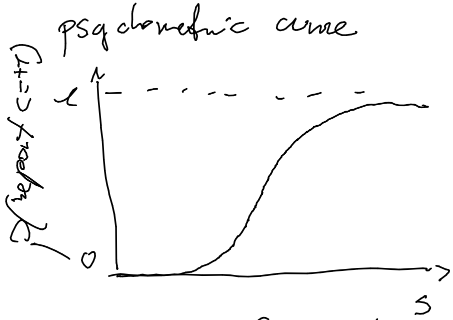
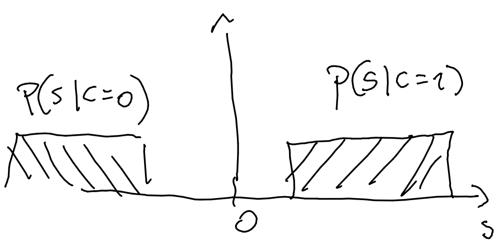
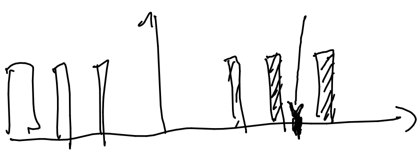
Classification

$c \to s \to x$



- $p(c=0) = p(c=1) = 1/2$
- $p(x|s)$ → gaussian
- $p(s|c)$ CCSD





$$p(c|x) = \frac{p(x|c)\,p(c)}{p(x)}$$

$s \to x$

$$p(x) = \sum_s p(x,s) = \sum_s p(x|s)\,p(s)$$

$c \to s \to x$

$$p(x|c) = \sum_s p(x,s|c)$$

$$= \sum_s p(x|c,s)\,p(s|c)$$

$p(x|c,s)$

Measurement only depends on the stimulus, <u>not</u> directly on the class

$$\Rightarrow \quad p(x|c,s) = p(x|s)$$

$$p(x|c) = \sum_s p(x|s)\,p(s|c)$$

$$p(x|c) = \int ds\; p(x|s)\,p(s|c)$$

$$p(x|c) = \int ds \; p(x|s)\, p(s|c)$$

② Inference

$$P(c|x) = \frac{p(x|c)\,p(c)}{p(x)}$$

$c = 0, 1$

$$d = \ln \frac{p(c=1|x)}{p(c=0|x)} = \ln \frac{p(x|c=1)}{p(x|c=0)} + \cancel{\ln \frac{p(c=1)}{p(c=0)}}$$

Class likelihoods: $\mathcal{L}(c;x)$

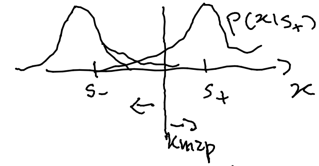
$$= p(x|c) = \int ds \; \underbrace{p(x|s)}_{\text{perceptual process}} \; \underbrace{p(s|c)}_{\text{experimental design}}$$

$$d = \ln \frac{\int ds\, p(x|s)\, p(s|c=1)}{\int ds\, p(x|s)\, p(s|c=0)} + \ln \frac{p(c=1)}{p(c=0)}$$

MAP rule: choose $C=1$ if $d>0$

K_{map}

Discrimination



$$d = \ln \frac{p(s_+|x)}{p(s_-|x)}$$

$$= \ln \frac{p(s_+)}{p(s_-)} + \frac{\Delta s}{\sigma^2}(x - \bar{s}) \Rightarrow d = a + bx$$

↳ $d = 0$ when $x = \bar{s} - \frac{\sigma^2}{\Delta s} \ln \frac{p(s_+)}{p(s_-)}$

↳ K_{map}

- write down d as a function of x
- set $d = 0$ and solve for x

→ we are interested in finding x such that

$$\boxed{\mathcal{L}(x;c=0) = \mathcal{L}(x;c=1)}$$

$$\int ds\, p(x|s)\, p(s|c=0) = \int ds\, p(x|s)\, p(s|c=1)$$

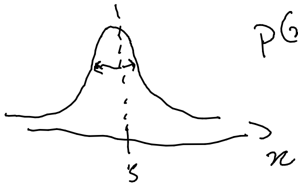
→ this value will be K_{map}

Special case (easy to solve by hand):

