Study Guide and Review

Choose the correct term from the list to complete each sentence.
axis of symmetry center
conic section conjugate axis
co-vertices degenerate conic
directrix eccentricity
ellipse foci
focus hyperbola
locus major axis
minor axis orientation
parabola parameter
parametric curve
parametric equation
transverse axis
vertex vertices

1. A _______ is a figure formed when a plane intersects a double−napped right cone.

SOLUTION:

conic section

2. A circle is the _______ of points that fulfill the property that all points be in a given plane and a specified distance from a given point.

SOLUTION:

locus

3. The _______ of a parabola is perpendicular to its axis of symmetry.

SOLUTION:

directrix

4. The co−vertices of a(n) _______ lie on its minor axis, while the vertices lie on its major axis.

SOLUTION:

ellipse

5. From any point on an ellipse, the sum of the distances to the _______ of the ellipse remains constant.

SOLUTION:

foci

6. The _______ of an ellipse is a ratio that determines how "stretched" or "circular" the ellipse is. It is found using the ratio $\frac{c}{a}$.

SOLUTION:

eccentricity

7. The _______ of a circle is a single point, and all points on the circle are equidistant from that point.

SOLUTION:

center

8. Like an ellipse, a _______ has vertices and foci, but it also has a pair of asymptotes and does not have a connected graph.

SOLUTION:

hyperbola

9. The equation for a graph can be written using the variables x and y, or using _______equations, generally using t or the angle θ.

SOLUTION:

parametric

10. The graph of $f(t) = (\sin t, \cos t)$ is a _______ with a shape that is a circle traced clockwise.

SOLUTION:

parametric curve

For each equation, identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

11. $(x + 3)^2 = 12(y + 2)$

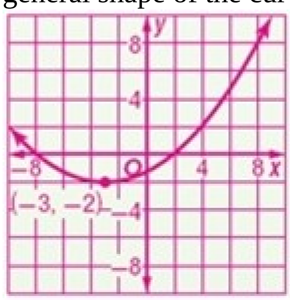
SOLUTION:

$(x + 3)^2 = 12(y + 2)$

The equation is in standard form and the squared term is x, which means that the parabola opens vertically. The equation is in the form $(x - h)^2 = 4p(y - k)$, so $h = -3$ and $k = -2$. Because $4p = 12$ and $p = 3$, the graph opens up. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: $(-3, -2)$	(h, k)
directrix: $y = -5$	$y = k - p$
focus: $(-3, 1)$	$(h, k + p)$
axis of symmetry: $x = -3$	$x = h$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



12. $(y - 2)^2 = 8(x - 5)$

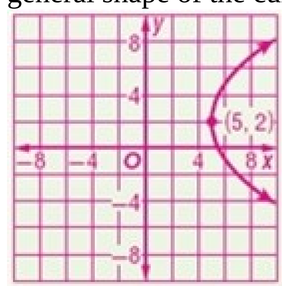
SOLUTION:

$(y - 2)^2 = 8(x - 5)$

The equation is in standard form and the squared term is y, which means that the parabola opens horizontally. The equation is in the form $(y - k)^2 = 4p(x - h)$, so $h = 5$ and $k = 2$. Because $4p = 8$ and $p = 2$, the graph opens to the right. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: $(5, 2)$	(h, k)
directrix: $x = 3$	$x = h - p$
focus: $(7, 2)$	$(h + p, k)$
axis of symmetry: $y = 2$	$y = k$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



13. $(x - 2)^2 = -4(y + 1)$

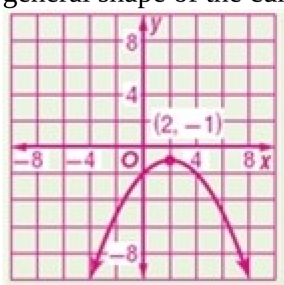
SOLUTION:

$(x - 2)^2 = -4(y + 1)$

The equation is in standard form and the squared term is x, which means that the parabola opens vertically. The equation is in the form $(x - h)^2 = 4p(y - k)$, so $h = 2$ and $k = -1$. Because $4p = -4$ and $p = -1$, the graph opens down. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: $(2, -1)$	(h, k)
directrix: $y = 0$	$y = k - p$
focus: $(2, -2)$	$(h, k + p)$
axis of symmetry: $x = 2$	$x = h$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



14. $(x - 5) = \frac{1}{12}(y - 3)^2$

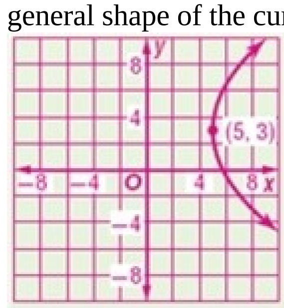
SOLUTION:

$(x - 5) = \frac{1}{12}(y - 3)^2$

The equation is in standard form and the squared term is y, which means that the parabola opens horizontally. The equation is in the form $(y - k)^2 = 4p(x - h)$, so $h = 5$ and $k = 3$. Because $4p = \frac{1}{12}$ and $p = \frac{1}{48}$, the graph opens to the right. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: $(5, 3)$	(h, k)
directrix: $x = 2\frac{47}{48}$	$x = h - p$
focus: $\left(5\frac{1}{48}, 3\right)$	$(h + p, k)$
axis of symmetry: $y = 3$	$y = k$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



Write an equation for and graph a parabola with the given focus F and vertex V.

15. $F(1, 1)$, $V(1, 5)$

SOLUTION:

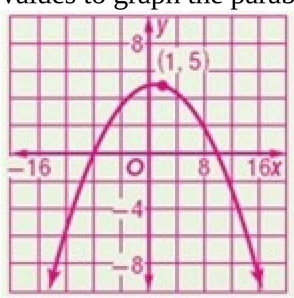
Because the focus and vertex share the same x-coordinate, the graph is vertical. The focus is $(h, k + p)$, so the value of p is $1 - 5$ or -4.

Because p is negative, the graph opens down. Write the equation for the parabola in standard form using the values of h, p, and k.

$$\begin{aligned} 4p(y-k) &= (x-h)^2 \\ 4(-4)(y-5) &= (x-1)^2 \\ -16(y-5) &= (x-1)^2 \end{aligned}$$

The standard form of the equation is $(x - 1)^2 = -16(y - 5)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



16. $F(-3, 6)$, $V(7, 6)$

SOLUTION:

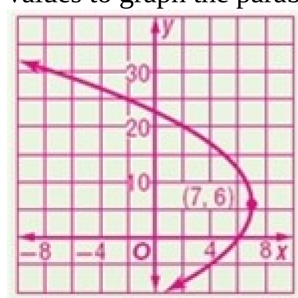
Because the focus and vertex share the same y-coordinate, the graph is horizontal. The focus is $(h + p, k)$, so the value of p is $-3 - 7$ or -10. Because p is negative, the graph opens to the left.

