APS1070

Foundations of Data Analytics and Machine Learning

Winter 2022

Week 10:

- *Polynomial Regression*
- *Optimization and Convexity*
- *Regularization*
- *Classification*
- *Neural Networks*

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Slide Attribution

These slides contain materials from various sources. Special thanks to the following authors:

- Lisa Zhang
- Roger Grosse
- Jason Riordon

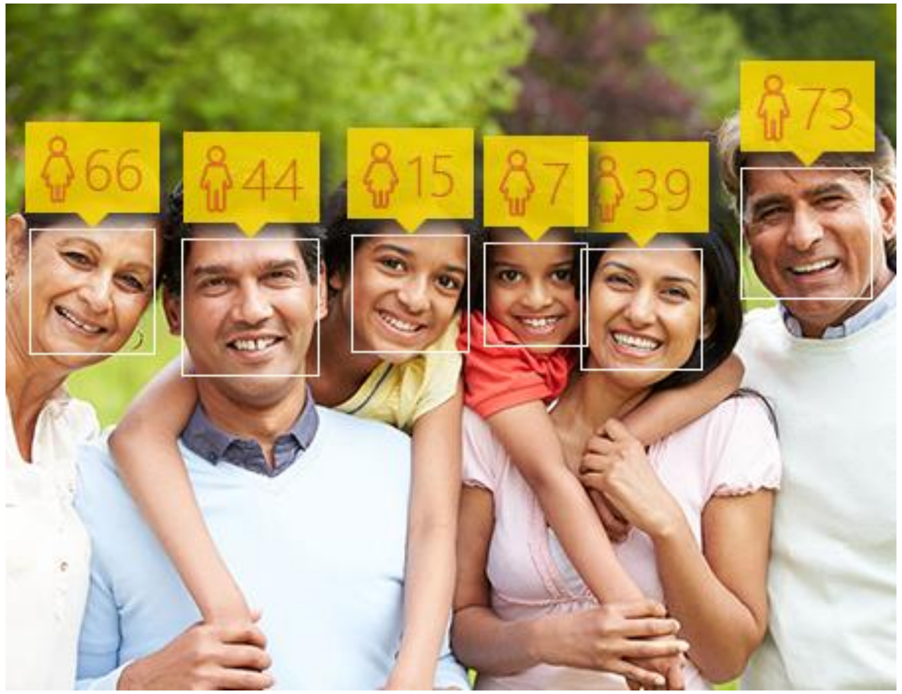
Last Time

- Linear Regression
  - Empirical Risk Minimization
  - Maximum Likelihood Estimation
    - Negative Log-likelihood

- Today we will continue with **nonlinear regression**.

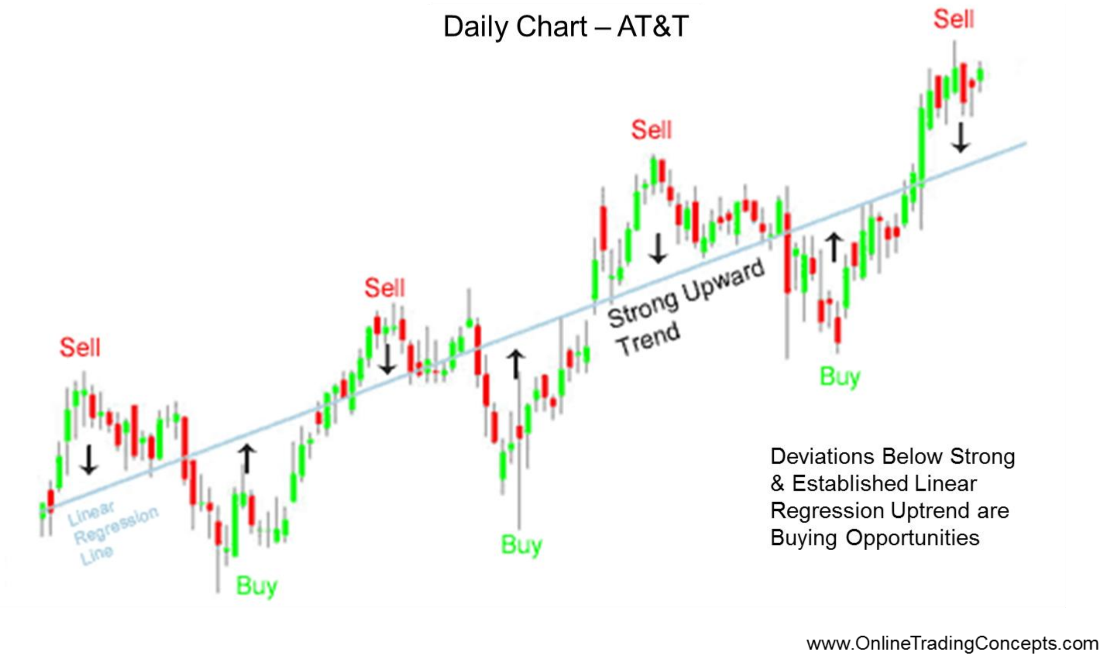
Nonlinear Regression

Age Prediction

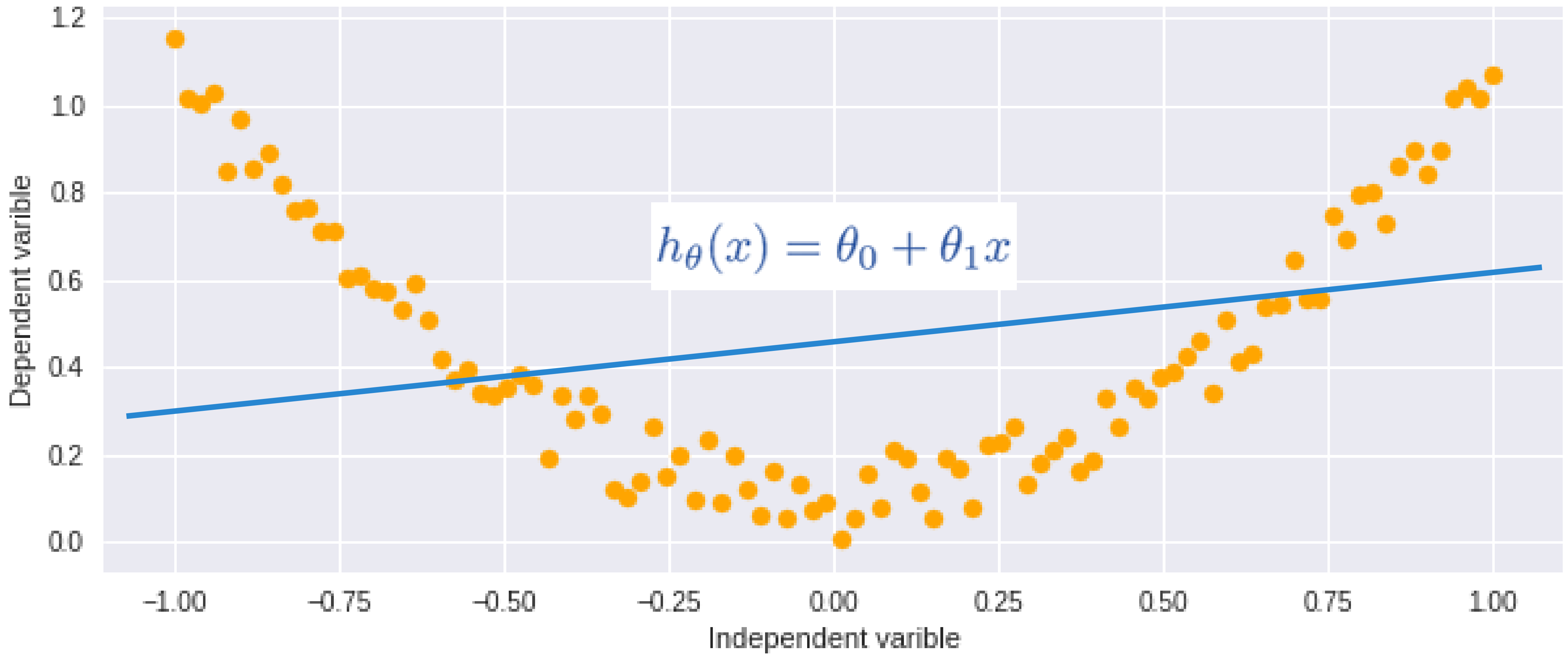


https://www.how-old.net/

Stock Market Prediction



Nonlinear Regression



Agenda

- Polynomial Regression
- Convexity and Optimization
- Logistic Regression
- Gradient Descent
- Regularization
- Multiclass Classification
- Neural Networks

Theme:
Nonlinear Regression (and Classification)

Nonlinear Regression

Recap: Linear Regression

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$

Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2N} \sum_{i=1}^{N} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal:

$$\left.\underset{\theta_0,\theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)\right\}$$

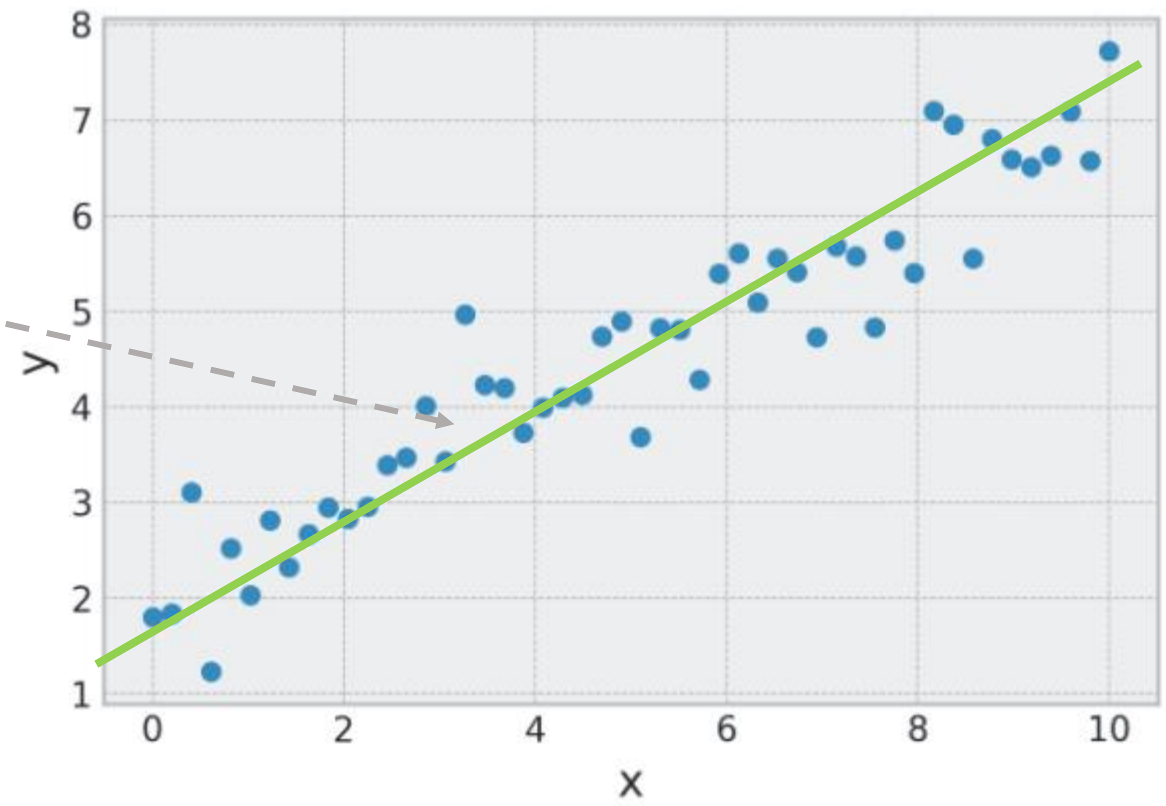
(1) direct solution

$$\frac{dJ}{d\theta} = 0$$

OR

(2) gradient descent

$$\theta \leftarrow \theta - \alpha \frac{\partial J}{\partial \theta}$$



Recap: Vectorization

Hypothesis:

$$h_\theta(\mathbf{X}) = \mathrm{X}\theta$$

Parameters:

$$\theta$$

Cost Function:

$$\mathcal{J}(\theta) = \frac{1}{2N} \|\boldsymbol{y} - \hat{\boldsymbol{y}}\|^2$$

Goal:

$$\left.\min_{\theta} J \right\}$$

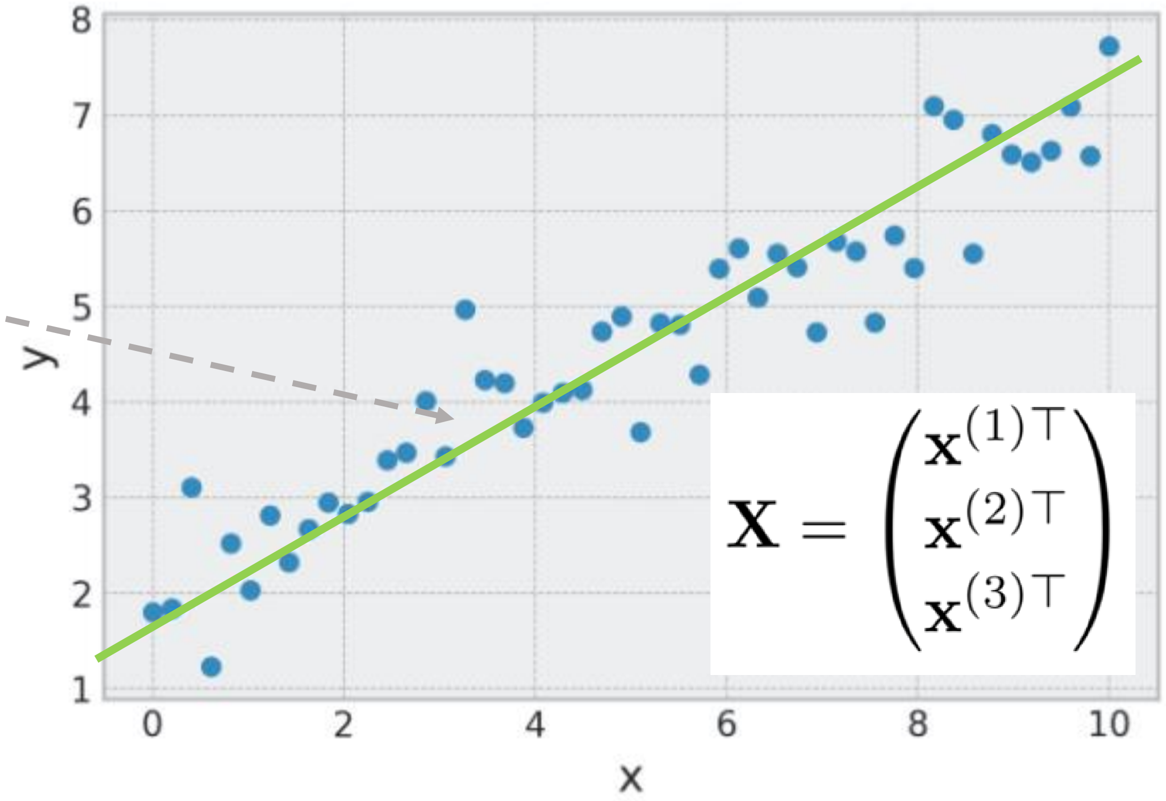
(1) direct solution

$$\frac{dJ}{d\theta} = 0$$

OR

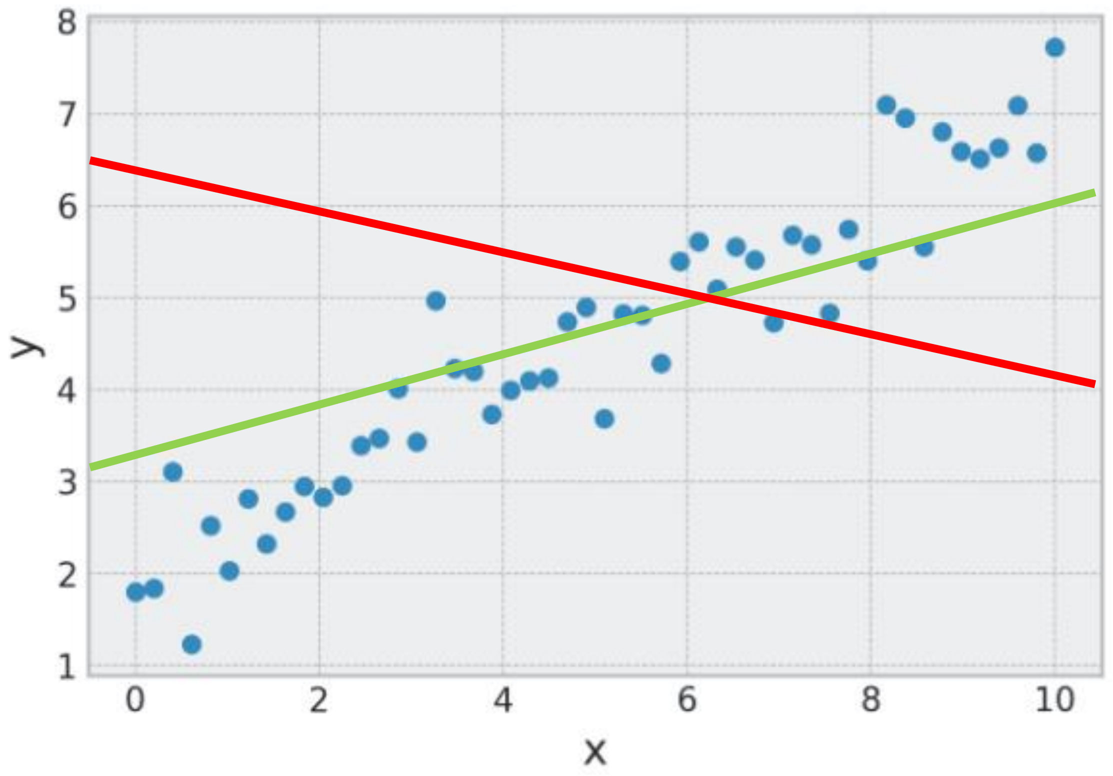
(2) gradient descent

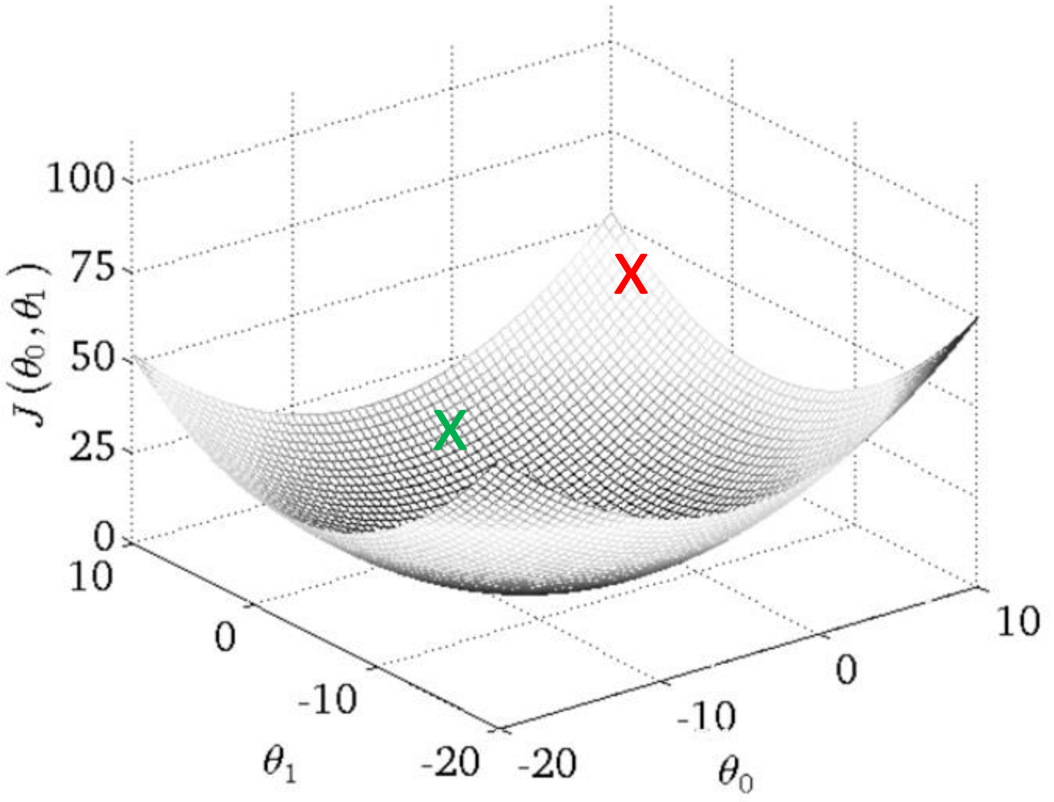
$$\theta \leftarrow \theta - \alpha \frac{\partial J}{\partial \theta}$$



$$\mathbf{X} = \begin{pmatrix} \mathbf{x}^{(1)\top} \\ \mathbf{x}^{(2)\top} \\ \mathbf{x}^{(3)\top} \end{pmatrix}$$

