2012.

一. (1) $u(t) = L(t)\dfrac{di(t)}{dt} + i(t)\dfrac{dL(t)}{dt}$

(2) $L(t) \geq 0$ 且 $L'(t) \geq 0$.

证明：同前

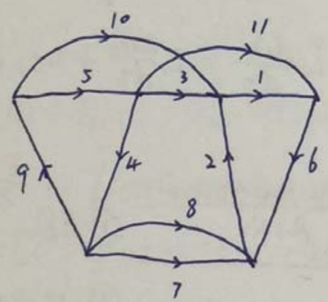
二. 解：(1)

$B_f = [-Q_t^T \quad 1_L]$

$Q_f = [1_t \quad -B_t^T]$

$$B_f = [-Q_t^T \quad 1_L] = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(2)



写回路，按着堆积即可

同向为正，反向为负.

三. 解：(1) CCCS 对原始不定导纳矩阵贡献：

$i_{43} = \beta i_{14} = \beta G_1(u_1 - u_4)$ 即为：

$$\begin{array}{c|cc} & 1 & 4 \\ \hline 4 & \beta G_1 & -\beta G_1 \\ 3 & -\beta G_1 & \beta G_1 \end{array} = \begin{array}{c|cc} & 1 & 4 \\ \hline 3 & -2 & 2 \\ 4 & 2 & -2 \end{array}$$

① 端子压缩：将多端网络的两个或更多端子连接在一起，形成一个端子，称为端子压缩：(行 1 列分别相加)

② 端子消除：

理想变压器 ~~耦合元件~~ 对原始不定导纳矩阵贡献：

$$\begin{array}{c|ccc} & 4 & 3 & 2 \\ \hline 4 & sC_2(1+2n+n^2) & -sC_2(1+n) & -nsC_2-n^2sC_2 \\ 3 & -sC_2(1+n) & sC_2 & nsC_2 \\ 2 & -(n^2+n)sC_2 & nsC_2 & n^2sC_2 \end{array} = \begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline 2 & 100s & 10s & -110s \\ 3 & 10s & s & -11s \\ 4 & -110s & -11s & 121s \end{array}$$

③ 端子接地：直接删掉对应行/列。

写不定导纳矩阵：

$$Y_i(s) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 100s & -90s & -10s \\ -2 & -90s & 81s & 9s+2 \\ 1 & -10s & 9s & s-1 \end{bmatrix}$$

(2) 写为定导纳矩阵（以3为参考节点）

$$Y_d(s)=\begin{bmatrix}1 & 0 & -1\\ 0 & 100s & -110s\\ -1 & -110s & 121s-2\end{bmatrix}$$

消掉节点4：
$$\begin{bmatrix}1+\frac{1}{121s-2} & -\frac{110s}{121s-2}\\ \frac{110s}{121s-2} & 110s-\frac{110^2s^2}{121s-2}\end{bmatrix}$$

整理得：
$$Y_{sc}(s)=\begin{bmatrix}\frac{121s-1}{121s-2} & -\frac{110s}{121s-2}\\ \frac{110s}{121s-2} & \frac{1210-220s}{121s-2}\end{bmatrix}$$
$$\begin{bmatrix}1+\frac{1}{s-1} & -\frac{ns}{s-1}\\ \frac{ns}{s-1} & n^2s-\frac{n^2s}{s-1}\end{bmatrix}$$

四 解：(1) 列写 u_{C1} 所在基本割集的KCL，i_{L2} 所在基本回路的KVL.

$$-i_s+(-i_{L2})+i_{R2}+i_{C2}+(-i_{C1})=0 \Rightarrow C_1\frac{du_{C1}}{dt}=C_2\frac{du_{C2}}{dt}-i_{L2}+i_{R2}-i_s$$

$$u_{L2}-u_{L1}-u_{C1}+u_s-R_1i_{R1}=0 \Rightarrow L_2\frac{di_{L2}}{dt}=L_1\frac{di_{L1}}{dt}+u_{C1}+R_1i_{R1}-u_s$$

$$\dot u_{C2}=\dot u_s-\dot u_{C1} \qquad \dot i_{L2}=-\dot i_s-\dot i_{L1}$$

$$i_{R1}=\frac{u_s-u_{C1}-R_2i_{L2}}{R_1+R_2} \qquad i_{R2}=\frac{u_s-u_{C1}+R_1i_{L2}}{R_1+R_2}$$

