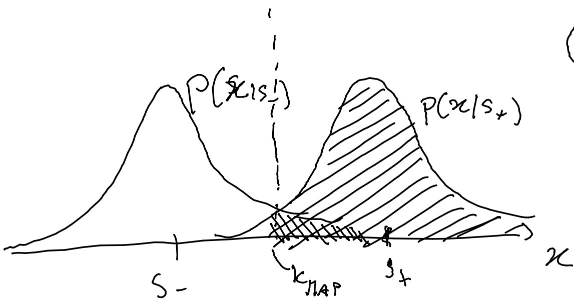
Binary choice tasks

↳ $\boxed{\text{discrimination}}$ $\{s_-, s_+\}$

Yes/no tasks

2AFC tasks

(1)



$$LPR = d = \ln \frac{P(s=s_+ \mid x)}{P(s=s_- \mid x)}$$

(2)

$$\hat{s}_{MAP} = \begin{cases} s_+ & \text{if } d>0 \\ s_- & \text{if } d<0 \end{cases}$$

$$\boxed{\begin{aligned} \bar{s} &= \frac{s_+ + s_-}{2} \\ \Delta s &= s_+ - s_- \end{aligned}}$$

$$d = \ln \frac{p(s_+)}{p(s_-)} + \frac{\Delta s}{\sigma^2}(x - \bar{s})$$

decision rule in x space:

report s_+ when $x > k_{map}$

$$k_{MAP} = \bar{s} \underbrace{- \frac{\sigma^2}{\Delta s} \ln \frac{p(s_+)}{p(s_-)}}_{\text{criterion shift}}$$

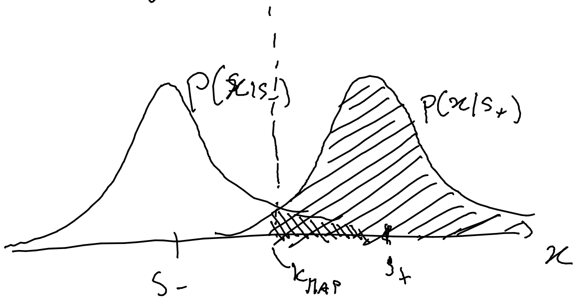
(3) Response distribution

$$P(\hat{s} = s_+ \mid s = s_+) = \text{////} = P(d>0 \mid s=s_+)$$

$$P(\hat{s} = s_+ \mid s = s_-) = \text{\\\\\\\\} = P(d>0 \mid s=s_-)$$

$$P(\hat{s}=s_+ \mid s=s_+) = P(d>0 \mid s=s_+) = P(x>\kappa_{MAP} \mid s=s_+)$$

- We're going to use a generic criterion κ, which may or may not be $=\kappa_{MAP}$



$$P(x>\kappa \mid s=s_+) = \int_{\kappa}^{+\infty} p(x \mid s=s_+)\,dx$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{x=\kappa}^{x=+\infty} \exp\left[-\frac{(x-s_+)^2}{2\sigma^2}\right] dx$$

Change of variables:

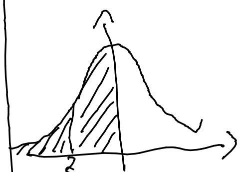
$$y = -\frac{x-s_+}{\sigma} \qquad \frac{dy}{dx} = -\frac{1}{\sigma} \Rightarrow dy = -\frac{dx}{\sigma} \qquad dx = -\sigma\,dy$$

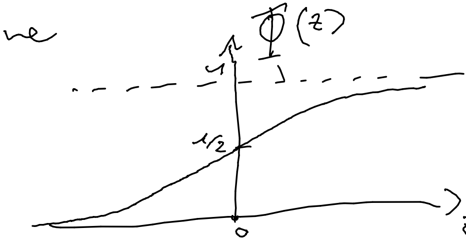
$$= \frac{1}{\sqrt{2\pi\cancel{\sigma^2}}} \int_{-\frac{\kappa-s_+}{\sigma}}^{-\infty} \exp\left[-\frac{y^2}{2}\right] (-\cancel{\sigma}) \cdot dy$$

$$= \int_{-\infty}^{\frac{s_+-\kappa}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{y^2}{2}\right] dy$$

$$= \Phi\left(\frac{s_+-\kappa}{\sigma}\right), \quad = P(x>\kappa \mid s=s_+)$$

where Φ is the standard normal cumulative





$$P(x>\kappa \mid s=s_-) = \Phi\left(\frac{s_- - \kappa}{\sigma}\right)$$

$$\kappa_{map} = \bar{s} - \frac{\sigma^2}{\Delta s} \ln\frac{P(s_+)}{P(s_-)}$$

$$P(x>\kappa_{map} \mid s=s_+) = P(\hat{s}=s_+ \mid s=s_+)$$

$$= \Phi\left(\frac{s_+}{\sigma} - \frac{1}{\sigma}\left[\bar{s} - \frac{\sigma^2}{\Delta s} \ln\frac{P(s_+)}{P(s_-)}\right]\right)$$

$$=$$

$$P(x > x_{MAP} \mid s = s_+) = P(\hat{s} = s_+ \mid s = s_+)$$

$$= \Phi\left(\frac{s_+}{\sigma} - \frac{1}{\sigma}\left[\bar{s} - \frac{\sigma^2}{\Delta s}\ln\frac{p(s_+)}{p(s_-)}\right]\right)$$

$$= \Phi\left(\frac{s_+}{\sigma} - \frac{s_+ + s_-}{2\sigma} + \frac{\sigma}{\Delta s}\ln\frac{p(s_+)}{p(s_-)}\right)$$

$$= \Phi\left(\frac{2s_+ - s_+ - s_-}{2\sigma} + \frac{\sigma}{\Delta s}\ln\frac{p(s_+)}{p(s_-)}\right)$$

$$= \Phi\left[\frac{\sigma}{\Delta s}\ln\frac{p(s_+)}{p(s_-)} + \frac{\Delta s}{2\sigma}\right]$$

$$\bar{s} = \frac{s_+ + s_-}{2}$$

$$\Delta s = s_+ - s_-$$

$$\frac{2s_+ - s_+ - s_-}{2\sigma} = \frac{s_+ - s_-}{2\sigma} = \frac{\Delta s}{2\sigma}$$

