Inflation Expectations and Macro Dynamics under Finite Horizon Planning

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October 5, 2022

[1] This does not express the view of the Federal Reserve Board or the Federal Reserve System.

- Macroeconomists have increasingly begun to incorporate behavioral elements into their models as an alternative to rational expectations.

- Evidence from (conensus) survey data on expectations:
  - Coibion & Gorodnichenko (2015): forecasts *underreact* to new information.
  - Angeletos, Huo, and Sastry (2020): underreaction is followed by *overreaction*.

- Gust, Herbst, Lopez-Salido (2021): the finite horizon planning (FHP) model of Woodford (2018) is an attractive alternative to RE (and some imperfect expectations) models for fitting key aggregate macro time series.

- This paper: Can the FHP model fit key facts on expectations? Yes!

Summary of Results

1. We derive analytical results for a simplified FHP model for the CG and AHS conditions for inflation expectations.
   - We show that these moments depend both on the parameters governing expectations formation and other structural parameters.

2. We estimate an FHP DSGE model:
   - Using data on inflation expectations, the FHP model remains preferable to alternatives in terms of statistical fit.
   - The FHP model is also consistent with CG and AHS conditions for inflation expectations. *Other models are not.*

I. Theory

A Simplified FHP Model

- k-level planner making a decision at date t only looks forward through period $t+k$. For any endogenous variable in periods $t+k-j$, Z_{t+k-j}, with $j = 0, 1, 2, ..., k$:

$$\mathbb{E}_t^k Z_{t+k-j} = E_t Z^j_{t+k-j}, \tag{1}$$

- As shown in Woodford (2018), under these assumptions, firms' price-setting behavior implies a log-linearized relationship for each period of the plan given by:

$$\pi^j_\tau = \beta E_\tau \pi^{j-1}_{\tau+1} + \kappa y_\tau, \tag{2}$$

  where $\tau = t + k - j$ denotes the planning period, and $1 < j \leq k$.

- Assume the output gap y_t follows an AR(1) process:

$$y_t = \rho y_{t-1} + e_t \tag{3}$$

Some algebra

Iterating forward, we have:

$$\pi_t^k = \kappa E_t \sum_{i=0}^{k-1} \beta^i y_{t+i} + \beta^k E_t \pi_{t+k}^0 \tag{4}$$

Firms use continuation value functions to assign value to events outside of their planning horizons:

$$\pi_{t+k}^0 = \kappa y_{t+k} + \beta(1-\theta) v_{pt}, \tag{5}$$

where v_{pt} is the *continuation value* to the plans of firms.

Firms to learn and update their beliefs based on *past experience*.

$$v_{pt+1} = (1-\gamma_p) v_{pt} + \gamma_p v_{pt}^e, \text{ with } v_{pt}^e = (1-\theta)^{-1} \pi_t^k. \tag{6}$$

Taking stock

$$\pi_t^k = \frac{1-(\beta\rho)^{k+1}}{1-\beta\rho}\kappa y_t + \beta^{k+1}(1-\theta_p)v_{pt}.$$

The relationship between FHP and RE is given by

$$\pi_t^k = [1-(\beta\rho)^{k+1}]\pi_t^{RE} + \beta^{k+1}(1-\theta)v_{pt} \tag{7}$$

because $\pi_t^{RE} = \lim_{k\to\infty} \pi_t^k = (1-\beta\rho)^{-1}\kappa y_t$.

Since $0 < 1-(\beta\rho)^{k+1} \leq 1$, inflation in the FHP model is less responsive to fluctuations in the output gap.

Longer-run beliefs about inflation depend on past inflation:

$$v_{pt} = \frac{\gamma_p}{1-\theta}\sum_{i=0}^{t-1}(1-\gamma_p)^i \pi_{t-1-i}^k. \tag{8}$$

Inflation in the FHP model displays an excess sensitivity to past inflation relative to the RE solution.