① 正负

② 给出电源作用

③ 输出方程：

整理得：
$$\dot u_{C1}=-u_{C1}-\frac{1}{2}i_{L2}+\frac{1}{2}\dot u_s+u_s-i_s$$
$$\dot i_{L2}=\frac{1}{2}u_{C1}-\frac{1}{4}i_{L2}-\frac{1}{2}\dot i_s-\frac{1}{2}u_s$$

写成矩阵形式：
$$\begin{bmatrix}\dot u_{C1}\\ \dot i_{L2}\end{bmatrix}=\begin{bmatrix}-1 & -\frac{1}{2}\\ \frac{1}{2} & -\frac{1}{4}\end{bmatrix}\begin{bmatrix}u_{C1}\\ i_{L2}\end{bmatrix}+\begin{bmatrix}\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}\begin{bmatrix}\dot u_s\\ \dot i_s\end{bmatrix}+\begin{bmatrix}1 & -1\\ -\frac{1}{2} & 0\end{bmatrix}\begin{bmatrix}u_s\\ i_s\end{bmatrix}$$

$-\frac{1}{4}$

(2)
$$i_{R1}=\frac{1}{R_1+R_2}u_s-\frac{1}{R_1+R_2}u_{C1}-\frac{R_2}{R_1+R_2}i_{L2}$$

$$u_{C2}=-u_{C1}+u_s$$

$$i_{L1}=-i_{L2}-i_s$$

$$\begin{bmatrix}i_{R1}\\ i_{L1}\\ u_{C2}\end{bmatrix}=\begin{bmatrix}-1 & -\frac{1}{2}\\ 0 & -1\\ -1 & 0\end{bmatrix}\begin{bmatrix}u_{C1}\\ i_{L2}\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}u_s\\ i_s\end{bmatrix}$$

$\boxed{五}$ 解：(1) $C\dfrac{du_C}{dt} = i_{R1} - i_{L1}$

$L_1\dfrac{di_{L1}}{dt} = -M\dfrac{di_{L2}}{dt} + u_C - R_2 i_{L1}$ $\Rightarrow$

$L_2\dfrac{di_{L2}}{dt} = -M\dfrac{di_{L1}}{dt} - R_3 i_{L2}$

$\dot{u}_C = -i_{L1} + i_{R1}$

$\dot{i}_{L1} = -\frac{1}{2}\dot{i}_{L2} + \frac{1}{2}u_C - \frac{1}{2}i_{L1}$

$\dot{i}_{L2} = -\dot{i}_{L1} - i_{L2}$

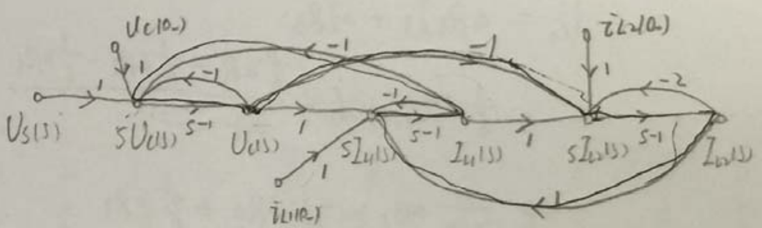
同时 $\boxed{i_{R1} = \dfrac{u_S - u_C}{R_1}}$ ☆

整理得：
$$\dot{u}_C = -u_C - i_{L1} + u_S$$
$$\dot{i}_{L1} = u_C - i_{L1} + i_{L2}$$
$$\dot{i}_{L2} = -u_C + i_{L1} - 2i_{L2}$$

写成矩阵形式：
$$\begin{bmatrix}\dot{u}_C\\ \dot{i}_{L1}\\ \dot{i}_{L2}\end{bmatrix} = \begin{bmatrix}-1 & -1 & 0\\ 1 & -1 & 1\\ -1 & 1 & -2\end{bmatrix}\begin{bmatrix}u_C\\ i_{L1}\\ i_{L2}\end{bmatrix} + \begin{bmatrix}1\\ 0\\ 0\end{bmatrix}u_S$$

