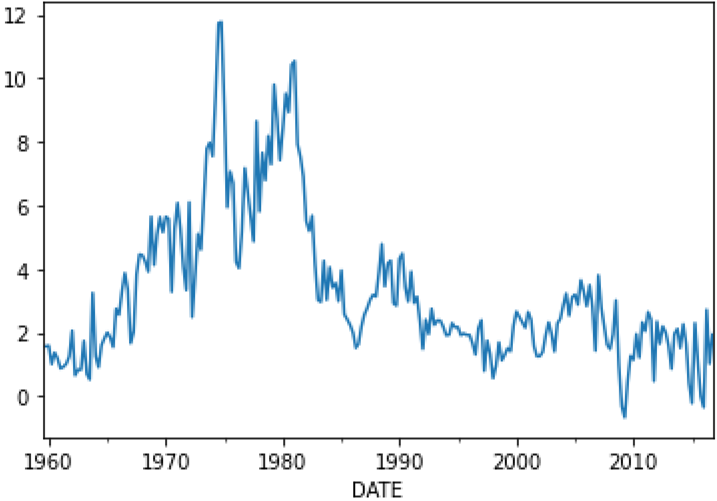
- The first one everyone learns is called `gradient descent`:

$$\theta' = \theta + \alpha \frac{dL(\hat{y}, y)}{d\theta}$$

- You can use this for OLS when $N > T$.

Example: Forecasting Inflation

Let's consider forecasting (GDP deflator) inflation.



Linear Regression

- Consider forecasting inflation using only it's lag and constant.
- Training sample: 1985-2000
- Testing sample: 2001-2015
- scikit-learn code

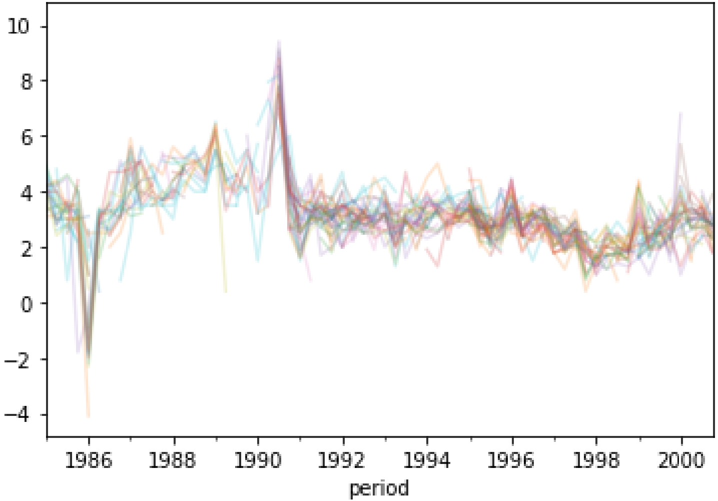
```
from sklearn.linear_model import LinearRegression
linear_model_univariate = LinearRegression()

train_start, train_end = ’1985’, ’2000’
inf[’inf_L1’] = inf.GDPDEF.shift(1)
inf = inf.dropna(how=’any’)
inftrain = inf[train_start:train_end]
Xtrain,ytrain = (inftrain.inf_L1.values.reshape(-1,1),
                 inftrain.inf)
fitted_ols = linear_model_univariate.fit(Xtrain,ytrain)
```

Many regressors

Let's add the individual spf forecasts to our regression.

```
/home/eherbst/miniconda3/lib/python3.8/site-packages/openpyx
  warn("""Cannot parse header or footer so it will be ignore
```



Estimating this in scikit learn is easy

```
spf_flatted_zero = spf_flat.fillna(0.)

spfX = spf_flatted_zero[train_forecasters][train_start:trai
spfXtrain = np.c_[Xtrain, spfX]

linear_model_spf = LinearRegression()
fitted_ols_spf = linear_model_spf.fit(spfXtrain,ytrain)
```

Table: Mean Squared Errors

	Train	Test
LS-univariate	0.59	2.28
LS-SPF	0	2.1

Regularization

We've got way too many variables – our model does horrible out of sample!

Their are many regularization techniques available for variable selection

Conventional: AIC, BIC

Alternative approach: Penalized regression.