Write the equation for the parabola in standard form using the values of h, p, and k.

$$\begin{aligned} (y-h)^2 &= 4p(x-k) \\ (y-6)^2 &= 4(-10)(x-7) \\ (y-6)^2 &= -40(x-7) \end{aligned}$$

The standard form of the equation is $(y - 6)^2 = -40(x - 7)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



17. $F(-2, -3)$, $V(-2, 1)$

SOLUTION:

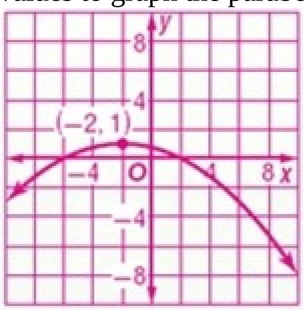
Because the focus and vertex share the same x-coordinate, the graph is vertical. The focus is $(h, k + p)$, so the value of p is $-3 - 1$ or -4. Because p is negative, the graph opens down.

Write the equation for the parabola in standard form using the values of h, p, and k.

$$\begin{aligned} 4p(y - k) &= (x - h)^2 \\ 4(-4)(y - 1) &= [x - (-2)]^2 \\ -16(y - 1) &= (x + 2)^2 \end{aligned}$$

The standard form of the equation is $(x + 2)^2 = -16(y - 1)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



18. $F(3, -4)$, $V(3, -2)$

SOLUTION:

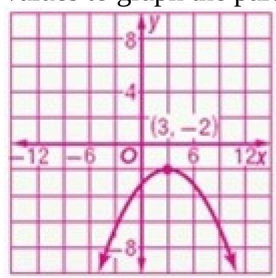
$F(3, -4)$, $V(3, -2)$
Because the focus and vertex share the same x-coordinate, the graph is vertical. The focus is $(h, k + p)$, so the value of p is $-4 - (-2)$ or -2. Because p is negative, the graph opens down.

Write the equation for the parabola in standard form using the values of h, p, and k.

$$\begin{aligned} 4p(y - k) &= (x - h)^2 \\ 4(-2)[y - (-2)] &= (x - 3)^2 \\ -8(y + 2) &= (x - 3)^2 \end{aligned}$$

The standard form of the equation is $(x - 3)^2 = -8(y + 2)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



Write an equation for and graph each parabola with focus F and the given characteristics.

19. $F(-4, -4)$; concave left; contains $(-7, 0)$

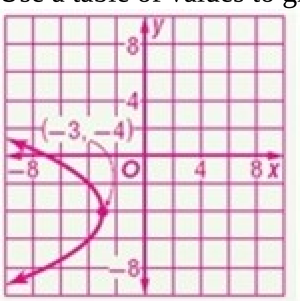
SOLUTION:

Because the parabola opens to the left, the vertex is $(-4 - p, -4)$. Use the standard form of the equation of a horizontal parabola and the point $(-7, 0)$ to find the equation.

$$\begin{aligned} 4p(x - h) &= (y - k)^2 \\ 4p[-7 - (-4 - p)] &= [0 - (-4)]^2 \\ 4p(-3 + p) &= 16 \\ p(-3 + p) &= 4 \\ p^2 - 3p &= 4 \\ p^2 - 3p - 4 &= 0 \\ (p - 4)(p + 1) &= 0 \\ p &= -1 \text{ or } 4 \end{aligned}$$

Because the parabola opens to the left, the value of p must be negative. Therefore, $p = -1$. The vertex is $(-3, -4)$ and the standard form of the equation is $(y + 4)^2 = -4(x + 3)$.

Use a table of values to graph the parabola.



20. $F(-1, 4)$; concave down; contains $(7, -2)$

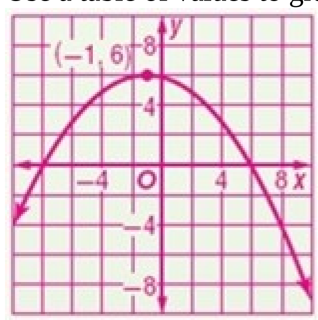
SOLUTION:

Because the parabola opens down, the vertex is $(-1, 4 - p)$. Use the standard form of the equation of a horizontal parabola and the point $(7, -2)$ to find the equation.

$$\begin{aligned} 4p(y - k) &= (x - h)^2 \\ 4p[-2 - (4 - p)] &= [7 - (-1)]^2 \\ 4p(-6 + p) &= 64 \\ p(-6 + p) &= 16 \\ p^2 - 6p &= 16 \\ p^2 - 6p - 16 &= 0 \\ (p - 8)(p + 2) &= 0 \\ p &= 8 \text{ or } -2 \end{aligned}$$

Because the parabola opens down, the value of p must be negative. Therefore, $p = -2$. The vertex is $(-1, 6)$, and the standard form of the equation is $(x + 1)^2 = -8(y - 6)$.

Use a table of values to graph the parabola.



21. $F(3, -6)$; concave up; contains (9, 2)

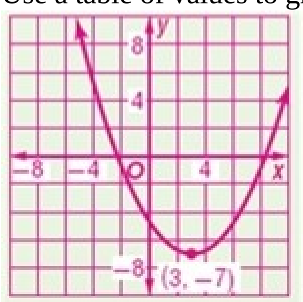
SOLUTION:

Because the parabola opens up, the vertex is $(3, -6 - p)$. Use the standard form of the equation of a horizontal parabola and the point (9, 2) to find the equation.

$$\begin{aligned} 4p(y-k) &= (x-h)^2 \\ 4p[2-(-6-p)] &= (9-3)^2 \\ 4p(8+p) &= 36 \\ p(8+p) &= 9 \\ p^2+8p &= 9 \\ p^2+8p-9 &= 0 \\ (p+9)(p-1) &= 0 \\ p &= -9 \text{ or } 1 \end{aligned}$$

Because the parabola opens up, the value of p must be positive. Therefore, $p = 1$. The vertex is $(3, -7)$, and the standard form of the equation is $(x-3)^2 = 4(y+7)$.

Use a table of values to graph the parabola.



Graph the ellipse given by each equation.

22. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

SOLUTION:

The ellipse is in standard form. The values of h and k are 0 and 0, so the center is at (0, 0).

