Lecture 3: Truth and Validity

MATH230

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Outline

1. Example
2. Argument
3. Truth Values
4. Valid Arguments

Example

"Thin is guilty," observed Watson, "because either Holmes is right and the vile Moriarty is guilty, or he (Holmes) is wrong and Thin did the job; but those scoundrels are either both guilty or both innocent; and, as usual, Holmes is correct".

Example from Richard Jeffrey, *Formal Logic: Its Scope and Limits*

Argument Structure

$$\frac{\begin{array}{c}\text{Proposition 1}\\ \text{Proposition 2}\\ \vdots\\ \text{Proposition n}\end{array}}{\text{Conclusion}}$$

Question: What makes for a “good argument”? What might we mean by a “good argument”?

Truth Values

It may either be the case that an atomic formula is true, or false.

A	A
T	1
F	0

There are precisely two truth values in *classical* propositional logic and each proposition must take *exactly one* of them: there are no contradictions and there are no other logical values. Note that there are other logics which relax these conditions.

Truth table: ($\neg$) Negation

A	$\neg A$
T	
F	

The truth of compound variables (wff with syntactic structure) will depend, ultimately, on the truth values of the atomic formulae from which it is built.

Truth table: ($\wedge$) Conjunction

A	B	$A \wedge B$
T	T	
T	F	
F	T	
F	F	

Truth table: ($\vee$) Disjunction

A	B	$A \vee B$
1	1	
1	0	
0	1	
0	0	

Truth table: ($\vee$) Disjunction

A	B	$A \vee B$
1	1	
1	0	
0	1	
0	0	

Your intuition may not agree with this. Perhaps $\vee$ should not be true when both A and B are true. We have another binary connective to associate to this meaning: *exclusive disjunction* and we denote it by $\oplus$ or $\veebar$.

Truth table: ($\rightarrow$) Implication

A	B	$A \rightarrow B$
1	1	
1	0	
0	1	
0	0	

Truth table: ($\rightarrow$) Implication

A	B	$A \rightarrow B$
1	1	
1	0	
0	1	
0	0	

Again, your intuition may resist here. Implication is used in this sense in mathematics, so we stick to it. See the reading at the end for a discussion on the different meanings of implication.

Example

Consider the wff $(A \to (B \land A))$ that involves more than one binary operation.

A	B	$B \land A$	$(A \to (B \land A))$
1	1		
1	0		
0	1		
0	0		

Valid Argument

Suppose Σ is the set of premises of an argument with γ as the conclusion.

Definition

We say an argument is valid if its conclusion is true in every case in which each of its premises are true.

If the argument for γ from Σ is valid, then we say γ is a *semantic consequence of* Σ and denote it $\Sigma \models \gamma$

Example: Modus Ponens

Show $\{A, (A \rightarrow B)\} \models B$

Example: Modus Ponens

Show $\{A, (A \to B)\} \models B$

A	B	$(A \to B)$
1	1	1
1	0	0
0	1	1
0	0	1

Example: Modus Ponens

Show $\{A, (A \to B)\} \vDash B$

A	B	$(A \to B)$
1	1	1
1	0	0
0	1	1
0	0	1

There is only one case in which both of the premises are true. In that case, the conclusion is also true.

A	B	$(A \to B)$
1	1	1

Counterexamples

Definition (Counterexample)

A counterexample to an argument is a case in which the premises are all true, but the conclusion is false.

Example: Affirming the Consequent

Show $\{B, (A \to B)\} \nvDash A$

A	B	$(A \to B)$
1	1	1
1	0	0
0	1	1
0	0	1

This time the one line of interest is the counterexample: A (the conclusion) is false, but the premises are true.

A	B	$(A \to B)$
0	1	1

Further Reading

Here are some recommended reading to follow up on the lecture content. They are all freely available online.

- L∃∀N, *Logic and Proof*. Sections: 3.1, 6.1, and 6.2
- Logic Matters, *An Introduction to Formal Logic*: p172 - 216