Estimating A Nonlinear DSGE: The Empirical Implications of the ZLB

Ed Herbst

November 18, 2020

This Paper

- In US, policy instrument has been at or near the effective lower bound (ELB) since the end of 2008.
- Question: What role did the ELB constraint play during and after the Great Recession (GR)?
- Answer: A sizeable one! ELB is responsible for:
  - Output: About 25 percent of contraction in GR. Absent ELB, recovery would have occured 1 year earlier.
  - Inflation: 0.3 ppt lower per year from 2008 - 2012.
  - *Caution: Posterior mean estimates, substantial uncertainty.*

How We Get These Numbers

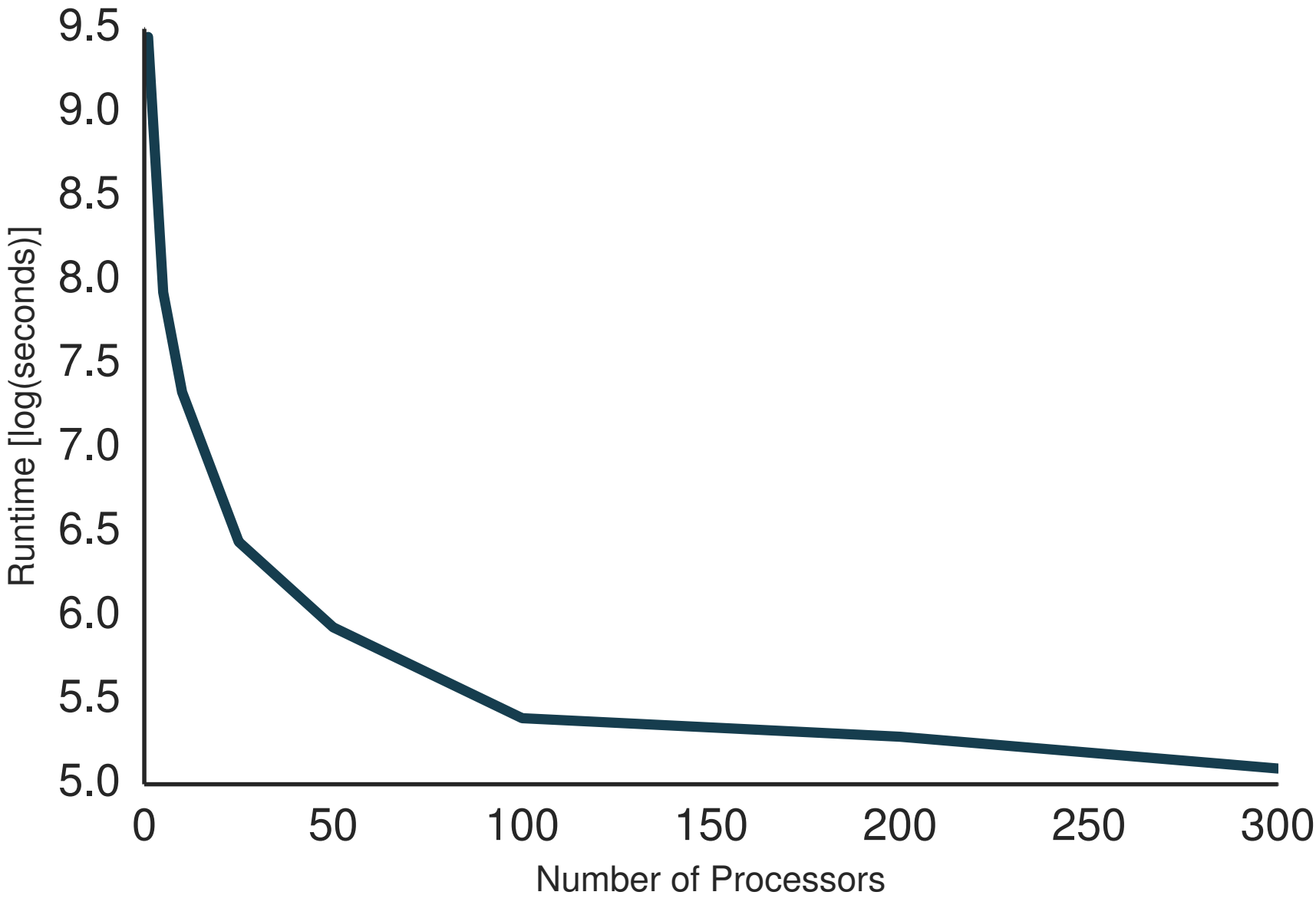
- Start with the fully nonlinear version of "medium scale" macroeconomic (DSGE) model along the lines of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) with parameters θ .
  - Explicitly takes into account the effect of future uncertainty about whether ELB will bind.
  - ELB systematically changes monqetary policy – and thus behavior of agents – even if ELB is never hit! See 2003-2004.

- *Technical Contribution:* estimating (using Bayesian methods) a nonlinear DSGE model in which a lower bound is occasionally binding.
  - solved via projection method similar to Christiano and Fisher (2000) and Judd et al. (2014)
  - estimated using a particle filter Markov Chain Monte Carlo algorithm

Solving the model

- Model has twelve state variables and five exogenous shocks.
- Solve for a minimum state variable solution for decision rules – approximated via Chebyshev polynomials.
- Use a fixed point algorithm [Judd et al. (2014)] to solve for the 96,390 coefficients that define our policy rules.
- This fixed point algorithm can be parallelized on computer cluster using a Message Passing Interface (MPI).
- (Note: In practice, never visit the second (deflationary) steady state.)

Gains to Parallelization of Model Solution



Estimating the Model

- For a given parameter value, model solution delivers a nonlinear decision rule for states of the model s_t [12x1] as a function of past states and structural shocks ϵ_t [5x1].

$$s_t = \Phi(s_{t-1}, \epsilon_t; \theta)$$

- States include variables we can't observe (like technology), so it's hard to estimate directly.

- Link model to subset of observed variables, yields nonlinear state space system:

$$\begin{aligned} s_t &= \Phi(s_{t-1}, \epsilon_t; \theta), \quad \epsilon_t \sim N(0, I), \\ y_t &= \Psi(s_t, \epsilon_t; \theta) + u_t, \quad u_t \sim N(0, \Sigma_u) \end{aligned}$$

- *Observables*: Output, consumption, and investment growth. GDP deflator. 3-month treasury bill.

Sample: 1983:Q1 to 2014:Q1 (includes ZLB period!)

Estimating the model

- Overall goal: obtained plausible values of parameters θ given the data Y.

- Bayesian approach: use the posterior distribution

$$p(\theta|Y) \propto \underbrace{p(Y|\theta)}_{\textit{likelihood}} \underbrace{p(\theta)}_{\textit{prior}}$$

- Problem 1: Can't access this posterior distribution easily!

