MAT235

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1 Recap

we've covered ch 10, 12, 13 before the break. Covered Parametric equaltions and polar coords, vectors of geometry, vector functions, and scalar functions (differential calculus)

Test 3 coveres chapter 15 and 16

2 Chapter 15: Integral Calculus

15.1 For single-variable calculus,

$$y = f(x)$$

Integral $\int_a^b f(x)\,dx$ represents the area under the curve, for some a and b.

This is most commonly created using Riemann sums

- Chop up the graph into equally sized partitions on the X axis
- Chose an xi which can either be a left point, right point, or a midpoint
- finally evaluate xi using the function f(x) and multiply it by the size of the parition
- in essence,

  $$A = \Sigma_0^n f(xi) * \Delta x$$

  represents the area under the curve estimate
- take the limit as n approaches 0, and it becomes the definition of an integral

Now assume:

$$z = f(x, y)$$

this creates a rectangular region underneath the curve, where $R = [a, b]X[c, d]$ which is equal to $(x, y) : a <= x <= bandc <= y <= d$

Calculate the volume under the surface of f(x,y) inside of R: Integral

$$\int \int f(x) \, dx \ f(x) \, dx$$

Subside R into smaller sub rectangles

$$R_i = [x_0, ...x_i]X[y_0, ...y_i]$$

$a = x_0 < x_1 < ... < x_n = b$ $c = y_0 < y_1 < ... < y_n = d$ Assuming each rectangle is equally spaced:

$$\delta x = (b - a)/n^{\delta} y = (d - c)/n$$

therefore, for each sub rectangle

$$\Delta A = \Delta x * \Delta y$$

with this knowledge, and definition:

- Choose an arbitrary point, $(x_j i, y_j i)$
- evaluate $f(x_j i, y_j i)$ -¿ height, $Vol = f(x_j i, y_j i)\Delta A$
- $$A = \Sigma_{i=0}^{n} \Sigma_{j=0}^{n} f(x_j i, y_j i) * \Delta A$$

  represents the area under the curve estimate

lets try another strategy - iterated integrals consider f(x,y) where R = [a, b] x [c, d]. $A(x) = \int_a^b f(x, y)dy$ considers 1 variable

$$\int_a^b \int_a^b f(x, y)dydx = \int \int_R f(x, y)dA$$

Fubini's theorem: if F is integrable on the rectangle R = [a, b] x [c, d], then

$$\int \int_R f'(x, y)dA = \int_c^d \int_a^b f(x, y)dydx = \int_a^b \int_c^d f(x, y)dxdy$$

essential for calculating 3rd dimension integrals

Example Exercise: evaluate $\int \int_R x - 3y^2 dA$ with R = $(x, y) : 0 <= x <= 2, 1 <= y <= 2$ or $[0, 2]x[1, 2]$

Example Exercise: evaluate $\int \int_R y * sin(x, y)dA$ with R = $[1, 2]x[0, \pi]$

let's consider a triple integral:

$$\int_e^f \int_c^d \int_a^b dV = (b-a)(d-c)(f-e)$$

but, for our purposes, we're just going to be focusing on double integrals.

Sean Tip: A lot of these integrals require integration by parts, and is something they can easily ask on a test to trip some students up. Review!

3 Vector Functions

A multi-variable function can take in 2 or 3 inputs and return 1 output, this is what we are familiar with, but a vector function can return multiple outputs. For example:

$$f(x, y) = (xy^2, x + y^2)$$

Parametrization

When f goes from a lower dimensional space to a higher dimensional space, we call f a Parametrization. We've dealt with 1 dimensional parametrization with functions that can be defined as t, the single parametric variable. Parametrizations are not unique, a circle can be defined as $f(t) =< Rsint, Rcost >=< x, y >$, $t \in [0, 2\pi]$ but also as $f(t) =< Rcost, Rsint >=< x, y >$, $t \in [0, 2\pi]$, where the only difference is between which direction the circle is created as t increases. For this course, the second definition is the one we would want to create.

2D Parametrization

$$f(a, v) : R^2 -> R^3$$

This parametrization takes in a 2d region and bends it in the 3rd dimension. this parametrization takes in a straight piece of paper of 2 variables and bends it into the 3D space. This parametrization takes in 2 inputs and outputs 3.

Example: Consider the plane $z = x + y + 10$ that is above the unit square on the xy-plane, we may choose the parametrization to be:

$$f(u, v) =< u, v, u + v + 10 >=< x, y, z > \;\; u \in [0, 1] \; v \in [0, 1]$$

This is called a natural parametrization as x and y are defined by using dummy variables, u and v. f takes on a point and gives out the same point with a new 3rd coordinate, or height, with the 3rd coordinate defined by z= u + v + 10. This lifts up the unit square from the xy=plane, onto the plane z = x + y + 10, however, notice the square is now slanted and also stretched. in this case, the plane z = x + y + 10 is flat, but in general, f lifts up a flat region in 2D into a curved surface in 3D with some stretching.

EX: find volume of the tetrahedron bounded by planes $x+2y+z=2, x=2y$

x = 0, z = 0

$$z = 2 - x - 2y$$

for x = 0, interesection with the xy-plane

$$0 = 2 - x - 2y$$

$$y = 1 - x/2$$

therfore, we get an integral like:

$$\int\int^{D} 2 - x - 2ydA$$

Type I: $x/2 <= y <= 1 - x/2$
Review this, and try with the example