Faculty of Computing, Engineering
& Sciences

Session 2012/13
Semester 2

STAFFORDSHIRE
UNIVERSITY

Module Code:	**CE62014-5**
Module Title:	**ENGINEERING MATHEMATICS APPLICATIONS 2**
Date:	**MONDAY 29TH APRIL 2013**
Time:	**14:00pm**
Duration:	**2 Hours**
Examiner:	**Prof. B. Burrows / Dr P.A. Lewis**
Extension:	**3549**

INSTRUCTIONS TO CANDIDATES:

This paper consists of three questions. You must answer **ALL QUESTIONS**.

The marks allocated for each question, or for its parts, are shown on the right.

Every question attempted should be clearly marked in the space provided on the front of the Answer Booklet.

Calculators are permitted.

YOU MAY LOSE MARKS IF YOU DO NOT SHOW ALL YOUR WORKING.

CANDIDATES WILL REQUIRE:

- Answer booklet
- Examination paper
- Formula Booklet
- An additional formula sheet is attached to the back of this paper

1. (a) Show that the eigenvalues of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

are -1, 0 and 1. Hence determine a linearly independent set of eigenvectors for this matrix.

[10 marks]

(b) The currrent, i, voltage v and charge q in a circuit satisfy the coupled first order differential equations

$$\begin{aligned} \frac{di}{dt} &= v \\ \frac{dv}{dt} &= -v \\ \frac{dq}{dt} &= i + q \end{aligned}$$

Using your results from (a) above determine the General Solutions for i, v and q. Describe all different possible long term behaviours of the solutions, using the following sets of initial conditions to illustrate your answer:

$$(i) \quad i(0) = -1, \;\; v(0) = 2, \;\; q(0) = 1$$

and

$$(ii) \quad i(0) = -1, \;\; v(0) = 2, \;\; q(0) = 0$$

[10 marks]

[Total: 20 marks]

2. (a) Use integration by parts to prove that

$$\int_0^{\pi} x^2 \cos(nx)dx = -\frac{2}{n}\int_0^{\pi} x \sin(nx)dx = \frac{2\pi}{n^2}(-1)^n$$

showing all your working.

[8 marks]

(b) Explain why $x^2 - \frac{\pi^2}{3}$ is an even function and hence expand

$$f(x) = x^2 - \frac{\pi^2}{3}, \quad -\pi \le x \le \pi, \quad f(x+2\pi) = f(x)$$

in a Fourier series giving explicit expressions for the coefficients. You may make use of the results in (a) above.

[9 marks]

(c) Write down the sum of the first 3 non-zero terms in the series and evaluate the numerical value of this sum when $x = 0$. Compare this with the exact value of the function.

[3 marks]

Total: 20 marks

3. (a) Sketch the graph of $f(t) = 3(u(t+2) - u(t-2))$.

[3 marks]

(b) Explain why $f(t)$ is even.

[1 marks]

(c) Show, by integration, that if $F(\omega) = \mathrm{FT}\{f(t)\}$ then

$$F(\omega) = 6\frac{\sin(2\omega)}{\omega}.$$

[4 marks]

(d) Hence, by using properties find the Fourier Transforms of the following:

$$(i)\ f(t+2) \qquad (ii)\ f(4t-5) \qquad (iii)\ e^{2jt} f(t) \qquad (iv)\ \frac{df}{dt} \quad (v)\ t f(t).$$

[10 marks]

(e) Evaluate the following two integrals

$$(i)\ \int_0^{\infty} x^2\, \delta(x+4)\, dx \qquad (i)\ \int_{-\infty}^{\infty} x^3\, \delta(x-4)\, dx$$

[2 marks]

Total: 20 marks

Eigenvalues & Eigenvectors

Determinants

2 by 2

If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then $\det(A) = |A| = ad - bc$

3 by 3

If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ then we can expand about **any** row or column

Row 1 : $a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$ **Column 2:** $-b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + e\begin{vmatrix} a & c \\ g & i \end{vmatrix} - h\begin{vmatrix} a & c \\ d & f \end{vmatrix}$

Inverse

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \dfrac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Properties of vectors

Let $u = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $v = \begin{pmatrix} f \\ g \\ h \end{pmatrix}$

Dot product: $u.v = af + bg + ch$

Length of vector: $|u| = \sqrt{a^2 + b^2 + c^2}$

Characteristic Equation

$$\det(A - \lambda I) = 0$$

Important Result

If P is a matrix whose columns contain the eigenvectors of a matrix A and D is a diagonal matrix containing the corresponding eigenvalues then

$$P^{-1}AP = D$$

Solution of Differential Equations

If $\dfrac{dy}{dt} = \lambda y$ then $y = Ae^{\lambda t}$

If $\dfrac{d^2y}{dt^2} + \omega^2 y = 0$ then $y = A\cos(\omega t) + B\sin(\omega t)$