Discrimination $\rightarrow$ Detection

$s_+ \quad s_-$

$\quad\quad \hookrightarrow 0$

$$d = \ln\frac{p(s_+)}{p(s=0)} + \frac{s_+}{\sigma^2}\left(x - \frac{s_+}{2}\right)$$

$$P(\hat{s} = s_+ \mid s = s_+) = \Phi\left(\frac{s_+}{2\sigma} + \frac{\sigma}{s_+}\ln\frac{p(s=s_+)}{p(s=0)}\right)$$

$\hookrightarrow$ HIT RATE

$$P(\hat{s} = s_+ \mid s = 0) = \Phi\left(-\frac{s_+}{2\sigma} + \frac{\sigma}{s_+}\ln\frac{p(s=s_+)}{p(s=0)}\right)$$

$\hookrightarrow$ FALSE-ALARM RATE

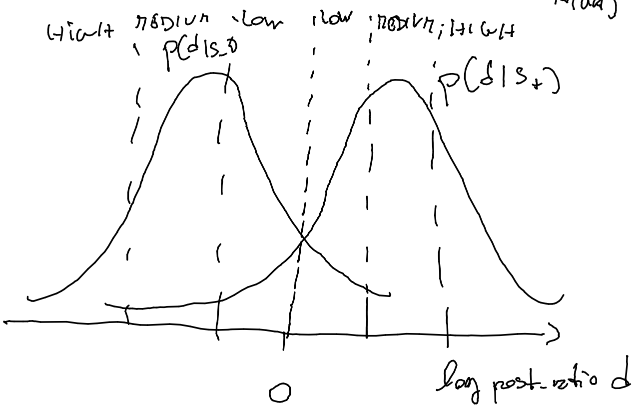
Back to discrimination..

d

$$\text{confidence} = |d| = \left| \ln \frac{p(s_+|x)}{p(s_-|s)} \right| =$$

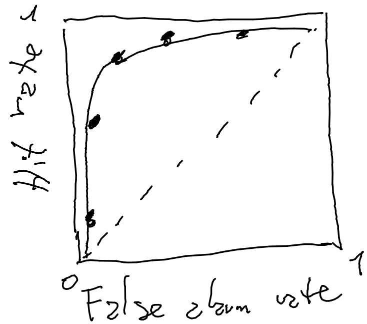
$$= \left| \ln \frac{p(x|s_+)}{p(x|s_-)} + \ln \frac{p(s_+)}{p(s_-)} \right|$$

Confidence rating: ask subject {LOW, MEDIUM, HIGH}



$$H(k) = p(d > k | s = s_+)$$

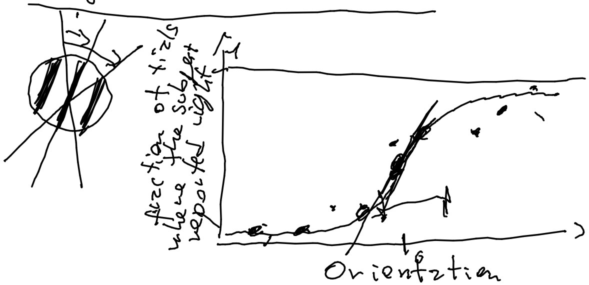
$$F(k) = p(d > k | s = s_-)$$



Receiver operating characteristic (ROC)

→ Signal detection theory

Binary classification task.



Psychometric curve

-> a summary of human (or animal) behavior that is plotted against a variable controlled by the experimenter.

(1) Generative model

$$C \to s \to x$$

$c \in \{0, 1\}$

$p(C=0) = p(C=1) = 1/2$

• Class-conditional stimulus distr. (CCSD)

$p(s|c)$

$$p(x, s, c) = p(c)\,\underbrace{p(s|c)}\,\overline{p(x|s)}$$

$x \to c$: $p(c|x)$

↳ marginalization.

a, b

$p(a, b)$

$$\boxed{p(a) = \sum_b p(a, b)}$$

s is discrete

$$p(x|c) = \sum_s p(x, s|c) = \sum_s p(x|s, c)\, p(s|c) = \sum_s p(x|s)\, p(s|c)$$

s is continuous

$$p(x|c) = \int ds \; p(x|s)\, p(s|c)$$

N / S hemisphere

W / L water / land

$p(N) = 1/2 \qquad p(W|N) = 0.6$

$p(S) = 1/2 \qquad p(W|S) = 0.8$

$$p(N, W) = p(N)\, p(W|N)$$

$$= 0.3$$

$p(N/S, W/L)$

$\hookrightarrow p(W/L)$

P_{joint}	N	S	
W	0.3	0.4	0.7
L	0.2	0.1	0.3
	1/2	1/2	

$$p(a) = \sum_b p(a,b)$$

$$p(a|c) = \sum_b p(a,b|c)$$

we want $p(x|c)$

$$C \to S \to x$$

$$p(x|c) = \sum_s p(x,s|c)$$

$$p(x,s|c) = p(x|s)\, p(s|c)$$