$a = \sqrt{9}$ or 3

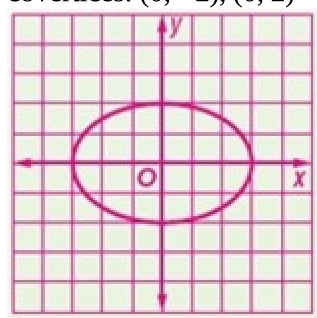
$b = \sqrt{4}$ or 2

$c = \sqrt{5} \approx 2.2$

orientation: horizontal

vertices: $(-3, 0)$, $(3, 0)$

covertices: $(0, -2)$, $(0, 2)$



23. $\dfrac{(x-3)^2}{16} + \dfrac{(y+6)^2}{25} = 1$

SOLUTION:

The ellipse is in standard form. The values of h and k are 3 and -6, so the center is at $(3, -6)$.

$a = \sqrt{25}$ or 5

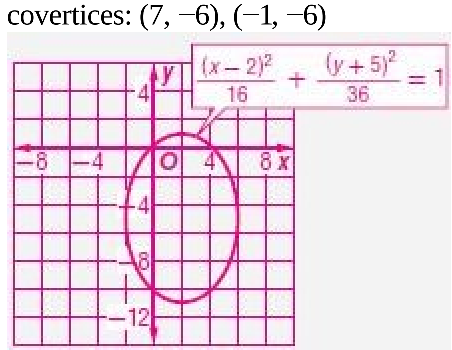
$b = \sqrt{16}$ or 4

$c = \sqrt{9} = 3$

orientation: vertical

vertices: $(3, -1)$, $(3, -11)$

covertices: $(7, -6)$, $(-1, -6)$



Write an equation for the ellipse with each set of characteristics.

24. vertices (7, –3), (3, –3); foci (6, –3), (4, –3)

SOLUTION:

The distance between the vertices is $2a$.

$$\begin{aligned} 2a &= \lvert 7 - 3 \rvert \\ a &= 2 \\ a^2 &= 4 \end{aligned}$$

The distance between the foci is $2c$.

$$\begin{aligned} 2c &= \lvert 6 - 4 \rvert \\ c &= 1 \\ c^2 &= 1 \end{aligned}$$

$$\begin{aligned} b^2 &= a^2 - c^2 \\ b^2 &= 3 \end{aligned}$$

The center is the midpoint of the vertices.

$$\frac{7+3}{2} = 5$$

center: (5, −3)

$$\frac{(x-5)^2}{4} + \frac{(y+3)^2}{3} = 1$$

25. foci (1, 2), (9, 2); length of minor axis equals 6

SOLUTION:

The length of the minor axis is $2b$. So, $b = 3$ and $b^2 = 9$.

The distance between the foci is $2c$.

$$\begin{aligned} 2c &= \lvert 9 - 1 \rvert \\ c &= 4 \\ c^2 &= 16 \end{aligned}$$

$$\begin{aligned} a^2 &= b^2 + c^2 \\ a^2 &= 25 \end{aligned}$$

The center is the midpoint of the foci.

$$\frac{9+1}{2} = 5$$

center: (5, 2)

$$\frac{(x-5)^2}{25} + \frac{(y-2)^2}{9} = 1$$

26. major axis (–4, 4) to (6, 4); minor axis (1, 1) to (1, 7)

SOLUTION:

The length of the minor axis is $2b$, so $b = 3$ and $b^2 = 9$.

The length of the major axis is $2a$, so $a = 5$ and $a^2 = 25$.

The center is the midpoint of the major axis.

$$\frac{4+6}{2} = 1$$

center: (1, 4)

$$\frac{(x-1)^2}{25} + \frac{(y-4)^2}{9} = 1$$

Write each equation in standard form. Identify the related conic.

27. $x^2 - 2x + y^2 - 4y - 25 = 0$

SOLUTION:

$$\begin{aligned} x^2 - 2x + y^2 - 4y - 25 &= 0 \\ x^2 - 2x + 1 + y^2 - 4y + 4 - 25 - 1 - 4 &= 0 \\ (x-1)^2 + (y-2)^2 - 30 &= 0 \\ (x-1)^2 + (y-2)^2 &= 30 \end{aligned}$$

The conic is a circle because the equation is of the form $(x - h)^2 + (y - k)^2 = r^2$.

28. $4x^2 + 24x + 25y^2 - 300y + 836 = 0$

SOLUTION:

$$\begin{aligned} 4x^2 + 24x + 25y^2 - 300y + 836 &= 0 \\ 4(x^2 + 6x) + 25(y^2 - 12y) + 836 &= 0 \\ 4(x^2 + 6x + 9) + 25(y^2 - 12y + 36) + 836 - 4(9) - 25(36) &= 0 \\ 4(x+3)^2 + 25(y-6)^2 &= 100 \\ \frac{(x+3)^2}{25} + \frac{(y-6)^2}{4} &= 1 \end{aligned}$$

The related conic is an ellipse because $a \neq b$ and the graph is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

29. $x^2 - 4x + 4y + 24 = 0$

SOLUTION:

$$\begin{aligned} x^2 - 4x + 4y + 24 &= 0 \\ x^2 - 4x + 4 + 4y + 24 - 4 &= 0 \\ (x-2)^2 + 4(y+5) &= 0 \\ (x-2)^2 &= -4(y+5) \end{aligned}$$

Because only one term is squared, the graph is a parabola.

Graph the hyperbola given by each equation.

30. $\frac{(y+3)^2}{30} - \frac{(x-6)^2}{8} = 1$

SOLUTION:

The equation is in standard form, and $h = 6$ and $k = -3$. Because $a^2 = 30$ and $b^2 = 8$, $a = \sqrt{30} \approx 5.5$ and $b = \sqrt{8}$. The values of a and b can be used to find c.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= (\sqrt{30})^2 + (\sqrt{8})^2 \\ c^2 &= 30 + 8 \\ c &= \sqrt{38} \approx 6.2 \end{aligned}$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

orientation: In the standard form of the equation, the x-term is being subtracted. Therefore, the orientation of the hyperbola is vertical.

center: $(h, k) = (6, -3)$

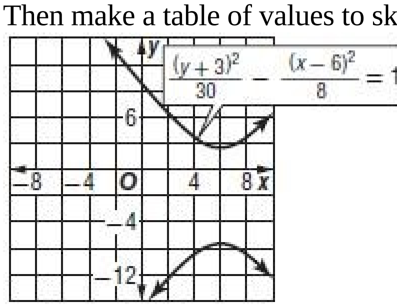
vertices: $(h, k \pm a) = (6, 2.5)$ and $(6, -8.5)$

foci: $(h, k \pm c) = (6, 3.2)$ and $(6, -9.2)$

asymptotes:

$$\begin{aligned} y - k &= \pm\tfrac{a}{b}(x - h) \\ y - (-3) &= \pm\frac{\sqrt{30}}{2\sqrt{2}}(x - 6) \\ y + 3 &= \pm\frac{\sqrt{15}}{2}(x - 6) \\ y + 3 &= \pm\left[\frac{\sqrt{15}}{2}x - 3\sqrt{15}\right] \\ y &= \pm\left[\frac{\sqrt{15}}{2}x - 3\sqrt{15}\right] - 3 \\ y &= \frac{\sqrt{15}}{2}x - 3\sqrt{15} - 3 \quad OR \\ y &= -\frac{\sqrt{15}}{2}x + 3\sqrt{15} - 3 \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