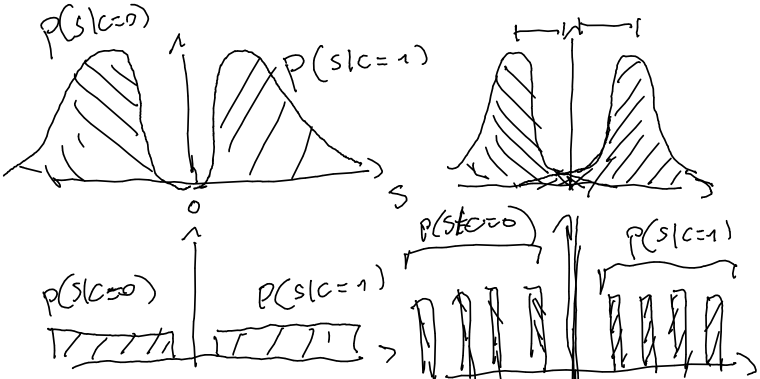
Assume

- prior is flat $p(c=0) = p(c=1)$
- measurement distr. is symmetric around s

$$p(x|s) = f(|x-s|)$$



- class-conditional stimulus distr. must be mirror-symmetric



$$P(s|c=0) = P(-s|c=1)$$

$\Rightarrow$ we will see that the $K_{map} = 0$

$$\mathcal{L}(c=0;x) = p(x|c=0) = \int ds\, p(x|s)\, p(s|c=0)$$

mirror symmetric

$$= \int ds\, p(x|s)\, p(-s|c=1)$$

But symmetric measurement means that

$$p(x|s) = f(|x-s|) = f(|-x+s|) = p(-x|-s)$$

$$= \int ds\, p(-x|-s)\, p(-s|c=1)$$

change of variable:

$s' = -s \quad ds' = -ds \quad \int_{-\infty}^{+\infty} ds = \int_{+\infty}^{-\infty} (-ds') = \int_{-\infty}^{+\infty} ds'$

$$= \int ds'\, p(-x|s')\, p(s'|c=1)$$

$$= \int ds\, p(-x|s)\, p(s|c=1)$$

$$= p(-x|c=1) = \mathcal{L}(c=1;-x)$$

$$\boxed{\mathcal{L}(c=1;x) = \mathcal{L}(c=0;x)}$$

$$\mathcal{L}(c=1;x) = \mathcal{L}(c=1;-x)$$

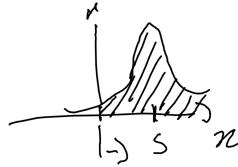
$\rightarrow$ this can only be when $x = -x \Rightarrow x = 0$

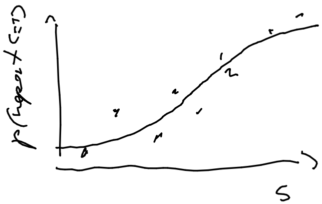
$$\boxed{K_{map} = 0}$$

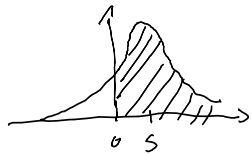
(1) (c) → (s) → (x)

(2) report $c=1$ when $x > 0$

(3) Response distribution.



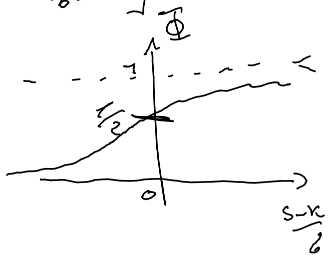




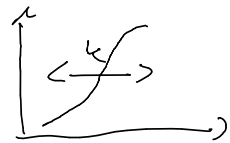
$$P(\hat{c}=1 \mid s) = \int_{\kappa}^{\infty} p(x \mid s)\,dx =$$

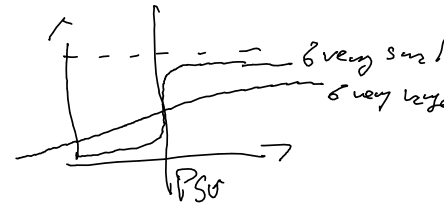
$$= \int_{\kappa}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-s)^2}{2\sigma^2}\right] dx$$

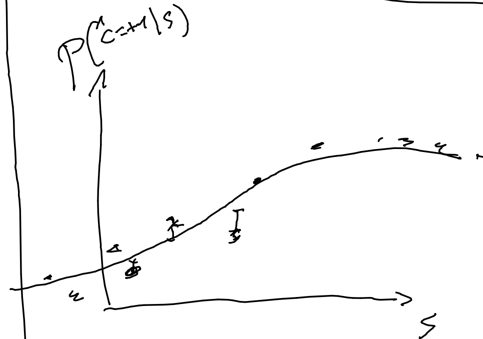
$$= \Phi\left(\frac{s-\kappa}{\sigma}\right)$$



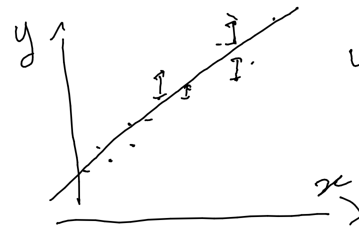
- κ is the point of subjective equivalence (PSE)
- $1/\sigma$ is the slope of the psychometric function







$$\Phi\left(\frac{s-\kappa}{\sigma}\right)$$



$y = ax + b$

dataset

$D = \{s_t, \hat{c}_t\}$

$P(D \mid \sigma)$