- Solution: Use Markov chain Monte Carlo (MCMC) methods to build a sequence $\{\theta^i\}_{i=1}^N$ which converges in distribution to posterior distribution.

Estimating the model, more

- Problem 2: Can't even get likelihood in closed form!
- Solution: Use a *particle filter*, as in Fernandez-Villaverde and Rubio-Ramirez (2007), to get likelihood estimate $\hat{p}(Y|\theta)$.
- Andrieu, Doucet, and Holenstein (2010): MCMC based on

$$\hat{p}(\theta|Y) \propto \hat{p}(Y|\theta)p(\theta).$$

  Converges to posterior of interest. See also Herbst and Schorfheide (2015).
- Still, poor particle filter tuning can lead to inefficient estimates.

Why is this a paper?

For a given θ it's hard to solve the model and it takes a lot of time.

Even given the model solution, likelihood estimation using the particle filter is slow and noisy.

How did we manage it?

- Parallelization of model solution algorithm.
- "Healthy" measurement error.
- Slight tweaks to bootstrap particle filter.
- **Parallelization of resampling step of particle filter tailored to FRB's computer cluster.**

What we're going to do

After estimating the model, w have a set of draws from our posterior $\{\theta^i\}_{i=1}^N$, $N = 1000$.

For each draw, generate (one draw) for smoothed states $\{\hat{s}_{t|T}\}_{t=1}^T$ and shocks $\{\hat{\varepsilon}_{t|T}\}_{t=1}^T$. This collection is the posterior distribution for states and shocks.

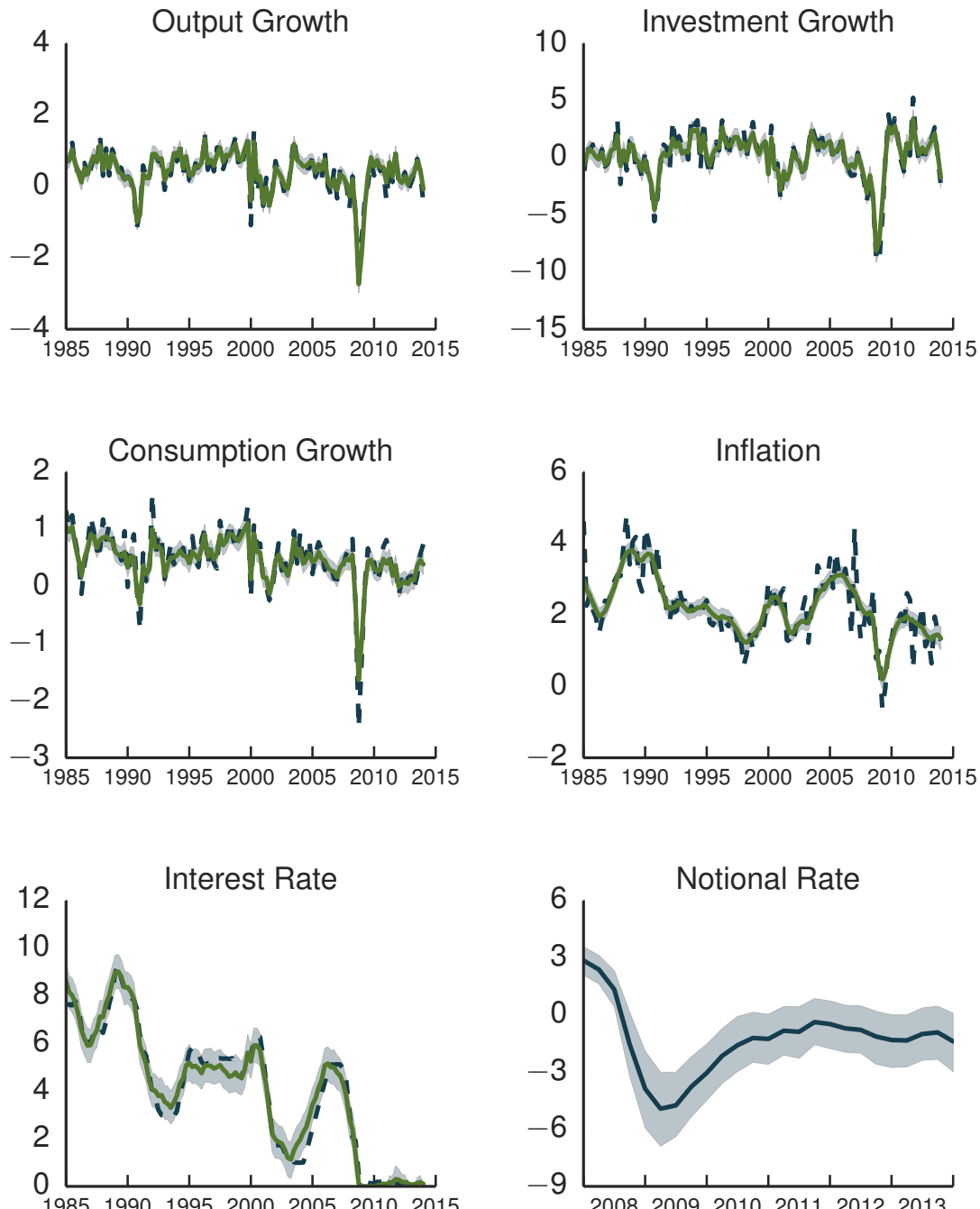
1. Assess how the distribution of states fits data (remember, measurement error!)

2. According to model, how likely was Great Recession?

3. What accounts for the Great Recession?

4. Using draws of shocks run counterfactual experiments: What was effect of ELB ?

Smoothed Estimates of Model Objects



Model Fit

- The model generally tracks the fluctuations in output, consumption, and investment growth, and generates contractions in these variables in all three of the recessions included in our sample period.