Study Guide and Review

31. $\frac{(x+7)^2}{18} - \frac{(y-6)^2}{36} = 1$

SOLUTION:

$$\frac{(x+7)^2}{18} - \frac{(y-6)^2}{36} = 1$$

The equation is in standard form, and $h = -7$ and $k = 6$. Because $a^2 = 18$ and $b^2 = 36$, $a = \sqrt{18} \approx 4.2$ and $b = 6$. The values of a and b can be used to find c.

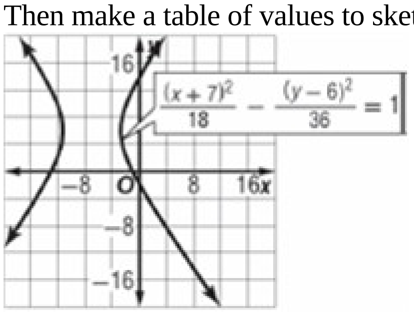
$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 18 + 36 \\ c^2 &= 54 \\ c &= \sqrt{54} \approx 7.3 \end{aligned}$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

orientation: In the standard form of the equation, the y–term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.
center: $(h, k) = (-7, 6)$
vertices: $(h \pm a, k) = (-11.2, 6)$ and $(-2.8, 6)$
foci: $(h \pm c, k) = (0.3, 6)$ and $(-14.3, 6)$
asymptotes:

$$\begin{aligned} y - k &= \pm\frac{b}{a}(x - h) \\ y - 6 &= \pm\frac{6}{3\sqrt{2}}(x - [-7]) \\ y - 6 &= \pm\sqrt{2}(x + 7) \\ y - 6 &= \pm\left[\sqrt{2}x + 7\sqrt{2}\right] \\ y &= \pm\left[\sqrt{2}x + 7\sqrt{2}\right] + 6 \\ y &= \left[\sqrt{2}x + 7\sqrt{2} + 6 \quad OR \\ y &= -\left[\sqrt{2}x - 7\sqrt{2} + 6 \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



32. $\frac{(y-1)^2}{4} - (x+1)^2 = 1$

SOLUTION:

$$\frac{(y-1)^2}{4} - (x+1)^2 = 1$$

The equation is in standard form, and $h = -1$ and $k = 1$. Because $a^2 = 4$ and $b^2 = 1$, $a = 2$ and $b = 1$. The values of a and b can be used to find c.

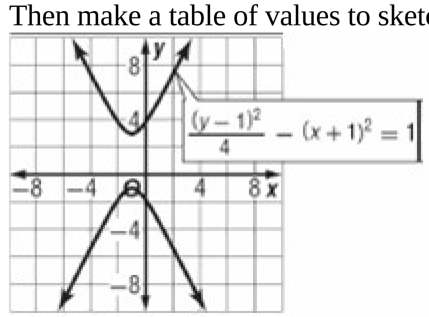
$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 4 + 1 \\ c^2 &= 5 \\ c &= \sqrt{5} \approx 2.2 \end{aligned}$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

orientation: In the standard form of the equation, the x–term is being subtracted. Therefore, the orientation of the hyperbola is vertical.
center: $(h, k) = (-1, 1)$
vertices: $(h, k \pm a) = (-1, 3)$ and $(-1, -1)$
foci: $(h, k \pm c) = (-1, 3.2)$ and $(-1, -1.2)$
asymptotes:

$$\begin{aligned} y - k &= \pm\frac{a}{b}(x - h) \\ y - 1 &= \pm\frac{2}{1}(x - [-1]) \\ y - 1 &= \pm 2(x + 1) \\ y - 1 &= \pm[2x + 2] \\ y &= \pm[2x + 2] + 1 \\ y &= 2x + 3 \quad OR \\ y &= -2x - 1 \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



33. $x^2 - y^2 - 2x + 4y - 7 = 0$

SOLUTION:

Convert the equation to standard form.

$$\begin{aligned} x^2 - y^2 - 2x + 4y - 7 &= 0 \\ x^2 - 2x + 1 - (y^2 - 4y + 4) - 7 - 1 + 4 &= 0 \\ (x-1)^2 - (y-2)^2 &= 4 \\ \frac{(x-1)^2}{4} - \frac{(y-2)^2}{4} &= 1 \end{aligned}$$

The equation is in standard form, and $h = 1$ and $k = 2$. Because $a^2 = 4$ and $b^2 = 4$, $a = 2$ and $b = 2$. The values of a and b can be used to find c.

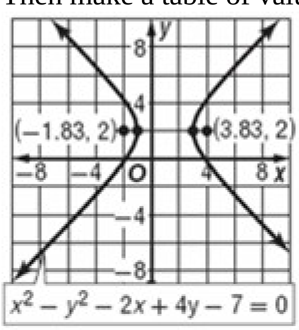
$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 4 + 4 \\ c^2 &= 8 \\ c &= \sqrt{8} \approx 2.8 \end{aligned}$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

orientation: In the standard form of the equation, the y–term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.
center: $(h, k) = (1, 2)$
vertices: $(h \pm a, k) = (-1, 2)$ and $(3, 2)$
foci: $(h \pm c, k) = (-1.8, 2)$ and $(3.8, 2)$
asymptotes:

$$\begin{aligned} y - k &= \pm\frac{b}{a}(x - h) \\ y - 2 &= \pm\frac{2}{2}(x - 1) \\ y - 2 &= \pm(x - 1) \\ y &= \pm[x + 1] + 2 \\ y &= x + 3 \quad OR \\ y &= -x + 1 \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



Write an equation for the hyperbola with the given characteristics.

34. vertices (7, 0), (−7, 0); conjugate axis length of 8

SOLUTION:

Because the y–coordinates of the vertices are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

The center is the midpoint of the segment between the vertices, or (0, 0). So, $h = 0$ and $k = 0$. The length of the conjugate axis of a hyperbola is $2b$. So, $2b = 8$, $b = 4$, and $b^2 = 16$. You can find a by determining the distance from a vertex to the center. One vertex is located at (7, 0), which is 7 units from (0, 0). So, $a = 7$.

