A Bitmapper's Companion

epilys

2021

an introduction
to basic bitmap
mathematics
and algorithms
with code
samples in **Rust**

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All non-screenshot figures were generated by hand in Inkscape unless otherwise stated.

The skull in the cover is a transformed bitmap of the skull in the 1533 oil painting by Hans Holbein the Younger, *The Ambassadors*, which features a floating distorted skull rendered in anamorphic perspective.

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Special Topics ▶ Computer Graphics ▶ Programming
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The source code is available here

https://github.com/epilys/bitmappers-companion

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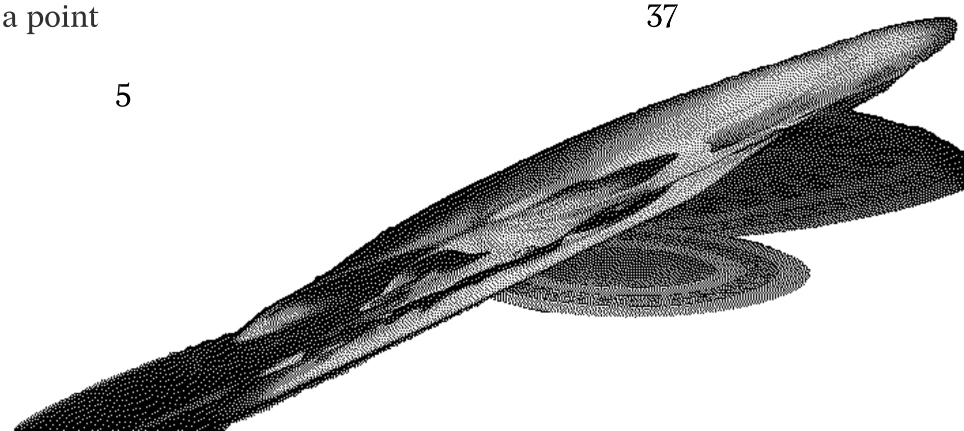

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Part I

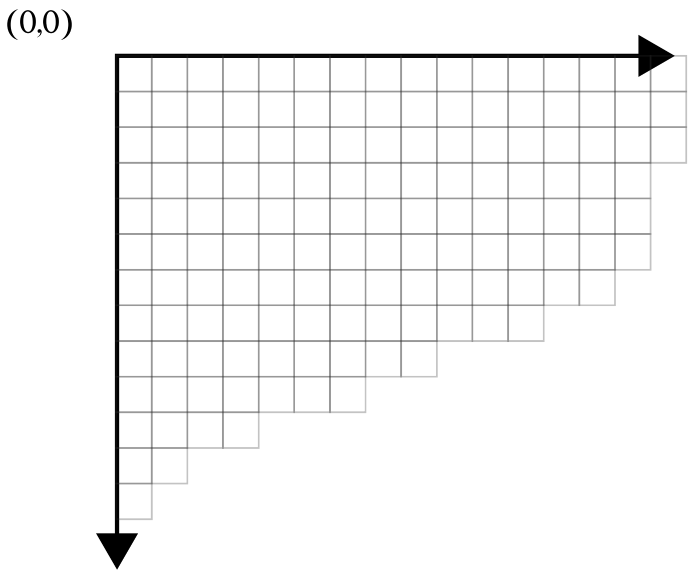
Introduction

Chapter 1
Data representation

intro

The data structures we're going to use is *Point* and *Image*. *Image* represents a bitmap, although we will use full RGB colors for our points therefore the size of a pixel in memory will be u8 instead of 1 bit.

We will work on the cartesian grid representing the framebuffer that will show us the pixels. The *origin* of this grid (i.e. the center) is at $(0, 0)$.



We will represent points as pairs of signed integers. When actually drawing them though, negative values and values outside the window's geometry will be ignored (clipped).