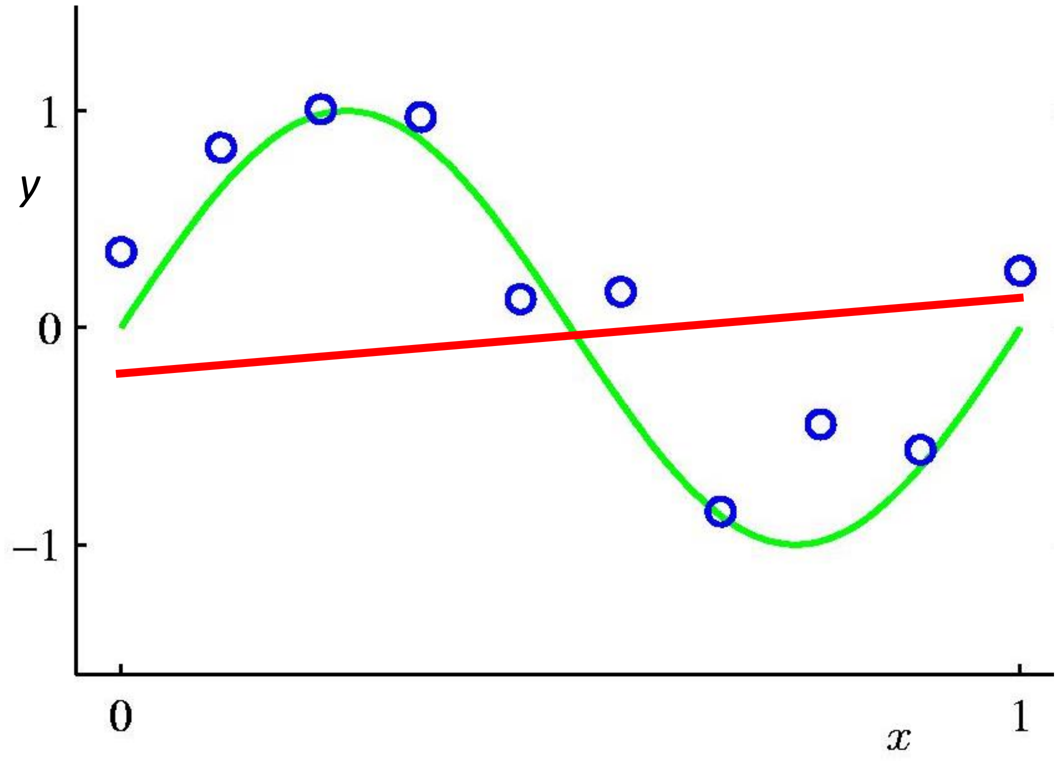
Recap: Convexity





Nonlinear Regression

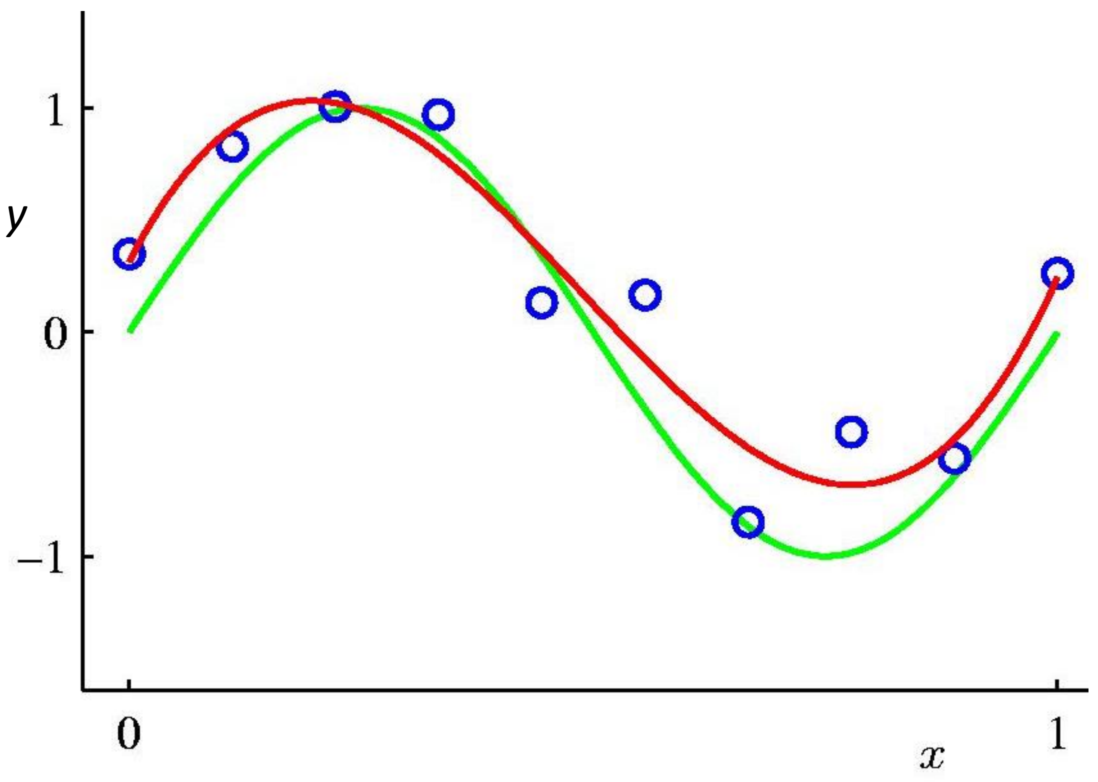
- Suppose we want to model the following data



Given noisy sample data we want to find a hypothesis for what generated the data

- Cannot be fit with a linear model…

Nonlinear Regression



- **One option** is to fit a low-degree polynomial:

$$\hat{y} = \theta_3 x^3 + \theta_2 x^2 + \theta_1 x + \theta_0$$

- This is known as **polynomial regression**

Q: Does this mean we have to derive a whole new algorithm?

Feature Mapping

- Implement a polynomial transformation (feature mapping) by replacing input with polynomial of increasing order:

$$\psi(x) = \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \end{pmatrix}$$

- Hence our hypothesis can be written as:

$$\hat{y} = \theta_3 x^3 + \theta_2 x^2 + \theta_1 x + \theta_0 (1)$$

$$\hat{y} = \boldsymbol{\theta}^{\mathbf{T}} \psi(x)$$

The derivations and algorithms from last lecture remain the same! Why?

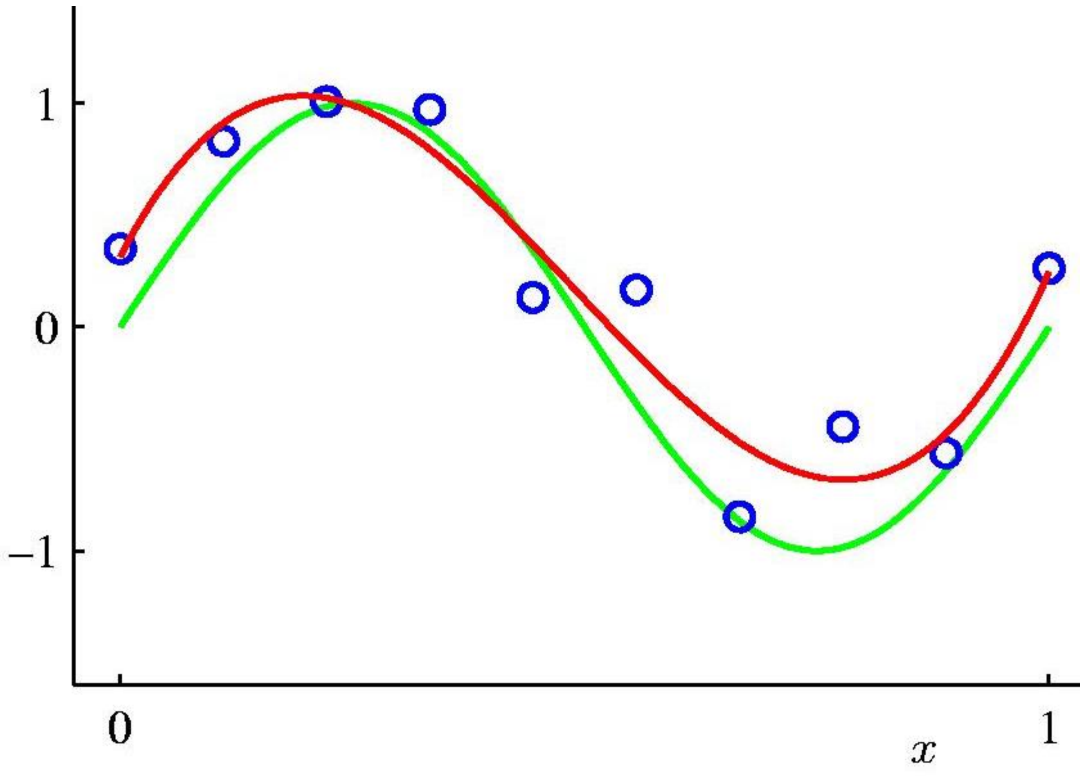
Polynomial Regression

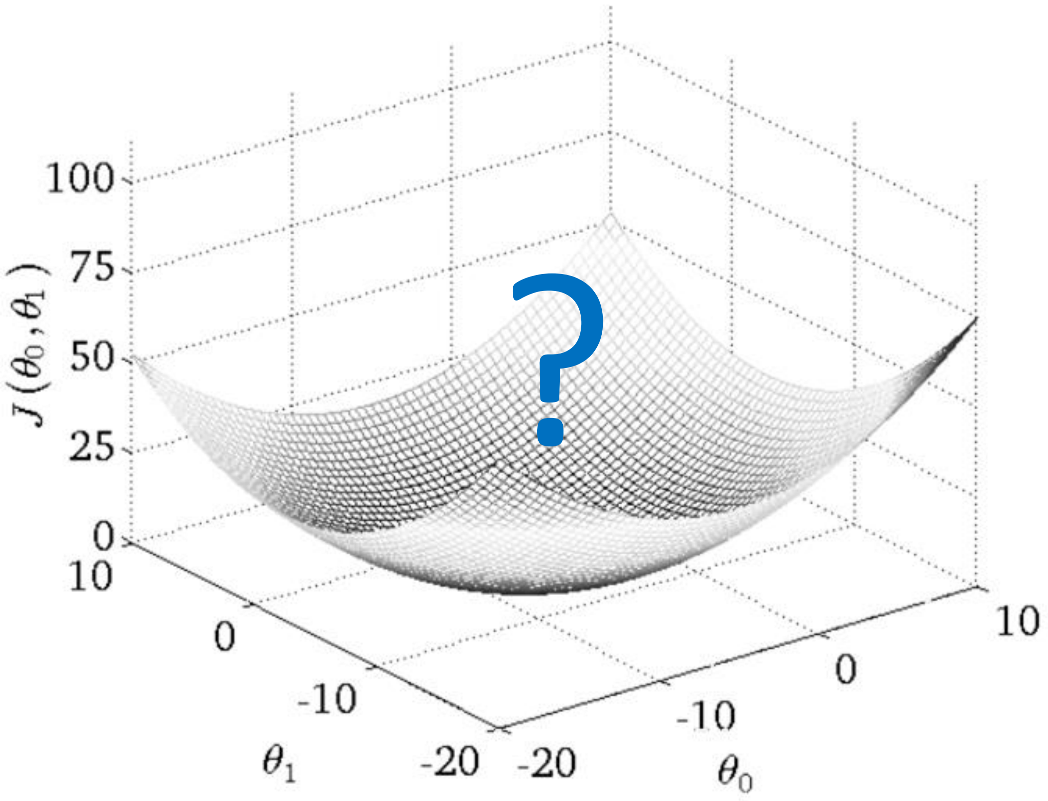
- This doesn't require changing the algorithm, just **pretend $\boldsymbol{\psi}(\boldsymbol{x})$ is the input vector**.

$$\hat{y} = \theta^{\mathrm{T}}\psi(x) \qquad \psi(x) = \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \end{pmatrix}$$

- Feature maps let us fit nonlinear models
- Before deep learning, most of the effort in building a practical machine learning system was **feature engineering.**

Q: Convexity of Polynomial Regression?





$$\hat{y} = \theta^{\mathrm{T}} \psi(x)$$

Direct Solution

- Polynomial regression is really a linear regression problem with some feature engineering.

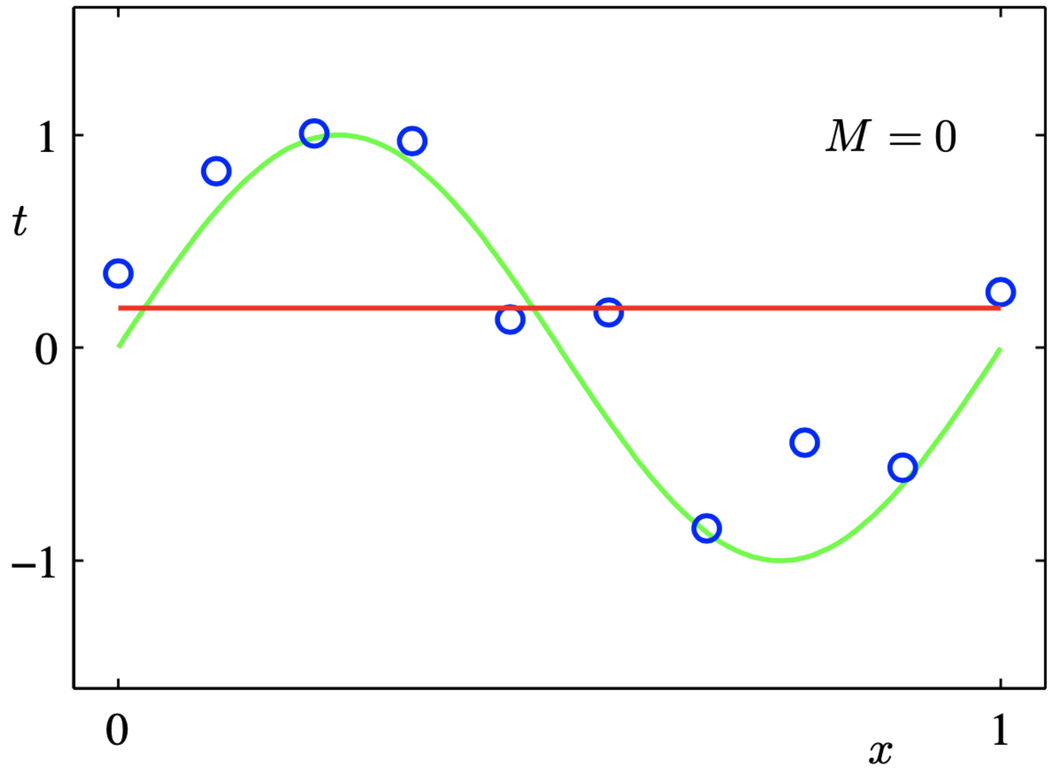
$$\hat{y} = \theta^{T}\psi(x)$$
$$\mathcal{J}(\theta) = \frac{1}{2N}\|\boldsymbol{y} - \hat{\boldsymbol{y}}\|^2 \xrightarrow{\text{minimize cost}} \theta = (\psi^{T}\psi)^{-1}\psi^{T}y$$

Requires that all columns are linearly independent

- Require that $\psi^{T}\psi \in \mathbb{R}^{D\times D}$ to be invertible. This is the case if and only if rank (ψ) = D.

Fitting Polynomial (M = 0)

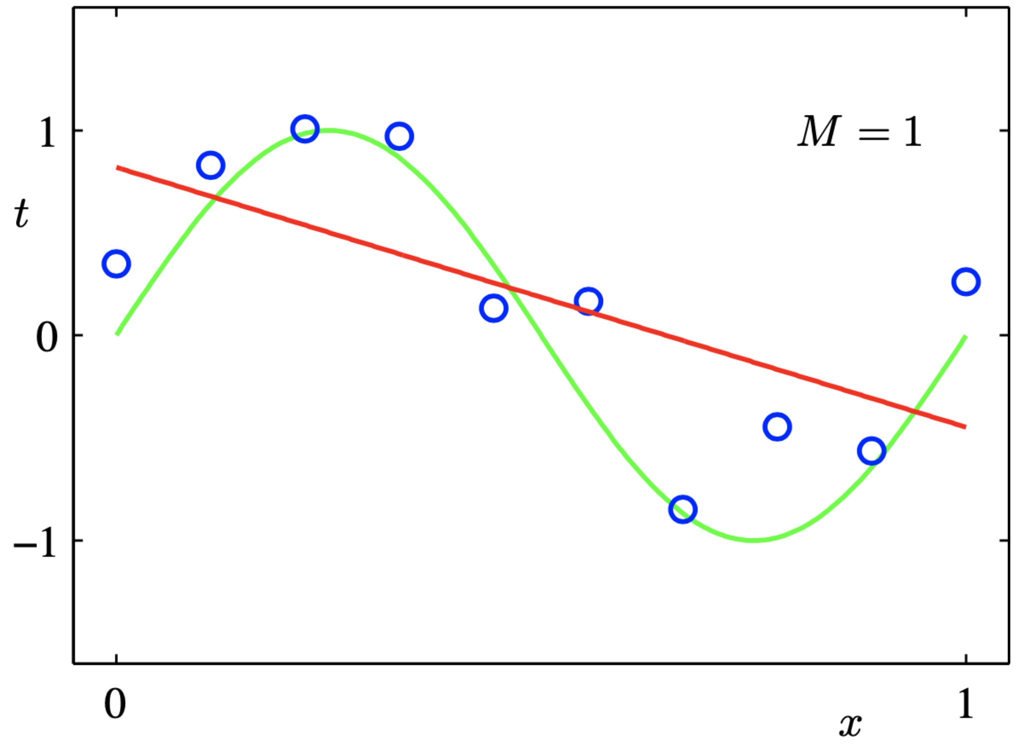
$$\hat{y} = w_0$$



-Pattern Recognition and Machine Learning, Christopher Bishop.

Fitting Polynomial (M = 1)

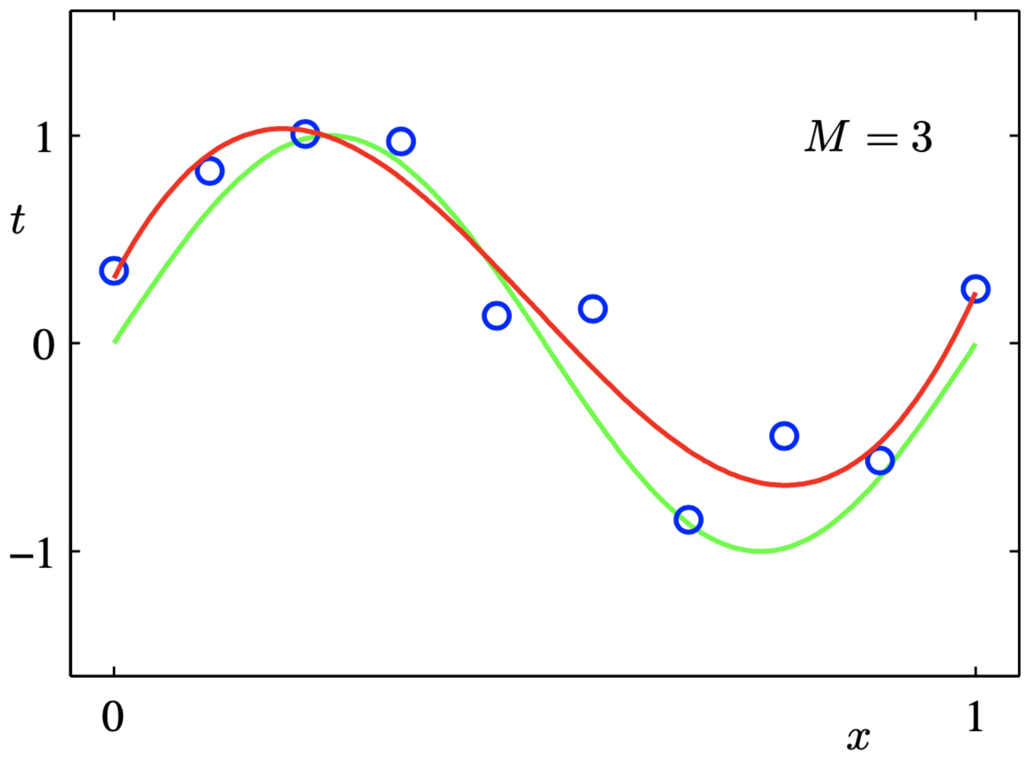
$$\hat{y} = w_0 + w_1 x$$



-Pattern Recognition and Machine Learning, Christopher Bishop.

Fitting Polynomial (M = 3)

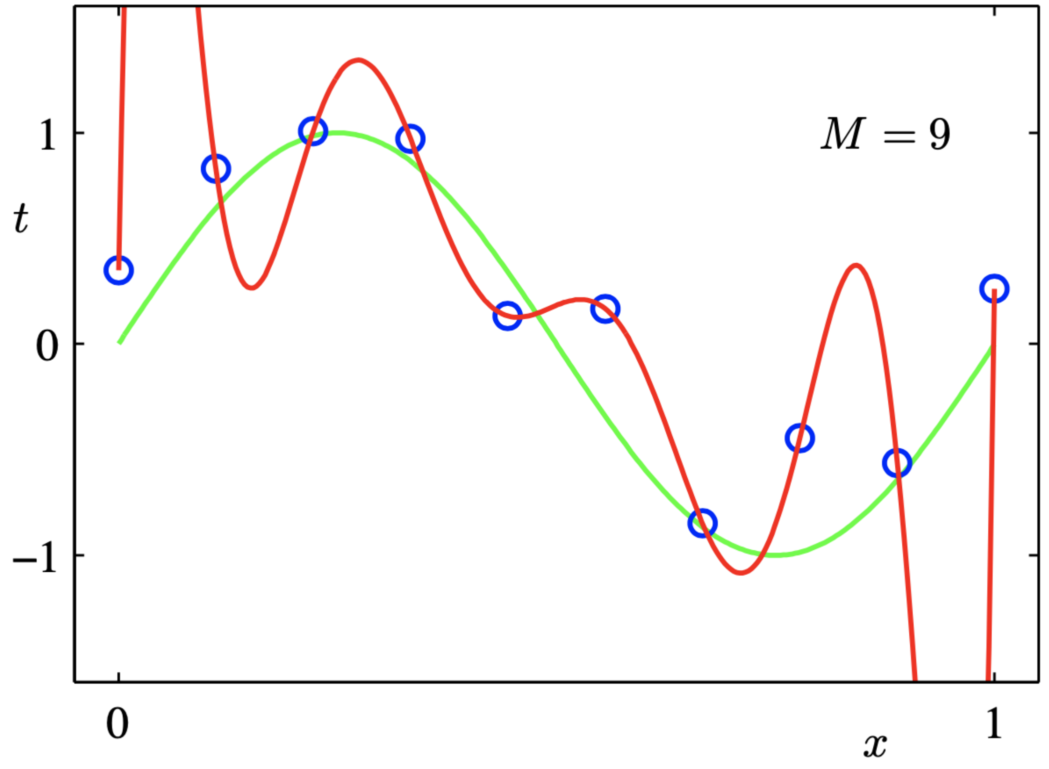
$$\hat{y} = w_0 + w_1 x + w_2 x^2 + w_3 x^3$$



-Pattern Recognition and Machine Learning, Christopher Bishop.