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{x^2}{49} - \frac{y^2}{16} = 1$.

35. foci (0, 5), (0, −5); vertices (0, 3), (0, −3)

SOLUTION:

Because the y−coordinates of the vertices are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-h)^2}{a^2}$ – $\frac{(y-k)^2}{b^2}$ = 1.

The center is the midpoint of the segment between the foci, or (0, 0). So, h = 0 and k = 0. You can find c by determining the distance from a focus to the center. One focus is located at (0, 5), which is 5 units from (0, 0). So, c = 5 and c^2 = 25. You can find a by determining the distance from a vertex to the center. One vertex is located at (0, 3), which is 3 units from (0, 0). So, a = 3 and a^2 = 9.

Now you can use the values of c and a to find b.

$$b^2 = c^2 - a^2$$
$$b^2 = 25 - 9$$
$$b^2 = 16$$
$$b = 4$$

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{x^2}{9} - \frac{y^2}{16}$ = 1.

36. foci (1, 15), (1, −5); transverse axis length of 16

SOLUTION:

Because the x−coordinates of the foci are the same, the transverse axis is vertical, and the standard form of the equation is $\frac{(y-h)^2}{a^2} - \frac{(x-k)^2}{b^2}$ = 1.

The center is the midpoint of the segment between the foci, or (1, 5). So, h = −1 and k = −5. You can find c by determining the distance from a focus to the center. One focus is located at (1, 15), which is 10 units from (1, 5). So, c = 10 and c^2 = 100. The length of the transverse axis is 16, so a = 8 and a^2 = 64.

Now you can use the values of c and a to find b.

$$b^2 = c^2 - a^2$$
$$b^2 = 100 - 64$$
$$b^2 = 36$$
$$b = 6$$

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{(y-5)^2}{64} - \frac{(x-1)^2}{36}$ = 1.

37. vertices (2, 0), (−2, 0); asymptotes $y = \pm\frac{3}{2}x$

SOLUTION:

Because the y−coordinates of the foci are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-k)^2}{a^2} - \frac{(y-h)^2}{b^2} = 1$.

The center is the midpoint of the segment between the vertices, or (0, 0).So, $h = 0$ and $k = 0$. To find a, determine the distance from a vertex to the center. One vertex is located at (2, 0), which is 2 units from (0, 0). So, $a = 2$ and $a^2 = 4$. Because the slopes of the asymptotes are $\pm\frac{b}{a}$, using the positive slope $\frac{3}{2}$, $b = 3$ and $b^2 = 9$.

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

Use the discriminant to identify each conic section.

38. $x^2 - 4y^2 - 6x - 16y - 11 = 0$

SOLUTION:

$$\begin{aligned} B^2 - 4AC &= 0 - 4(1)(-4) \\ &= 16 \end{aligned}$$

The discriminant is greater than 0, so the conic is a hyperbola.

39. $4y^2 - x - 40y + 107 = 0$

SOLUTION:

$$\begin{aligned} B^2 - 4AC &= 0 - 0(4) \\ &= 0 \end{aligned}$$

The discriminant is 0, so the conic is a parabola.

40. $9x^2 + 4y^2 + 162x + 8y + 732 = 0$

SOLUTION:

$$\begin{aligned} B^2 - 4AC &= 0 - 4(9)(4) \\ &= -144 \end{aligned}$$

The discriminant is less than 0 and $A \neq C$, so the conic is an ellipse.

Use a graphing calculator to graph the conic given by each equation.

41. $x^2 - 4xy + y^2 - 2y - 2x = 0$

SOLUTION:

$$\begin{aligned} x^2 - 4xy + y^2 - 2y - 2x &= 0 \\ y^2 + (-4x - 2)y + x^2 - 2x &= 0 \end{aligned}$$

$$y = \frac{-(-4x-2) \pm \sqrt{(-4x-2)^2 - 4(1)(x^2 - 2x)}}{2(1)}$$

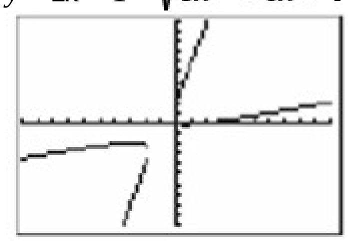
$$y = \frac{4x + 2 \pm \sqrt{16x^2 + 16x + 4 - 4x^2 + 8x}}{2}$$

$$y = \frac{4x + 2 \pm \sqrt{12x^2 + 24x + 4}}{2}$$

$$y = \frac{4x + 2 \pm \sqrt{4(3x^2 + 6x + 1)}}{2}$$

$$y = \frac{4x + 2 \pm 2\sqrt{3x^2 + 6x + 1}}{2}$$

$$y = 2x + 1 \pm \sqrt{3x^2 + 6x + 1}$$



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42. $x^2 - 3xy + y^2 - 3y - 6x + 5 = 0$

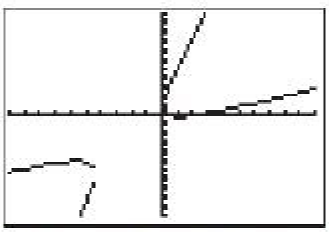
SOLUTION:

$$\begin{aligned} x^2 - 3xy + y^2 - 3y - 6x + 5 &= 0 \\ y^2 + (-3x - 3)y + x^2 - 6x + 5 &= 0 \end{aligned}$$

$$y = \frac{-(-3x-3) \pm \sqrt{(-3x-3)^2 - 4(1)(x^2 - 6x + 5)}}{2(1)}$$

$$y = \frac{3x + 3 \pm \sqrt{9x^2 + 18x + 9 - 4x^2 + 24x - 20}}{2}$$

$$y = \frac{3x + 3 \pm \sqrt{5x^2 + 42x - 11}}{2}$$



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43. $2x^2 + 2y^2 - 8xy + 4 = 0$