Consider the loss function:

$$L(\hat{y}, y) = \frac{1}{2T}\sum_{t=1}^{T}(f(x_t;\theta) - y)^2 + \lambda\sum_{i=1}^{N}\left[(1-\alpha)|\theta_i| + \alpha|\theta_i|^2\right].$$

This is called elastic net regression. When $\lambda = 0$, we're back to OLS.

Many special cases.

Ridge Regression

The ridge regression Hoerl and Kennard (2004) is special case where $\alpha = 1$.

Long (1940s) used in statistics and econometrics.

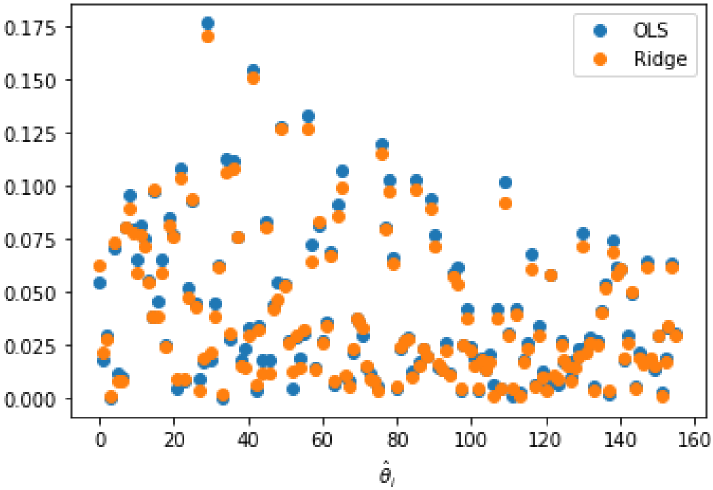
This is sometimes called (or is a special case of) "Tikhonov regularization"

It's an L2 penalty, so it's won't force parameters to be exactly zero.

Can be formulatd as Bayesian linear regression.

```
from sklearn.linear_model import Ridge
fitted_ridge = Ridge().fit(spfXtrain,ytrain)
```

LS vs Ridge ($\lambda = 1$) Coefficients



Lasso Regression

Set $\alpha = 0$

- This is an $L1$ penalty – forces small coefficients to exactly 0.

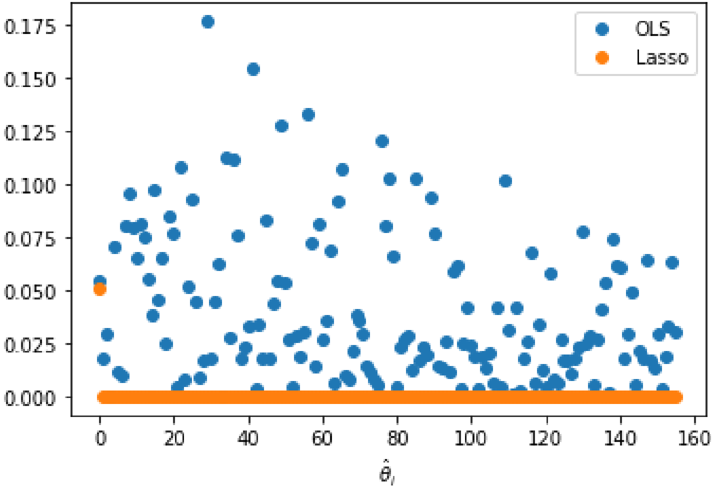
- Greatly reduces model complexity.

- Can you give economic interpretation to the parameters?

- Bayesian interpetation: Laplace prior in θ.

```
from sklearn.linear_model import Lasso
fitted_lasso = Lasso().fit(spfXtrain,ytrain)
```

LS vs Lasso ($\lambda = 1$) Coefficients



Picking λ

- Use “rule of thumb” given subject matter.
- Tradition validation: Use many λ on your test sample, Assess accuracy of each on validation sample, pick one which gives minimum loss.
- Cross validation:
  1. Divide sample in K parts:
  2. For each $k \in K$, pick λ_k, fit model *using the $K - k$ sample*.
  3. Plot Loss against λ_k, pick λ which yields minimum.
- Chernozhukov et al. (2018) derive “Oracle” properties for LASSO, pick λ based on this.