`src/lib.rs`:

This code file is a PDF attachment

```
pub type Point = (i64, i64);
pub type Line = (i64, i64, i64);

pub const fn from_u8_rgb(r: u8, g: u8, b: u8) -> u32 {
    let (r, g, b) = (r as u32, g as u32, b as u32);
    (r << 16) | (g << 8) | b
}
pub const AZURE_BLUE: u32 = from_u8_rgb(0, 127, 255);
pub const RED: u32 = from_u8_rgb(157, 37, 10);
pub const WHITE: u32 = from_u8_rgb(255, 255, 255);
```

```
pub const BLACK: u32 = 0;

pub struct Image {
    pub bytes: Vec<u32>,
    pub width: usize,
    pub height: usize,
    pub x_offset: usize,
    pub y_offset: usize,
}

impl Image {
    pub fn new(width: usize, height: usize, x_offset: usize, y_offset: usize) -> Self;
    pub fn magick_open(path: &str, x_offset: usize, y_offset: usize) -> Result<Self,
 ↪  Box<dyn Error>>;
    pub fn from_xbm(path: &str, x_offset: usize, y_offset: usize) -> Result<Self, Box<dyn
 ↪  Error>>;
    pub fn draw(&self, buffer: &mut Vec<u32>, fg: u32, bg: Option<u32>, window_width:
 ↪  usize);
    pub fn draw_outline(&mut self);
    pub fn clear(&mut self);
    pub fn plot(&mut self, x: i64, y: i64);
    pub fn get(&mut self, x: i64, y: i64) -> u32;
    pub fn plot_ellipse(
        &mut self,
        (xm, ym): (i64, i64),
        (a, b): (i64, i64),
        quadrants: [bool; 4],
        _wd: f64,
    );
    pub fn plot_line_width(&mut self, point_a: Point, point_b: Point, wd: f64);
    pub fn flood_fill(&mut self, mut x: i64, y: i64);
}
```

An RGB color with coordinates (r,g,b) where r,g,b : `u8` values is represented as a `u32` number with the red component shifted 16 bits to to the left, the green component 8 bits, and the final 8 bits are the blue component. It's essentially laying the r,g,b values sequentially and forming a 32 bit value out of three 8 bit values.

Our `Image::plot(x,y)` function sets the (x,y) pixel to black. To do that we set the element `y * width + x` of the `Image's` buffer to the black color as RGB.

Chapter 2

Displaying pixels to your screen

intro

src/bin/introduction.rs:

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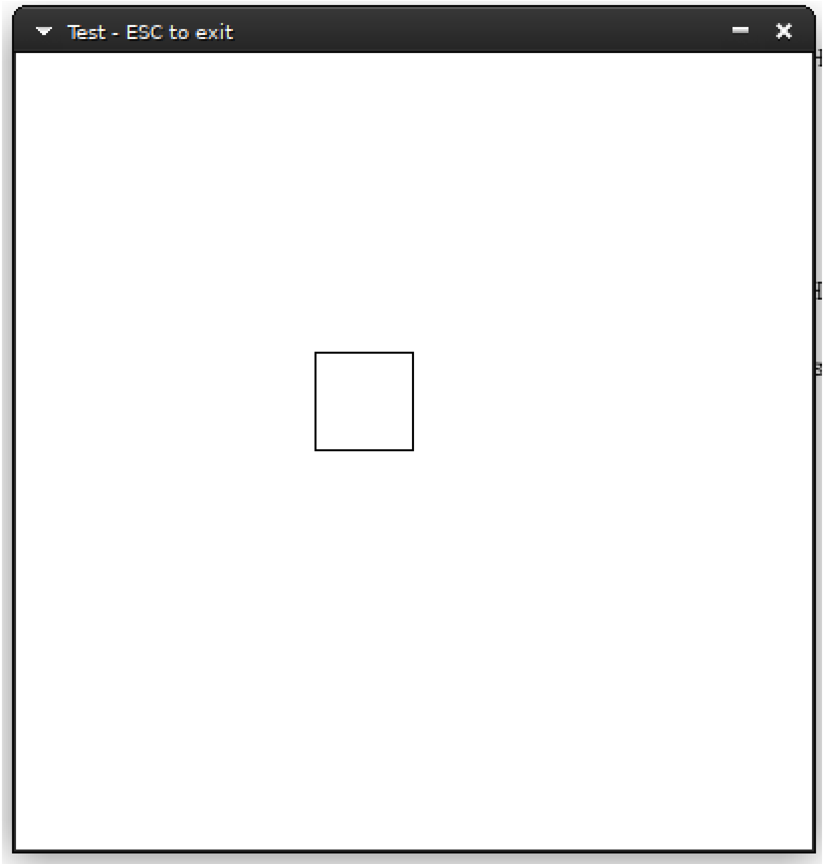
A way to display an *Image* is to use the `minifb` crate which allows you to create a window and draw pixels directly on it. Here's how you could set it up:

```
use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

fn main() {
    let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
    let mut window = Window::new(
        "Test - ESC to exit",
        WINDOW_WIDTH,
        WINDOW_HEIGHT,
        WindowOptions {
            title: true,
            //borderless: true,
            //resize: false,
            //transparency: true,
            ..WindowOptions::default()
        },
    )
    .unwrap();

    // Limit to max ~60 fps update rate
    window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

    let mut image = Image::new(50, 50, 150, 150);
    image.draw_outline();
    image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

    while window.is_open()
         && !window.is_key_down(Key::Escape)
         && !window.is_key_down(Key::Q) {
        window
            .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
            .unwrap();
        let millis = std::time::Duration::from_millis(100);
        std::thread::sleep(millis);
    }
}
```