Forecasting

A firm with a planning horizon of length $k > 0$ has a one-step ahead forecast given by:

$$\mathbb{E}_t^k \pi_{t+1} = [1 - (\beta\rho)^k] \frac{\kappa\rho}{1 - \beta\rho} y_t + \beta^k (1 - \theta) v_{pt}. \tag{9}$$

Let $\mathbb{F}_{t+1}^k = \pi_{t+1}^k - \mathbb{E}_t^k \pi_{t+1}$ denote the one-step inflation forecast error, defined as the difference between realized and expected inflation.

$$\mathbb{F}_{t+1}^k = \left[\beta^{k+1} \gamma_p A(k) + \rho(\beta\rho)^k\right] \kappa y_t - \beta^k \left[1 - \beta(1 - \widetilde{\gamma}_p)\right] (1 - \theta) v_{pt} + O_{t+1}. \tag{10}$$

where the parameters $A(k) = \frac{1-(\beta\rho)^{k+1}}{1-\beta\rho}$ and $\widetilde{\gamma}_p = \gamma_p(1 - \beta^{k+1})$.

The AHS property

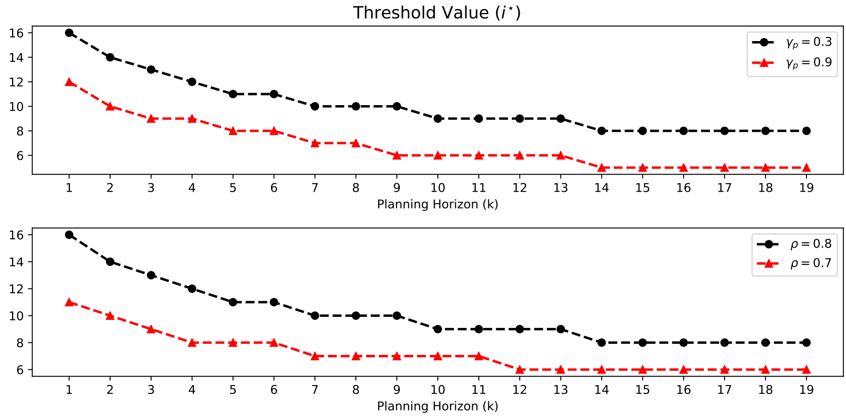
Define $\mathbb{F}^k_{t+1} = \pi^k_{t+1} - \mathbb{E}^k_t \pi_{t+1}$.

Proposition

(IRFs of Inflation Forecasts and Forecast Errors). Let $\frac{\partial \mathbb{E}^k_{t+i} \pi_{t+1+i}}{\partial e_t}$ *and* $\frac{\partial \mathbb{F}^k_{t+1+i}}{\partial e_t}$ *for* $i \geq 0$ *be the impulse response functions for a firm's one-step ahead inflation forecast and forecast error, respectively.*

1. *Without learning:* $\frac{\partial \mathbb{E}^k_{t+i} \pi_{t+1+i}}{\partial e_t} \geq 0$ *and* $\frac{\partial \mathbb{F}_{t+1+i}}{\partial e_t} \geq 0$, $\forall i \geq 0$ *and* $k > 0$.
2. *With learning: If* $\gamma_p \leq \frac{1-\rho}{1-\beta^{k+1}}$, *there is a threshold forecast horizon,* $i^\star$, *such that:*
   2.1 $\frac{\partial \mathbb{E}^k_{t+i} \pi_{t+1+i}}{\partial e_t} \geq 0$ *for* $i \geq 0$,
   2.2 $\frac{\partial \mathbb{F}_{t+1}}{\partial e_t} > 0$ *and* $\frac{\partial \mathbb{F}_{t+1+i}}{\partial e_t} < 0$ *for* $i \geq i^\star$,

Delayed Overreaction of Inflation Forecasts in the FHP Model



The CG property

In the FHP model, a firm's inflation forecast revision can be defined as:

$$\mathbb{R}_t^k = \left[\mathbb{E}_t^k - \mathbb{E}_{t-1}^k\right] \pi_{t+1}.$$

Proposition (Forecast Error and Revision Correlation)