Fitting Polynomial (M = 9)

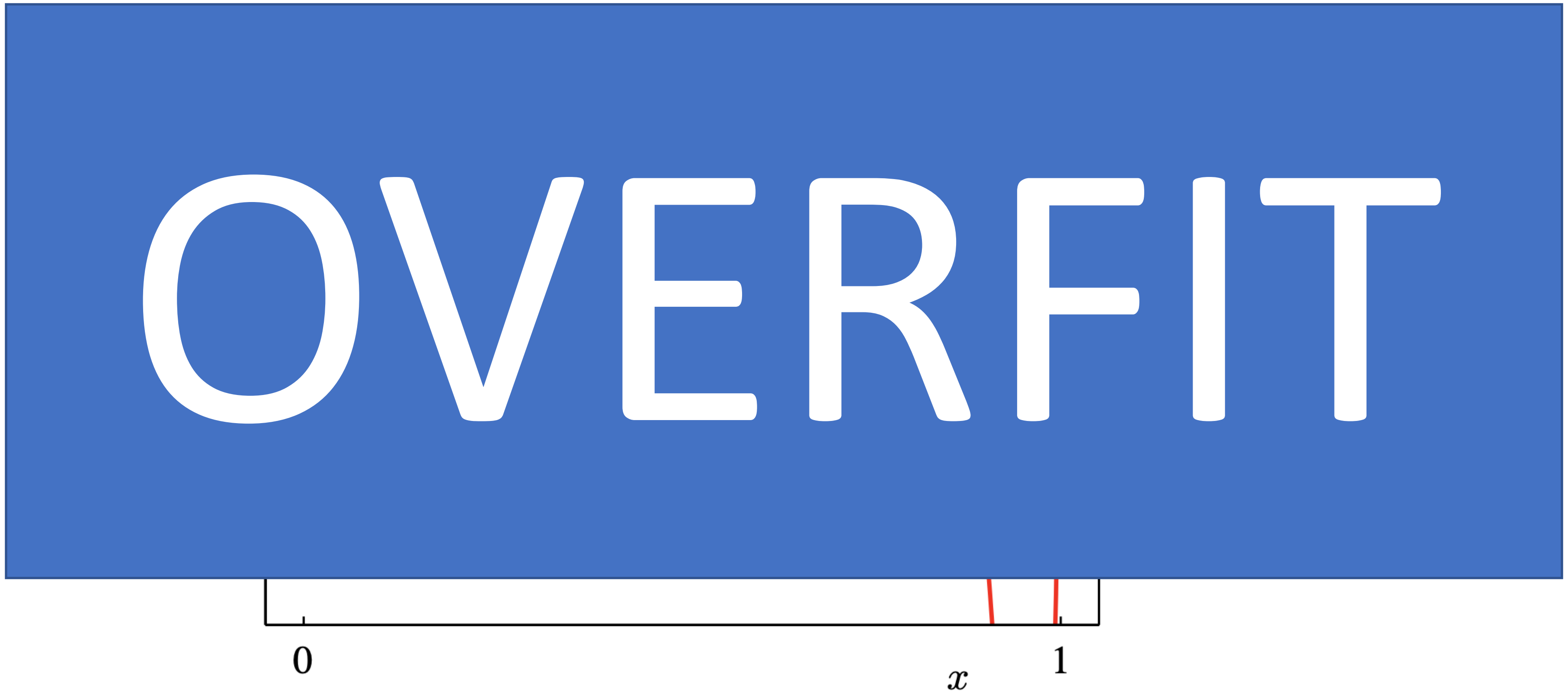
$$\hat{y} = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + \ldots + w_9 x^9$$



-Pattern Recognition and Machine Learning, Christopher Bishop.

Fitting Polynomial (M = 9)

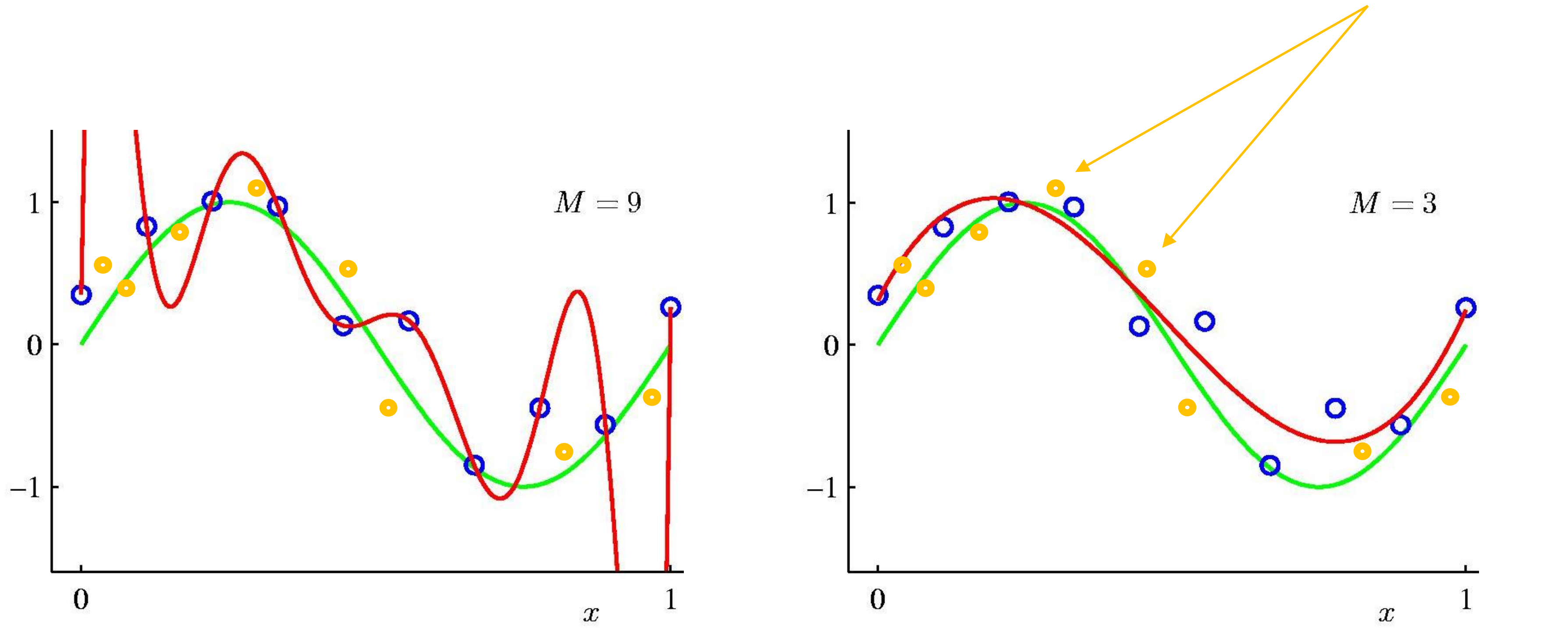
$$\hat{y} = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + \ldots + w_9 x^9$$



-Pattern Recognition and Machine Learning, Christopher Bishop.

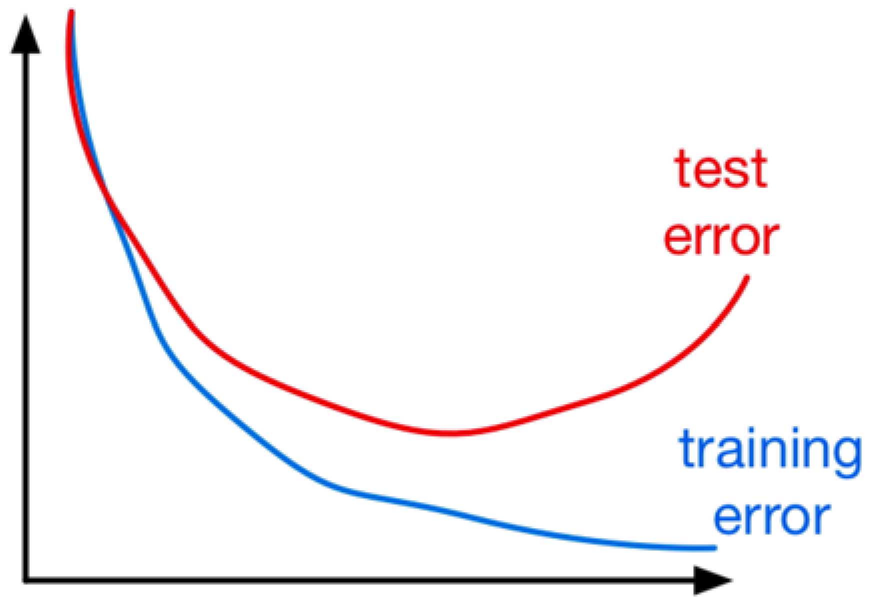
Generalize to New Samples

- We could give the hypothesis a higher complexity, or capacity to fit the data, but this may not generalize well to **new samples**



Generalization

- Training and test error as a function of # parameters:

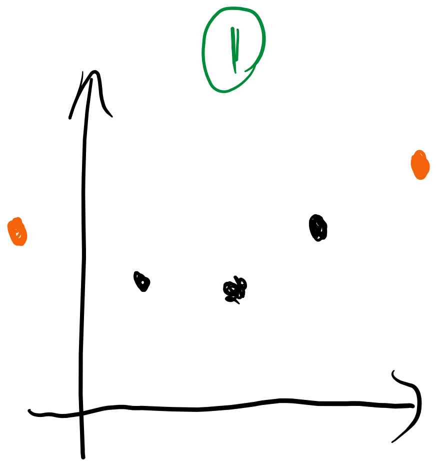


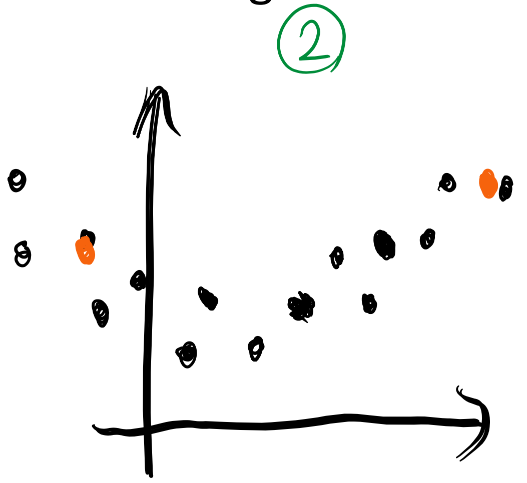
parameters
(model complexity)

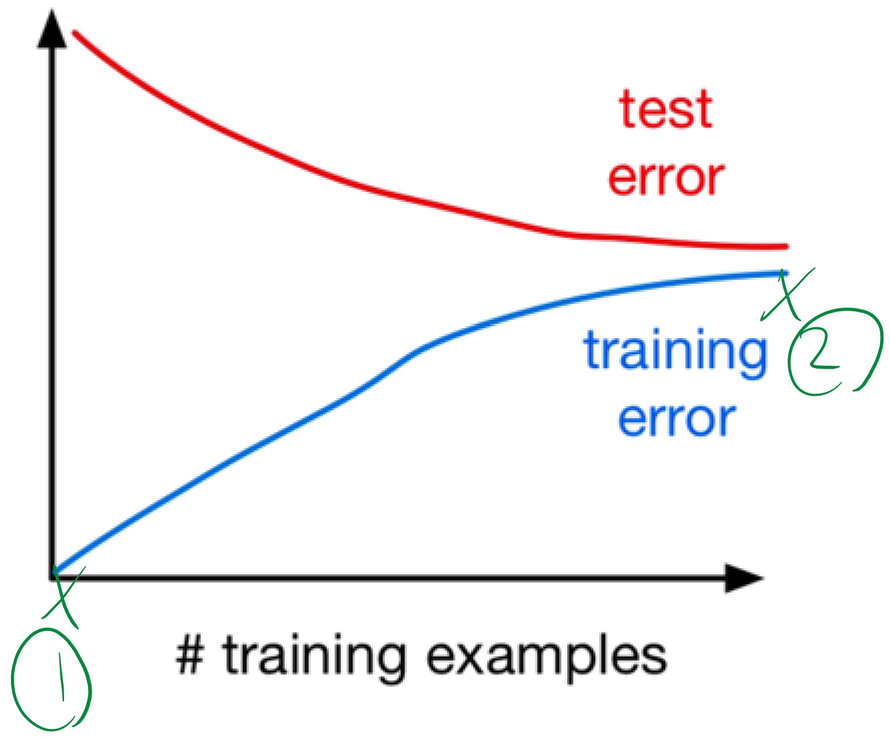
Generalization

- Training and test error as a function of # training examples:

Fixed model: Polynomial of degree 2







Training error: Zero

Test error: >>> 0

Their difference: Huge

Training error: Some nonzero value a

Test error: a+ε

Their difference: ε

Regularization

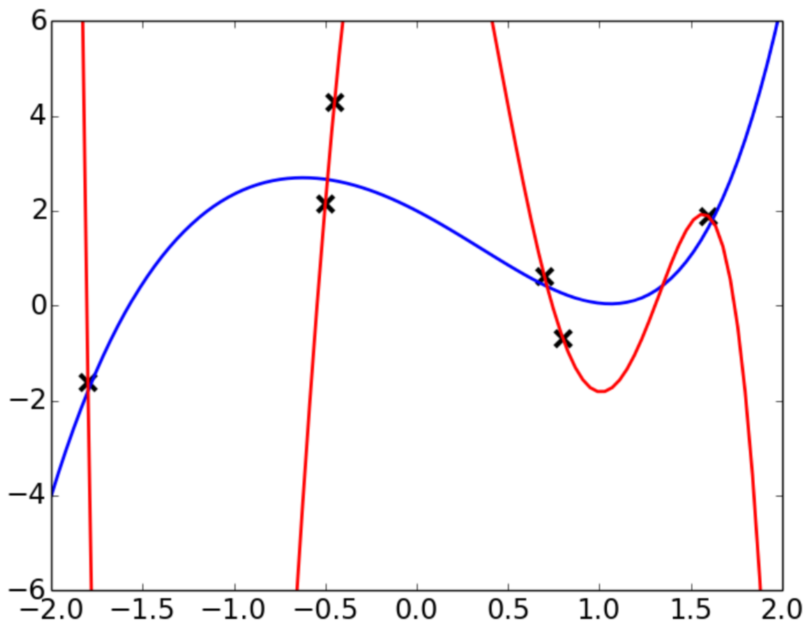
- The degree of the polynomial is a hyperparameter, just like k in KNN. We can tune it using a validation set.
- But restricting the size of a model is a crude solution, since you'll never be able to learn a more complex model, even if the data support it.

- **Another approach:** keep the model flexible, but regularize it
    - Regularizer: a function that quantifies how much we prefer one hypothesis vs another

L^2 Regularization

Observation:

polynomials that overfit often have large coefficients



$$y = 0.1x^5 + 0.2x^4 + 0.75x^3 - x^2 - 2x + 2$$

$$y = -7.2x^5 + 10.4x^4 + 24.5x^3 - 37.9x^2 - 3.6x + 12$$

L² Regularization

- **Another reason** we want parameters (weights) to be small:
  - Suppose inputs x_1 and x_2 are nearly identical for all training examples. The following two hypotheses make nearly the same predictions:

$$\boldsymbol{\theta}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \boldsymbol{\theta}_2 = \begin{pmatrix} -9 \\ 11 \end{pmatrix}$$

  - But the second model might make weird predictions if the test distribution is slightly different (e.g. x_1 and x_2 match less closely).

L^2 Regularization

- We can **encourage the parameters to be small** by adding an L^2 penalty (regularizer) to our cost function:

$$\frac{1}{2}\|\boldsymbol{\theta}\|^2 = \frac{1}{2}\sum_j \theta_j^2$$

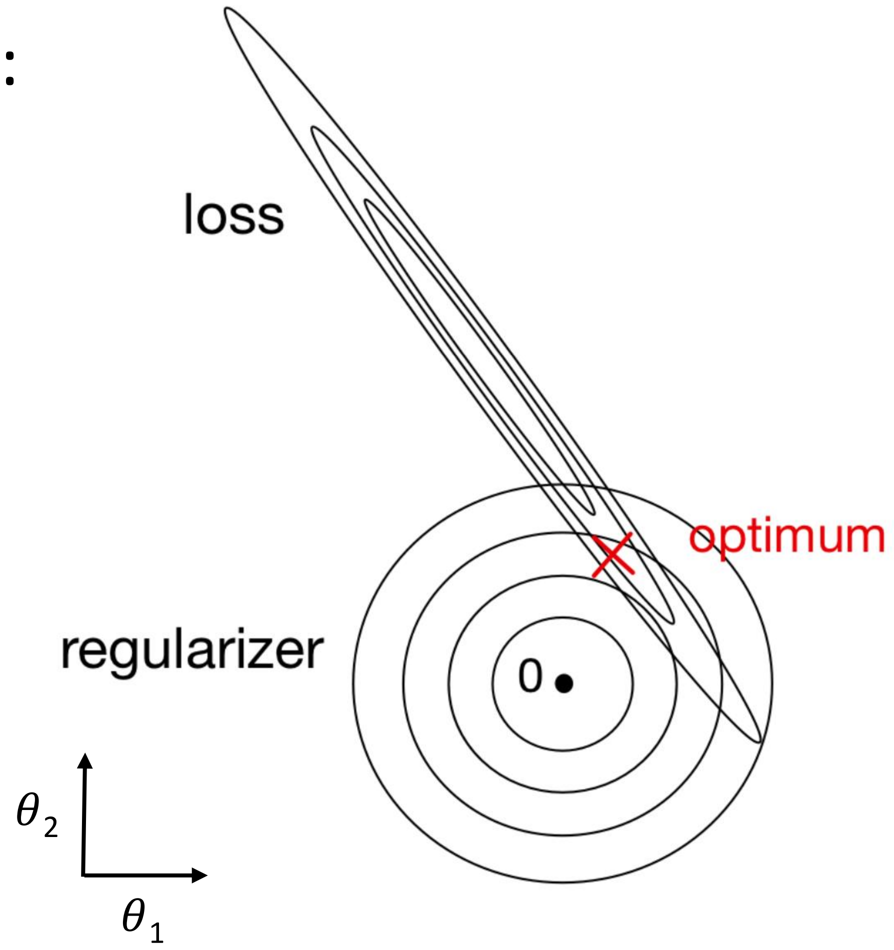
hyperparameter to be tuned

$$\mathcal{J}_{reg} = \mathcal{J} + \frac{\lambda}{2}\sum_j \theta_j^2$$

- The regularized cost function makes a tradeoff between fit to the data and the norm of the weights vector.

L^2 Regularization

- The geometric picture:



L^2 Regularization

$$\mathcal{J}(w,b) = \frac{1}{2N}\sum_{i=1}^{N}\left(y^{(i)} - t^{(i)}\right)^2$$

- We can encourage the weights to be small by choosing as our regularizer the L^2 penalty.

$$\mathcal{R}(\mathbf{w}) = \tfrac{1}{2}\|\mathbf{w}\|^2 = \frac{1}{2}\sum_j w_j^2.$$

  - Note: to be pedantic, the L^2 norm is Euclidean distance, so we're really regularizing the *squared* L^2 norm.

- The regularized cost function makes a tradeoff between fit to the data and the norm of the weights.

$$\mathcal{J}_{\text{reg}} = \mathcal{J} + \lambda\mathcal{R} = \mathcal{J} + \frac{\lambda}{2}\sum_j w_j^2$$

- Here, λ is a hyperparameter that we can tune using a validation set.

