

H1 CA2

Qn	Solutions	
1	$\frac{d}{dx} \ln(5 - 6x^2) = -\frac{12x}{5 - 6x^2}$	

Qn	Solutions	
2	From G.C, the numerical value of gradient of D at point $x = 1$ is 2.	
	Equation of tangent at D is: $y - 0 = 2(x - 1)$ $y = 2x - 2$	

Qn	Solutions	
3(a)	$\int \frac{(3x^2 - 1)^2}{x} dx$ $= \int \frac{9x^4 - 6x^2 + 1}{x} dx$ $= \int 9x^3 - 6x + \frac{1}{x} dx$ $= \frac{9}{4}x^4 - 3x^2 + \ln x + c$	
3(b)	$\int \frac{1}{2\sqrt{1 - \pi x}} dx$ $= \frac{1}{2} \int (1 - \pi x)^{-\frac{1}{2}} dx$ $= \frac{1}{2} \frac{(1 - \pi x)^{\frac{1}{2}}}{\frac{1}{2}(-\pi)} + c$ $= \frac{(1 - \pi x)^{\frac{1}{2}}}{-\pi} + c$	

Qn	Solutions	
4	$R = \int_0^2 e^{1-\frac{1}{2}x} + 3x \, dx$ $= \left[\frac{e^{1-\frac{1}{2}x}}{-\frac{1}{2}} + \frac{3x^2}{2} \right]_0^2$ $= \left[\frac{e^{1-\frac{1}{2}(2)}}{-\frac{1}{2}} + \frac{3(2)^2}{2} \right] - \left[\frac{e^{1-\frac{1}{2}(0)}}{-\frac{1}{2}} + \frac{3(0)^2}{2} \right]$ $= 4 + 2e$ $p = 4, q = e$ <p>Since $e^{1-\frac{1}{2}x} = m - 3x$ has no real roots, $m < 1.2494$ $m < 1.25$ (to 3 s.f.)</p>	
Qn	Solutions	
5(i)	<p>Curve surface area = $\pi r \times \frac{4\pi}{r^2} = \frac{4\pi^2}{r}$</p> <p>Area of rectangle = $2r \times \frac{4\pi}{r^2} = \frac{8\pi}{r}$</p> <p>Total surface area of the trash bin = $\frac{8\pi}{r} + \frac{4\pi^2}{r} + \frac{1}{2}\pi r^2$</p>	
(ii)	$A = \frac{8\pi}{r} + \frac{4\pi^2}{r} + \frac{1}{2}\pi r^2$ $\frac{dA}{dr} = -\frac{8\pi}{r^2} - \frac{4\pi^2}{r^2} + \pi r$ <p>To minimize surface area of the trash bin, $\frac{dA}{dr} = 0$</p> $-\frac{8\pi}{r^2} - \frac{4\pi^2}{r^2} + \pi r = 0$ $8\pi + 4\pi^2 = \pi r^3$ $r^3 = 4\pi + 8$	
	<p>[Hence]</p> <p>Ratio of diameter of semi-circular surface to height of the trash bin</p> $= 2r : \frac{4\pi}{r^2}$ $= r^3 : 2\pi$ $= 4\pi + 8 : 2\pi$ $= 2\pi + 4 : \pi$	

Qn	Solution													
6(i)	$C = (35)(500) + (0.95)(5)x - 30e^{0.01x}$ $\therefore C = 17500 + 4.75x - 30e^{0.01x}$													
6(ii)	$\frac{dC}{dx} = 4.75 - 30(0.01)e^{0.01x}$ $= 4.75 - 0.3e^{0.01x}$ <p>For maximum or minimum, $\frac{dC}{dx} = 0$.</p> $4.75 - 0.3e^{0.01x} = 0$ $x = 276.212$ <p>When $x = 276.212$, $C = \\$18337.01$.</p>													
	<p>Using 1st derivative test,</p> <table><tr><td>x</td><td>276.10</td><td>276.212</td><td>276.23</td></tr><tr><td>$\frac{dC}{dx}$</td><td>0.0053045</td><td>0</td><td>-</td></tr><tr><td>Slope</td><td>/</td><td>-</td><td>\</td></tr></table> <p>OR</p> <p>Using 2nd derivative test,</p> $\frac{d^2C}{dx^2} = -0.003e^{0.01x} < 0 \quad (\text{since } e^{0.01x} > 0)$ <p>Hence maximum C.</p>	x	276.10	276.212	276.23	$\frac{dC}{dx}$	0.0053045	0	-	Slope	/	-	\	
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$\frac{dC}{dx}$	0.0053045	0	-											
Slope	/	-	\											