SOLUTION:

$$\begin{aligned} 2x^2 + 2y^2 - 8xy + 4 &= 0 \\ 2y^2 + (-8x)y + 2x^2 + 4 &= 0 \end{aligned}$$

$$y = \frac{-(-8x) \pm \sqrt{(-8x)^2 - 4(2)(2x^2 + 4)}}{2(2)}$$

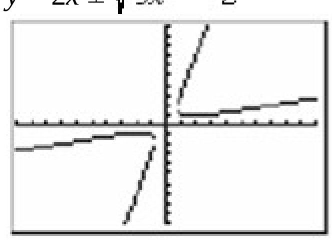
$$y = \frac{8x \pm \sqrt{64x^2 - 16x^2 - 32}}{4}$$

$$y = \frac{8x \pm \sqrt{48x^2 - 32}}{4}$$

$$y = \frac{8x \pm \sqrt{16(3x^2 - 2)}}{4}$$

$$y = \frac{8x \pm 4\sqrt{3x^2 - 2}}{4}$$

$$y = 2x \pm \sqrt{3x^2 - 2}$$



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44. $3x^2 + 9xy + y^2 = 0$

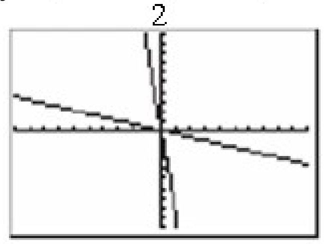
SOLUTION:

$$\begin{aligned} 3x^2 + 9xy + y^2 &= 0 \\ y^2 + (9x)y + 3x^2 &= 0 \end{aligned}$$

$$y = \frac{-9x \pm \sqrt{(9x)^2 - 4(1)(3x^2)}}{2(1)}$$

$$y = \frac{-9x \pm \sqrt{81x^2 - 12x^2}}{2}$$

$$y = \frac{-9x \pm \sqrt{69x^2}}{2}$$



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45. $4x^2 - 2xy + 8y^2 - 7 = 0$

SOLUTION:

$$\begin{aligned} 4x^2 - 2xy + 8y^2 - 7 &= 0 \\ 8y^2 + (-2x)y + 4x^2 - 7 &= 0 \end{aligned}$$

$$y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(8)(4x^2 - 7)}}{2(8)}$$

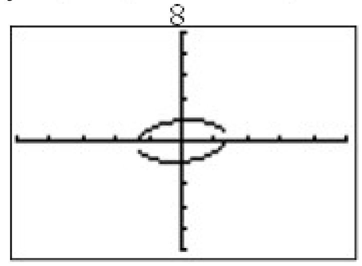
$$y = \frac{2x \pm \sqrt{4x^2 - 128x^2 + 224}}{16}$$

$$y = \frac{2x \pm \sqrt{224 - 124x^2}}{16}$$

$$y = \frac{2x \pm \sqrt{4(56 - 31x^2)}}{16}$$

$$y = \frac{2x \pm 2\sqrt{56 - 31x^2}}{16}$$

$$y = \frac{x \pm \sqrt{56 - 31x^2}}{8}$$



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Write each equation in the $x'y'$–plane for the given value of θ. Then identify the conic.

46. $x^2 + y^2 = 4$; $\theta = \frac{\pi}{4}$

SOLUTION:

Find the equations for x and y.

$x = x' \cos\theta - y' \sin\theta$

$x = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'$

$y = x' \sin\theta + y' \cos\theta$

$y = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$

Substitute into the original equation.

$$\begin{aligned} x^2 + y^2 &= 4 \\ \left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right)^2 + \left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y\right)^2 &= 4 \\ \frac{1}{2}(x')^2 - x'y' + \frac{1}{2}(y')^2 + \frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2 &= 4 \\ (x')^2 + (y')^2 &= 4 \end{aligned}$$

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(1)(1) \\ &= 0 - 4 \\ &= -4 \end{aligned}$$

$B^2 - 4AC$ is less than 0, $B = 0$, and $A = C$, so the conic is a circle.

47. $x^2 - 2x + y = 5$; $\theta = \frac{\pi}{3}$

SOLUTION:

Find the equations for x and y.

$x = x' \cos\theta - y' \sin\theta$

$x = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y'$

$y = x' \sin\theta + y' \cos\theta$

$y = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y'$

Substitute into the original equation.

$$\begin{aligned} x^2 - 2x + y &= 5 \\ \left(\frac{1}{2}x' - \frac{\sqrt{3}}{2}y'\right)^2 - 2\left(\frac{1}{2}x' - \frac{\sqrt{3}}{2}y'\right) + \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' &= 5 \\ \frac{1}{4}(x')^2 - \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2 - x' + \sqrt{3}y' + \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' &= 5 \\ (x')^2 - 2\sqrt{3}x'y' + 3(y')^2 - 4x' + 4\sqrt{3}y' + 2\sqrt{3}x' + 2y' &= 20 \\ (x')^2 - 2\sqrt{3}x'y' + 3(y')^2 + (2\sqrt{3} - 4)x' + (4\sqrt{3} + 2)y' &= 20 \end{aligned}$$

$$\begin{aligned} B^2 - 4AC &= \left(-2\sqrt{3}\right)^2 - 4(1)(3) \\ &= 12 - 12 \\ &= 0 \end{aligned}$$

$B^2 - 4AC = 0$, so the conic is a parabola.

48. $x^2 - 4y^2 = 4$; $\theta = \frac{\pi}{2}$

SOLUTION:

Find the equations for x and y.

$x = x' \cos \theta - y' \sin \theta$
$x = x' \cos 90 - y' \sin 90$
$x = -y'$

$y = x' \sin \theta + y' \cos \theta$
$y = x' \sin 90 + y' \cos 90$
$y = x'$

Substitute into the original equation.

$$\begin{aligned} (-y')^2 - 4(x')^2 &= 4 \\ (y')^2 - 4(x')^2 &= 4 \end{aligned}$$

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(1)(-4) \\ &= 0 + 16 \\ &= 16 \end{aligned}$$

$B^2 - 4AC$ is greater than 0, so the conic is a hyperbola.

49. $9x^2 + 4y^2 = 36$; $\theta = 90°$

SOLUTION:

$9x^2 + 4y^2 = 36$, $\theta = \frac{\pi}{2}$

Find the equations for x and y.

$x = x' \cos \theta - y' \sin \theta$
$x = x' \cos 90 - y' \sin 90$
$x = -y'$

$y = x' \sin \theta + y' \cos \theta$
$y = x' \sin 90 + y' \cos 90$
$y = x'$

Substitute into the original equation.

$9(-y')^2 + 4(x')^2 = 36$

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(9)(4) \\ &= 0 - 72 \\ &= -72 \end{aligned}$$

$B^2 - 4AC$ is less than 0, $B = 0$, and $A \neq C$, so the conic is an ellipse.

Sketch the curve given by each pair of parametric equations over the given interval.