Let $\beta_{CG} = \frac{cov(\mathbb{R}_t^k, \mathbb{F}_{t+1}^k)}{var(\mathbb{R}_t^k)}$ *denote the univariate regression coefficient from regressing the one-step ahead forecast error on the forecast revision in the FHP model.*

1. *Without learning: If* $\rho > 0$*, then* $\beta_{CG} > 0$*, for any finite planning horizon* $k > 0$*.*
2. *With learning: If* $\rho = 0$ *and* $\gamma_p < \frac{1-\beta}{1-\beta^{k+1}}$*, then* $\beta_{CG} > 0$*.*

- With learning: $\beta_{CG} > 0$ if firms do not update their longer-run beliefs too quickly.

Table: Predictability Regression Results For FHP Model

	β_{CG}	
	$\rho = 0$	$\rho = 0.8$
$\gamma_p = 0.3$	-0.01	0.66
$\gamma_p = 0.9$	-0.03	0.73

NOTE: Population coefficient from a regression of one-step ahead inflation forecast errors on forecast revision. For the remaining parameters, $k = 6$, $\beta = 0.99$, and $\kappa = 0.05$.

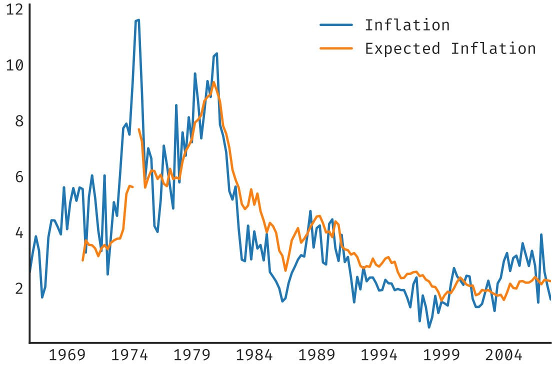
II. Empirics

Why we need system-based estimation

- We extend to the estimated DSGE of Gust, Herbst, Lopez-Salido (2021) to include data on inflation expectations.
- The is a methodological departure from CG (2015) and AHS (2020).
- unlike (some) other models of imperfect expectations formation, predictability properties depend on a broader set of structural parameters
- our analysis also allows to understand the extent to which matching the predictability of inflation expectations is also consistent with overall time series fit of inflation, output, short-term interest rates and inflation expectations.

Inflation Expectations

We use the mean forecast for four-quarter-ahead (GDP deflator) inflation expectations from the Survey of Professional Forecasters (SPF).



Assume that this is a noisy measure of model consistent inflation expectations.

- We also use data on output growth, (GDP deflator) inflation, and the federal funds rate.
- Our sample is from 1966-2007.
- Denote the standard macro data as Y and expected inflation data as F.
- Measure of overall fit *log marginal data density*:

$$\log p(Y, F) = \log p(Y) + \log p(F|Y)$$

Some Estimation Results

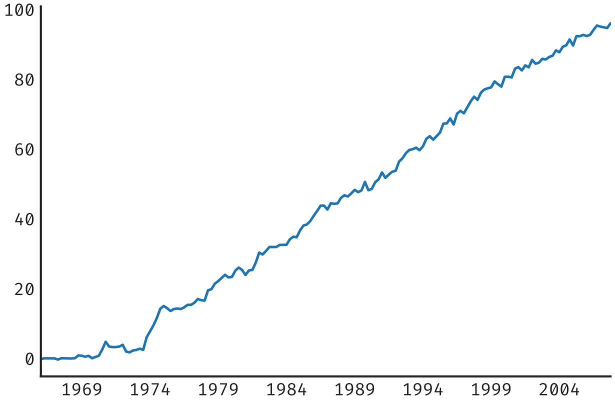
Even with inflation expectations observed, a lot of evidence for low k.