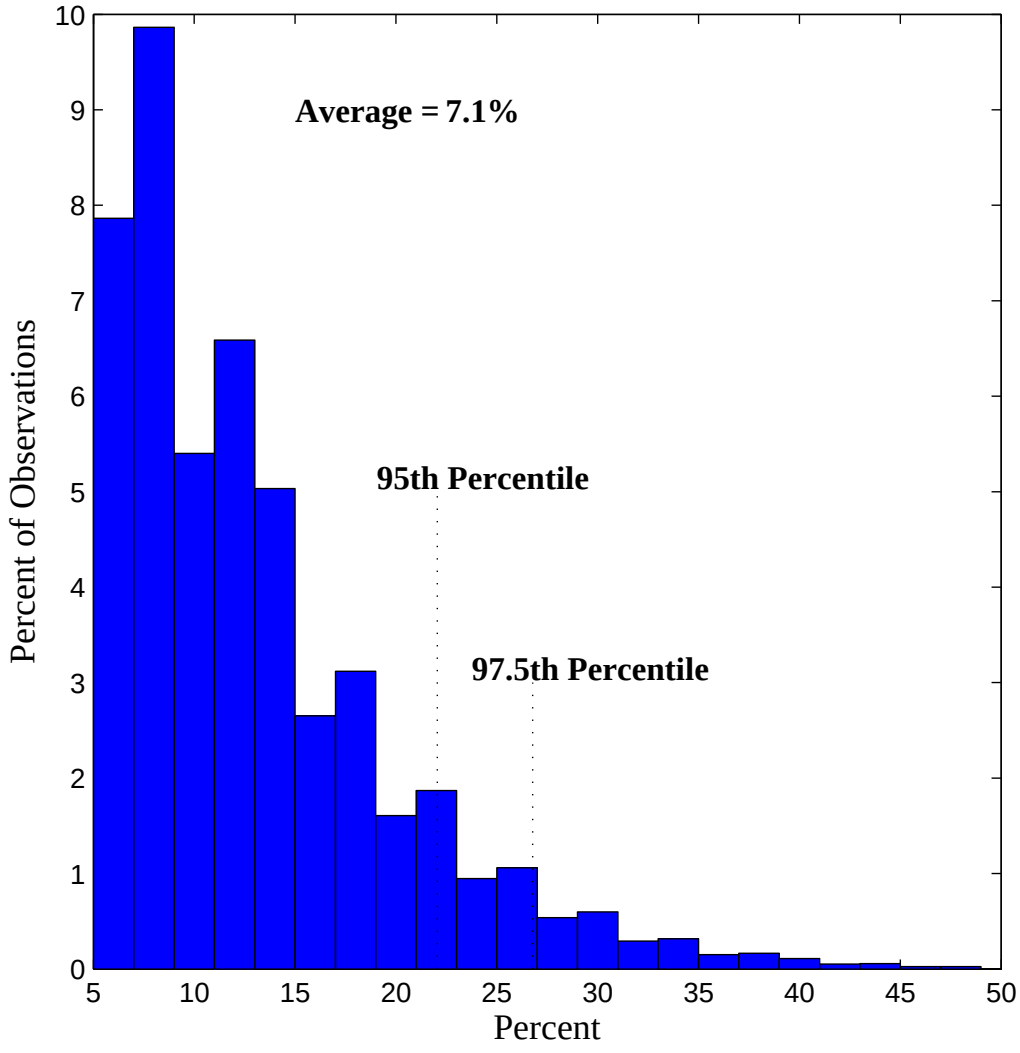
- For the Great Recession, the model accounts for the sharp falls in output and investment that occurred and modestly understates the fall in consumption.

- From 2009 onwards, the notional interest rate is well below zero. It falls to about minus 5.5 percent in the first half of 2009, gradually moves up to about minus 1 percent at the end of 2012, and then hovers around that level through 2014:Q4.

How Likely was ELB Spell?

- Simulate data from the model using parameter draws from the posterior distribution. Compute statistics about frequency of ELB episodes.
- On average, the model implies that there is a 7 percent probability of the nominal rate being at the ELB.
- In our sample, the corresponding statistic in the data is 17 percent.
- But, our simulations show that the sampling distribution of this statistic has a long right tail.
- $\Longrightarrow$ it is difficult to estimate this probability with much

Distribution of the Probability of Hitting the Zero Lower Bound



Question: What shocks drove the economy to the ELB?

Model has five candidates

- marginal efficiency of investment shocks (JPT 2011), disturbance to the financial sector's ability to channel savings into investment;
- risk premium shocks directly affects the spread between the risk-free rate and the return on risky assets;
- Total factor productivity: technology shock;
- Government spending shocks;

Shocks, continued

- Look at standardized risk premium shock, standardized MEI shock, and technology shock from 2008-2013.

- Also look at model-implied equity premium

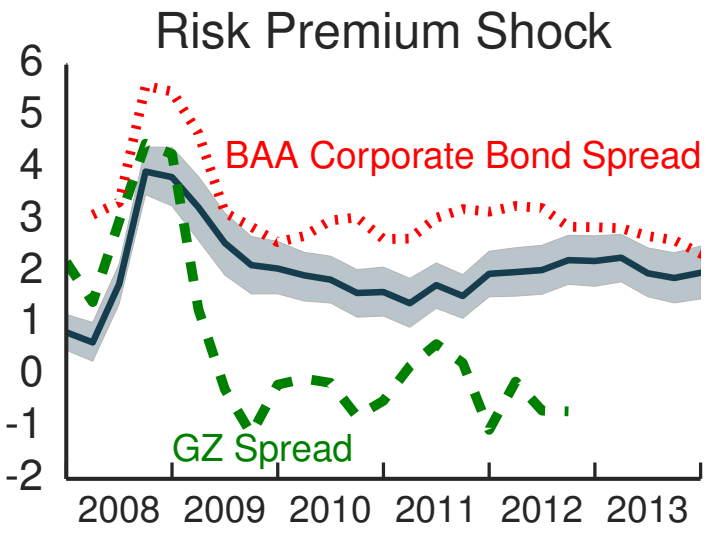
$$\frac{E_t R^k_{t+1}}{R_t} = \frac{\tilde{r}^k_{t+1} + q_{t+1}(1-\delta)}{R_t q_t},$$

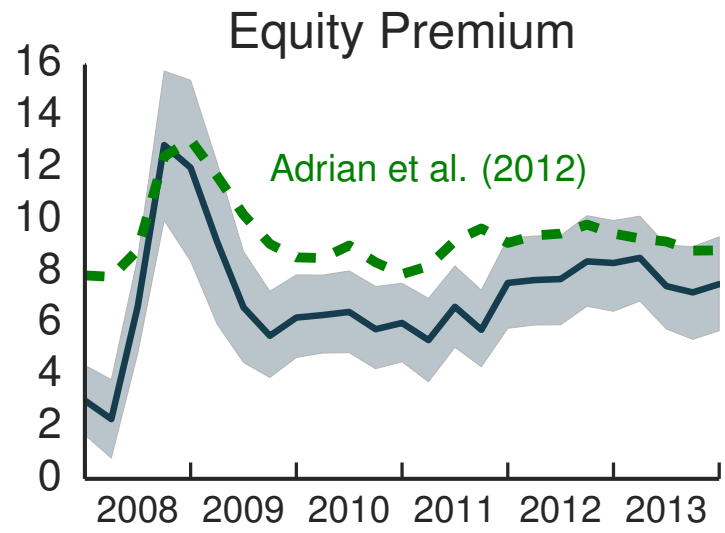
  $\tilde{r}^k_{t+1}$ = user cost of capital.

