

## Complex Numbers Tutorial 9B: Polar and Exponential Forms Solutions

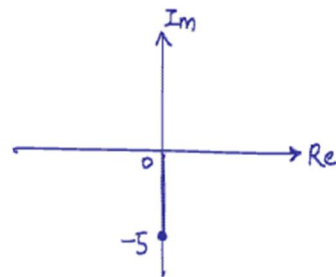
### Basic Mastery Questions

$$\begin{aligned} \text{i)} \quad |1 + \sqrt{3}i| &= \sqrt{1+3} = 2 \\ \arg(1 + \sqrt{3}i) &= \tan^{-1}\sqrt{3} = \frac{\pi}{3} \\ \therefore 1 + \sqrt{3}i &= 2 \left[ \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right] \# \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad |1 - i| &= \sqrt{1+1} = \sqrt{2} \\ \arg(1 - i) &= \tan^{-1}(-1) = -\frac{\pi}{4} \\ \therefore 1 - i &= \sqrt{2} \left[ \cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right] \# \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad \left| \frac{(1 + \sqrt{3}i)^4}{(1 - i)^6} \right| &= \frac{|1 + \sqrt{3}i|^4}{|1 - i|^6} = \frac{2^4}{2^3} = 2 \\ \arg\left[\frac{(1 + \sqrt{3}i)^4}{(1 - i)^6}\right] &= 4\arg(1 + \sqrt{3}i) - 6\arg(1 - i) \\ &= 4\left(\frac{\pi}{3}\right) - 6\left(-\frac{\pi}{4}\right) = \frac{17\pi}{6} \equiv \frac{5\pi}{6} \\ \therefore \frac{(1 + \sqrt{3}i)^4}{(1 - i)^6} &= 2\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) \# \end{aligned}$$

$$\begin{aligned} 2i) \quad 5\left(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}\right)^5 &= 5(0 - i)^5 \\ &= 5(-i)^5 \\ &= -5i \# \end{aligned}$$



ii)

$$\begin{aligned} &\left( \sin\left(-\frac{\pi}{3}\right) + i \cos\left(-\frac{\pi}{3}\right) \right)^6 \\ &= \left( \cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right) \right)^6 \\ &= \left( \cos\left(\frac{5\pi}{6}\right) + i \sin\left(\frac{5\pi}{6}\right) \right)^6 \\ &= \cos 5\pi + i \sin 5\pi = -1 \end{aligned}$$