L^2 Regularization

- Recall the gradient descent update:

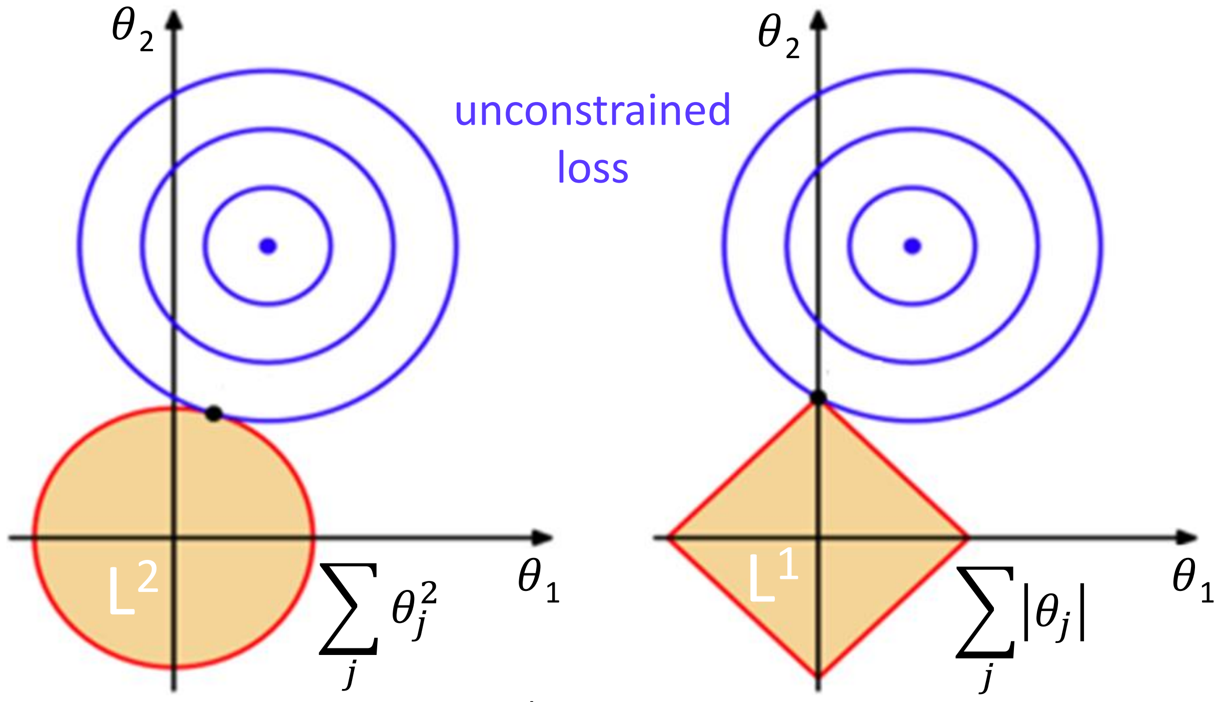
$$\mathbf{w} \leftarrow \mathbf{w} - \alpha \frac{\partial \mathcal{J}}{\partial \mathbf{w}}$$

- The gradient descent update of the regularized cost has an interesting interpretation as weight decay:

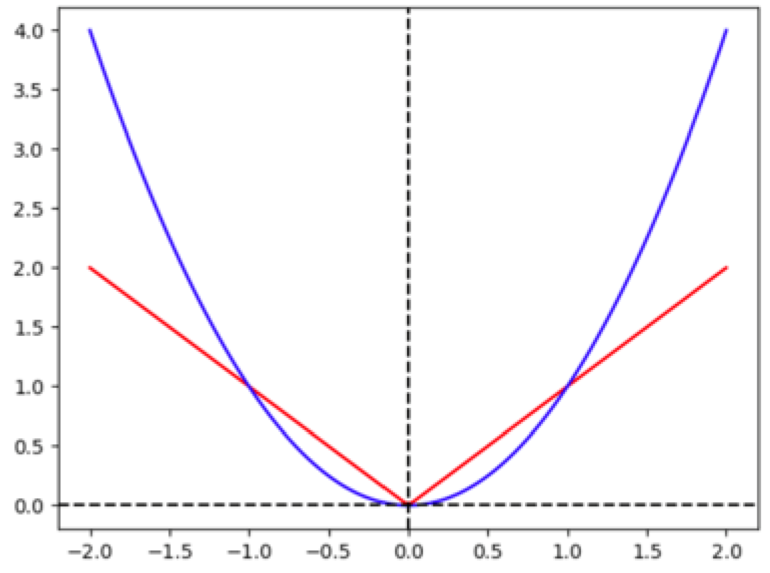
$$\begin{aligned}\mathbf{w} &\leftarrow \mathbf{w} - \alpha \left(\frac{\partial \mathcal{J}}{\partial \mathbf{w}} + \lambda \frac{\partial \mathcal{R}}{\partial \mathbf{w}} \right) \\ &= \mathbf{w} - \alpha \left(\frac{\partial \mathcal{J}}{\partial \mathbf{w}} + \lambda \mathbf{w} \right) \\ &= (1 - \alpha\lambda)\mathbf{w} - \alpha \frac{\partial \mathcal{J}}{\partial \mathbf{w}}\end{aligned}$$

L^1 vs L^2 Regularization

- The L^1 norm (or sum of absolute values) is another regularizer that encourages weights to be exactly zero.



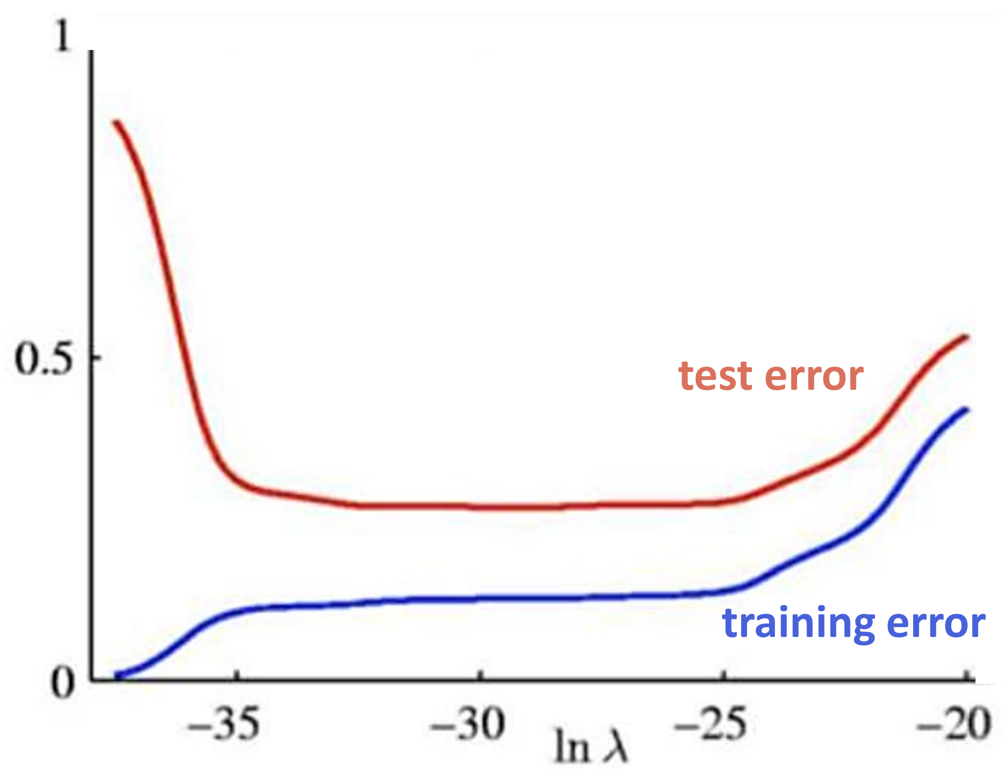
Source: Bishop PRML



L1-regularization tends to push parameters to zero

Generalization

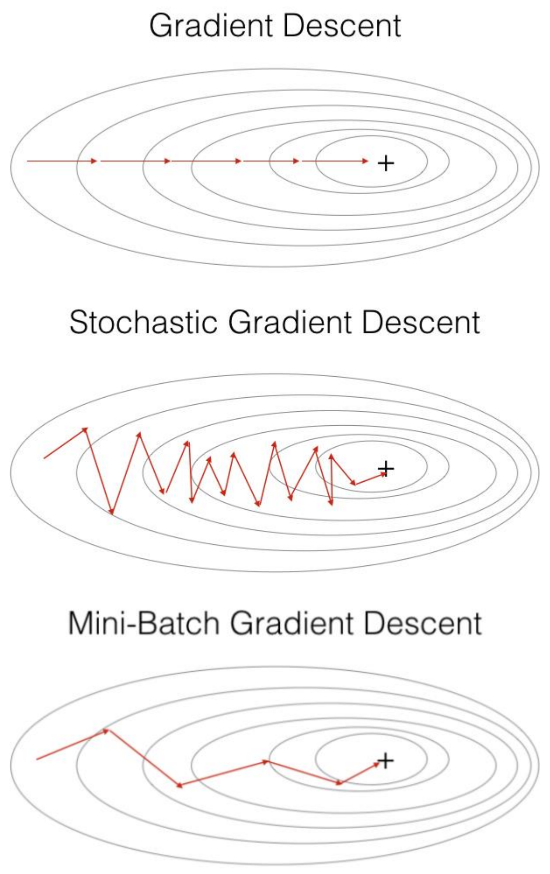
- Training and test error as a function of regularization parameter λ:



	$\ln\lambda = -\infty$	$\ln\lambda = -18$	$\ln\lambda = 0$
θ_0	0.35	0.35	0.13
θ_1	232.37	4.74	-0.05
θ_2	-5321.83	-0.77	-0.06
θ_3	48568.31	-31.97	-0.05
θ_4	-231639.30	-3.89	-0.03
θ_5	640042.26	55.28	-0.02
θ_6	-1061800.52	41.32	-0.01
θ_7	1042400.18	-45.95	-0.00
θ_8	-557682.99	-91.53	0.00
θ_9	125201.43	72.68	0.01

Ineffective Batch Size

- Q: What happens if the batch size is too small? Too large?
- **Too large:**
  - Computationally expensive
  - Average loss might not change very much as batch size grows
- **Too small:**
  - We optimize a (possibly very) different loss function at each iteration
  - Noisy
  - SGD may actually take longer

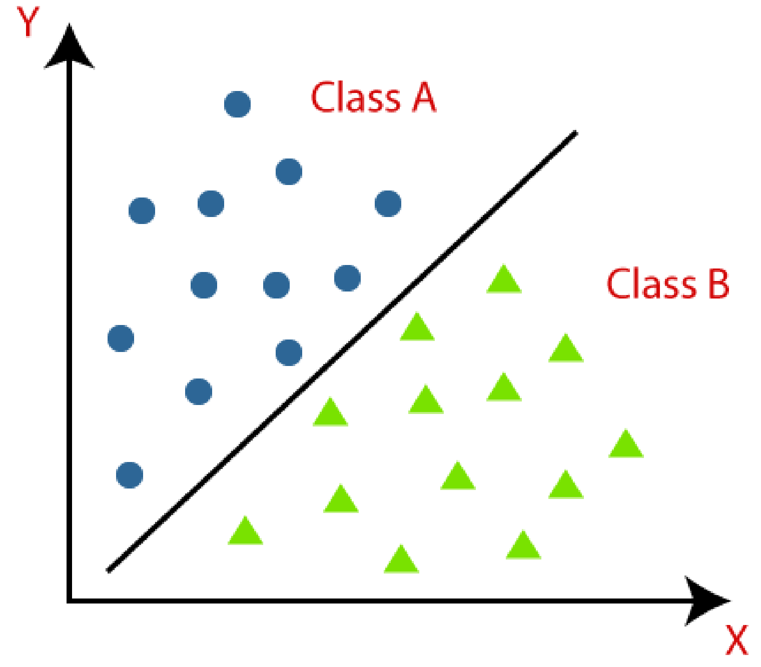


Classification

Overview

- **Classification:** predicting a discrete-valued target
- **Binary classification:** number of target values is 2 (binary-valued)

- Examples:
  - predict where a patient has a disease given presence or absence of various symptoms
  - classify e-mails as spam or non-spam
  - predict whether a financial transaction is fraudulent



Binary Classification

- We can start with our linear function of x, but now we introduce a threshold :

$$\mathbf{z} = \boldsymbol{\theta}^{\mathrm{T}}\mathbf{x} + b$$

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq r \\ 0 & \text{if } z < r \end{cases}$$

Binary Classification

- Eliminating the threshold
  - Can make the threshold *r* = 0 without loss of generality.

$$\boldsymbol{\theta}^{\mathrm{T}}\mathbf{x} + b \geq 0 \longleftarrow \boldsymbol{\theta}^{\mathrm{T}}\mathbf{x} + \underbrace{b - r}_{\triangleq b'} \geq 0$$

- Eliminating the bias
  - Add a dummy feature x_0 which always takes the value 1. The weight θ_0 is equivalent to the bias.

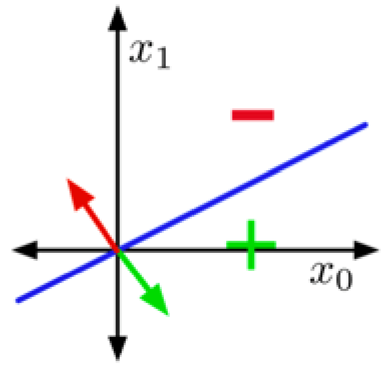
$$\mathbf{z} = \boldsymbol{\theta}^{\mathrm{T}}\mathbf{x} \qquad \hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

The Geometric Picture

- Here we're visualizing the logical NOT example
- Training examples are points
- Hypothesis are half-spaces whose boundaries pass through the origin
- This boundary is the decision boundary
  - In 2-D it's a line, but think of it as a hyperplane

- If the training examples can be separated by a linear decision rule, they are linearly separable.

- Input Space, or Data Space:



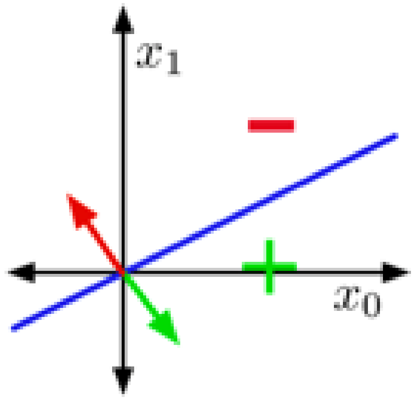
NOT

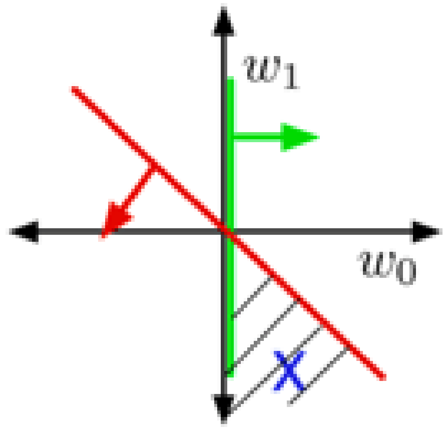
x_0	x_1	t
1	0	1
1	1	0

The Geometric Picture

- Hypotheses are points
- Training examples are half-spaces whose boundaries pass through the origin
- The region satisfying all the constraints is the feasible region; if this region is nonempty, the problem is feasible

Weight Space



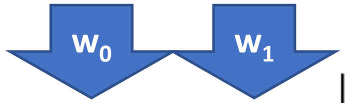


$$w_0 > 0$$

$$w_0 + w_1 < 0$$

How do you optimize?

Example 1



NOT

x_0	x_1	t
1	0	1
1	1	0

Example 2

AND

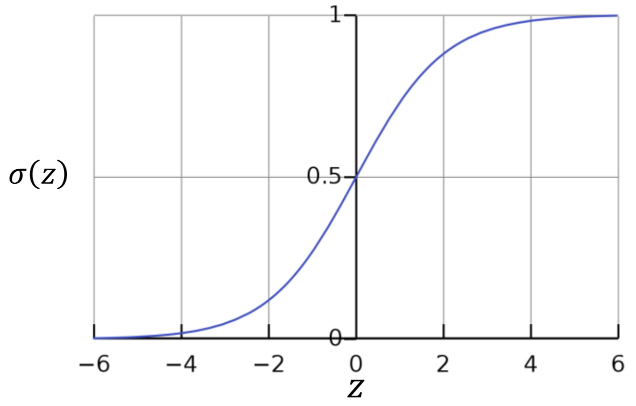
x_0	x_1	x_2	t
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

How do you optimize?

Logistic Regression

$$\mathbf{z = \theta^T x}$$

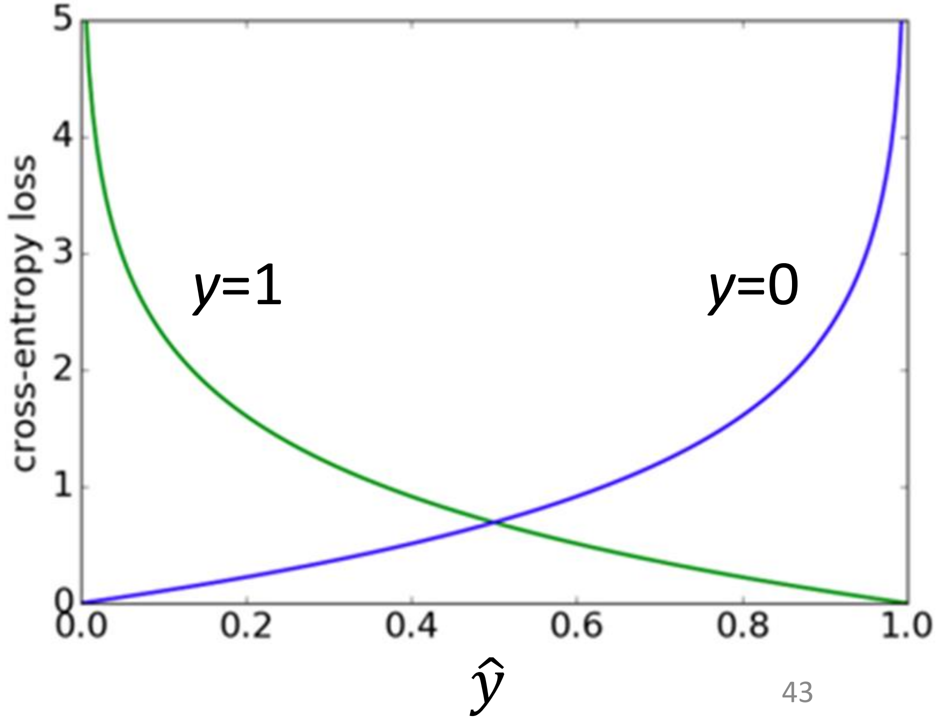
$$\hat{y} = \sigma(z) = \frac{1}{1+e^{-z}}$$



- Because the prediction $\hat{y} \epsilon$ [0, 1], we can interpret it as the estimated probability that the label is positive (y = 1).
- **Cross-entropy loss captures** this intuition:

$$\mathcal{L}_{CE}(\hat{y}, y) = \begin{cases} -\log \hat{y} & \text{if } y = 1 \\ -\log(1-\hat{y}) & \text{if } y = 0 \end{cases}$$

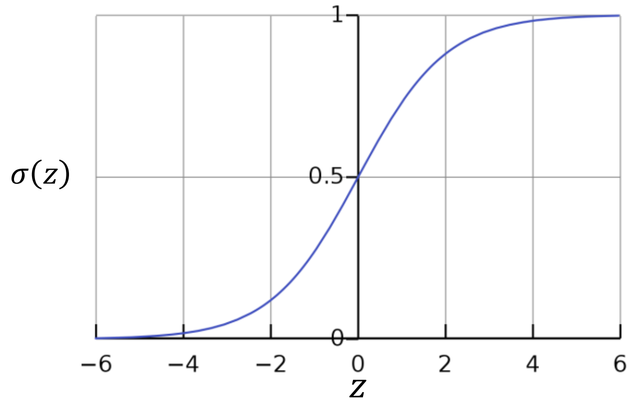
$$= -\mathrm{y} \log \hat{y} - (1-\mathrm{y}) \log(1-\hat{y})$$



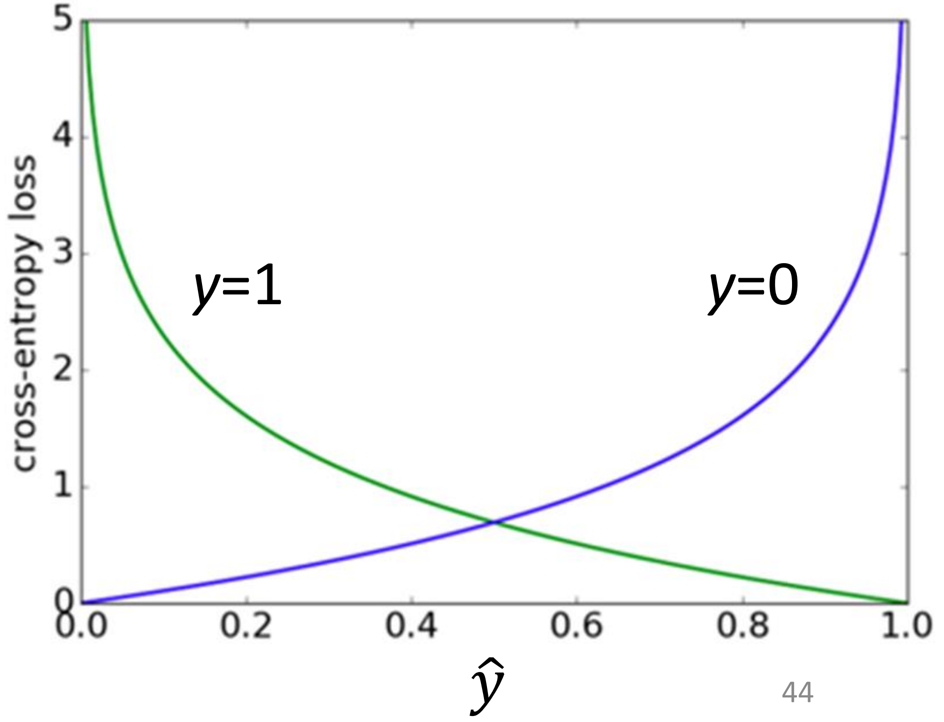
Logistic Regression with Cross-entropy Loss

$$\mathbf{z} = \boldsymbol{\theta}^{\mathrm{T}}\mathbf{x}$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$



- Being 99% confident in the wrong answer is much more wrong than being only 90% confident.



$$\mathcal{L}_{CE}(\hat{y}, y) = -\mathrm{y} \log \hat{y} - (1 - \mathrm{y}) \log(1 - \hat{y})$$

Logistic Regression with Cross-entropy Loss

- **Computation Problem:**

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

- If $\hat{y} = \sigma(z)$ gets too close to 0 or 1, it may cause very subtle and hard-to-find bugs:

$$\hat{y} = \sigma(z) \qquad \Rightarrow \hat{y} \approx 0$$

$$\mathcal{L}_{CE}(\hat{y}, y) = -\mathrm{y} \log \hat{y} - (1 - \mathrm{y}) \log(1 - \hat{y}) \quad \Rightarrow \text{computes} \log 0$$

