Inflation Expectations and Macro Dynamics under Finite Horizon Planning

Chris Gust, Ed Herbst, David Lopez-Salido
Federal Reserve Board[1]

June 18, 2022

[1] This does not express the view of the Federal Reserve Board or the Federal Reserve System.

- Macroeconomists have increasingly begun to incorporate behavioral elements into their models as an alternative to rational expectations.

- Evidence from (conensus) survey data on expectations:
  - Coibion & Gorodnichenko (2015): forecasts *underreact* to new information.
  - Angeletos, Huo, and Sastry (2020): underreaction is followed by *overreaction*.

- Gust, Herbst, Lopez-Salido (2021): the finite horizon planning (FHP) model of Woodford (2018) is an attractive alternative to RE (and some imperfect expectations) models for fitting key aggregate macro time series.

- This paper: Can the FHP model fit key facts on expectations? Yes!

Summary of Results

1. We derive analytical results for a simplified FHP model for the CG and AHS conditions for inflation expectations.
   - We show that these moments depend both on the parameters governing expectations formation and other structural parameters.

2. We estimate an FHP DSGE model:
   - Using data on inflation expectations, the FHP model remains preferable to alternatives in terms of statistical fit.
   - The FHP model is also consistent with CG and AHS conditions for inflation expectations. *Other models are not.*

I. Theory

A Simplified FHP Model

- k-level planner making a decision at date t only looks forward through period $t+k$. For any endogenous variable in periods $t+k-j$, Z_{t+k-j}, with $j=0,1,2,...,k$:

$$\mathbb{E}_t^k Z_{t+k-j} = E_t Z^j_{t+k-j}, \tag{1}$$

- As shown in Woodford (2018), under these assumptions, firms' price-setting behavior implies a log-linearized relationship for each period of the plan given by:

$$\pi^j_\tau = \beta E_\tau \pi^{j-1}_{\tau+1} + \kappa y_\tau, \tag{2}$$

where $\tau = t+k-j$ denotes the planning period, and $1 < j \leq k$.

- Assume the output gap y_t follows an AR(1) process:

$$y_t = \rho y_{t-1} + e_t \tag{3}$$

- Iterating forward, we have:

$$\pi_t^k = \kappa E_t \sum_{i=0}^{k-1} \beta^i y_{t+i} + \beta^k E_t \pi_{t+k}^0 \tag{4}$$

  Aggregate inflation depends on expected of output gap, and the expected inflation rate at the end of the planning horizon.
- Firms use continuation value functions to assign value to events outside of their planning horizons (i.e., the longer-run from their viewpoint).

$$\pi_{t+k}^0 = \kappa y_{t+k} + \beta(1-\theta) v_{pt}, \tag{5}$$

  where v_{pt} is the (log-linearized) *continuation value* to the plans of firms.
- Firms to learn and update their beliefs based on *past experience*. In this case, the value function v_{pt} evolves according to:

$$v_{pt+1} = (1-\gamma_p) v_{pt} + \gamma_p v_{pt}^e, \text{ with } v_{pt}^e = (1-\theta)^{-1} \pi_t^k. \tag{6}$$

Taking stock

- One can show that dynamic of inflation under FHP can be written as:

$$\pi_t^k = [1 - (\beta\rho)^{k+1}]\pi_t^{RE} + \beta^{k+1}(1-\theta)v_{pt} \tag{7}$$

- Since $0 < 1 - (\beta\rho)^{k+1} \leq 1$, inflation in the FHP model is less responsive to fluctuations in the output gap. This muted responsiveness of inflation is a function of ρ and k.
- The second deviation from the RE solution is that firm's longer-run beliefs about inflation, as discussed above, depend on past inflation:

$$v_{pt} = \frac{\gamma_p}{1-\theta}\sum_{i=0}^{t-1}(1-\gamma_p)^i \pi_{t-1-i}^k. \tag{8}$$

- Inflation in the FHP model displays an excess sensitivity to past inflation relative to the RE solution.

