Celestini Project 2019

Section E

i). If we have n bins and m balls and we throw balls into bins independently at random, so that each ball is equally likely to fall into any of the bins.
Then using linearity of expectations: the expected number of bins is given by the formula :
n*[(n-1)/n]^m

Now here n = 50 and m=100 then expectation = 50*[(50-1)/50]^100 = 6.630

ii). Solution and answer:
a) Here probability = 10^-10 and no. of trials =1000
Let the random variable Xi be 1 if it is an erroneous bit , and 0 otherwise
Thus P{Xi=1} = 10^-10
The no. of erroneous bits in a block of 1000 bits is thus given by N = sum over i=1 to 1000 (Xi)
= 1000*10^-10
Ans = 10^-7

b) Using poisson's distribution, here n= 1000; p= 10^-10
Lambda = n*p = 10^-7
To find 10 or more errors, we will subtract the probability for upto 9 errors from 1
Approximate probability is given by : e^-lambda*(lambda^k)/k!
Therefore , the probability in our case will be 1 - (e^-(10^-7))*[(10^-7)/1!+(10^-14)/2!+....+(10^-63)/9!) = which is approximately equal to 0.9999, since the latter terms in the bracket are negligible.

iii). Using binomial distribution in this case , here n=30 , r=30, probability of success = probability of getting a spade + probability of getting a queen = 13/52 +3/52 since there is a queen in spades too
p=16/52 ; q= 36/52
the fact that if she wins she gains $4 and if she fails she loses $1 we get her winnings after 30 nights to be equal 30 *4*p - 30*1*q
= 30*(64-36)/52 which is approximately $16.