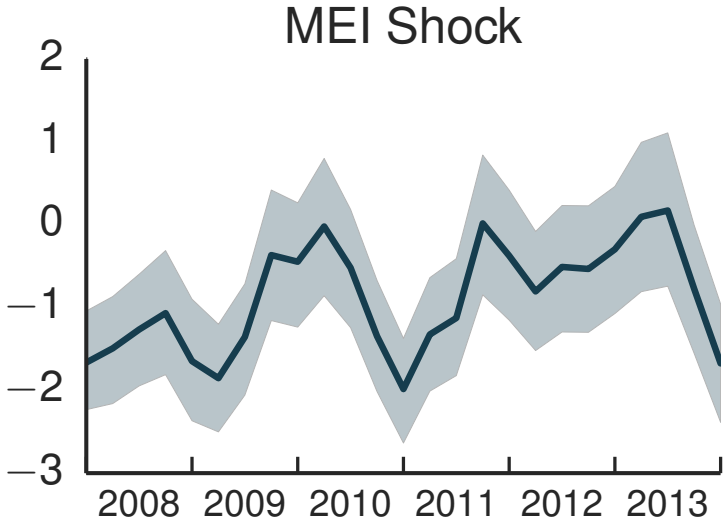
- Express technology as deviation from its 2008:Q1 deterministic growth path.

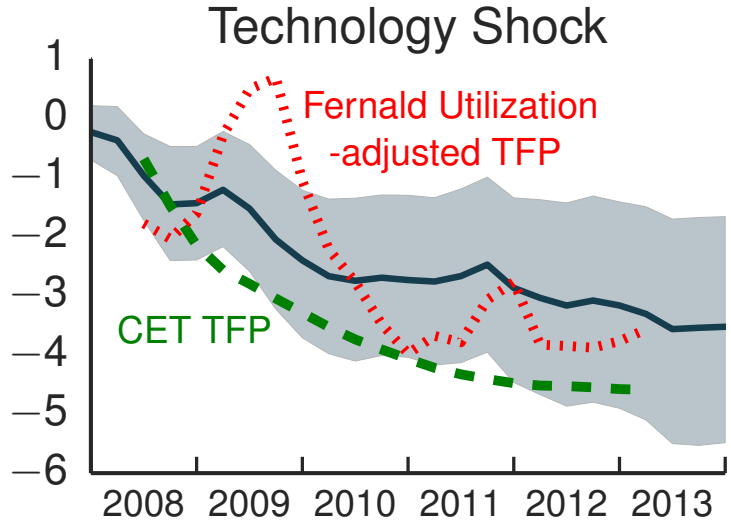
- Compare to measures found in elsewhere in literature.

Shocks over the Great Recession and beyond









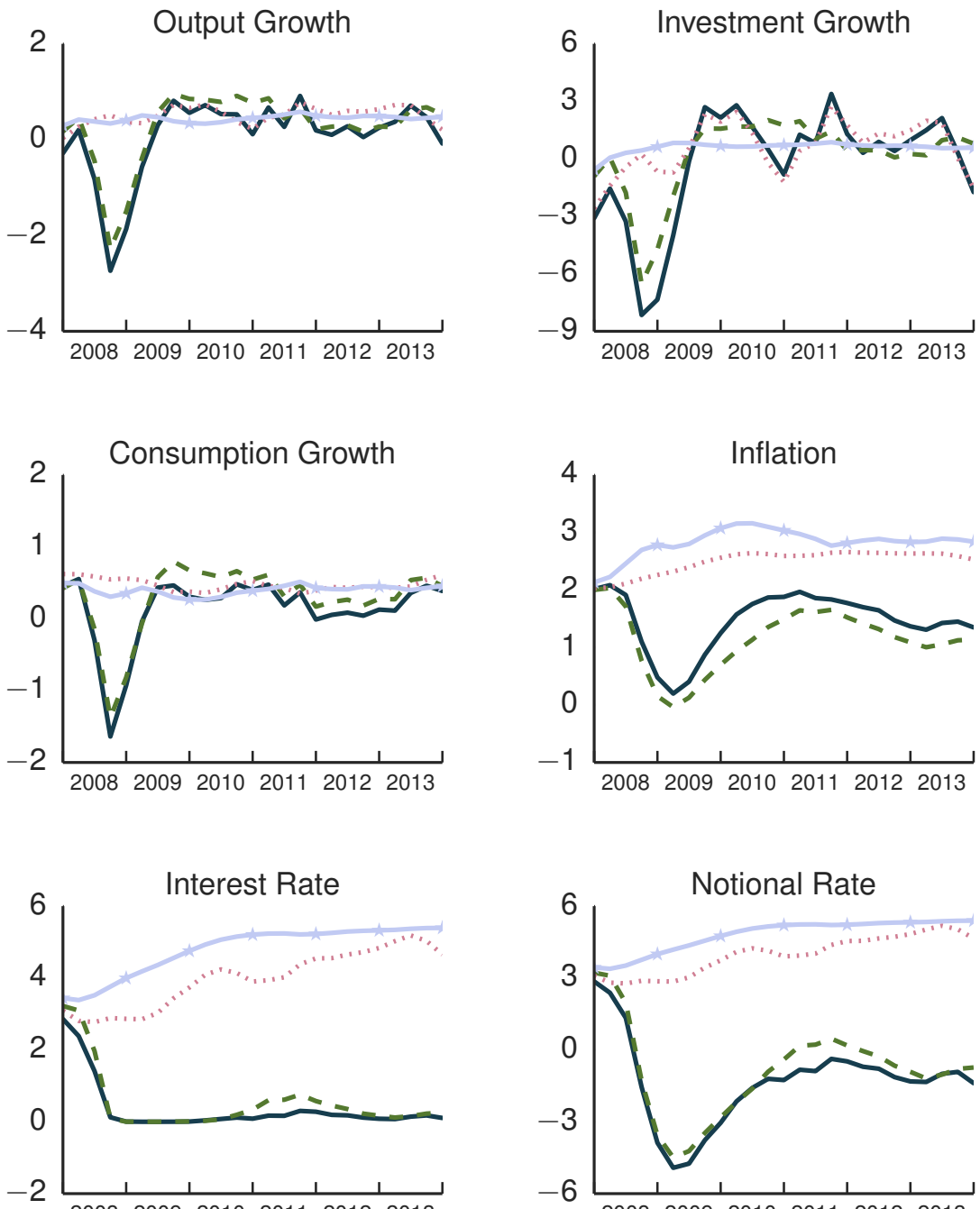
Which shocks were the most important?

- It's hard to do a variance decomposition with the nonlinear mode – Let's do something a bit easier instead.