- Instead, we combine the activation function and the loss into a single logistic-cross-entropy function.

$$\mathcal{L}_{LCE}(\sigma(z), y) = \mathrm{y} \log(1 + e^{-z}) + (1 - \mathrm{y}) \log(1 + e^{z})$$

Example:

$$\mathcal{L}_{CE}(\hat{y}, y) = -\mathrm{y} \log \hat{y} - (1 - \mathrm{y}) \log(1 - \hat{y})$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\mathcal{L}_{LCE}(\sigma(z), y) = \mathrm{y} \log(1 + e^{-z}) + (1 - \mathrm{y}) \log(1 + e^{z})$$

Weight Updates

- Comparison of gradient descent updates:
  - Linear regression:

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \frac{\alpha}{N}\sum_{i=1}^{N}(\hat{y}^{(i)} - y^{(i)})\mathbf{x}^{(i)}$$

  - Logistic regression:

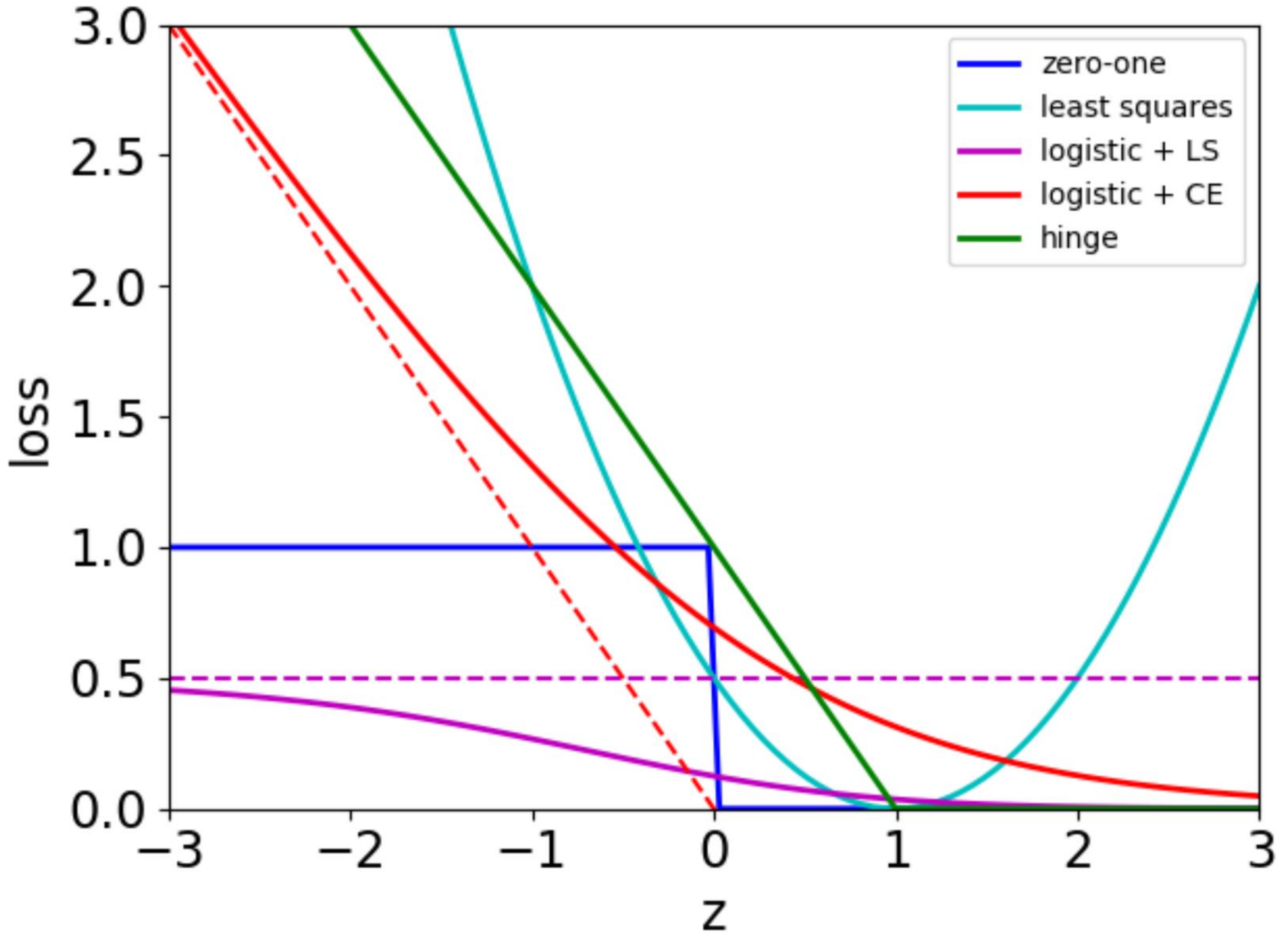
$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \frac{\alpha}{N}\sum_{i=1}^{N}(\hat{y}^{(i)} - y^{(i)})\mathbf{x}^{(i)}$$

$$z = \boldsymbol{\theta}^{\mathrm{T}}\mathbf{x} + b$$

Using activation function

$$\hat{y} = \frac{1}{1 + e^{-z}}$$

Loss Summary



Multiclass Classification

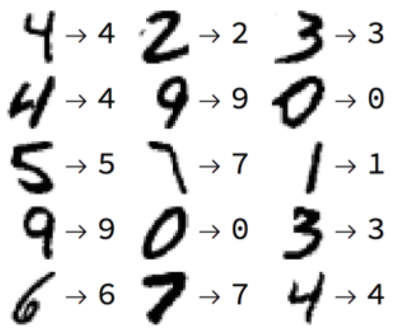
Multiclass Classification

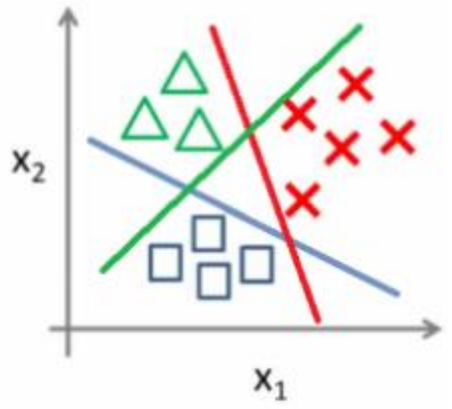
- What about classification task with more than two categories?

- Targets form a discrete set {1, …, K}
- It's often more convenient to represent them as one-hot vectors or a one-of-K encoding:

$$y = [0,\ldots\ldots 0, 1, 0, \ldots\ldots 0]$$

entry k is a 1, all others are 0





Multiclass Classification

- Now there are *D* input dimensions and *K* output dimensions, so we need *K x D* parameters, which we arrange as a matrix $\boldsymbol{\theta}$.
- Also, we have a *K*-dimensional vector **b** of biases.
- Linear predictions:

$$z_k = \sum_j \theta_{kj} x_j + b_k$$

- Vectorized:

$$z = \boldsymbol{\theta}\mathbf{x} + \mathbf{b}$$

Activation Function

- A natural activation function to use is the softmax function, a multivariable generalization of the logistic (sigmoid) function:

$$\hat{y}_k = \mathrm{softmax}(z_1, \ldots, z_K)_k = \frac{e^{z_k}}{\sum_{k'} e^{z_{k'}}}$$

Softmax makes differences larger – pushes values close to 1 to 1, values close to 0 to 0

- The input are called the logits.
- Outputs are positive and sum to 1 (interpreted as probabilities)

Loss Function

- If a model outputs a vector of class probabilities, we can use cross-entropy as the loss function:

Binary

$$\mathcal{L}_{CE}(\hat{y}, y) = -\mathrm{y} \log \hat{y} - (1 - \mathrm{y}) \log(1 - \hat{y})$$

Multi-class

$$\mathcal{L}_{CE}(\hat{\mathbf{y}}, \mathbf{y}) = -\sum_{k=1}^{K} y_k \log \hat{y}_k$$

$$= -\mathbf{y}^T \log \hat{\mathbf{y}}$$

Softmax Regression

- Softmax regression:

$$z = \boldsymbol{\theta}\mathbf{x} + \mathbf{b}$$

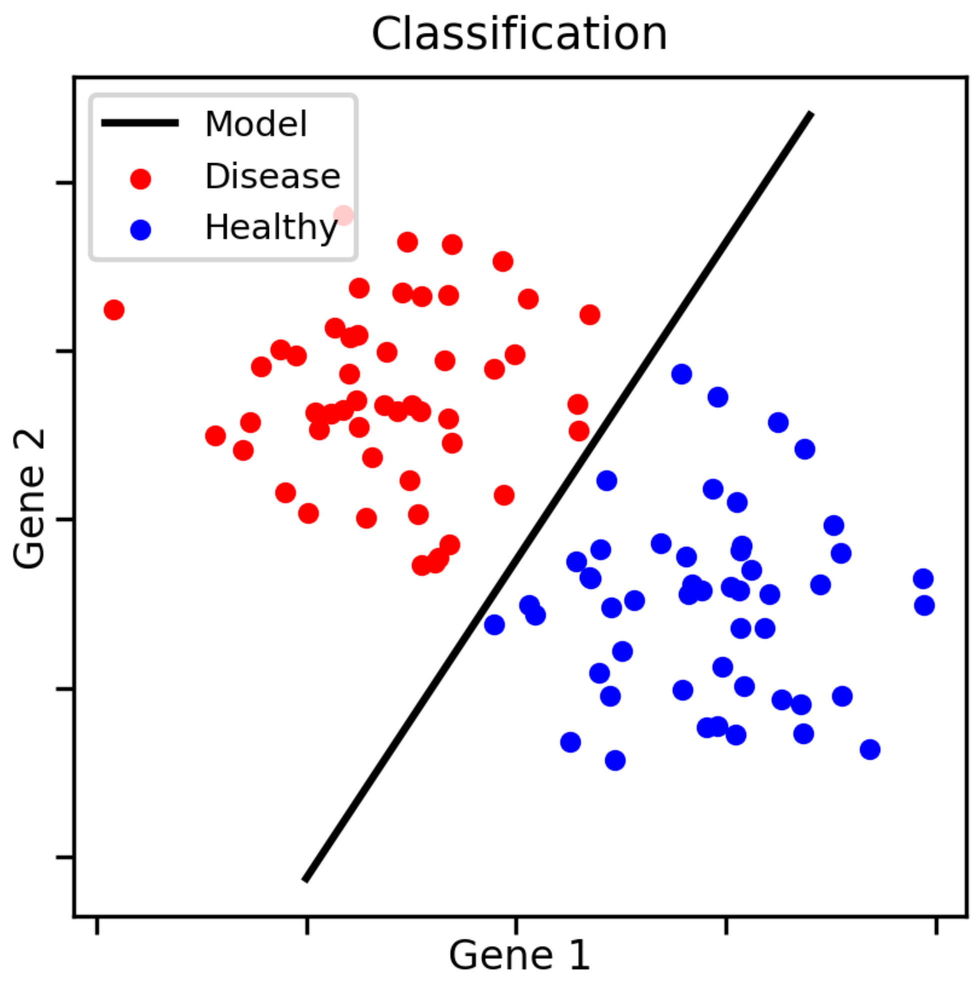
$$\hat{\mathbf{y}} = \text{softmax}(\mathbf{z})$$

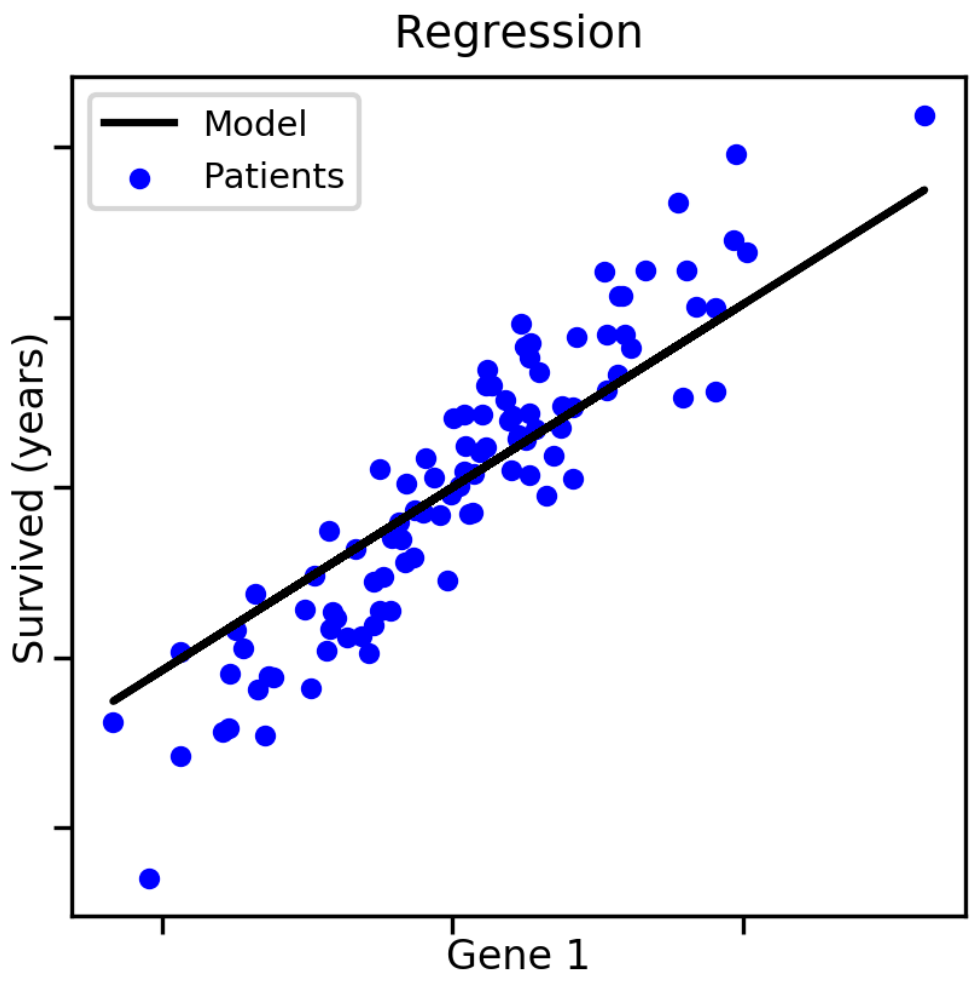
$$\mathcal{L}_{CE}(\hat{\mathbf{y}}, \mathbf{y}) = -\mathbf{y}^T \log \hat{\mathbf{y}}$$

- Gradient descent updates:

$$\frac{\partial \mathcal{L}_{CE}}{\partial \mathbf{z}} = \hat{\mathbf{y}} - \mathbf{y}$$

Summary


Classification
Model
Disease
Healthy
Gene 2
Gene 1


Regression
Model
Patients
Survived (years)
Gene 1

Practice in Google Colab

Neural Networks

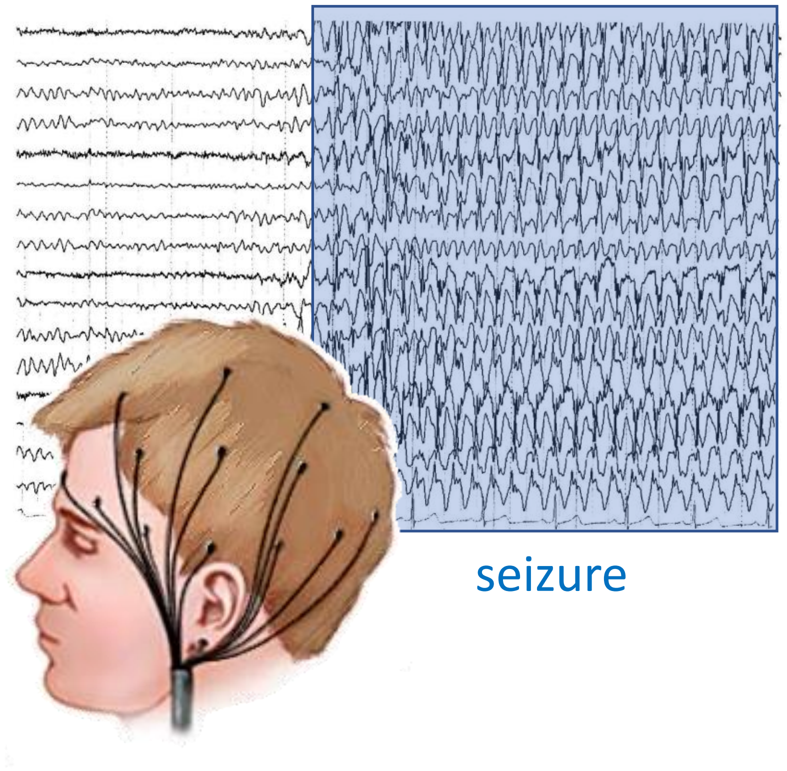
Challenges with Feature Maps

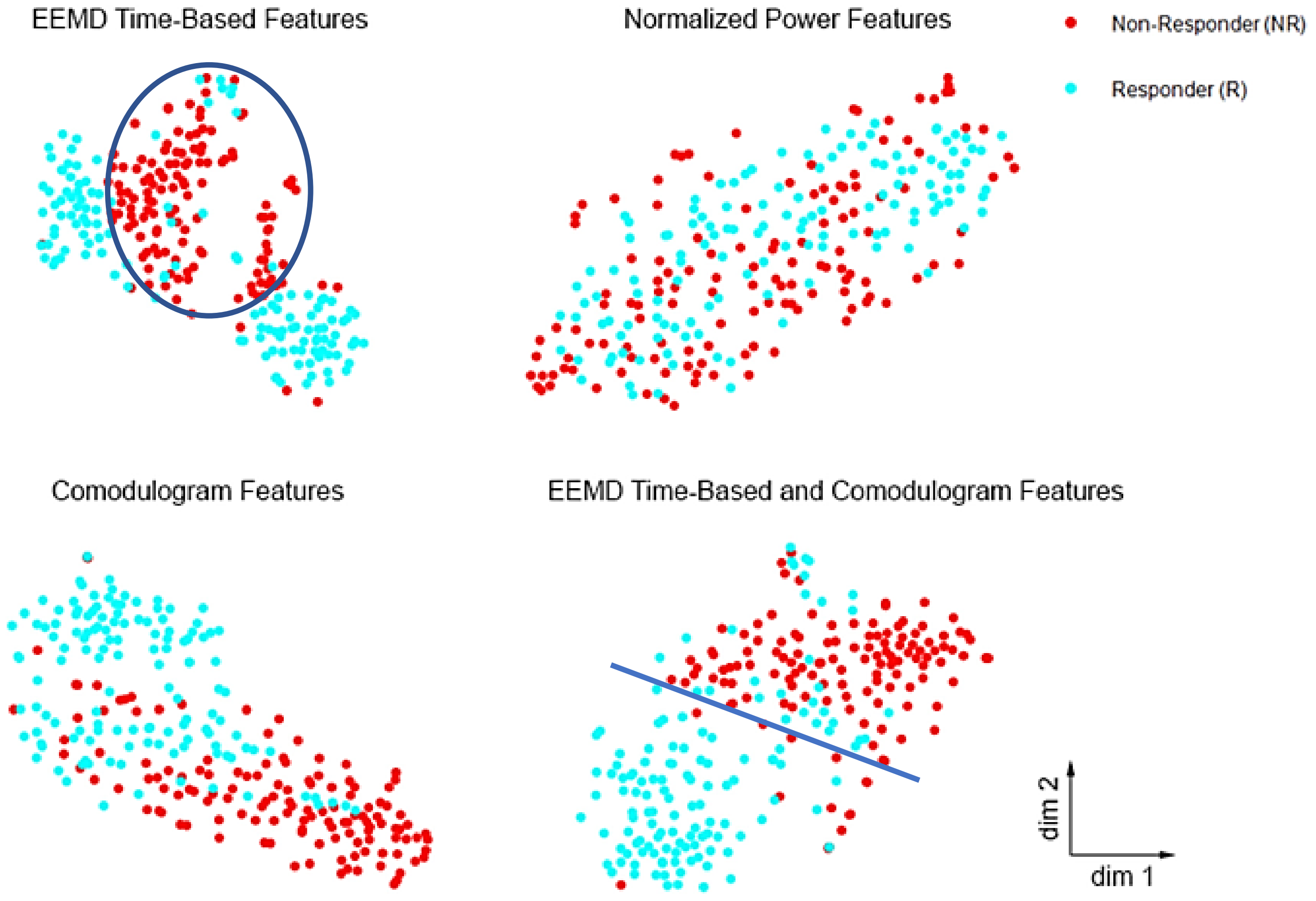
- Feature maps can be useful for solving nonlinear regression and classification problems.

- Have several limitations:
  - The feature maps must be selected in advance
  - Not always easy to pick a good feature map and can take a long time to craft
  - In high dimensions the feature representations can explode

Example of Feature Engineering:

Objective: Predict **responders** from **non-responders** given raw electroencephalogram data.