(2) 拉氏变换：

$$sU_C(s) = -U_C(s) - I_{L1}(s) + U_S(s) + u_C(0_-)$$
$$sI_{L1}(s) = U_C(s) - I_{L1}(s) + I_{L2}(s) + i_{L1}(0_-)$$
$$sI_{L2}(s) = -U_C(s) + I_{L1}(s) - 2I_{L2}(s) + i_{L2}(0_-)$$



$U_S(s) = \dfrac{3}{s}$ （$\varepsilon(t)$ 的拉氏变换为 $\frac{1}{s}$.）☆

(3) 图的行列式 $\Delta = 1 - [(-s^{-1}) + (-s^{-1}) + (-2s^{-1}) + (-s^{-2}) + s^{-2} \overset{+s^{-3}}{\wedge}] + (-s^{-1})(s^{-2}) + (-s^{-1})(-3s^{-1}) + (-s^{-1})(-2s^{-1})$

$(-s^{-2})(-2s^{-1}) - (-s^{-1})(-s^{-1})(-2s^{-1}) = 1 + 4s^{-1} \overset{-2s^{-3}}{\cancel{}} + 3s^{-2} + 2s^{-2} + 4s^{-3} = 1 + 4s^{-1} + 5s^{-2} + 2s^{-3}$

$P_{m1} = s^{-2}$

$\Delta_{m1} = 1 + 2s^{-1}$

$P_{m2} = -s^{-3}$

$\Delta_{m2} = 1$

$$T_1 = \frac{I_{L1}(s)}{U_S(s)} = \frac{s^{-2} + 2s^{-3} - s^{-3}}{1 + 4s^{-1} + 5s^{-2} + 2s^{-3}} = \frac{s+1}{s^3 + 4s^2 + 5s + 2} \ *$$

(4) 由 $u_C(0_-)$ 至 $U_C(s)$：$P_{1(11)} = s^{-1}$，$\Delta_{1(11)} = 1 - [(-s^{-1}) + (-2s^{-1})] - s^{-2} + (-s^{-1})(-2s^{-1}) = 1 + 3s^{-1} + s^{-2}$

由 $u_C(0_-)$ 至 $I_{L1}(s)$：$P_{1(21)} = s^{-2}$，$\Delta_{1(21)} = 1 + 2s^{-1}$ $P_{2(21)} = -s^{-3}$ $\Delta_{2(21)} = 1$

由 $u_C(0_-)$ 至 $I_{L2}(s)$：$P_{1(31)} = s^{-3}$，$\Delta_{1(31)} = 1$ $P_{2(31)} = -s^{-2}$ $\Delta_{2(31)} = 1 + s^{-1}$

$i_{L1}(0_-)$ $P_{1(12)} = -s^{-2}$，$\Delta_{1(12)} = 1 + 2s^{-1}$； $P_{1(22)} = s^{-1}$，$\Delta_{1(22)} = 1 + 3s^{-1} + 2s^{-2}$； $P_{2(32)} = s^{-3}$ $\Delta_{2(32)} = 1$ $P_{1(32)} = s^{-2}$，$\Delta_{1(32)} = 1 + s^{-1}$；

$i_{L2}(0_-)$ $P_{1(13)} = -s^{-3}$，$\Delta_{1(13)} = 1$； $P_{1(23)} = s^{-2}$，$\Delta_{1(23)} = 1 + s^{-1}$； $P_{1(33)} = s^{-1}$，$\Delta_{1(33)} = 1 + 2s^{-1} + 2s^{-2}$ *

$$\Phi(s) = \begin{bmatrix} \dfrac{s^{-1}+3s^{-2}+s^{-3}}{1+4s^{-1}+5s^{-2}+s^{-3}} & \dfrac{-s^{-1}}{1+4s^{-1}+5s^{-2}+s^{-3}} & \dfrac{-s^{-3}}{1+4s^{-1}+5s^{-2}+s^{-3}} \\ \dfrac{s^{-2}+2s^{-3}}{1+4s^{-1}+5s^{-2}+s^{-3}} & \dfrac{s^{-1}+3s^{-2}}{1+4s^{-1}+5s^{-2}+s^{-3}} & \dfrac{s^{-2}+s^{-3}}{1+4s^{-1}+5s^{-2}+s^{-3}} \\ \dfrac{s^{-3}}{1+4s^{-1}+5s^{-2}+s^{-3}} & \dfrac{s^{-2}+s^{-3}}{1+4s^{-1}+5s^{-2}+s^{-3}} & \dfrac{s^{-1}+2s^{-2}+2s^{-3}}{1+4s^{-1}+5s^{-2}+s^{-3}} \end{bmatrix}$$