The AHS property

Let $\mathbb{F}^k_{t+1} = \pi^k_{t+1} - \mathbb{E}^k_t \pi_{t+1}$.

Proposition (Dynamic Overshooting)

(IRFs of Inflation Forecasts and Forecast Errors). Let $\frac{\partial \mathbb{E}^k_{t+i} \pi_{t+1+i}}{\partial e_t}$ *and* $\frac{\partial \mathbb{F}^k_{t+1+i}}{\partial e_t}$ *for* $i \geq 0$ *be the impulse response functions for a firm's one-step ahead inflation forecast and forecast error, respectively.*

1. *Without learning:* $\frac{\partial \mathbb{E}^k_{t+i} \pi_{t+1+i}}{\partial e_t} \geq 0$ *and* $\frac{\partial \mathbb{F}_{t+1+i}}{\partial e_t} \geq 0$, $\forall i \geq 0$ *and* $k > 0$.
2. *With learning: If* $\gamma_p \leq \frac{1-\rho}{1-\beta^{k+1}}$, *there is a threshold forecast horizon,* $i^\star$, *such that:*
   - 2.1 $\frac{\partial \mathbb{E}^k_{t+i} \pi_{t+1+i}}{\partial e_t} \geq 0$ *for* $i \geq 0$,
   - 2.2 $\frac{\partial \mathbb{F}_{t+1}}{\partial e_t} > 0$ *and* $\frac{\partial \mathbb{F}_{t+1+i}}{\partial e_t} < 0$ *for* $i \geq i^\star$,

The CG property

In the FHP model, a firm's inflation forecast revision can be defined as:

$$\mathbb{R}_t^k = \left[\mathbb{E}_t^k - \mathbb{E}_{t-1}^k\right] \pi_{t+1}.$$

Proposition (Forecast Error and Revision Correlation)

Let $\beta_{CG} = \frac{cov(\mathbb{R}_t^k, \mathbb{F}_{t+1}^k)}{var(\mathbb{R}_t^k)}$ *denote the univariate regression coefficient from regressing the one-step ahead forecast error on the forecast revision in the FHP model.*

1. *Without learning: If* $\rho > 0$*, then* $\beta_{CG} > 0$*, for any finite planning horizon* $k > 0$*.*
2. *With learning: If* $\rho = 0$ *and* $\gamma_p < \frac{1-\beta}{1-\beta^{k+1}}$*, then* $\beta_{CG} > 0$*.*

Table: Predictability Regression Results For FHP Model

	β_{CG}	
	$\rho = 0$	$\rho = 0.8$
$\gamma_p = 0.3$	-0.01	0.66
$\gamma_p = 0.9$	-0.03	0.73

NOTE: Population coefficient from a regression of one-step ahead inflation forecast errors on forecast revision. For the remaining parameters, $k = 6$, $\beta = 0.99$, and $\kappa = 0.05$.

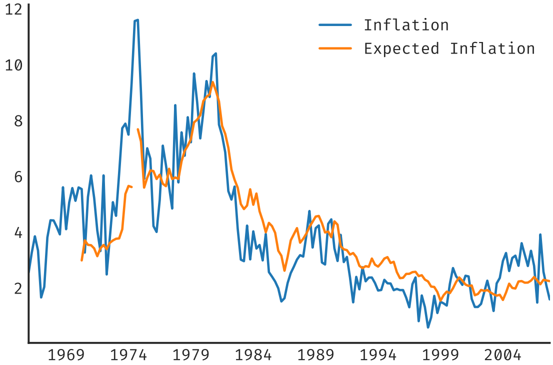
II. Empirics

Why we need system-based estimation

- We extend to the estimated DSGE of Gust, Herbst, Lopez-Salido (2021) to include data on inflation expectations.
- The is a methodological departure from CG (2015) and AHS (2020).
- unlike (some) other models of imperfect expectations formation, predictability properties depend on a broader set of structural parameters
- our analysis also allows to understand the extent to which matching the predictability of inflation expectations is also consistent with overall time series fit of inflation, output, short-term interest rates and inflation expectations.