Table: Log MDDs as function of k.

horizon	log p(Y)	log p(F given Y)
k = 0	-720.42	-56.02
k = 1	-716.54	-49.37
k = 2	-718.91	-49.53
k = 3	-721.46	-48.95
k = 4	-723.52	-48.46

Adding inflation expectations leads to more evidence in favor of larger k.

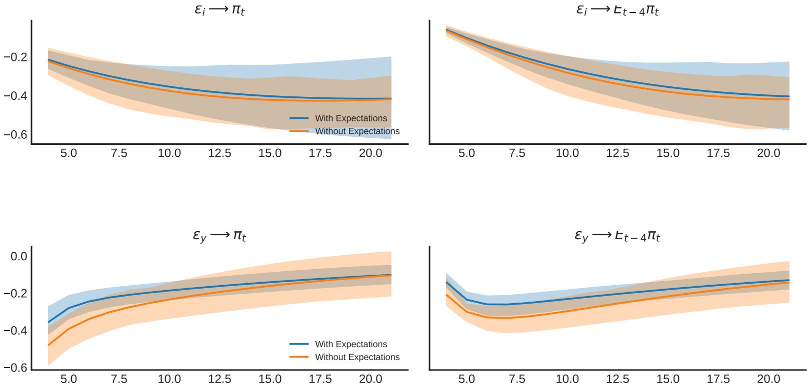
$\log p(F|Y)$ relative to the Hybrid New Keynesian Model



Parameters are remarkably stable

	With Expectations Data		Without Expectations Data	
	Mean	[05, 95]	Mean	[0, 95]
		Learning		height
γ	0.50	[0.33, 0.67]	0.47	[0.30, 0.64]
γ_f	0.16	[0.13, 0.20]	0.20	[0.14, 0.29]
		Endogenous Propagation		height
κ	0.03	[0.02, 0.04]	0.03	[0.02, 0.04]
σ	2.71	[1.96, 3.59]	2.67	[1.92, 3.54]
		Monetary Policy Rule		height
Φ_π	0.96	[0.71, 1.26]	0.98	[0.72, 1.28]
Φ_y	0.90	[0.60, 1.31]	0.89	[0.59, 1.29]
$\Phi_{\pi L}R$	1.92	[1.56, 2.32]	1.84	[1.49, 2.23]
$\Phi_{yL}R$	0.13	[0.04, 0.25]	0.13	[0.04, 0.25]
		Persistence of Exogenous Processes		height

And so are impulse responses



Posterior predictive checks

- Let $\tilde{y}$ denote a random variable as distinguished from the realized value y.
- The *posterior predictive distribution* is given by:

$$p(\tilde{y}|y) = \int p(\tilde{y}|\theta)p(\theta|y)d\theta. \tag{11}$$

- A natural "test" is to reject the model if some important feature of the data lies far in a tail of the predictive distribution.

CG and AHS as Posterior Predictive Checks

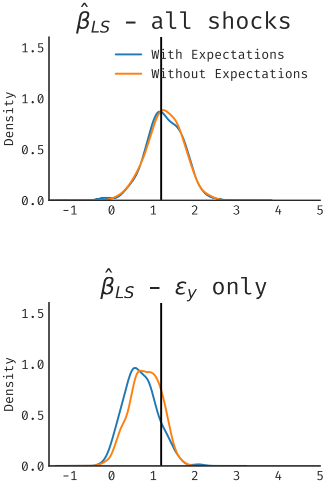
Posterior Predictive Checks For $i = 1, \ldots, N$:

