Lecture 1: Welcome

Logic, Automata, and Computability

MATH230

Te Kura Pāngarau | School of Mathematics and Statistics
Te Whare Wānanga o Waitaha | University of Canterbury

Outline

Welcome to MATH230

Lecturer: Robert Culling (Rob)

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Feel free to come and chat about the course anytime.

Tutor: Kerry Manson

Tutorials are on Monday evenings every week.

Hand-in select problems that Wednesday.

Class Representative: ? Could be you ?

Who is taking this course?

What is your major? Why have you taken this course?

What do you hope to get out of this course?

What Will We Study?

Mathematics, and metamathematics, that came into focus in the 1800s and blossomed in the 1900s.

Work that was motivated by the following questions about mathematics:

- Is mathematics consistent?
- Can we determine/prove all true statements?
- What does it mean to give a proof?
- Is there a fixed process/procedure/algorithm that we can give a statement to tell us whether or not the statement is true?

Hilbert's Program called for an axiomatic formalisation of mathematics so that these questions *about* mathematics could be answered precisely.

What Will We Study?

In order to be precise about these issues mathematicians chose to write mathematics in a precise language: first order predicate logic.

With mathematics written in a particular language it is easier to say what exactly we mean by a proof.

Furthermore the precise language is able to be encoded into numerals. This enabled mathematicians to write down specifically what one might mean by "an effective procedure". Thus mathematicians could get into a position to give answers to the possibility of a "universal proof procedure".

What Will We Study?

As we follow this story we will talk about the following topics:

- Propositional logic.
- First order logic.
- First order theories of arithmetic.
- Theories of computation.
- Limits of computation and mathematics.

Contemporary Influence

This subject, the foundations of mathematics, lives on today in many different forms. For example

- Research of mathematicians from around the world.
- Philosophy.
- Interactive proof checkers.
- Computers and computer science.

In particular researchers at Microsoft have developed the open source project L∃∀N. This is an interactive theorem prover.

This project boasts an ever growing library of definitions, theorems, and proofs of mathematics in a formal language. Your first tutorial will introduce you to this programming language via the Natural Numbers Game.

Report

We can only cover so much in lectures. These topics have a vast history and many interesting contemporary ideas. As part of your assessment you will be writing a report on some idea, person, machine, model, logic, that we just don't have time to cover.

- Logic
  - Non-standard logics
  - Different proof theories
  - Different semantics
  - Philosophy
- Models of Computation
  - Elementary cellular automata
  - Conway's game of Life
  - Register machines
  - Ada Lovelace and the first computer program
- People, changing of ideas, theorem checkers/provers etc.

Come and chat with me to finalise your topic.

Assessment

- Report 20%
- Tutorials 10% (Hand-in 10 out of the 12).
- Test 15%.
- Assignments (Turing machines) 20%.
- Examination 35% (With 40% hurdle).

All official course information can be found on Learn.

Expectations

University guidelines suggest spending at least 10 hours per week on a 15 point course.

- 3 hours on lectures actively taking notes
  (Double-speed is not good!)
- 1 hour in tutorial
- 6 hours extra to study. What can you do?

Expectations of me: You can expect me to be available to help in my office in appointed times or when we have planned a meeting. I want you to do well and I will make sure I am here to help that happen.

Further Reading

Throughout the course I will recommend you read certain sections of two texts: both of which are freely available online at the following links.

- L∃∀N: Logic and Proof.
  @ https://leanprover.github.io/logic_and_proof/index.html
- Logic Matters: An Introduction to Formal Logic.
  @ https://www.logicmatters.net/ifl/
- The Stanford Encyclopedia of Philosophy.

I encourage you to find your own extra resources online and in the libraries on campus. However, beware that formal logic has a lot of symbols and authors don't always agree on which symbols to use for which concepts. So, notation may differ from ours. Always use our notation in assessments.