Source: Colic et al, 2016

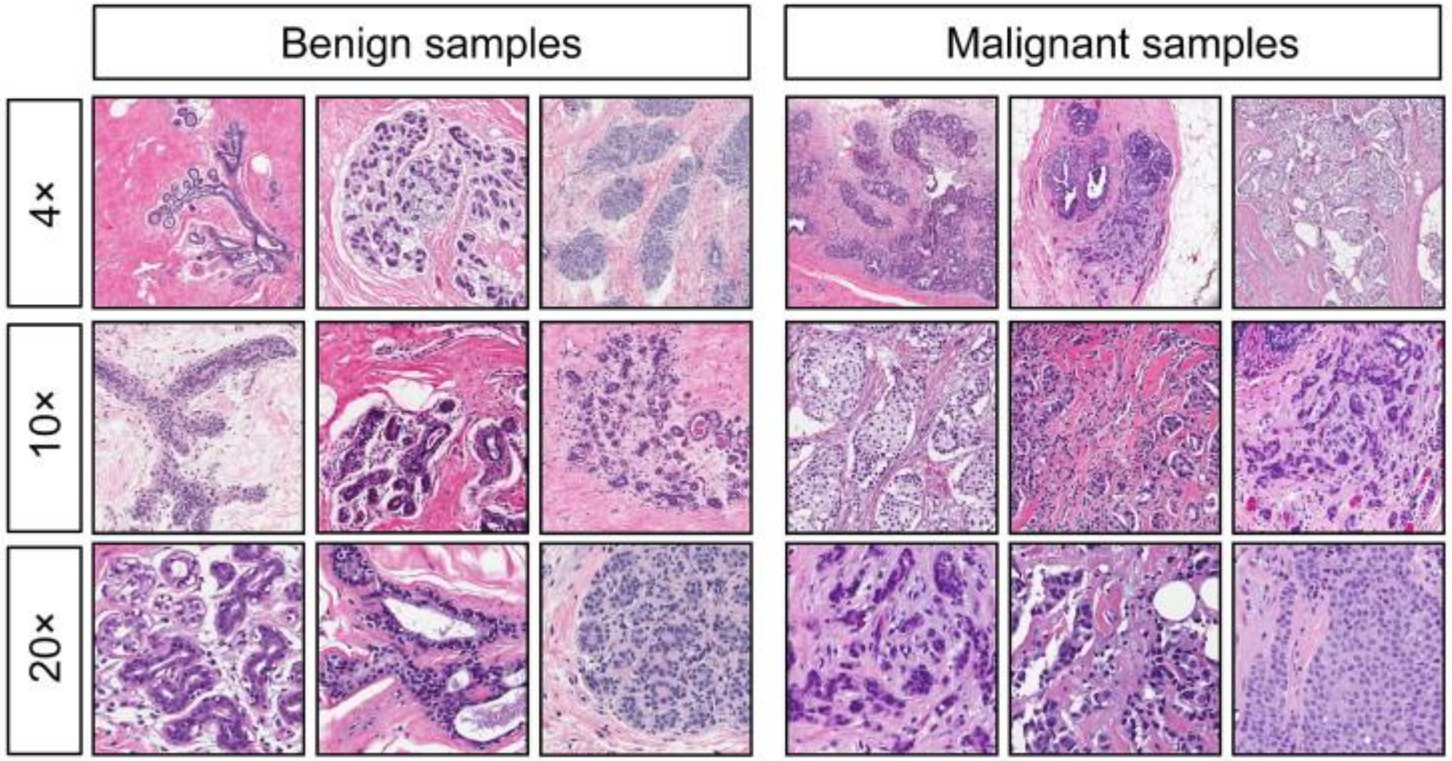
Challenges with Feature Maps

We need an algorithm that can learn good features for nonlinear regression and classification.

Motivating Example: Tumor Classification

Objective: Classify an image of a biopsy as cancerous (malignant) or benign (tumor)

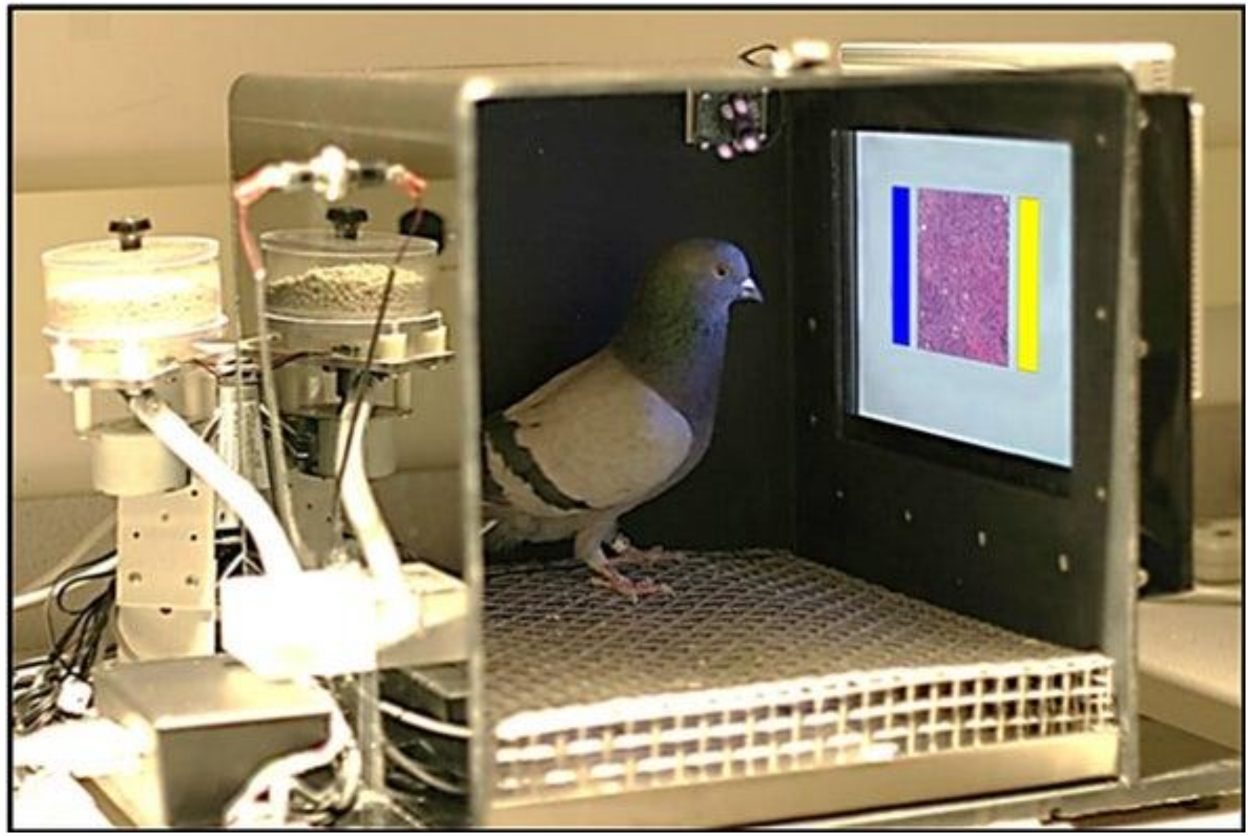
- Pathologists/radiologists train for years to do this!
- How would you solve this problem?



Source: Levenson et al., 2015

Maybe We Can Use Pigeons

- **Train a Pigeon!**
- A 2015 study suggests that the common pigeon can reliably distinguish between benign versus malignant tumors and, in doing so, could help researchers develop better cancer screening technologies.



Source: Levenson et al., 2015

Maybe We Can Use Pigeons

Training Algorithm:

1. Show an image of a magnified biopsy to pigeon
2. Pigeon pecks at one of two answer buttons on sides for malignant/benign
3. Correct classifications are rewarded with food pellets

Three correct trials with benign images follow
(the yellow button is correct).

Video: https://www.youtube.com/watch?v=flzGjnJLyS0

Why are we talking about pigeons?

We need to answer similar questions in training a pigeon/artificial neural network:

- How will we reward correct responses?
- How do we train the neural network efficiently?
- How do we know the pigeon **didn’t just memorize** the images we showed it?
- What are the ethics of trusting a pigeon to detect cancer?

How do Pigeons Work?

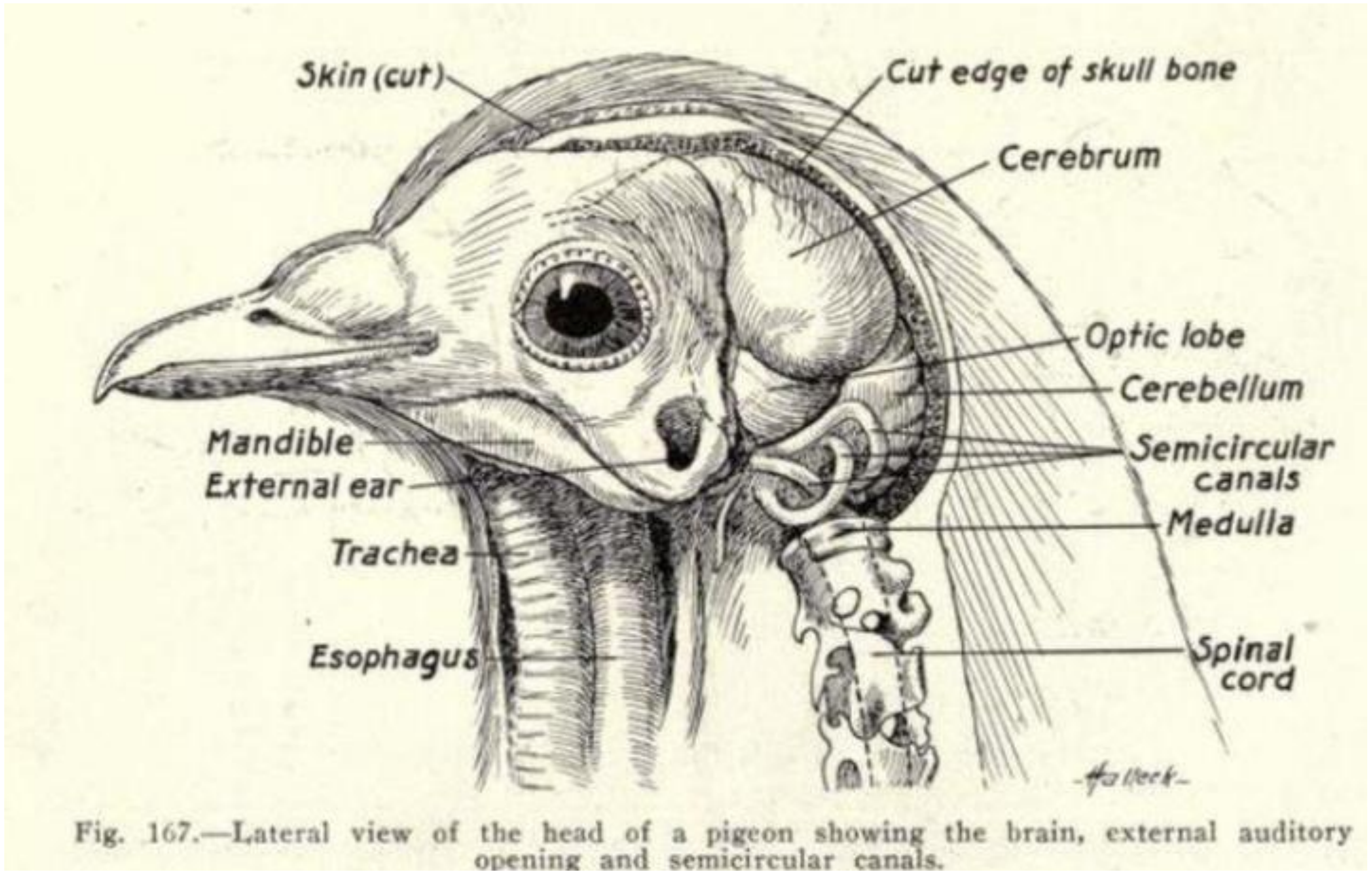
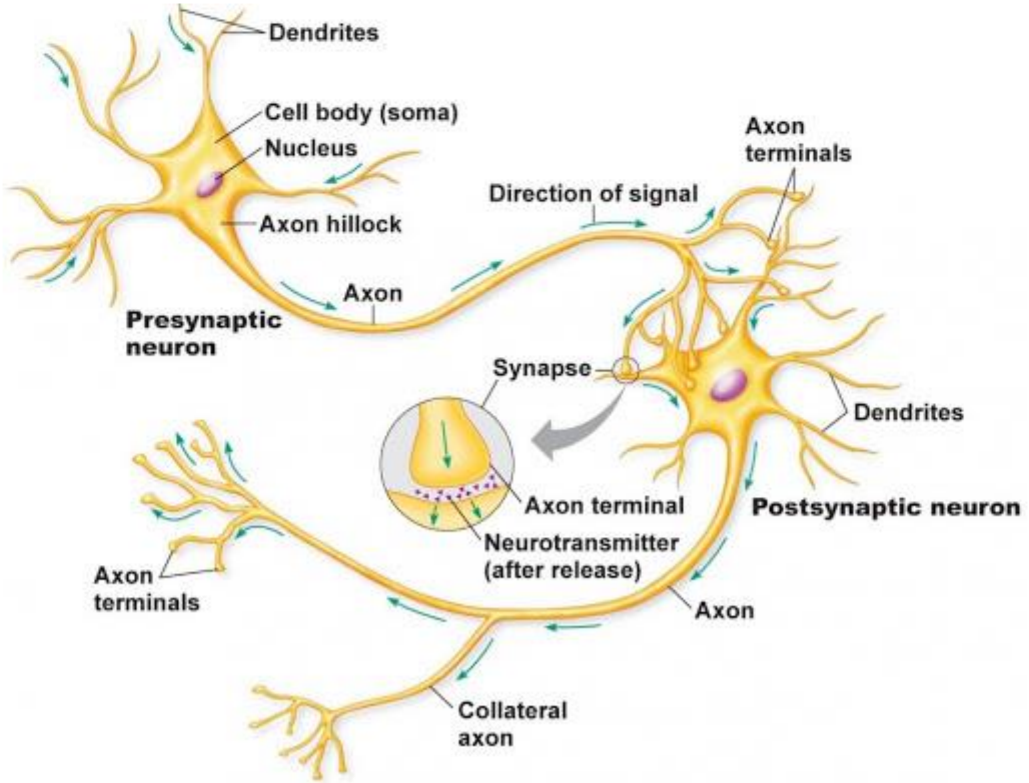


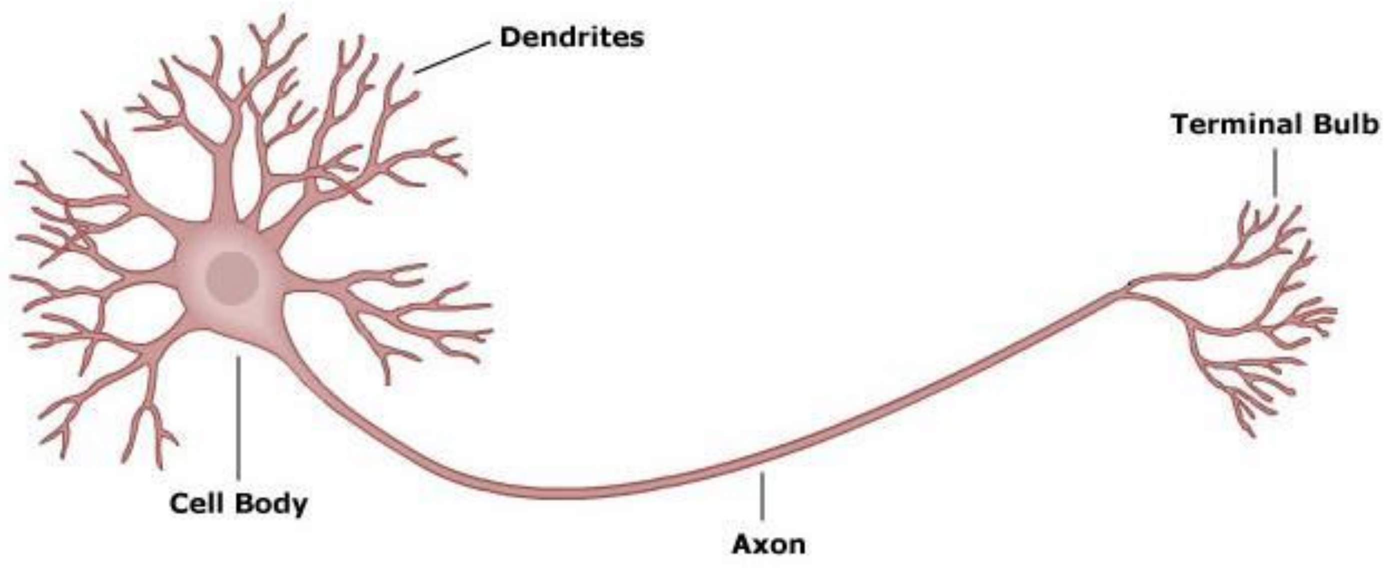
Fig. 167.—Lateral view of the head of a pigeon showing the brain, external auditory opening and semicircular canals.



Source: Letsmaketech

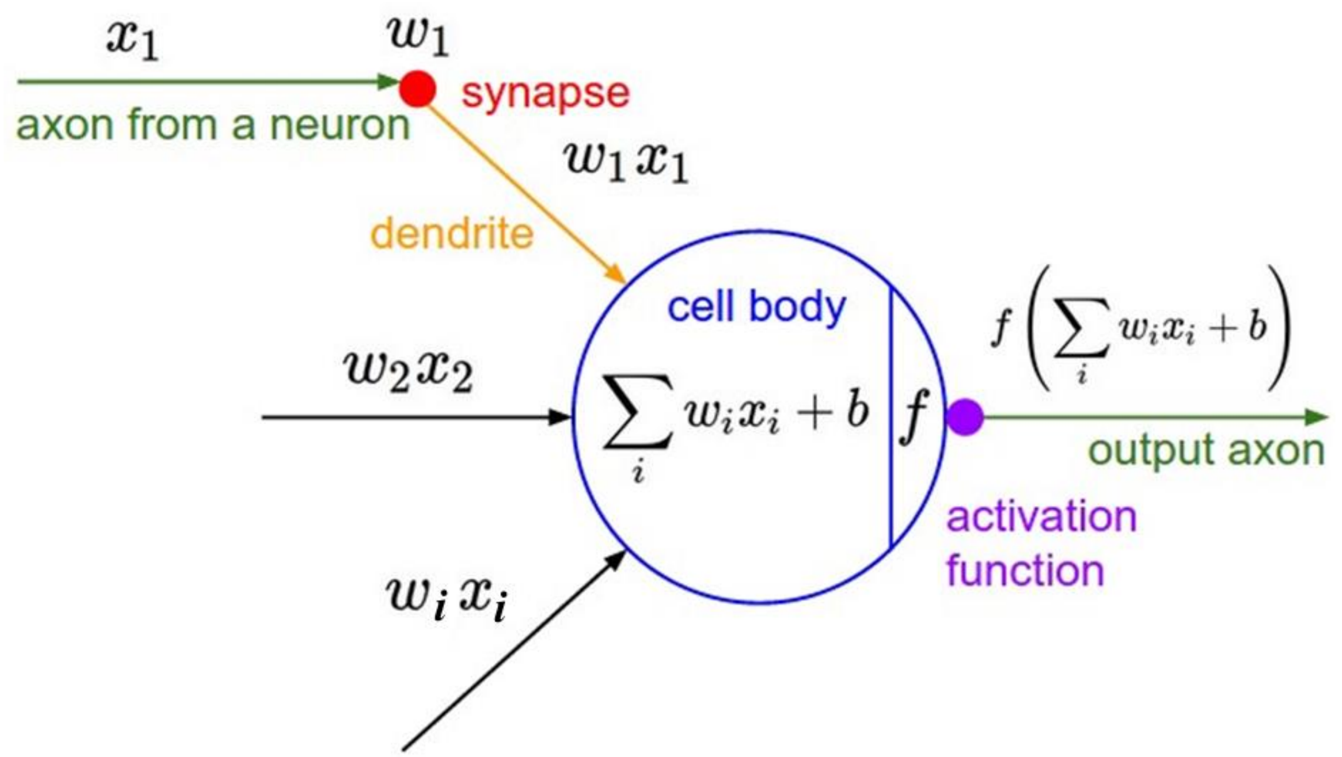
Simplified Neuron Anatomy

- **Dendrites:** are connected to other cells that provide information.
- **Cell body:** consolidates information from dendrites.
- **Axon:** an extension from the cell body that passes information to other neurons.
- **Synapse:** the area where the axon of one neuron and the dendrite of another connect.