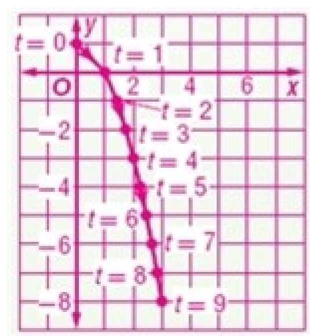
50. $x = \sqrt{t}$, $y = 1 - t$; $0 \le t \le 9$

SOLUTION:

Make a table of values for $0 \le t \le 9$.

t	x	y
0	0	1
1	1	0
2	1.4	−1
3	1.7	−2
4	2	−3
5	2.2	−4
6	2.4	−5
7	2.6	−6
8	2.8	−7
9	3	−8

Plot the (x, y) coordinates for each t−value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t moves from 0 to 9.



51. $x = t + 2, y = t^2 - 4; -4 \le t \le 4$

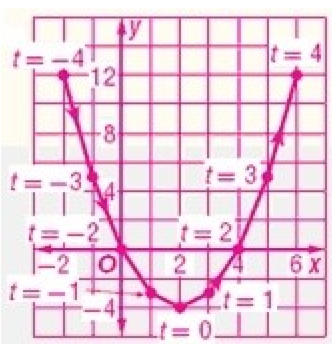
SOLUTION:

Make a table of values for $-4 \le t \le 4$.

$x = t + 2, y = t^2 - 4; -4 \le t \le 4$

t	x	y
−4	−2	12
−3	−1	5
−2	0	0
−1	1	−3
0	2	−4
1	3	−3
2	4	0
3	5	5
4	6	12

Plot the (x, y) coordinates for each t–value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t moves from −4 to 4.



Write each pair of parametric equations in rectangular form. Then graph the equation.

52. $x = t + 5$ and $y = 2t - 6$

SOLUTION:

$x = t + 5$ and $y = 2t - 6$

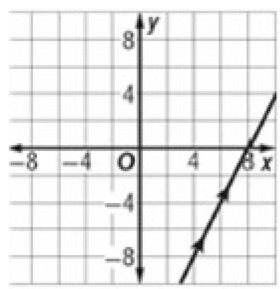
Solve for t.

$$\begin{aligned} x &= t + 5 \\ x - 5 &= t \end{aligned}$$

Substitute into the other equation.

$y = 2t - 6$
$y = 2(x - 5) - 6$
$y = 2x - 16$

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases. Since substituting larger values for t in $x = t + 5$ produces increasing values of x, the orientation moves from left to right.



53. $x = 2t$ and $y = t^2 - 2$

SOLUTION:

$x = 2t$ and $y = t^2 - 2$

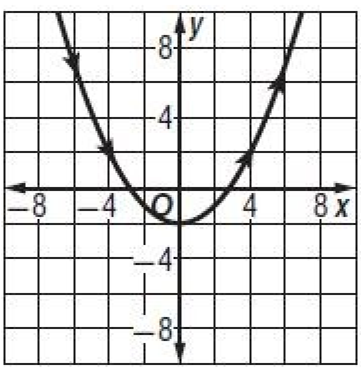
Solve for t.

$$\begin{aligned} x &= 2t \\ \frac{1}{2}x &= t \end{aligned}$$

Substitute into the other equation.

$$\begin{aligned} y &= t^2 - 2 \\ y &= \left(\frac{1}{2}x\right)^2 - 2 \\ y &= \frac{1}{4}x^2 - 2 \end{aligned}$$

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases. Since substituting larger values for t in $x = t + 5$ produces increasing values of x, the orientation moves from left to right.



54. $x = t^2 + 3$ and $y = t^2 - 4$

SOLUTION:

$x = t^2 + 3$ and $y = t^2 - 4$

Solve for t.

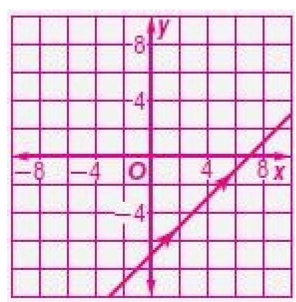
$$\begin{aligned} x &= t^2 + 3 \\ x - 3 &= t^2 \\ \pm\sqrt{x-3} &= t \end{aligned}$$

Substitute into the other equation.

$$\begin{aligned} y &= t^2 - 4 \\ &= (\pm\sqrt{x-3})^2 - 4 \\ &= x - 3 - 4 \\ &= x - 7 \end{aligned}$$

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases.

Since substituting larger values for t in $x = t^2 + 3$ produces increasing values of x, the orientation moves from left to right.



55. $x = t^2 - 1$ and $y = 2t + 1$

SOLUTION:

$x = t^2 - 1$ and $y = 2t + 1$

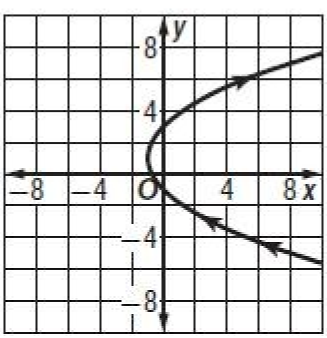
Solve for t.

$$\begin{aligned} y &= 2t + 1 \\ y - 1 &= 2t \\ \frac{y-1}{2} &= t \end{aligned}$$

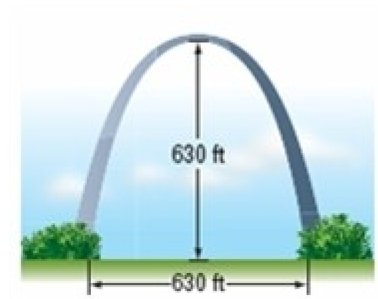
Substitute into the other equation.

$$\begin{aligned} x &= t^2 - 1 \\ x &= \left(\frac{y-1}{2}\right)^2 - 1 \\ x + 1 &= \frac{(y-1)^2}{4} \\ 4(x+1) &= (y-1)^2 \end{aligned}$$

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases. Since substituting larger values for t in $y = 2t + 1$ produces increasing values of x, the orientation moves from bottom to top.



56. **MONUMENTS** The St. Louis Arch is in the shape of a *catenary*, which resembles a parabola.
a. Write an equation for a parabola that would approximate the shape of the arch.
b. Find the location of the focus of this parabola.



SOLUTION:

a. With the vertex at the origin, the parabola goes through (0, 0), (315, −630), and (−315, −630). Use a system of equations to determine a, b, and c in $y = ax^2 + bx + c$.

$$\begin{aligned} &0a + 0b + 0c = 0 \\ &(315)^2a + 315b + 0c = -630 \\ &(-315)^2a - 315b + 0c = -630 \end{aligned}$$

$c = 0$

$$\begin{aligned} 99{,}225a + 315b &= -630 \\ \underline{+\,99{,}225a - 315b} &= \underline{-630} \\ 198{,}450a &= -1260 \\ a &= \frac{2}{315} \end{aligned}$$

Substitute and solve for b.

$$\begin{aligned} 99{,}225 \cdot \frac{2}{315} + 315b &= -630 \\ 630b &= -630 \\ b &= -1 \end{aligned}$$

The equation is $y = -\frac{2}{315}x^2 + 630$.

b. Convert the equation to standard form.

$$\begin{aligned} y &= -\frac{2}{315}x^2 + 630 \\ y - 630 &= -\frac{2}{315}x^2 \\ -\frac{315}{2}(y - 630) &= x^2 \end{aligned}$$

The value of p is $-\frac{315}{2} \div 4$ or −39.375. The focus of the parabola is 630 − 39.375 or 590.625 feet above the ground.

