Faculty of Computing, Engineering & Technology

Session 2011/12
Semester 2

STAFFORDSHIRE UNIVERSITY

Module Code:	CE62014-5/CE61018-4
Module Title:	**Engineering Mathematics with Applications 2/Applied Mathematical Techniques**
Date:	**Tuesday 8th May**
Time:	**2.00pm-4.00pm**
Duration:	**2 hours**
Examiner:	**Brian Burrows/Pat Lewis**
Extension:	**3549**

INSTRUCTIONS TO CANDIDATES:

This paper consists of three questions. You must answer **ALL** questions.

The marks allocated for each question, or for its parts, are shown on the right.

All personal details must be completed at the top of every Answer Booklet. Please ensure that the corner of the booklet is folded down and sealed.

Every question attempted should be clearly marked in the space provided on the front of the Answer Booklet.

Calculators are permitted.

YOU MAY LOSE MARKS IF YOU DO NOT SHOW ALL YOUR WORKING.

CANDIDATES WILL REQUIRE:

- Answer booklet
- Examination paper
- Formula booklet
- An additional formula sheet is attached to the back of this paper

1. (a) Show that the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & -1 & 1 \end{pmatrix}$$

are -1, 0 and 1. Hence determine a linearly independent set of eigenvectors for this matrix.

[12 marks]

(b) The components of velocity u, v and w of an object satisfy the coupled first order differential equations

$$\begin{aligned} \frac{du}{dt} &= u + w \\ \frac{dv}{dt} &= -2v + 2w \\ \frac{dw}{dt} &= -v + w \end{aligned}$$

Using your results from (a) above determine the General Solutions for u, v and w. Describe all different possible behaviours of the solutions.

[8 marks]

[Total: 20 marks]

2. (a) Using the techniques of integration by parts show that:

i.

$$\int_{-1}^{0} (1 + x) \cos(n\pi x) dx = \frac{(1 - (-1)^n)}{n^2\pi^2}, \quad n \neq 0$$

ii.

$$\int_{0}^{1} (1 - x) \cos(n\pi x) dx = \frac{(1 - (-1)^n)}{n^2\pi^2}, \quad n \neq 0$$

iii.

$$\int_{-1}^{0} (1 + x) \sin(n\pi x) dx = \frac{-1}{n\pi}$$

iv.

$$\int_{0}^{1} (1 - x) \sin(n\pi x) dx = \frac{1}{n\pi}$$

[12 marks]

(b) Hence find the Fourier series for the function which is defined in one period by

$$(u(x + 1) - u(x))(1 + x) + \mu(u(x) - u(x - 1))(1 - x), \quad -1 \leq x \leq 1$$

[6 marks]

Explain the significance of the two special cases when (a) $\mu = 1$ and (b) $\mu = -1$.

[2 marks]

[Total: 20 marks]

3. Given the result

$$FT\{f(t)\} = FT\{e^{-t^2}\} = \pi e^{-\frac{\omega^2}{4}} = F(\omega)$$

show, without using tables, how this can be used to find the Fourier Transform of $e^{-(t-b)^2}$

[6 marks]

Use the property tables of Fourier Transforms with the function $f(t)$ defined above to find

(a) $FT\{f(t-3)\}$

(b) $FT\{f(3t-5)\}$

(c) $FT^{-1}\{j\omega F(\omega)\}$

(d) $FT^{-1}\{F(\omega-3)\}$

[8 marks]

A filter can be represented by a frequency function $\phi(\omega) = 3\delta(\omega-4)$ where the input and output signals are represented by the frequency functions $X(\omega)$ and $Y(\omega)$, where $Y(\omega) = \phi(\omega)X(\omega)$. Given that $X(\omega) = F(\omega)$ above find the time-dependent output $y(t)$ whose Fourier Transform is $Y(\omega)$.

[6 marks]

[Total: 20 marks]

Eigenvalues & Eigenvectors

Determinants

2 by 2

If $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then $\det(A) = |A| = ad - bc$

3 by 3

If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ then we can expand about any row or column

Row 1 : $a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$ **Column 2:** $-b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + e\begin{vmatrix} a & c \\ g & i \end{vmatrix} - h\begin{vmatrix} a & c \\ d & f \end{vmatrix}$

Inverse

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \dfrac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Properties of vectors

Let $u = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $v = \begin{pmatrix} f \\ g \\ h \end{pmatrix}$

Dot product: $u.v = af + bg + ch$

Length of vector: $|u| = \sqrt{a^2 + b^2 + c^2}$

Characteristic Equation

$$\det(A - \lambda I) = 0$$

Important Result

If P is a matrix whose columns contain the eigenvectors of a matrix A and D is a diagonal matrix containing the corresponding eigenvalues then

$$P^{-1}AP = D$$

Solution of Differential Equations

If $\dfrac{dy}{dt} = \lambda y$ then $y = Ae^{\lambda t}$

If $\dfrac{d^2 y}{dt^2} + \omega^2 y = 0$ then $y = A\cos(\omega t) + B\sin(\omega t)$