- Starting in 2007:Q4, we take the estimated state as an initial value, and simulate the economy forward feeding in the smoothed values of one of the shock processes, assuming that the values of the other shocks are set to zero.

- Look at differences in trajectories of model-implied

Shock Contributions



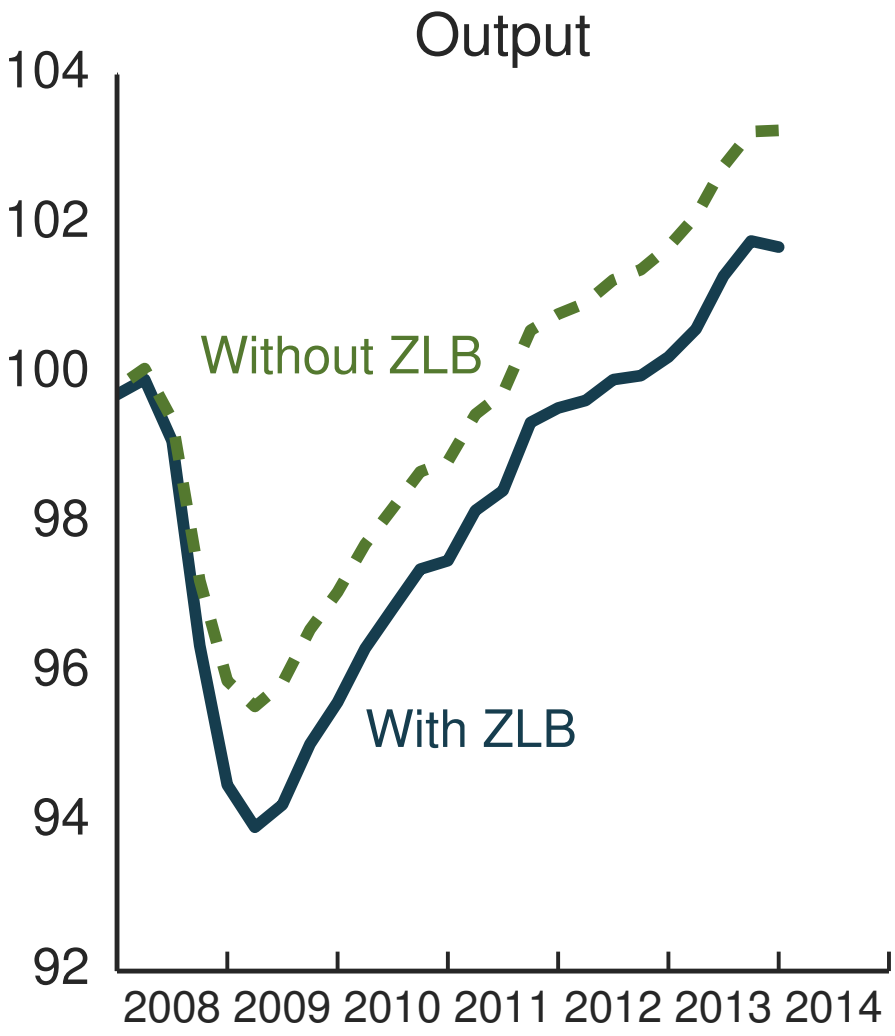
Contributions to Great Recession

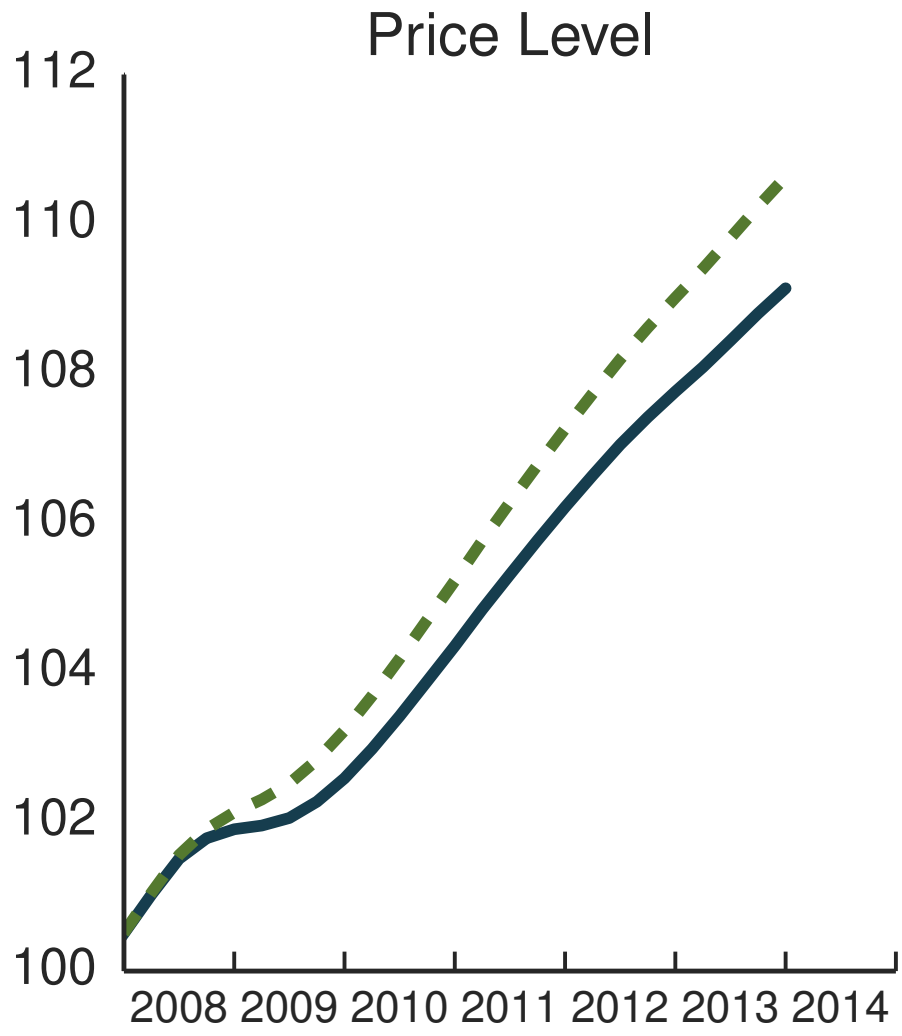
- The large contraction in output, consumption and investment growth is mostly explained by the risk premium shock.
- This shock also generates a somewhat larger fall in inflation than under the baseline path.
- These disinflationary effects are offset by the upward pressure on inflation induced by the fall in the level of technology.
- The risk premium shock pushes the notional interest rate well below zero (to about -5.5 percent).

How Much Did The ELB Matter?