$$= \frac{1}{s^3+4s^2+5s+2}\begin{array}{c|c} & \begin{matrix} u_C & i_{L1} & i_{L2} \end{matrix} \\ \hline \begin{matrix} U_C \\ I_{L1} \\ I_{L2} \end{matrix} & \begin{bmatrix} s^2+3s+1 ✓ & -s-2 & -1 \\ s+2 ✓ & s^2+3s+2 ✓ & s+1 ✓ \\ -s-1 ✓ & s+2 ✓ & s^2+2s+2 ✓ \end{bmatrix} \end{array}$$

$$\frac{1}{(s+1)^2(s+2)}\begin{bmatrix} s^2+3s+1 & -s-2 & -1 \\ s+1 & s^2+3s+2 & s+1 \\ -s & s+2 & s^2+2s+2 \end{bmatrix}$$

$\boxed{(5)}$ $U_C(s) = \Phi_{11}(s)u_C(0_-) + \Phi_{12}i_{L1}(0_-) + \Phi_{13}i_{L2}(0_-) = \dfrac{2s^2+6s+2}{s^3+4s^2+5s+1}$

$+ T_S U_S(s)$.

$s(s+2)^2+(s+1)$

六、解：(1) 列写节点方程：

$$(G_1+G_2)U_{n1}-G_2U_{n2}=I_s$$

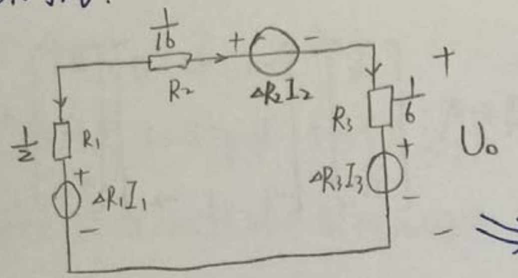
$$-G_2U_{n1}+(G_2+G_3)U_{n2}=0$$

★原网络节点电压

解：$U_{n1}=\frac{11}{3}$ $U_{n2}=1$ $I_1=\frac{U_{n1}}{R_1}=\frac{\frac{11}{3}}{2}=\frac{11}{6}$ $I_2=\frac{U_{n1}-U_{n2}}{R_2}=\frac{\frac{11}{3}-1}{16}=\frac{1}{6}$

★U_o应包含新增电压源！

增量网络：



$$I_3=\frac{U_{n2}}{R_3}=\frac{1}{6}$$

$$\Rightarrow \Delta I(R_1+R_2+R_3)=\Delta R_1 I_1-\Delta R_2 I_2-\Delta R_3 I_3$$

★新网络 { 节点电压 等（易于求解即可）

$$U_o=\Delta R_3 I_3+\Delta I R_3$$

$$=\frac{1}{6}\Delta R_3+6\times\frac{\frac{11}{6}\Delta R_1-\frac{1}{6}\Delta R_2-\frac{1}{6}\Delta R_3}{24}=\frac{1}{6}\Delta R_3+\frac{11}{24}\Delta R_1-\frac{1}{24}\Delta R_2-\frac{1}{24}\Delta R_3$$

$$=\frac{11}{24}\Delta R_1-\frac{1}{24}\Delta R_2+\frac{1}{8}\Delta R_3$$

$$\therefore\ \hat{S}_{R_1}^{U_o}=\frac{11}{24}=0.4583$$

$$S_{R_2}^{\cancel{\Delta u_o}}=-\frac{1}{24}=-0.04167$$

$$\hat{S}_{R_3}^{U_o}=\frac{1}{8}=0.125$$

$$\hat{S}_{R_1}^{T}=\frac{\partial T}{\partial R_1}=\frac{\partial}{\partial R_1}\left(\frac{U_o}{I_s}\right)=\frac{1}{I_s}\hat{S}_{R_1}^{U_o}=0.2292$$

$\Delta I(R_1R_2$