Inflation Expectations

We use the mean forecast for four-quarter-ahead (GDP deflator) inflation expectations from the Survey of Professional Forecasters (SPF).



Assume that this is a noisy measure of model consistent inflation expectations.

- We also use data on output growth, (GDP deflator) inflation, and the federal funds rate.
- Our sample is from 1966-2007.
- Denote the standard macro data as Y and expected inflation data as F.
- Measure of overall fit *log marginal data density*:

$$\log p(Y, F) = \log p(Y) + \log p(F|Y)$$

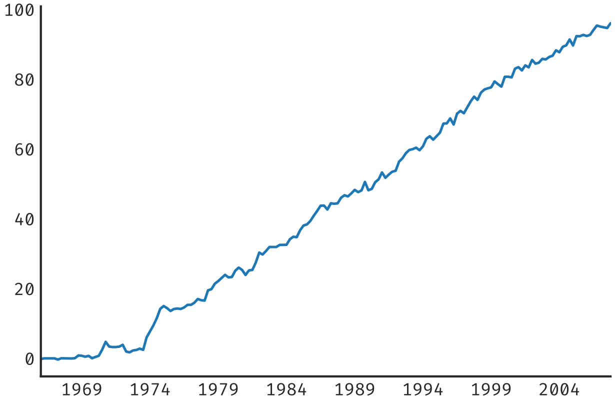
Some Estimation Results

Even with inflation expectations observed, a lot of evidence for low k.

Table: Log MDDs as function of k.

horizon	log p(Y)	log p(F given Y)
$k = 0$	-720.42	-56.02
$k = 1$	-716.54	-49.37
$k = 2$	-718.91	-49.53
$k = 3$	-721.46	-48.95
$k = 4$	-723.52	-48.46

$\log p(F|Y)$ relative to the Hybrid New Keynesian Model

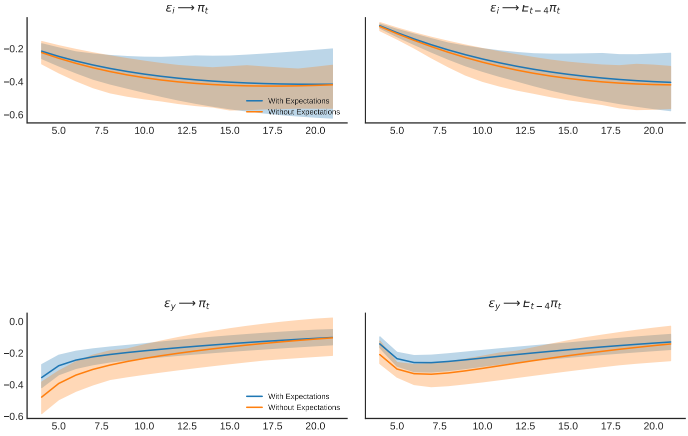


Parameters are remarkably stable

Table: FHP MODEL $k = 1$: KEY PARAMETER ESTIMATES

	With Exp. Data		Without Exp. Data	
Horizon and Learning Parameters				
γ	0.48	[0.32, 0.65]	0.45	[0.30, 0.62]
γ_f	0.20	[0.16, 0.24]	0.22	[0.14, 0.30]
Endogenous Propagation				
κ	0.02	[0.01, 0.04]	0.03	[0.01, 0.05]
σ	2.67	[1.86, 3.60]	2.75	[1.93, 3.69]
Persistence of Exogenous Process				
ρ_ξ (demand)	0.87	[0.79, 0.94]	0.87	[0.79, 0.93]
ρ_i (monetary policy)	0.95	[0.91, 0.99]	0.95	[0.91, 0.99]
ρ_y (supply)	0.36	[0.30, 0.43]	0.46	[0.31, 0.61]
ρ_F (Exp. meas. errors)	0.93	[0.89, 0.98]		

And so are impulse responses



Posterior predictive checks

- Let $\tilde{y}$ denote a random variable as distinguished from the realized value y.
- The *posterior predictive distribution* is given by:

$$p(\tilde{y}|y) = \int p(\tilde{y}|\theta)p(\theta|y)d\theta. \tag{9}$$

- A natural "test" is to reject the model if some important feature of the data lies far in a tail of the predictive distribution.