- Use joint posterior distribution of (smoothed) structural shocks and parameters.
- Feed these shocks into the model without effective lower bound on interest rate.
- Look at difference between actual outcomes and counterfactual ones.

Mean Estimates (Please forgive the lack of uncertainty bands!)

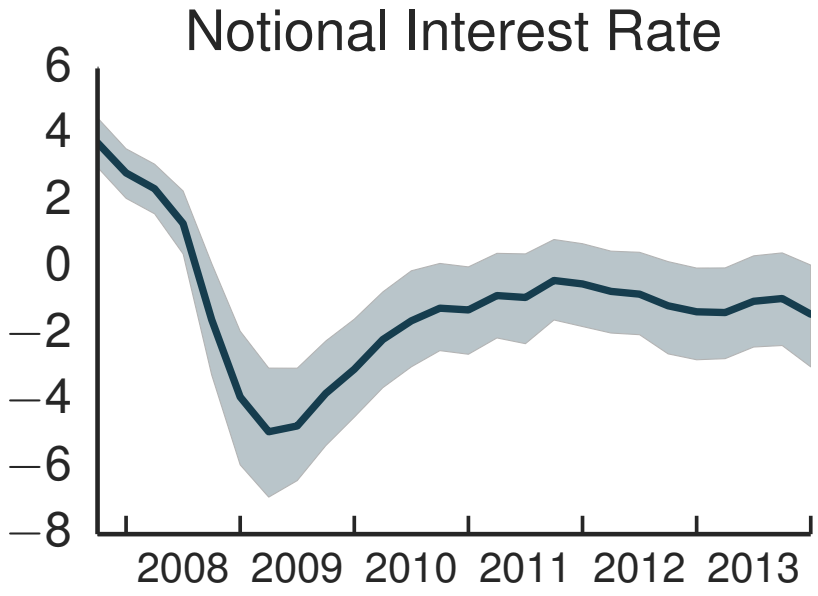


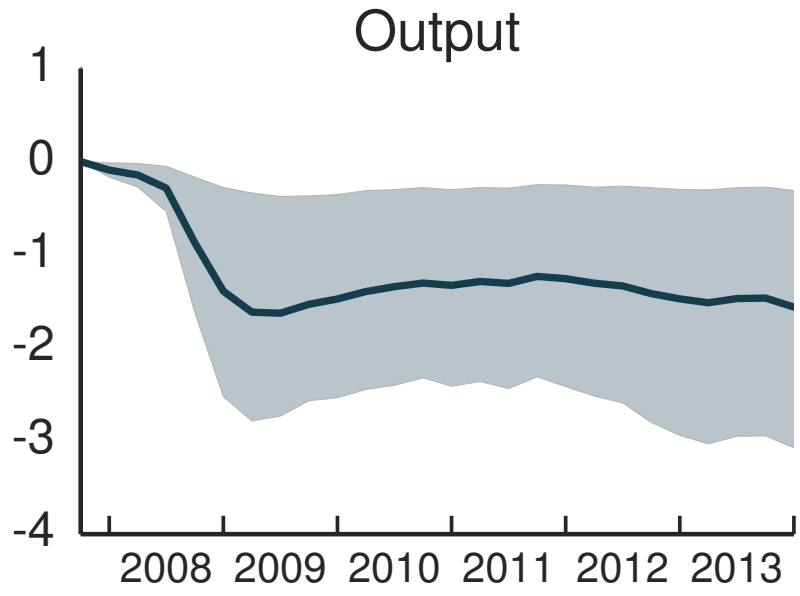


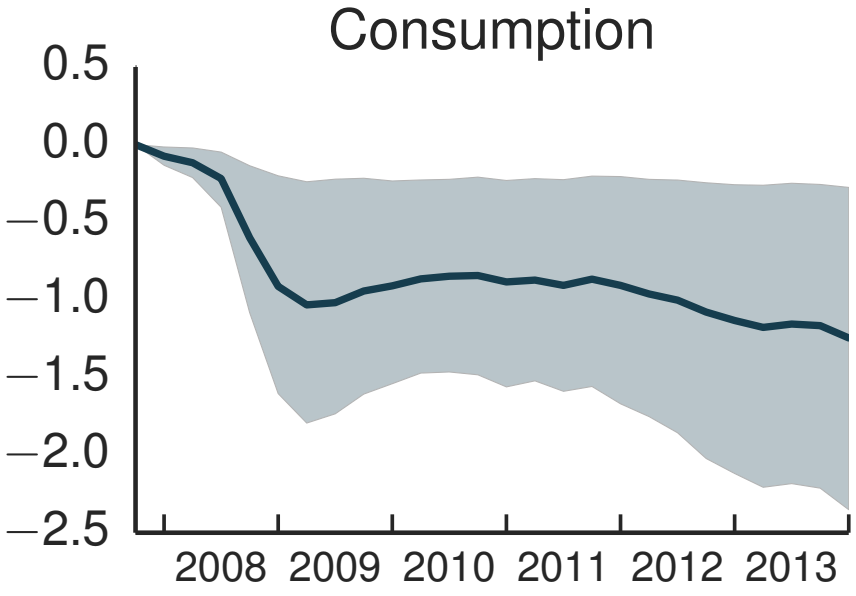
ELB was significant

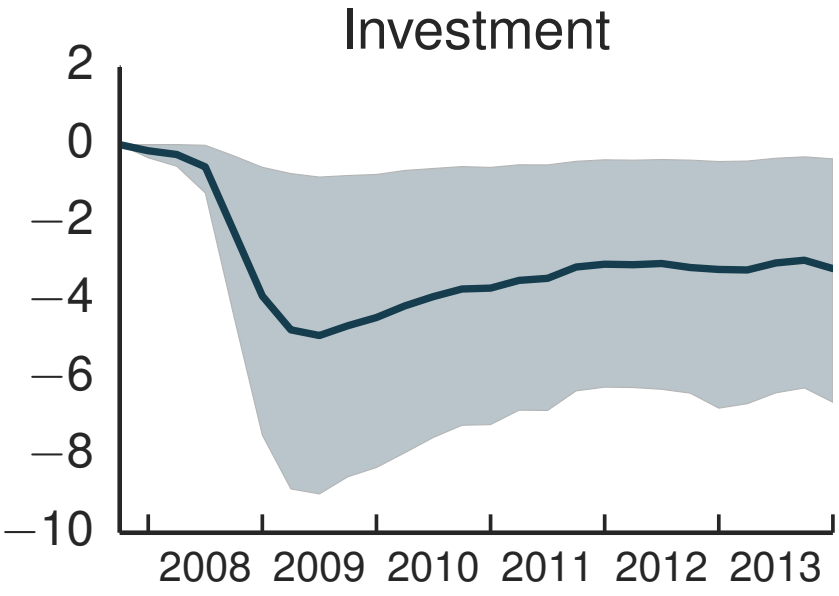
- With ELB, in 2009:Q2 output was about 6 percent below its level in 2007:Q4.
- Without ELB, output would have fallen by only 4.5 percent.
- $\Longrightarrow$ 25 percent of output drop in 2009 due to ELB.
- 68 percent interval covers values between 6 to 46 percent.
- At mean estimates, average output was 1.2 percent below counterfactual through 2013.
- Inflation would have been 0.3 pct points, higher on average