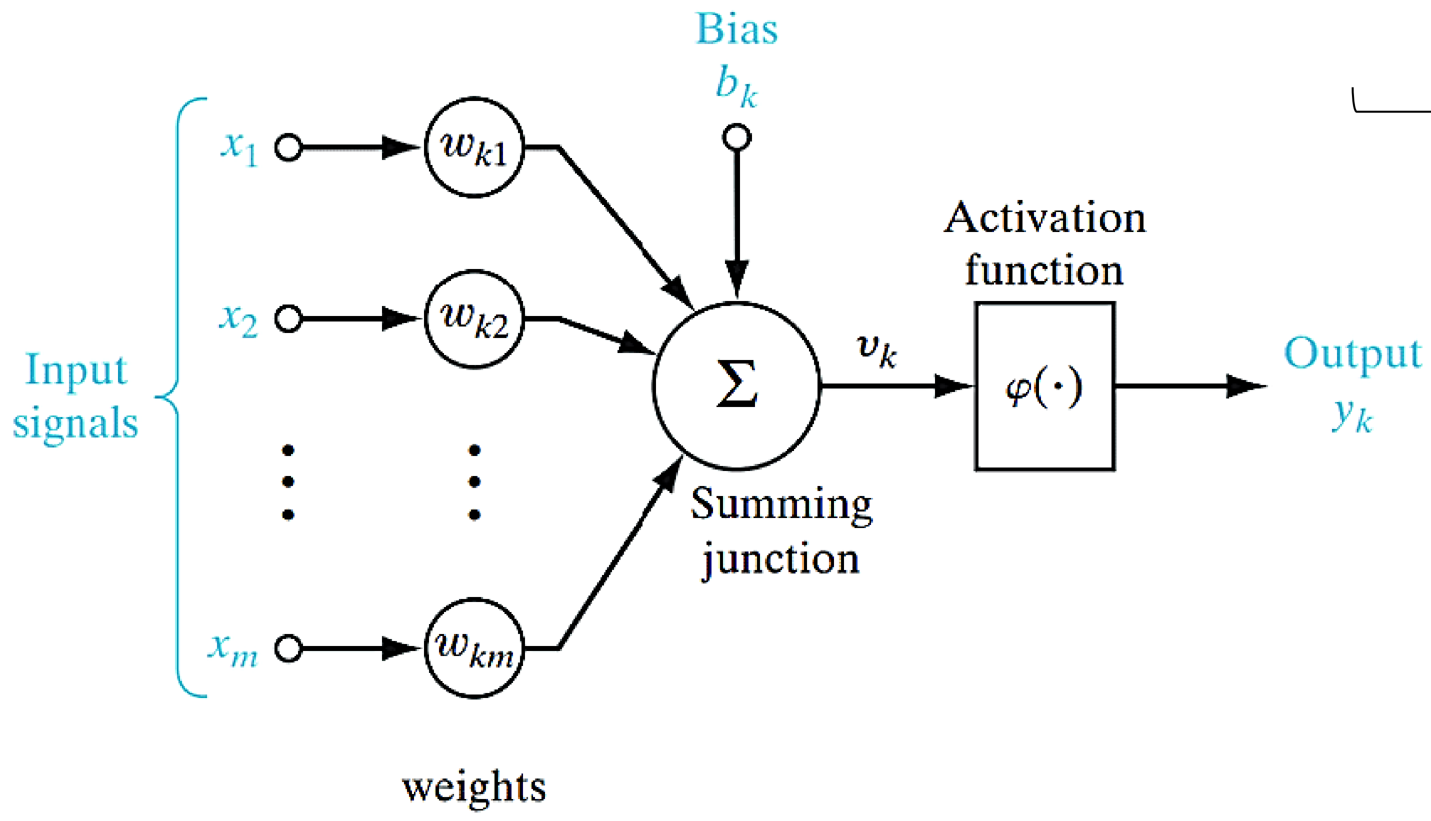


Artificial Neural Network

- Maybe we’re note quite ready for pigeon doctors, but we can use the next best thing… an artificial pigeon (**artificial neural network)**



Artificial Neural Network

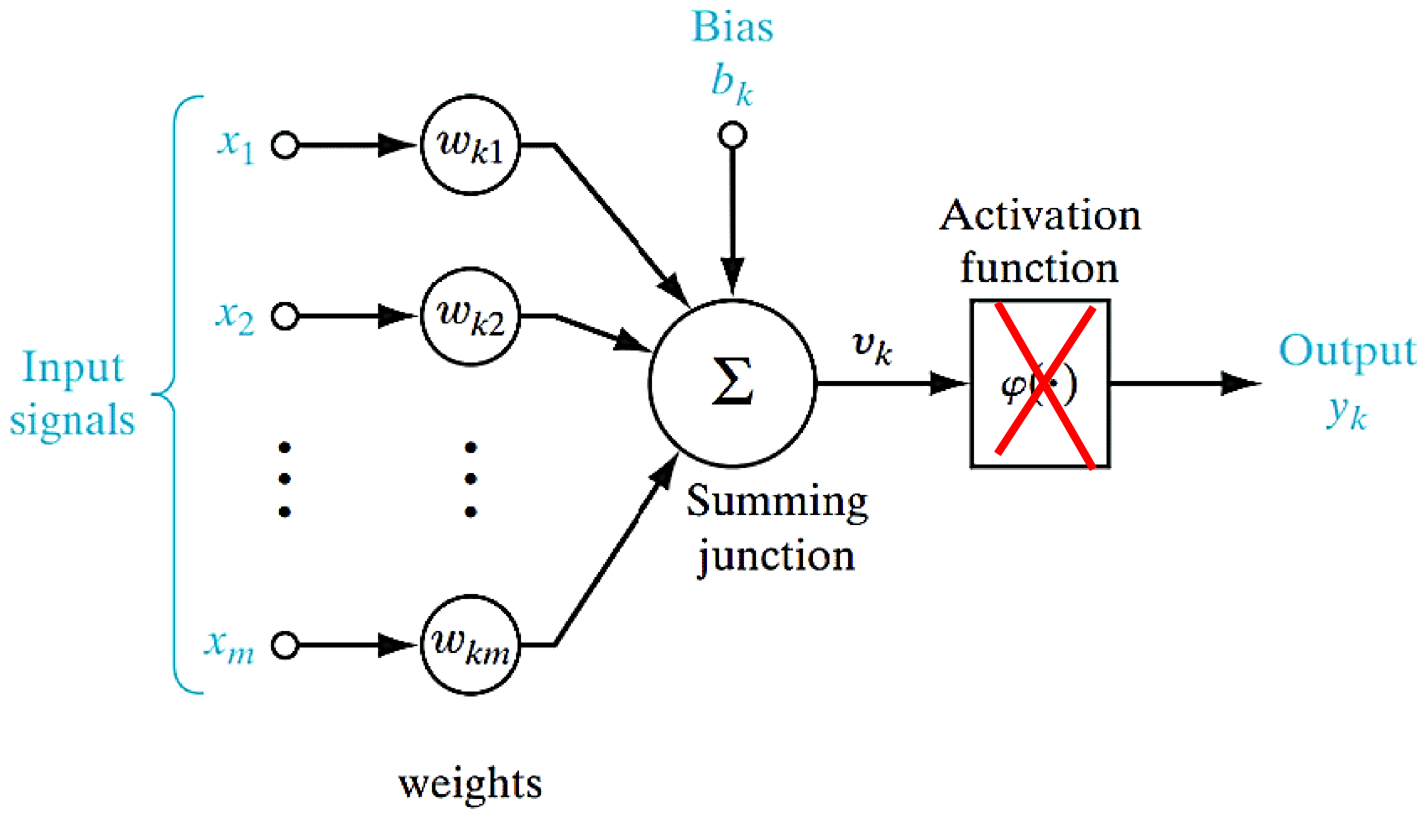


$$\hat{y} = \varphi\underbrace{\left(\sum_{i=0}^{p} w_i x_i = \mathrm{w}^T \mathrm{x}\right)}$$

does this look familiar?

Artificial Neural Network

$$y = \varphi\left(\sum_{i=0}^{p} w_i x_i = \mathrm{w}^T \mathrm{x} + \mathrm{b}\right)$$



Logistic Regression!

What will we have if we exclude the activation function?

Training / Learning Parameters

- In order to train an ANN we should define the error on our predictions.
- This is the same as with linear regression and logistic regression:

Mean Squared Error
(regression)

$$\frac{1}{2N}\sum_{n=1}^{N}||\hat{y}_n - y_n||^2$$

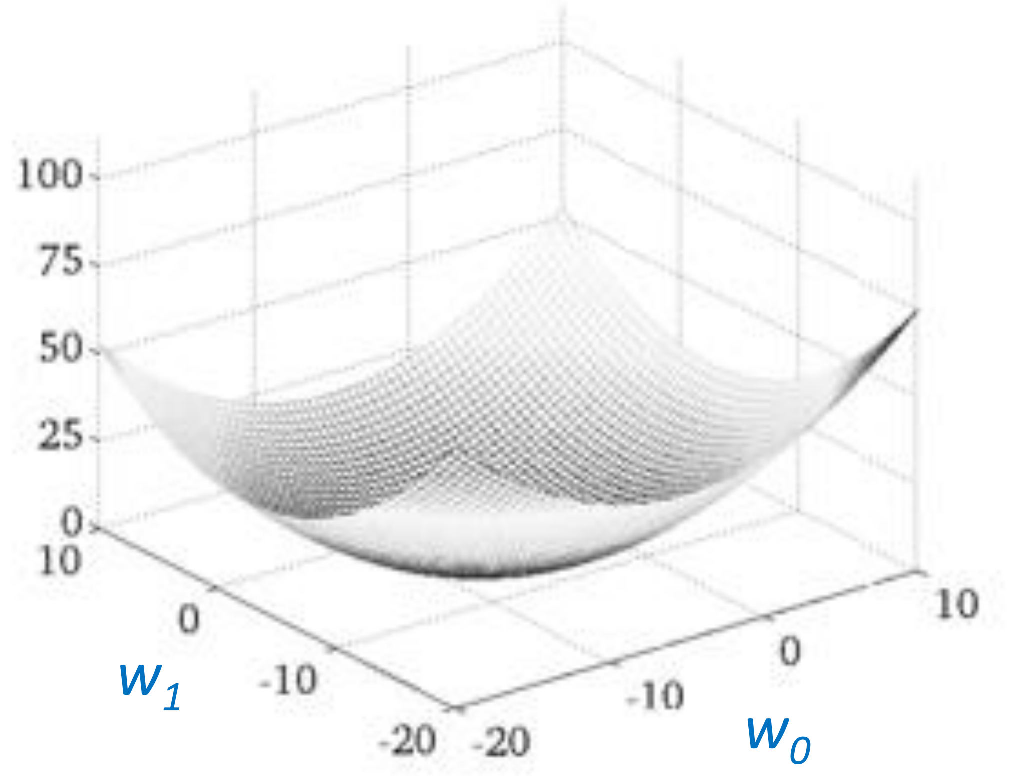
Cross-Entropy Loss
(classification)

$$-\frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{K} y_{n,k}\log(\hat{y}_{n,k})$$

implement gradient descent to learn parameters!

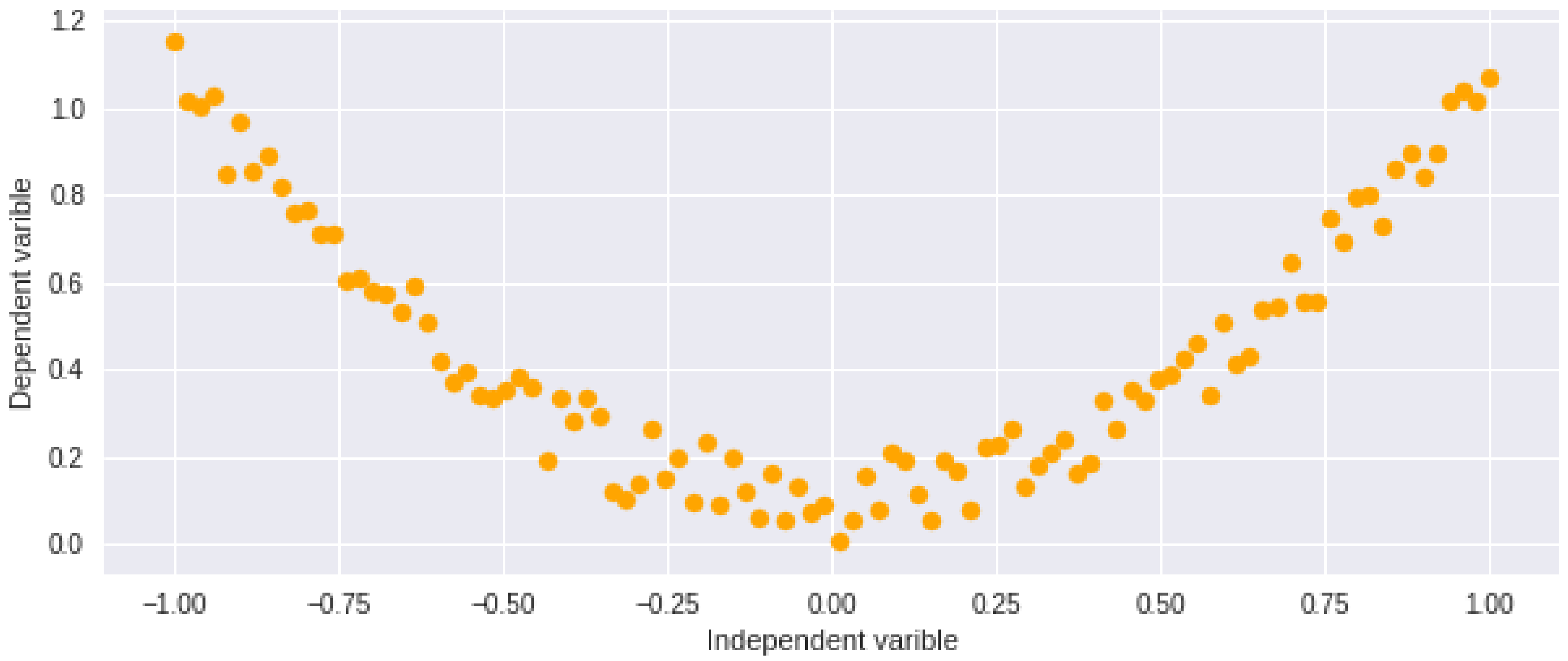
Gradients

- For this simple version of neural networks all the gradient calculations are the same as we've seen earlier…
- It probably wouldn't be a surprise to find out that to train this simple ANN is a convex problem (not true for all ANNs).

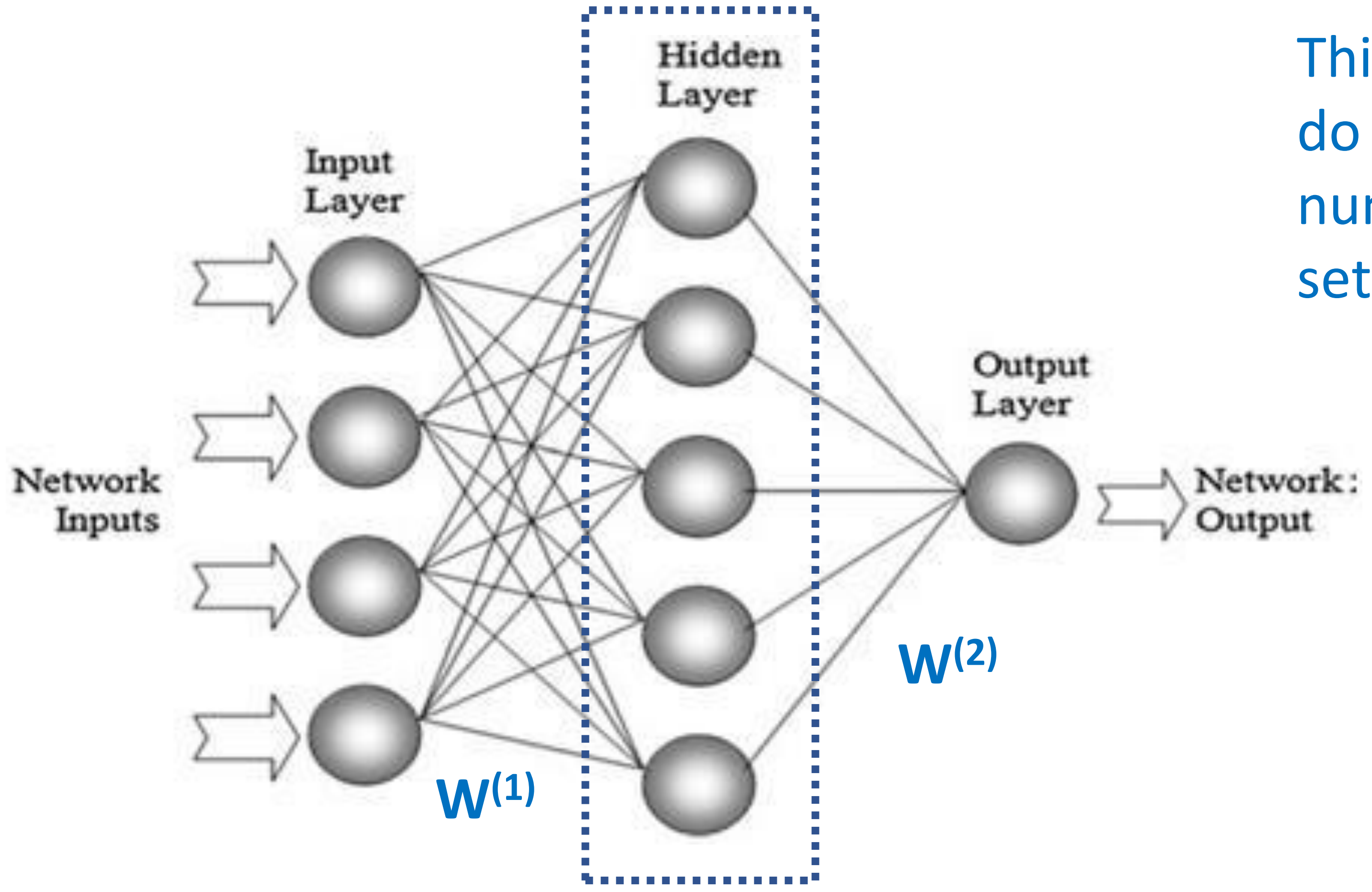


parameters (w) can be multidimensional

Achieving Nonlinearity



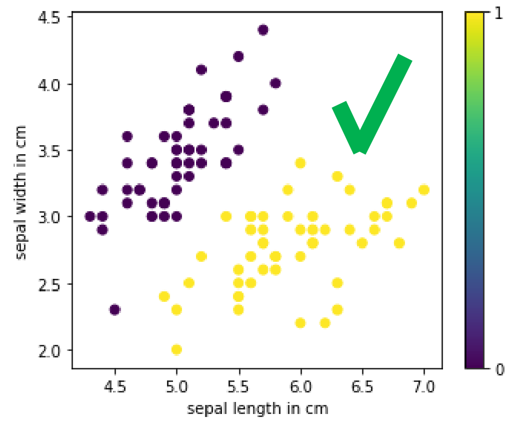
Add More Neurons?



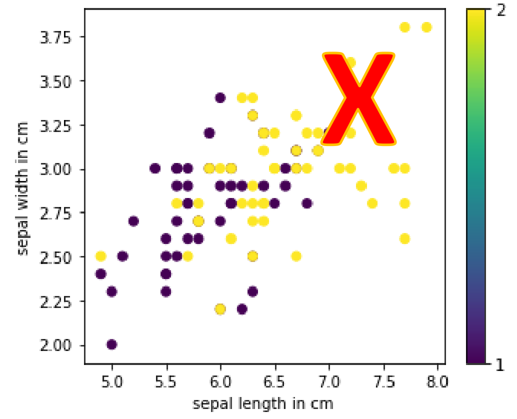
This is a 2-layer neural network. We do not count the input layer, so the number of layers equal number of sets of weights and biases.

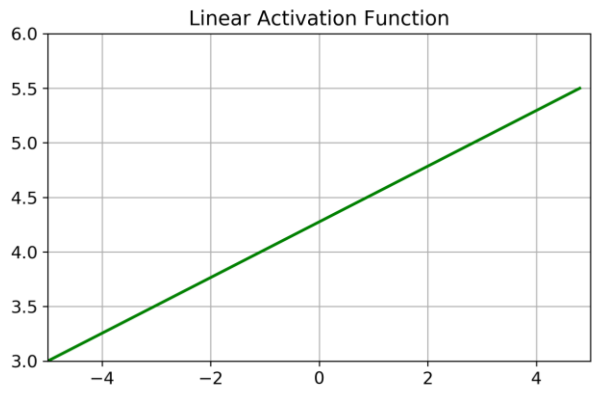
2-layer ANN with Linear Activation

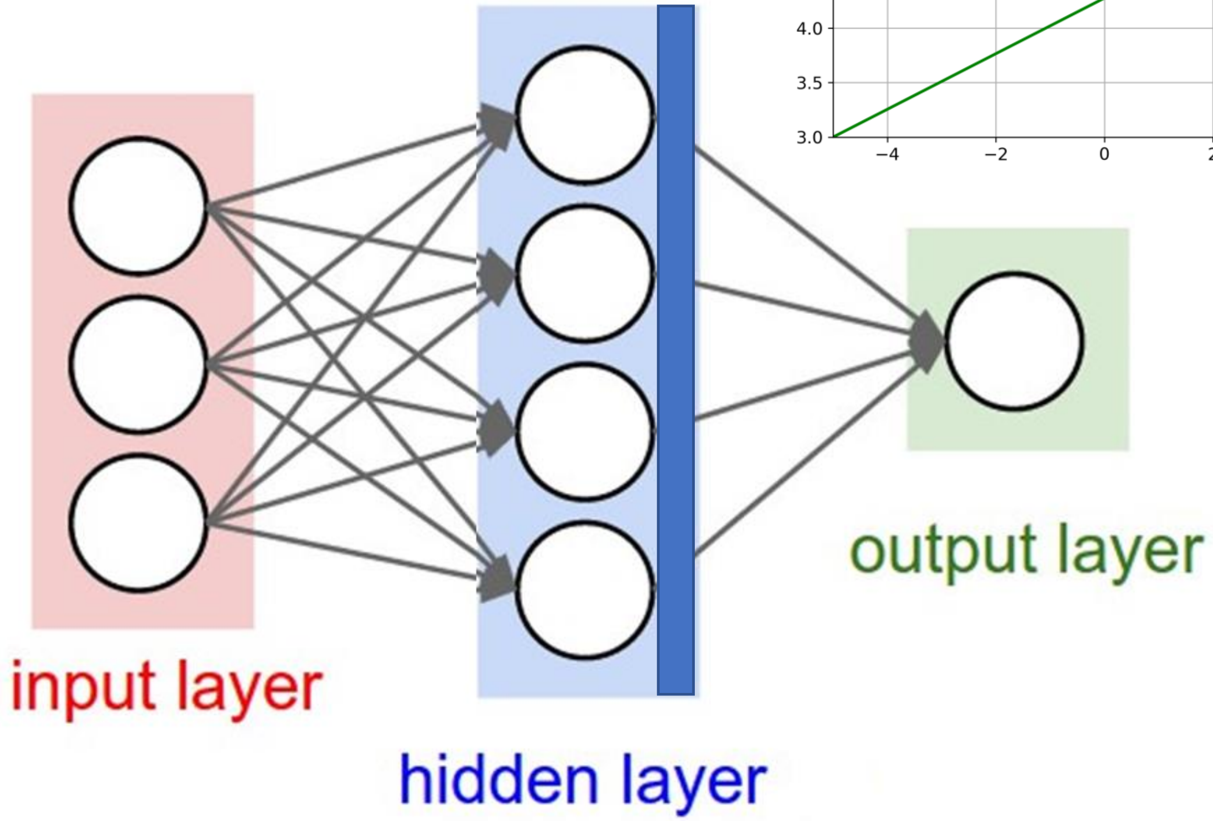
linear



nonlinear







Expressive Power

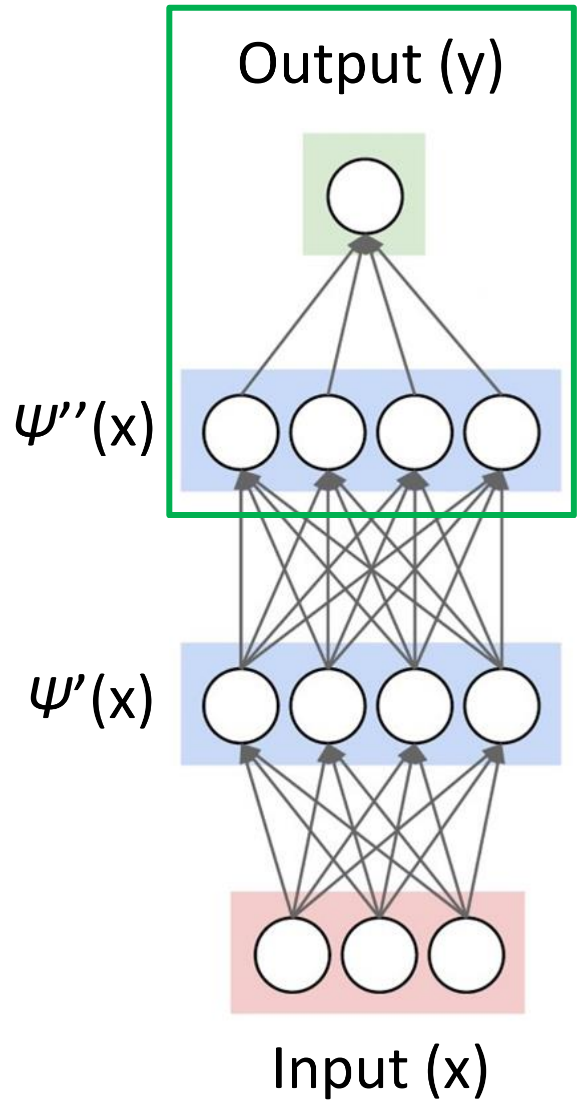
- Adding more linear layers does not help increase the capacity of the model.
- Any sequence of linear layers can be equivalently represented with a single linear layer.