Table: Mean Squared Error, $\lambda = 1$

Method	Train	Train
Least Squares (Univariate)	0.35	0.71
Least Squares (SPF)	0.0	0.68
Least Squares (SPF-Ridge)	0.0003	0.67
Least Squares (SPF-Lasso)	0.59	0.96

Support Vector Machines

- While Elastic net, lasso, and ridge were designed around regularization, other machine learning techniques are designed to fit more flexible models.
- Support Vector Machines are typically used in classification problems.
- Essentially SVM constructs a seperating hyperplane (hello 2nd basic welfare theorem), to optimally seperate ("classify") points.
- What's cool about the support vector machine is that you can use a kernel trick, so your hyperplanes need not correspond to lines in euclidean space.
- For regression, the hyperplane will be prediction.

Support Vector Machines

- Explicit form:

$$f(x;\theta) = \sum_{i=1}^{n} \theta_i h(x) + \theta_0.$$

- ϵ insensitive loss: function does not penalize predictions which are in an ϵ of the, otherwise the penalized by a factor relatid to C (comes from dual problem).

- Key choice here: the choice of kernel

- ϵ, C chosen by (cross) validation.

Estimating Support Vector Machine

```
from sklearn.svm import SVR
fitted_svm = SVR().fit(Xtrain,ytrain)
```

Table: Mean Squared Error, $\Lambda = 1$

Method	Train	Train
Least Squares (Univariate)	0.35	0.71
Least Squares (SVM)	0.06	0.73

Other Popular ML Techniques

- Forests: partition feature space, fit individual models condition on subsample.
- Obviously, you can partition ad infinitum to obtain perfect predictions.
- Random forest: pick random subspace/set; average over these random models.
- Long literature in econometrics about model averaging Bates and Granger (1969).
- Further refinements: bagging, boosting.

Neural Networks

Let's construct a hypothesis function using a neural network.

Suppose that we have N features in x_t.

(Let $x_{0,t}$ be the intercept.)

Neural Networks are modeled after the way neurons work in a brain as basical computational units.

- Inputs (dendrites) channeled to outputs (axons)
- Here the input is x_t and the output is $f(x_t;\theta)$.
- The neuron maps the inputs to outputs using a (nonlinear) activation function g.
- By adding layers of neurons, we can create very (arbitrary) complex prediction models (all "logical gates").

Neural Networks, continued

Drop the t subscript. Consider:

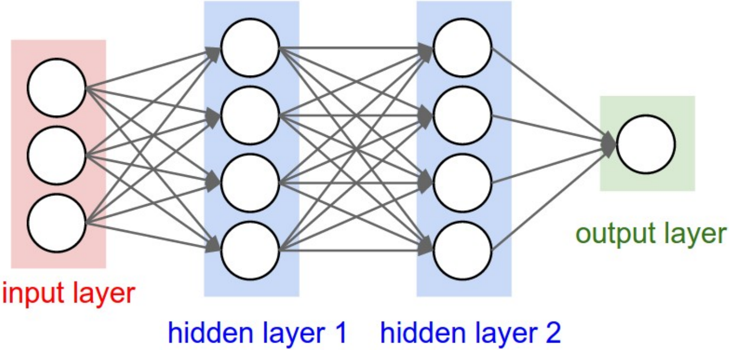
$$\begin{bmatrix} x_0 \\ \vdots \\ x_N \end{bmatrix} \to [\ ] \to f(x;\theta)$$

a_i^j activation of unit i in layer j.
β^j matrix of weights controlling function mapping layer j to layer $j+1$.

$$\begin{bmatrix} x_0 \\ \vdots \\ x_N \end{bmatrix} \to \begin{bmatrix} a_0^2 \\ \vdots \\ a_N^2 \end{bmatrix} \to f(x;\theta)$$

.

Neural Networks in a figure



Neural Networks Continued

If $N = 2$ and our neural network has 1 hidden layer.

$$\begin{aligned} a_1^2 &= g(\theta_{10}^1 x_0 + \theta_{11}^1 x_1 + \theta_{12}^1 x_2) \\ a_2^2 &= g(\theta_{20}^1 x_0 + \theta_{21}^1 x_1 + \theta_{22}^1 x_2) \\ f(x;\theta) &= g(\theta_{10}^2 a_0^2 + \theta_{11}^2 a_1^2 + \theta_{12}^2 a_2^2 \qquad (1) \\ & \qquad (2) \end{aligned}$$

(a_0^j is always a constant ("bias") by convention.)

Matrix of coefficients θ^j sometimes called weights

Depending on g, f is highly nonlinear in x! Good and bad . . .

