MAT235

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1 Recap

we've covered ch 10, 12, 13 before the break. Covered Parametric equaltions and polar coords, vectors of geometry, vector functions, and scalar functions (differential calculus)

Test 3 coveres chapter 15 and 16

2 Chapter 15: Integral Calculus

15.1 For single-variable calculus,

$$y = f(x)$$

Integral $\int_a^b f(x)\,dx$ represents the area under the curve, for some a and b.

This is most commonly created using Riemann sums

- Chop up the graph into equally sized partitions on the X axis
- Chose an xi which can either be a left point, right point, or a midpoint
- finally evaluate xi using the function f(x) and multiply it by the size of the parition
- in essence,
  $$A = \Sigma_0^n f(xi) * \Delta x$$
  represents the area under the curve estimate
- take the limit as n approaches 0, and it becomes the definition of an integral

Now assume:

$$z = f(x, y)$$

this creates a rectangular region underneath the curve, where $R = [a, b]X[c, d]$ which is equal to $(x, y) : a <= x <= bandc <= y <= d$

Calculate the volume under the surface of f(x,y) inside of R: Integral

$$\int \int f(x)\,dx\ f(x)\,dx$$

Subside R into smaller sub rectangles

$$R_i = [x_0, ...x_i]X[y_0, ...y_i]$$

$a = x_0 < x_1 < ... < x_n = b$ $c = y_0 < y_1 < ... < y_n = d$ Assuming each rectangle is equally spaced:

$$\delta x = (b - a)/n^{\delta}y = (d - c)/n$$

therefore, for each sub rectangle

$$\Delta A = \Delta x * \Delta y$$

with this knowledge, and definition:

- Choose an arbitrary point, $(x_j i, y_j i)$
- evaluate $f(x_j i, y_j i)$ -¿ height, $Vol = f(x_j i, y_j i)\Delta A$
- $$A = \Sigma_{i=0}^{n}\Sigma_{j=0}^{n} f(x_j i, y_j i) * \Delta A$$

  represents the area under the curve estimate

lets try another strategy - iterated integrals consider f(x,y) where R = [a, b] x [c, d]. $A(x) = \int_a^b f(x,y)dy$ considers 1 variable

$$\int_a^b \int_a^b f(x,y)dydx = \int \int_R f(x,y)dA$$

Fubini's theorem: if F is integrable on the rectangle R = [a, b] x [c, d], then

$$\int \int_R f'(x,y)dA = \int_c^d \int_a^b f(x,y)dydx$$

essential for calculating 3rd demension integrals

Example Exercise: evaluate $\int \int_R x - 3y^2 dA$ with R = $(x,y) : 0 <= x <= 2, 1 <= y <= 2$ or $[0,2]x[1,2]$

Example Exercise: evaluate $\int \int_R y * sin(x,y)dA$ with R = $[1,2]x[0,\pi]$