Notional Interest Rate, Plus Differences in Y, C, and I









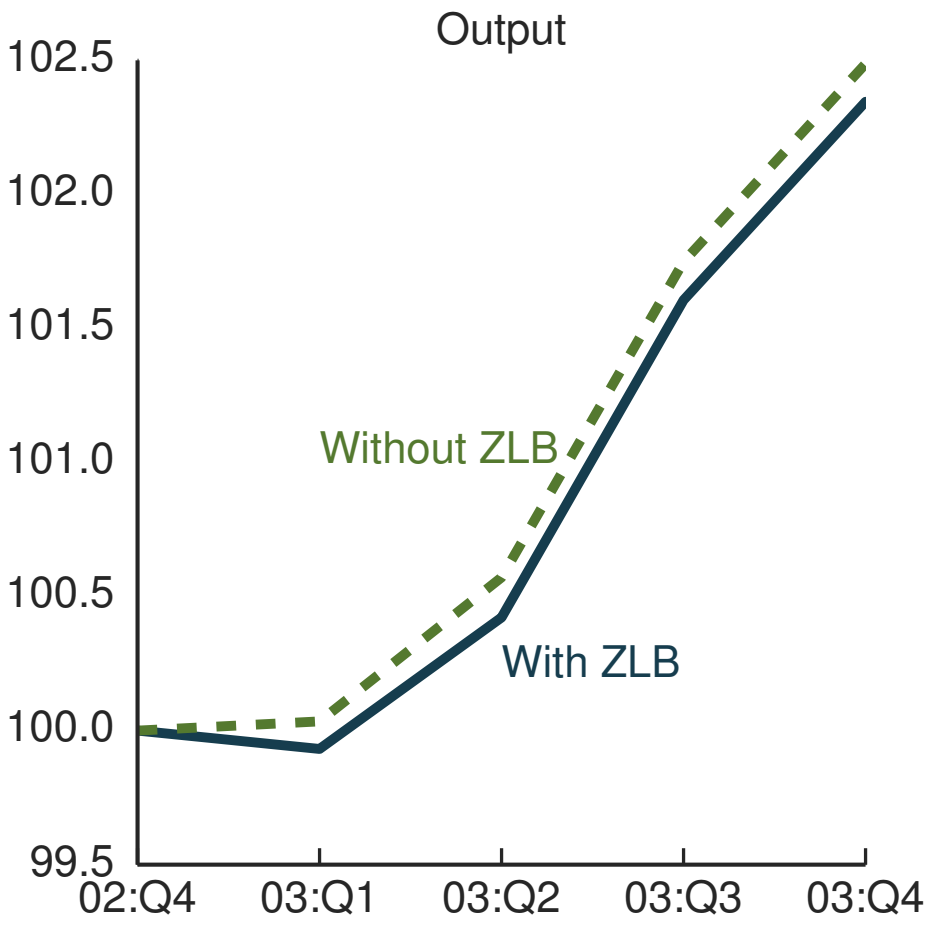
More Counterfactuals

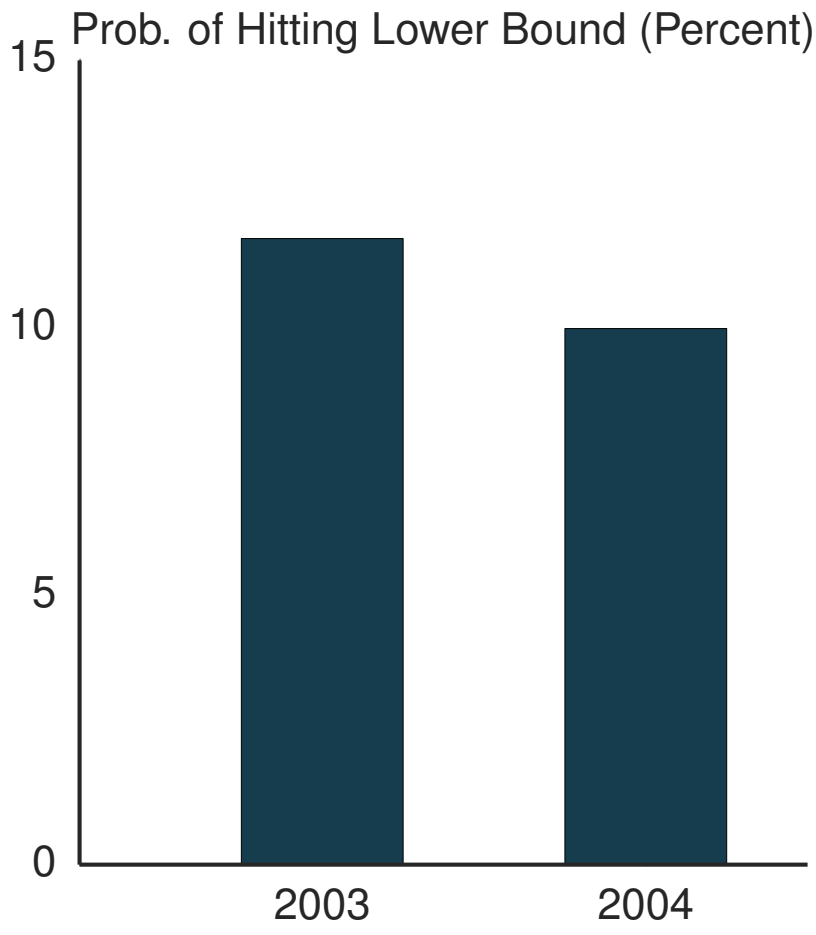
- The 2003-2004 deflationary scare: Policy rate doesn't hit the ELB, but comes close.

  {"*.. [W]e face new challenges in maintaining price stability, specifically to prevent inflation from falling too low. . . [T]here is an especially pernicious, albeit remote, scenario in which inflation turns negative. . . engendering a corrosive deflationary spiral. . .*"} *(Alan Greenspan before the House Committee on Financial Services, July 15, 2003)*

- Does constraint make a difference? YES (a small one).

Effect of ELB, 2003-2004





Effect of ELB, 2003 - 2004

- at the end of 2002, taking into the economy's current state and the estimated policy rule, private sector agents believed there was a 12 percent chance that the nominal interest rate would fall to its lower bound during 2003.