Which activation function?

name	
linear	θx
sigmoid	$1/(1 + e^{-\theta x}$
tanh	$tanh(\theta x)$
rectified linear unit	$max(0, \theta x)$
$\ldots$	$\ldots$

How to pick $g \ldots$?

- Dependent on problem: prediction vs classification.
- Think about derivate of cost/loss wrt deep parameters.
- Trial and error

How to estimate this model.

Just like any other ML model: minimize the loss!

Gradient descent needs a derivative

back propagation algorithm

Application: Nakamura (2005)

- Nakamura (2005) considers (GDP deflator) inflation forecasting with a neural network.

- Model has 1 hidden layer, and uses a hyperbolic tangent activation function

- Can be explicitly written as:

$$\hat{\pi}_{t+h} = w_{2,1} \tanh(w'_{1,1} x_t + b_{1,1}) + w_{2,2} \tanh(w'_{1,2} x_t + b_{1,2}) + b_{2,1}$$

- x_t is a vector of $t-1$ variables. For simplicity, I'll consider $x_{t-1} = \pi_{t-1}$.

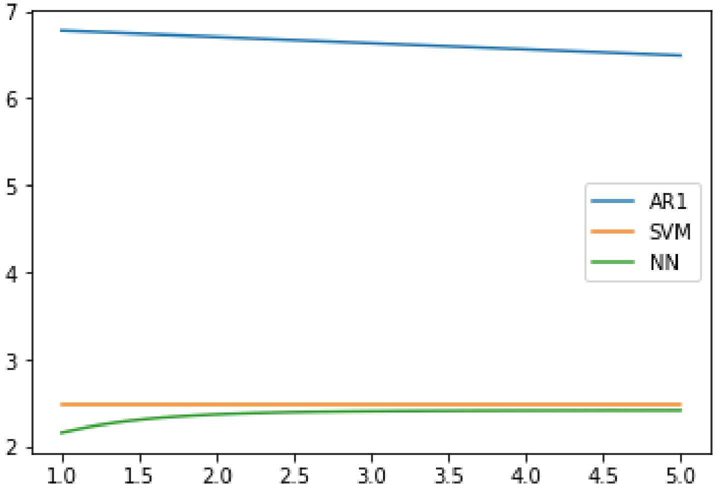
scikit-learn code

```
from sklearn.neural_network import MLPRegressor

NN = MLPRegressor(hidden_layer_sizes=(2,),
                  activation=’tanh’,
                  alpha=1e-6,
                  max_iter=10000,
                  solver=’lbfgs’)

fitted_NN = NN.fit(Xtrain,ytrain)
```

Neural Network vs. AR(1): Predicted Values



What is the “right” method to use

You might have guessed. . .

Wolpert and Macready (1997): A universal learning algorithm does *cannot* exist.

Need prior knowledge about problem. . .

This is has been present in econometrics for a very long time. . .

There’s no free lunch!.

References I

Athey, S. (2018): "The Impact of Machine Learning on Economics," in *The Economics of Artificial Intelligence: An Agenda*, National Bureau of Economic Research, Inc.

Bates, J. M., and C. W. J. Granger. (1969): "The Combination of Forecasts," *Or*, 20, 451.

Chernozhukov, V., W. K. Härdle, C. Huang, and W. Wang. (2018): "Lasso-Driven Inference in Time and Space," *Ssrn Electronic Journal*, .

Hastie, T., R. Tibshirani, and J. Friedman. (2009): "The Elements of Statistical Learning," *Springer Series in Statistics*, .

Hoerl, A. E., and R. W. Kennard. (2004): "Ridge Regression," *Encyclopedia of Statistical Sciences*, .

Mullainathan, S., and J. Spiess. (2017): "Machine Learning: An Applied Econometric Approach," *Journal of Economic Perspectives*, 31, 87 106.

Nakamura, E. (2005): "Inflation Forecasting Using a Neural Network," *Economics Letters*, 86, 373 378.

References II

VARIAN, H. R. (2014): "Big Data: New Tricks for Econometrics," *Journal of Economic Perspectives*, 28, 3 28.

WOLPERT, D., AND W. MACREADY. (1997): "No Free Lunch Theorems for Optimization," *Ieee Transactions on Evolutionary Computation*, 1, 67 82.