1. *Construct* $\tilde{y}$*.* Draw $\theta^i \sim p(\theta | Y, F)$, and simulate a single trajectory of $\tilde{y} = \{\Delta y_t, \pi_t, i_t, \mathbb{E}_t^k[\bar{\pi}_{t+4}], \mathbb{E}_{t-1}^k[\bar{\pi}_{t+4}]\}_{t=1}^T$, where $T = 168$, the length of the actual observables.
2. *Construct* $\mathcal{S}_{AHS}(\tilde{y})$ as the point estimates of the impulse response coefficents of $\bar{\pi}_t$ and $\mathbb{E}_{t-4}^k[\bar{\pi}_t]$ to a "supply" shock in a VAR(4) model for $\left[\Delta y_t, \pi_t, i_t, \mathbb{E}_t^k[\bar{\pi}_{t+4}]\right]$. The "supply" shock is identified as the shock that maximizes the forecast error variance in inflation over the medium term.
3. *Construct* $\mathcal{S}_{CG}(\tilde{y})$. Estimate the regression model,

$$\bar{\pi}_t - \mathbb{E}_{t-4}^k[\bar{\pi}_t] = \alpha + \beta \left(\mathbb{E}_{t-4}^k[\bar{\pi}_t] - \mathbb{E}_{t-5}^k[\bar{\pi}_t] \right) + u_t. \qquad (12)$$

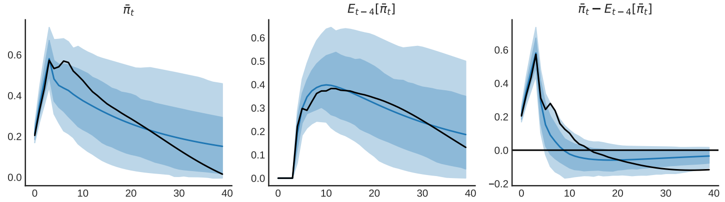
   Store the OLS point esitmate of β.

CG Coefficients



AHS

- AHS construct their shock using a SVAR identification: by maximizing the FEVD at "business cycle frequencies." We do the same:



- Note, however, we can also do this conditional on our (DSGE) identified shocks.

AHS

We can also count the proportion of trajectories from our posterior predictive checks which satisfy the *dynamic overshooting property*.

Shock	$\mathbb{P}$(Dynamic Overshooting)	$mean(i*)$	$std(i*)$
AHS	0.86	10.09	5.08
Supply	0.93	9.52	3.50
Monetary Policy	0.52	30.26	8.43
Demand	0.68	22.83	5.44

Like the CG property, the significant heterogeneity across shocks.

But in general, the FHP model is consistent with the AHS evidence.

II. Other Models

Sticky Information

Mankiw and Reis (2002);

- Infrequent updating of information set of firms (government by parameter λ.

$$\pi_t = (1-\lambda)\lambda^{-1} mc_t + \mathbb{E}^{\lambda}_{t-1}\left[\pi_t + \Delta mc_t\right], \qquad (13)$$

- Can show that in population $\beta_{CG} = \lambda/(1-\lambda)$.

- After a surprise, expectations will eventually to RE; no AHS property.

- We model price-setting firms as having sticky information in an NK model additional endogenous persistence mechanisms: inflation indexation and habit formation.