- ZLB binding imply higher future real policy rates $\Longrightarrow$ scenarios are also characterized by downward shifts in the distributions of outcomes in aggregate spending and output.

- the estimated level of output is about 0.2 percent lower with ELB.

- uncertainty about the course of monetary policy can have a tangible economic impact

Forward Guidance and the Estimated Policy Rule

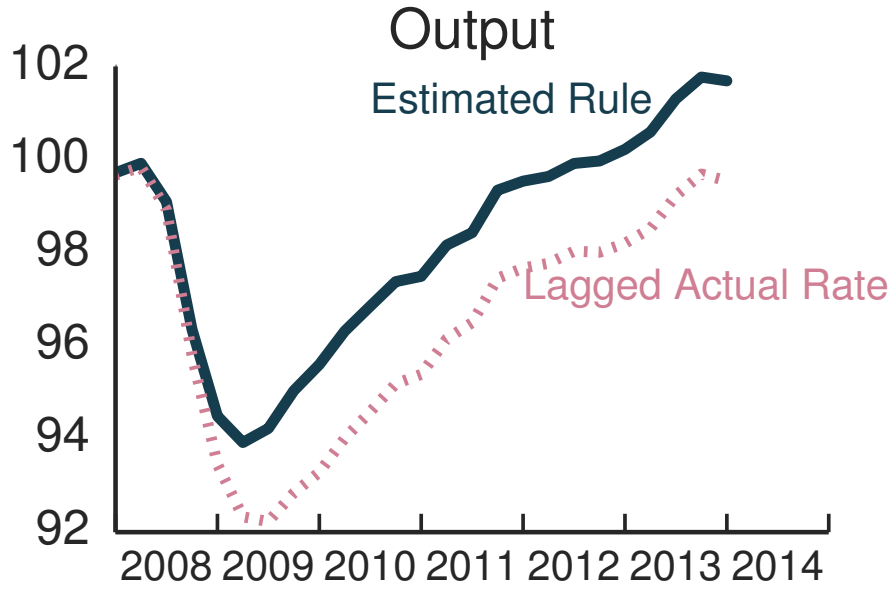
- In both the 2003 and during the GR, forward guidance about the policy rate played a prominent role in FOMC communications.

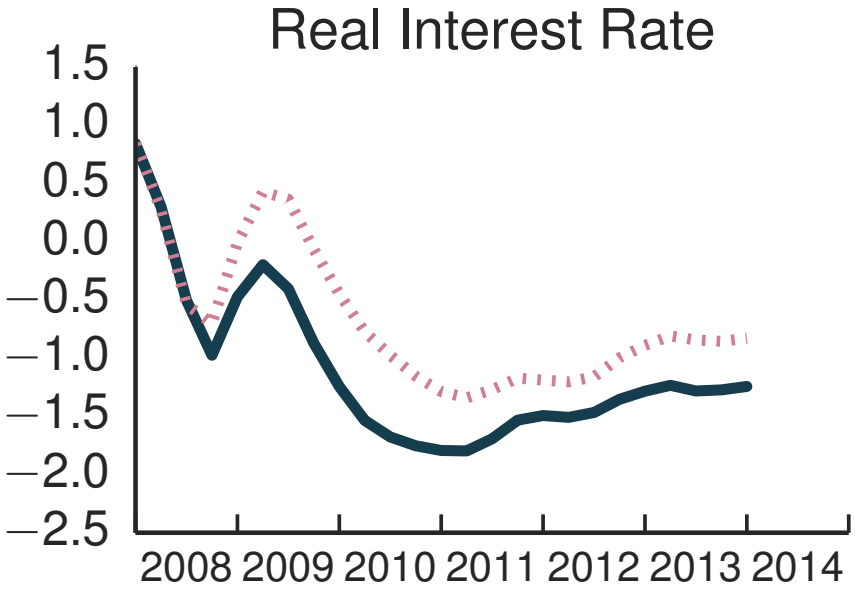
- FOMC, late 2008: “weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time.”

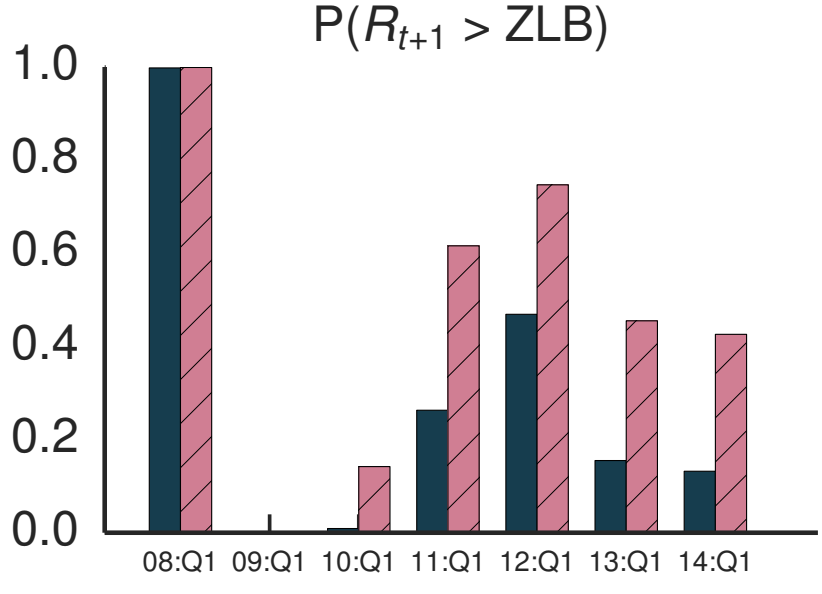
- Capture forward guidance through the state-contingent path of rates implied by our estimated interest-rate rule.

- The presence of the lagged notional rate, in particular, is an important element of a strategy that seeks to maintain future rates at a low level.

Effect of Interest Rate Smoothing







Forward Guidance and the Estimated Policy Rule

- Estimated rule is more accomodative than the alternative rule, because agents known rule will remain at the ELB for longer and in a wider set of circumstances.

- under the estimated rule, private sector agent's beliefs the probability that the nominal rate would 'liftoff' next quarter is always lower than under the alternative rule

- Under both of these rules the nominal interest rate is estimated to remain at the zero lower bound through the end of the sample

- the inclusion of the lagged notional rate captures the forward-guidance language used by the FOMC at the time

Conclusion

We did:

- We solved and estimated (using Bayesian techniques) a fully nonlinear DSGE with an occasionally binding constraint (ELB).
- Quantified the effect of the ELB – it's big!

More to do:

- Econometrics of nonlinear models.
- Nonlinear policy rules.
- Other unconventional policy.