CG and AHS as Posterior Predictive Checks

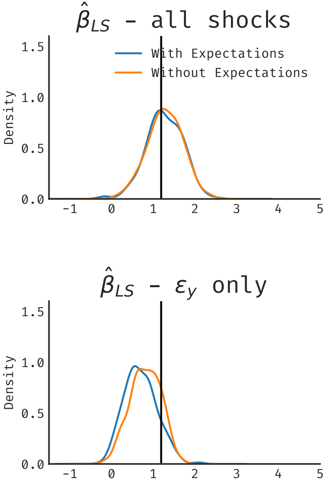
Posterior Predictive Checks For $i = 1, \ldots, N$:

1. *Construct* $\tilde{y}$. Draw $\theta^i \sim p(\theta | Y, F)$, and simulate a single trajectory of $\tilde{y} = \{\Delta y_t, \pi_t, i_t, \mathbb{E}_t^k[\bar{\pi}_{t+4}], \mathbb{E}_{t-1}^k[\bar{\pi}_{t+4}]\}_{t=1}^T$, where $T = 168$, the length of the actual observables.
2. *Construct* $\mathcal{S}_{AHS}(\tilde{y})$ as the point estimates of the impulse response coefficents of $\bar{\pi}_t$ and $\mathbb{E}_{t-4}^k[\bar{\pi}_t]$ to a "supply" shock in a VAR(4) model for $\left[\Delta y_t, \pi_t, i_t, \mathbb{E}_t^k[\bar{\pi}_{t+4}]\right]$. The "supply" shock is identified as the shock that maximizes the forecast error variance in inflation over the medium term.
3. *Construct* $\mathcal{S}_{CG}(\tilde{y})$. Estimate the regression model,

$$\bar{\pi}_t - \mathbb{E}_{t-4}^k[\bar{\pi}_t] = \alpha + \beta \left(\mathbb{E}_{t-4}^k[\bar{\pi}_t] - \mathbb{E}_{t-5}^k[\bar{\pi}_t] \right) + u_t. \quad (10)$$

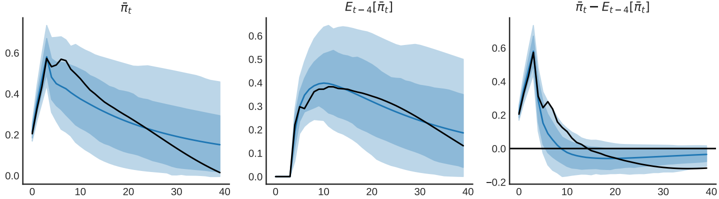
   Store the OLS point esitmate of β.

CG Coefficients



AHS

- AHS construct their shock using a SVAR identification: by maximizing the FEVD at "business cycle frequencies." We do the same:



- Note, however, we can also do this conditional on our (DSGE) identified shocks.

AHS

- We can also count the proportion of trajectories from our posterior predictive checks which satisfy the *dynamic overshooting property*.

Shock	$\mathbb{P}$(Dynamic Overshooting)	$mean(i*)$	$std(i*)$
AHS	0.86	10.09	5.08
Supply	0.93	9.52	3.50
Monetary Policy	0.52	30.26	8.43
Demand	0.68	22.83	5.44

- Like the CG property, the significant heterogeneity across shocks.
- But in general, the FHP model is consistent with the AHS evidence.

Conclusion

- FHP model does a good job at matching key moments related into inflation expectation (revision) predictability while fitting the aggregate time series of inflation expectations well.
- In ongoing work, we show that this performance is very difficult to achieve under alternative models of expectations formation, like diagnostic expectations.
- Thanks!