Faculty of Computing, Engineering & Technology

Session 2010/11
Semester 2

STAFFORDSHIRE UNIVERSITY

Module Code:	**CE62014-5**
Module Title:	**Engineering Mathematics 2**
Date:	**Tuesday 3rd May 2011**
Time:	**10:00am – 12:00pm**
Duration:	**2 Hours**
Examiner:	**Prof B. Burrows /Dr.P.A. Lewis**
Extension:	**3549/3420**

INSTRUCTIONS TO CANDIDATES:

This paper consists of **THREE** questions. You must answer **ALL** questions.

The marks allocated for each question, or for its parts, are shown on the right.

All personal details must be completed at the top of every Answer Booklet. Please ensure that the corner of the booklet is folded down and sealed.

CANDIDATES WILL REQUIRE:

- Answer booklet
- Examination paper
- Mathematics Formula Booklet

1. (a) Show that the eigenvalues of the matrix

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$$

are -2, 0 and 5. Hence determine a linearly independent set of eigenvectors for this matrix.

[10 marks]

(b) The currents i_1, i_2 and i_3 satisfy the coupled first order differential equations

$$\begin{aligned} \frac{di_1}{dt} &= 2i_1 + 3i_3 \\ \frac{di_2}{dt} &= -2i_2 \\ \frac{di_3}{dt} &= 2i_1 + 3i_3 \end{aligned}$$

Using your results from (a) above determine the General Solutions for the currents. Describe the different possible behaviours that the currents can exhibit and illustrate your answer by considering the following two sets of intial conditions:

$$(i) \quad i_1(0) = 6, \;\; i_2(0) = 3, \;\; i_3(0) = -4$$

and

$$(ii) \quad i_1(0) = 7, \;\; i_2(0) = 3, \;\; i_3(0) = 2$$

[10 marks]

2. The periodic function $f(x)$ of period 2 is defined in one period by

$$f(x) = x(u(x) - u(x-1)) + (u(x-1) - u(x-2))$$

(a) Sketch the function in $0 \leq x \leq 2$.

[2 marks]

(b) Show that

$$I_n = \int_0^1 x \sin(n\pi x) + \int_1^2 \sin(n\pi x) dx = -\frac{1}{n\pi}$$

and that

$$J_n = \int_0^1 x \cos(n\pi x) + \int_1^2 \cos(n\pi x) dx = \frac{(-1)^n - 1}{n^2\pi^2}$$

[12 marks]

(c) Use the expressions for I_n and J_n to write down the Fourier series for $f(x)$.

[4 marks]

(d) What does the series converge to at $x = 0$?

[2 marks]

3. The functions $f(t)$ and $g(t)$ are defined by

$$f(t) = \exp(-2t)\,u(t), \quad g(t) = \frac{1}{2h}(u(t+h) - u(t-h))$$

(a) Find *from first principles* the Fourier transforms of $tf(t)$ and $f(t)$ and verify the result

$$FT\{tf(t)\} = j\frac{d}{d\omega}FT\{f(t)\}$$

[8 marks]

(b) Show that the Fourier transform of $g(t)$ is $G(\omega) = \mathrm{sinc}(\omega h)$ and use the tables of Fourier transforms to find:

i. $FT\{g(t+2)\}$
ii. $FT\{g(t-2)\}$
iii. $FT\{g(3t+2)\}$
iv. $FT^{-1}\{G(2\omega+3)\}$ and
v. $FT\{\exp(jt)g(t)\}$.

[8 marks]

(c) As $h \to 0$ we have $g(t) \to \delta(t)$. What is the limit of $G(\omega)$ as $h \to 0$?

[2 marks]

(d) Evaluate

$$\int_{-\infty}^{0} \omega H(\omega)d\omega$$

where $H(\omega) = \delta(\omega+3) + \delta'(\omega-2)$.

[2 marks]

Eigenvalues & Eigenvectors

Determinants

2 by 2

If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then $\det(A) = |A| = ad - bc$

3 by 3

If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ then we can expand about **any** row or column

Row 1 : $a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$ **Column 2:** $-b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + e\begin{vmatrix} a & c \\ g & i \end{vmatrix} - h\begin{vmatrix} a & c \\ d & f \end{vmatrix}$

Inverse

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \dfrac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Properties of vectors

Let $u = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $v = \begin{pmatrix} f \\ g \\ h \end{pmatrix}$

Dot product: $u.v = af + bg + ch$

Length of vector: $|u| = \sqrt{a^2 + b^2 + c^2}$

Characteristic Equation

$$\det(A - \lambda I) = 0$$

Important Result

If P is a matrix whose columns contain the eigenvectors of a matrix A and D is a diagonal matrix containing the corresponding eigenvalues then

$$P^{-1}AP = D$$

Solution of Differential Equations

If $\dfrac{dy}{dt} = \lambda y$ then $y = Ae^{\lambda t}$

If $\dfrac{d^2y}{dt^2} + \omega^2 y = 0$ then $y = A\cos(\omega t) + B\sin(\omega t)$