

MAT235

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1 Recap

we've covered ch 10, 12, 13 before the break. Covered Parametric equations and polar coords, vectors of geometry, vector functions, and scalar functions (differential calculus)

Test 3 covers chapter 15 and 16

2 Chapter 15: Integral Calculus

15.1 For single-variable calculus,

$$y = f(x)$$

Integral $\int_a^b f(x) dx$ represents the area under the curve, for some a and b.

This is most commonly created using Riemann sums

- Chop up the graph into equally sized partitions on the X axis
- Chose an x_i which can either be a left point, right point, or a midpoint
- finally evaluate x_i using the function $f(x)$ and multiply it by the size of the partition
- in essence,

$$A = \sum_0^n f(x_i) * \Delta x$$

represents the area under the curve estimate

- take the limit as n approaches ∞ , and it becomes the definition of an integral

Now assume:

$$z = f(x, y)$$

this creates a rectangular region underneath the curve, where $R = [a, b] \times [c, d]$ which is equal to $(x, y) : a \leq x \leq b \text{ and } c \leq y \leq d$

Calculate the volume under the surface of $f(x,y)$ inside of R : Integral

$$\int \int f(x) dx \quad f(x) dx$$

Subside R into smaller sub rectangles

$$R_i = [x_0, \dots x_i] \times [y_0, \dots y_i]$$

$a = x_0 < x_1 < \dots < x_n = b$ $c = y_0 < y_1 < \dots < y_n = d$ Assuming each rectangle is equally spaced:

$$\delta x = (b - a)/n \quad \delta y = (d - c)/n$$

therefore, for each sub rectangle

$$\Delta A = \Delta x * \Delta y$$

with this knowledge, and definition:

- Choose an arbitrary point, (x_j, y_j)
- evaluate $f(x_j, y_j)$ - height, $Vol = f(x_j, y_j) \Delta A$
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$$A = \sum_{i=0}^n \sum_{j=0}^n f(x_j, y_j) * \Delta A$$

represents the area under the curve estimate

lets try another strategy - iterated integrals consider $f(x,y)$ where $R = [a, b] \times [c, d]$. $A(x) = \int_a^b f(x, y) dy$ considers 1 variable

$$\int_a^b \int_a^b f(x, y) dy dx = \int \int_R f(x, y) dA$$

Fubini's theorem: if F is integrable on the rectangle $R = [a, b] \times [c, d]$, then

$$\int \int_R f(x, y) dA = \int_c^d \int_a^b f(x, y) dy dx = \int_a^b \int_c^d f(x, y) dx dy$$

essential for calculating 3rd dimension integrals

Example Exercise: evaluate $\int \int_R x - 3y^2 dA$ with $R = (x, y) : 0 \leq x \leq 2, 1 \leq y \leq 2$ or $[0, 2] \times [1, 2]$

Example Exercise: evaluate $\int \int_R y * \sin(x, y) dA$ with $R = [1, 2] \times [0, \pi]$

let's consider a triple integral:

$$\int_e^f \int_c^d \int_a^b dV = (b-a)(d-c)(f-e)$$

but, for our purposes, we're just going to be focusing on double integrals.

Sean Tip: A lot of these integrals require integration by parts, and is something they can easily ask on a test to trip some students up. Review!