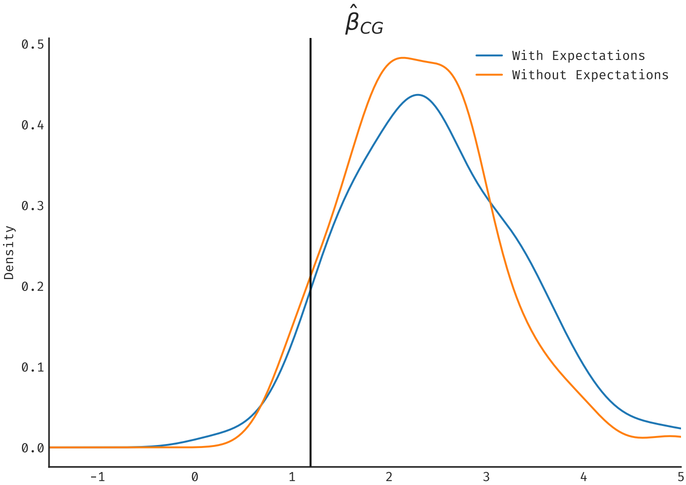
Sticky Information, Results

Table: Log MDDs

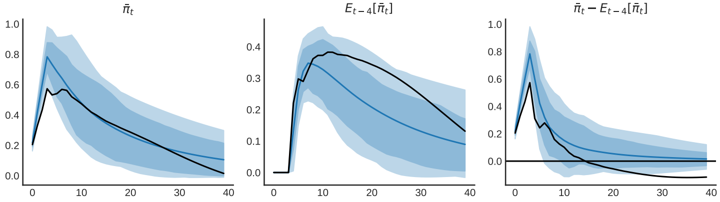
horizon	log p(Y)	log p(F given Y)
FHP $k = 1$	-716.54	-49.37
FHP $k = 4$	-723.52	-48.46
Sticky Information	-753.73	50.58

- The Sticky Information model fits the standard macroeconomic data poorly.
- The conditional fit of the expectations data is on par with the FHP model.

CG in the Sticky Information Model



AHS in the Sticky Information Model



- In the SI model the forecast error rises in response to the AHS "inflation" shock.
- the median impulse response of the forecast error in the SI model does not turn into an overreaction, and instead monotonically converges back to zero

Diagnostic Expectations

Bordalo et al. (2018); Bianchi et al. (2022)

- Agents expectations can overly influenced by past events
- DE Expectations Operator with J period reference:

$$\mathbb{E}_t^{\theta} X_{t+1}^{RE} = E_t X_{t+1}^{RE} + \theta \left[E_t X_{t+1}^{RE} - \sum_{j=1}^{J} \alpha_j E_{t-j} X_{t+1}^{RE} \right], \quad (14)$$

with $\theta \geq 0$ and α such that $\sum_{j=1}^{J} \alpha_j = 1$.

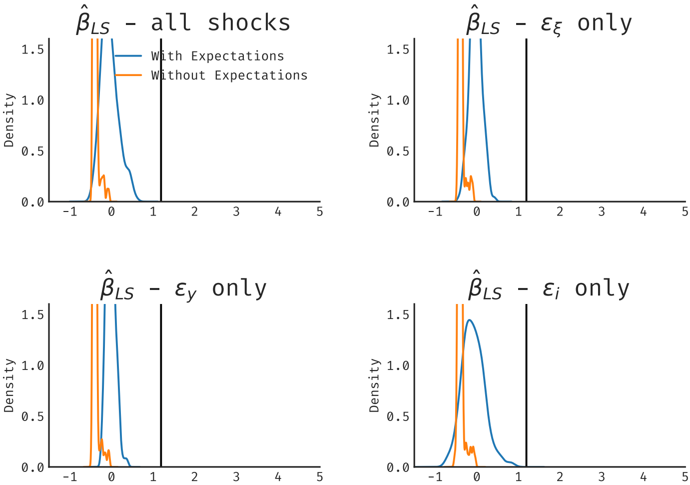
- When $J > 1$, the decisions of household and firm can be time inconsistent.
- We model DE for $J = 1$ NK model additional endogenous persistence mechanisms: inflation indexation and habit formation.

Diagnostic Expectations, Results

horizon	log p(Y)	log p(F given Y)
FHP k = 1	-716.54	-49.37
FHP k = 4	-723.52	-48.46
Diagnostic Expectations ($J = 1$)	-732.0	-152.27

- The FHP model out performs the DE model, particularly for the expectations data.
- The estimated value of θ is very close to zero when inflation expectations are added to the dataset, suggesting the model is essentially the RE model.

Diagnostic Expectations, CG Result



Conclusion

- FHP model does a good job at matching key moments related into inflation expectation (revision) predictability while fitting the aggregate time series of inflation expectations well.
- In ongoing work, we show that this performance is very difficult to achieve under alternative models of expectations formation, like diagnostic expectations.
- Thanks!