$$\hat{\mathbf{y}} = \underbrace{\mathbf{W}^{(1)}\mathbf{W}^{(2)}\mathbf{W}^{(3)}}\mathbf{x}$$

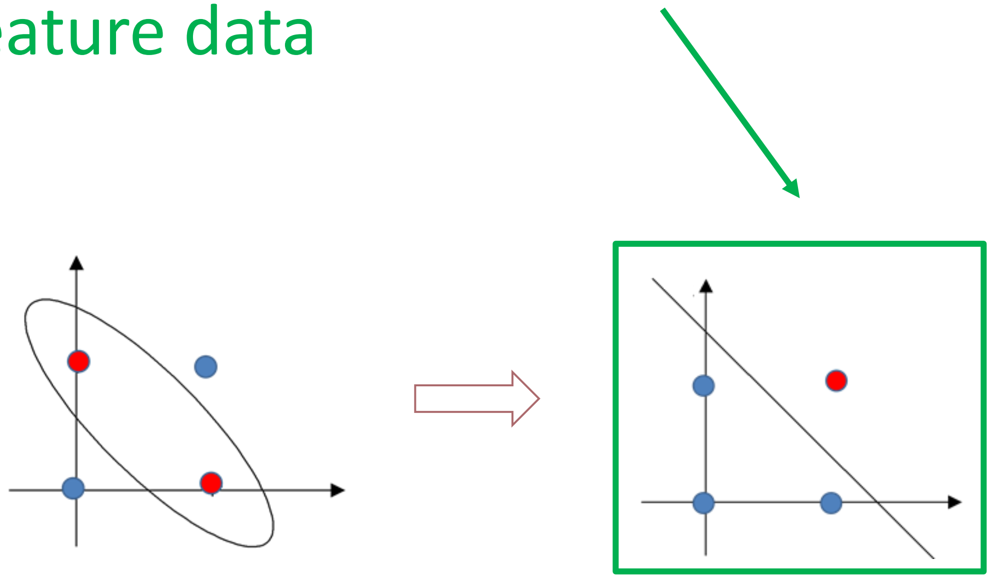
$$\mathbf{W}'\mathbf{x}$$

- Deep linear networks are no more expressive than **linear regression!**

Need an Activation Function

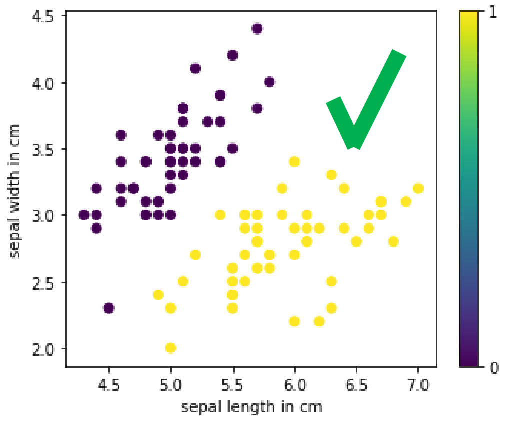


- Neural Networks can be viewed as a **way of learning features**
- The goal being that the final layer is presented with linearly separable feature data

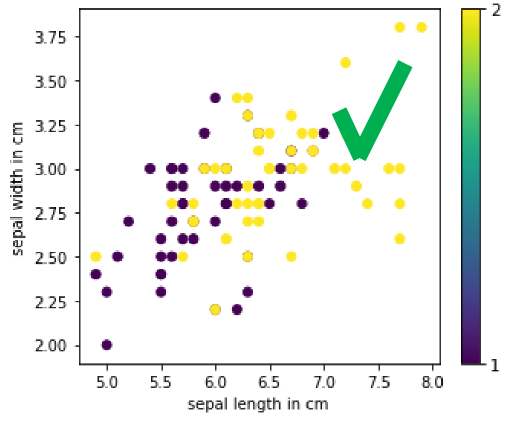


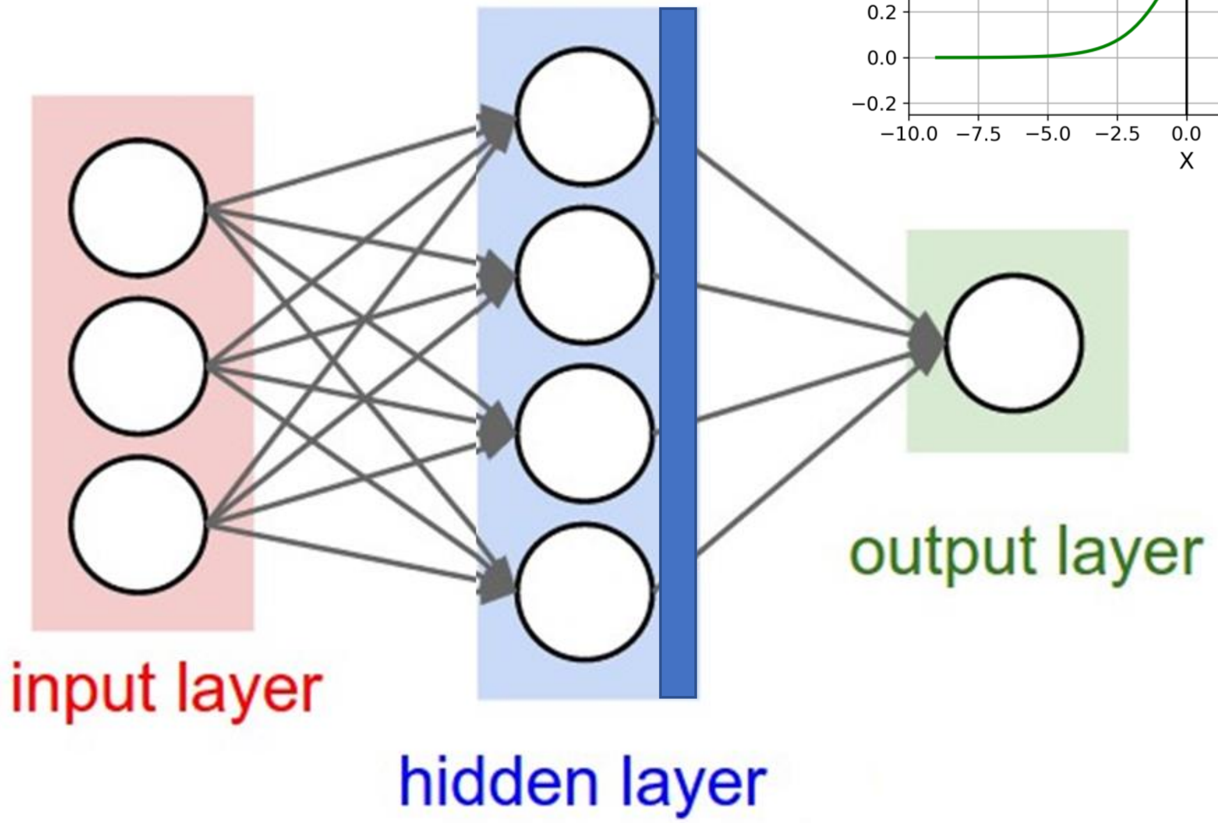
2-layer ANN with Nonlinear Activation

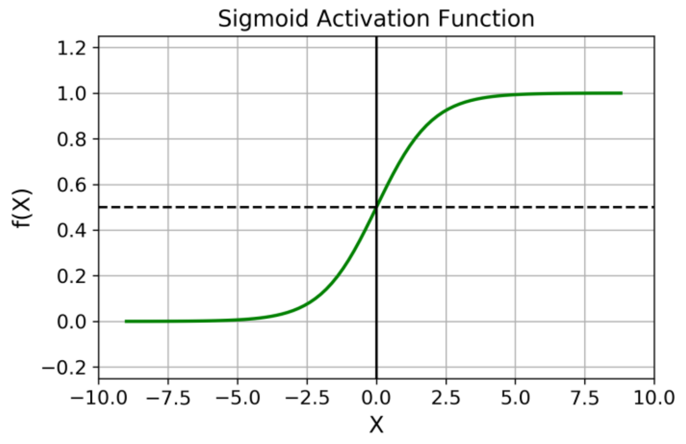
linear



nonlinear





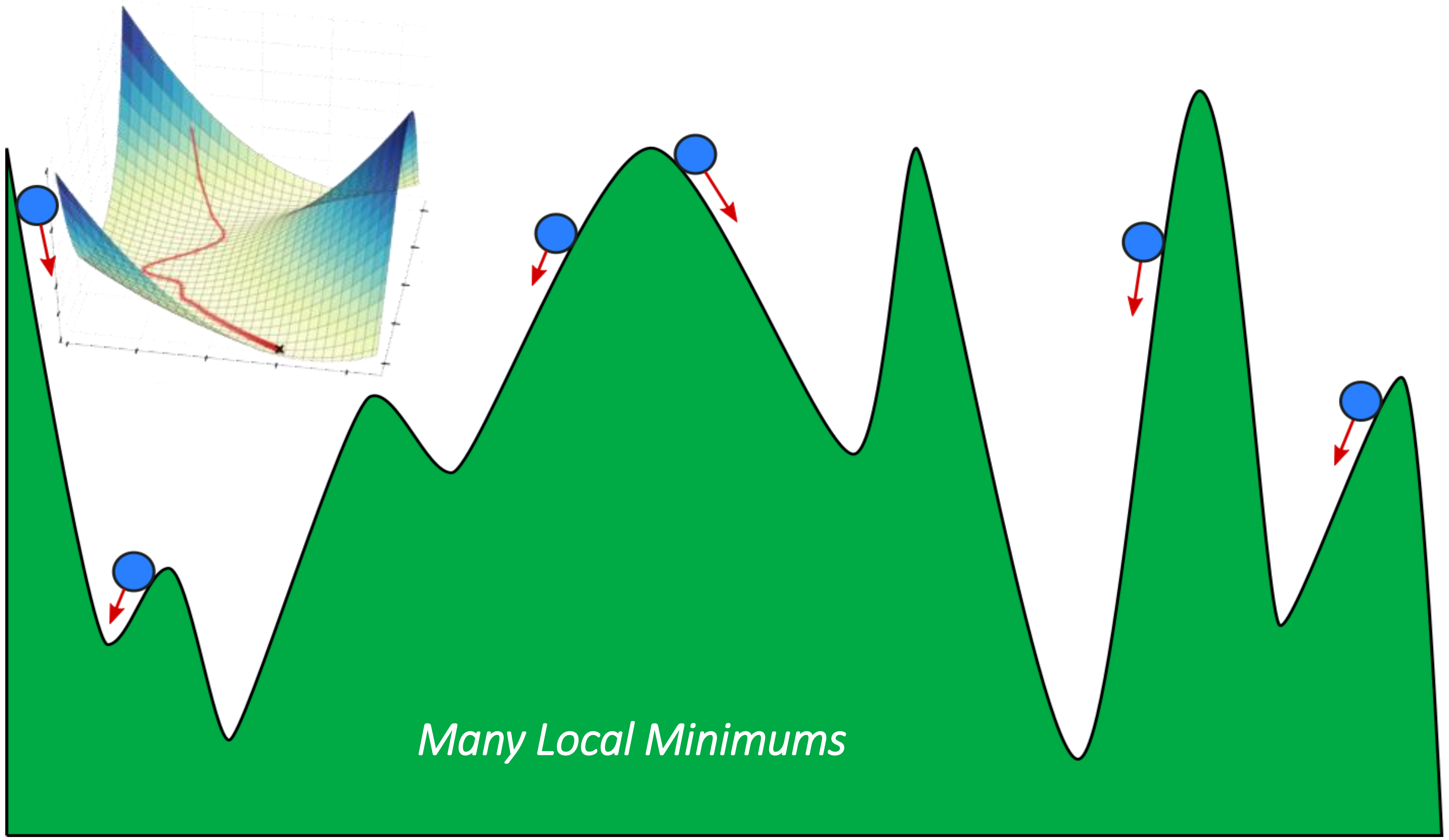


Expressive Power

- Multilayer feed-forward neural nets with nonlinear activation functions are **universal function approximators.**

- They can approximate any function arbitrarily well, but this comes at a cost…

Cost Function is Non-Convex!



Nonlinear activations introduce non-convex surface!

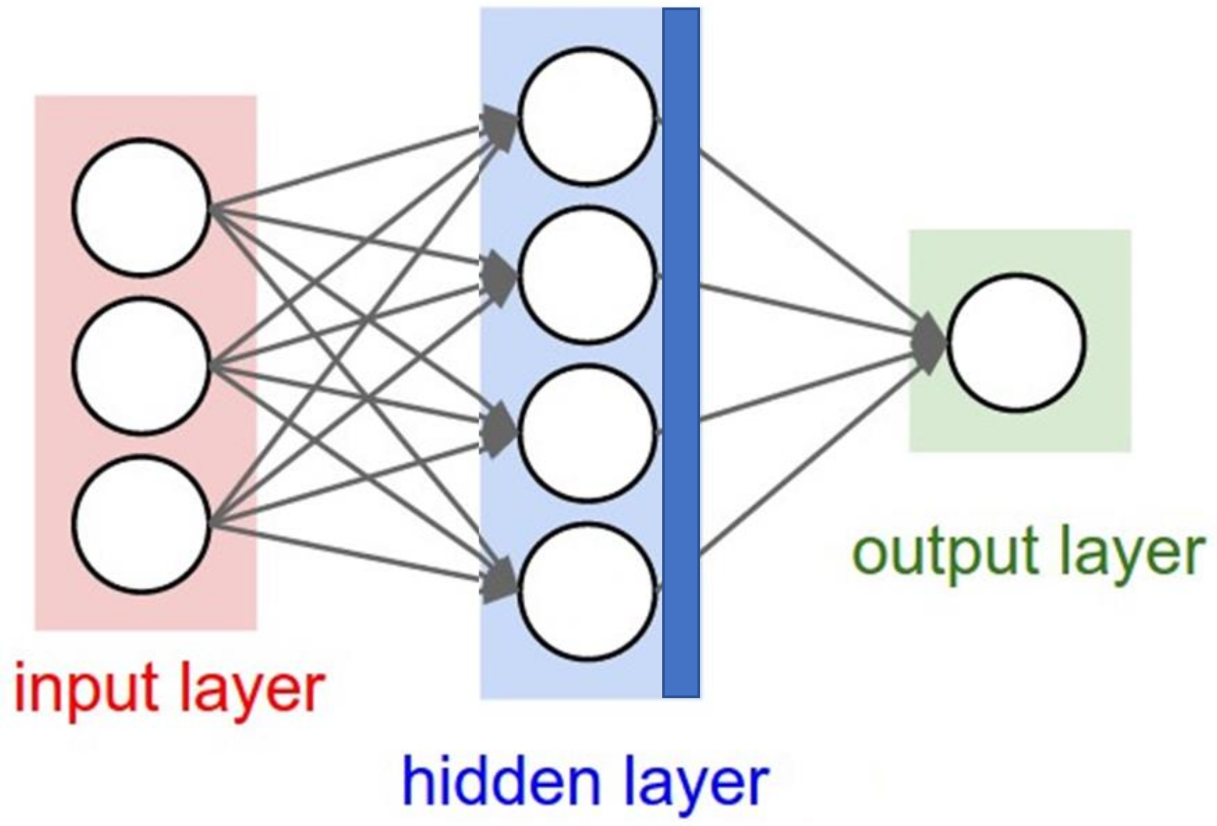
Tuning Neural Networks

Changing ANN Architecture:

- Number of hidden units
- Weights
- Activation Functions

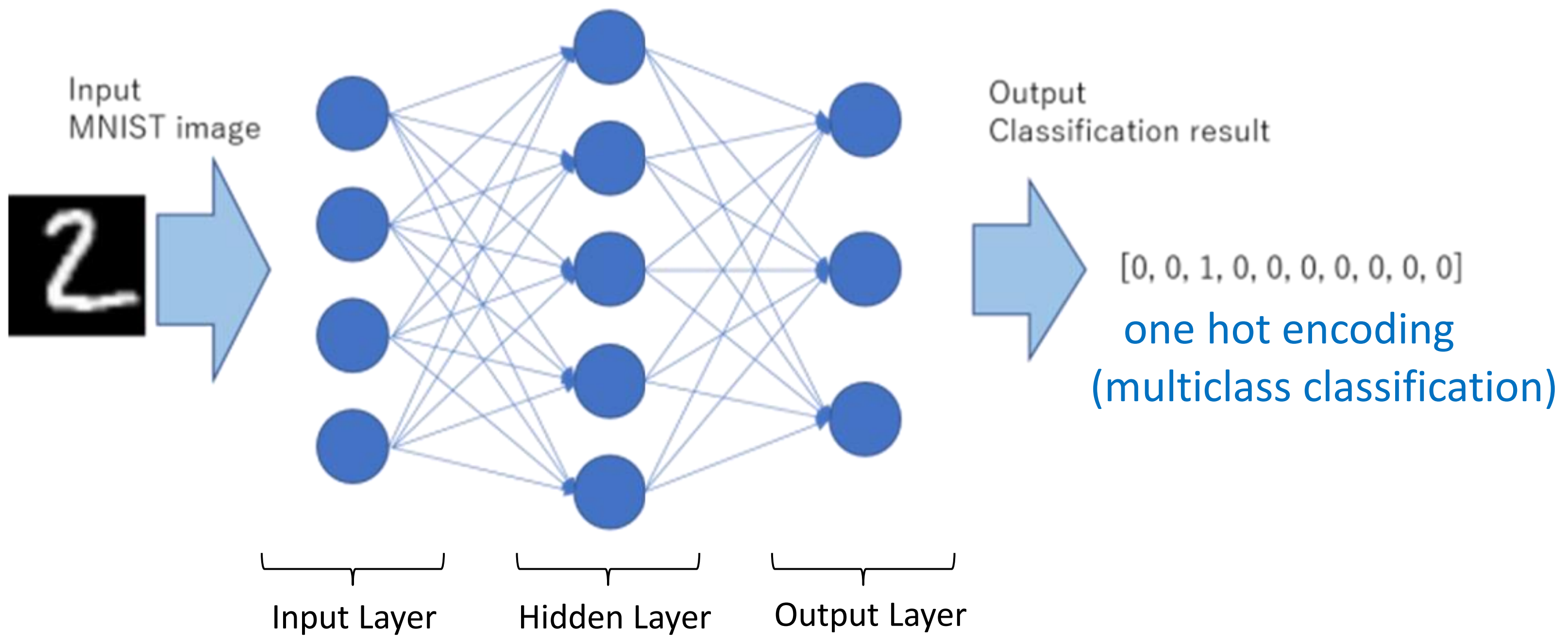
Applying Different Training Techniques:

- Number of iterations
- Learning rate and adaptive learning rate
- Momentum
- Batching of Data
- Regularization
- Dropout
- Feature Augmentation
- Many more…



Take-home Exercise: Discuss on Piazza

Q: Determine the gradients for a 2-layer artificial neural network with sigmoid activation on the hidden and output layers. The error is computed using mean squared error loss.



Next Time

- Week 10 Q&A session: Thursday and Friday
  - Project 4 is due on April 1st

- Week 11 Lecture – Deep Learning (and More)
  - Neural Network Architectures
  - Automatic Differentiation
  - Discrete Optimization