57. **WATER DYNAMICS** A rock dropped in a pond will produce ripples of water made up of expanding concentric circles. Suppose the radii of the circles expand at 3 inches per second.

a. Write an equation for the circle 10 seconds after the rock is dropped in the pond. Assume that the point where the rock is dropped is the origin.

b. One concentric circle has equation $x^2 + y^2 = 225$. How many seconds after the rock is dropped does it take for the circle to have this equation?

SOLUTION:

a. After 10 seconds, the radius of the circle will be 3 · 10 or 30 inches. With the center at the origin, the equation for the circle is $x^2 + y^2 = 900$.

b. In this equation, $r^2 = 225$, so $r = 15$. Since the radii are expanding by 3 inches per second, it will take 15 ÷ 3 or 5 seconds.

58. **ENERGY** Cooling towers at a power plant are in the shape of a hyperboloid. The cross section of a hyperboloid is a hyperbola.

a. Write an equation for the cross section of a tower that is 50 feet tall and 30 feet wide.

b. If the ratio of the height to the width of the tower increases, how is the equation affected?

SOLUTION:

a. The height of the cross section is 50 feet, which is $2a$. So, $a = 25$ and $a^2 = 625$. The width of the cross section is 30 feet, which is $2b$. So, $b = 15$ and $b^2 = 225$.

The transverse axis is vertical, so the equation for the cross section is $\frac{x^2}{225} - \frac{y^2}{625} = 1$.

b. Sample answer: The ratio of the denominator associated with y to the denominator associated with x will increase.

59. **SOLAR DISH** Students building a parabolic device to capture solar energy for cooking marshmallows placed at the focus must plan for the device to be easily oriented. Rotating the device directly toward the Sun's rays maximizes the heat potential.

a. After the parabola is rotated 30° toward the Sun, the equation of the parabola used to create the device in the $x'y'$–plane is $y' = 0.25(x')^2$. Find the equation of the parabola in the xy–plane.

b. Graph the rotated parabola.

SOLUTION:

a. $y' = 0.25(x')^2$, $\theta = 30$

Use the rotation formulas for x' and y' to find the equation of the rotated conic in the xy-plane.

$x' = x \cos \theta + y \sin \theta$

$x' = \frac{\sqrt{3}}{2}x + \frac{1}{2}y$

$y' = y \cos \theta - x \sin \theta$

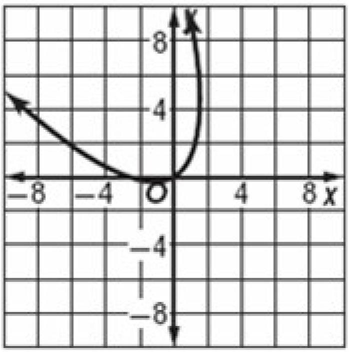
$y' = \frac{\sqrt{3}}{2}y - \frac{1}{2}x$

Substitute these values into the original equation.

$$y' = 0.25(x')^2$$
$$\frac{\sqrt{3}}{2}y - \frac{1}{2}x = 0.25\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y\right)^2$$
$$\frac{\sqrt{3}}{2}y - \frac{1}{2}x = 0.25\left(\frac{3}{4}x^2 + \frac{\sqrt{3}}{2}xy + \frac{1}{4}y^2\right)$$
$$\frac{\sqrt{3}}{2}y - \frac{1}{2}x = \frac{3}{16}x^2 + \frac{\sqrt{3}}{8}xy + \frac{1}{16}y^2$$
$$8\sqrt{3}y - 8x = 3x^2 + 2\sqrt{3}xy + y^2$$
$$0 = 3x^2 + 2\sqrt{3}xy + y^2 + 8x - 8\sqrt{3}y$$

b. Solve for y.

$$y^2 + (2\sqrt{3}x - 8\sqrt{3})y + 3x^2 + 8x = 0$$
$$y = \frac{8\sqrt{3} - 2\sqrt{3}x \pm \sqrt{(2\sqrt{3}x - 8\sqrt{3})^2 - 4(3x^2 + 8x)}}{2}$$
$$y = \frac{8\sqrt{3} - 2\sqrt{3}x \pm \sqrt{12x^2 - 96x + 192 - 12x^2 - 32x}}{2}$$
$$y = \frac{8\sqrt{3} - 2\sqrt{3}x \pm \sqrt{192 - 128x}}{2}$$



60. **GEOMETRY** Consider $x_1(t) = 4 \cos t, y_1(t) = 4 \sin t, x_2(t) = 4 \cos 2t$, and $y_2(t) = 4 \sin 2t$.

a. Compare the graphs of the two sets of equations; x_1 and y_1; and x_2 and y_2.

b. Write parametric equations for a circle of radius 6 that complete its graph in half the time of $x_1(t)$ and $y_1(t)$.

c. Write the equations from part **b** in rectangular form.

SOLUTION:

a. Sample answer: They are both the graph of a circle of radius 4, with equations $x_2(t)$ and $y_2(t)$ traveling around the circle twice as fast.

b. Use a coefficient equal to the length of the radius. To complete it in half the time, replace t with $\frac{1}{2}t$.

The equations are $x(t) = 6\cos\left(\frac{1}{2}t\right)$, $y(t) = 6\sin\left(\frac{1}{2}t\right)$.

c. Solve the equations for sin θ and cos θ. Then use a trigonometric identity.

$$x = 6\cos\left(\frac{1}{2}t\right)$$
$$\frac{x}{6} = \cos\left(\frac{1}{2}t\right)$$

$$y = 6\sin\left(\frac{1}{2}t\right)$$
$$\frac{y}{6} = \sin\left(\frac{1}{2}t\right)$$

$$\sin^2\left(\frac{1}{2}t\right) + \cos^2\left(\frac{1}{2}t\right) + = \left(\frac{y}{6}\right)^2 + \left(\frac{x}{6}\right)^2$$
$$1 = \frac{y^2}{36} + \frac{x^2}{36}$$
$$x^2 + y^2 = 36$$