Running this will show you something like this:



By drawing each individual pixel with the `Image::plot` and `Image::plot_color` functions, we can draw any possible RGB picture of the buffer size. In this book's chapters, we will usually calculate pixels by using discrete calculations of each pixels as integers, or by using rational values (with 64 bit floating point representation) and then calculating their integer values with the `floor` function. This can also be done by casting an `f64` type to `i64` with `as`:

```
let val: f64 = 5.5;
let val: i64 = val as i64;
assert_eq!(5i64, val);
```

intro

Chapter 3
Bits to byte pixels

If we worked with 1 bit images (black and white) it could be a more space-efficient representation to store the pixels as bits: 8 pixels in 1 byte. For this book we accept that our images can have RGB colors. The `xbm` format stores pixels like that, and we might wish to convert them to our representation.

Let's define a way to convert bit information to a byte vector:

```
pub fn bits_to_bytes(bits: &[u8], width: usize) -> Vec<u32> {
    let mut ret = Vec::with_capacity(bits.len() * 8);
    let mut current_row_count = 0;
    for byte in bits {
        for n in 0..8 {
            if byte.rotate_right(n) & 0x01 > 0 {
                ret.push(BLACK);
            } else {
                ret.push(WHITE);
            }
            current_row_count += 1;
            if current_row_count == width {
                current_row_count = 0;
                break;
            }
        }
    }
    ret
}
```

Chapter 4
Loading graphics files in Rust

intro

The book's library includes a method to load `xbm` files on runtime (see *Including `xbm` files in* ***Rust*** for including them in your binary at compile time). If your system has `ImageMagick` installed and the commands `identify` and `magick` are in your `PATH` environment variable, you can use the `Image::magick_open` method:

```
impl Image {
    ...
    pub fn magick_open(path: &str, x_offset: usize, y_offset: usize) -> Result<Self,
↪   Box<dyn Error>>;
    ...
}
```

It simply converts the image file you pass to it to raw bytes using the invocation `magick convert` *`path`* `RGB:-` which prints raw RGB content to `stdout`.

If you have another way to load pictures such as your own code or a picture format library crate, all you have to do is convert the pixel information to an `Image` whose definition we repeat here:

```
pub struct Image {
    pub bytes: Vec<u32>,
    pub width: usize,
    pub height: usize,
    pub x_offset: usize,
    pub y_offset: usize,
}
```

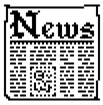
Chapter 5

Including `xbm` files in **Rust**

The end of this chapter includes a short **Rust** *program to automatically convert `xbm` files to equivalent* **Rust** *code.*

`xbm` files are C source code files that contain the pixel information for an image as macro definitions for the dimensions and a static `char` array for the pixels, with each bit column representing a pixel. If the width dimension doesn't have 8 as a factor, the remaining bit columns are left blank/ignored.

They used to be a popular way to share user avatars in the old internet and are also good material for us to work with, since they are small and numerous. The following is such an image:



Then, we can convert the `xbm` file from C to **Rust** with the following transformations:

```
#define news_width 48
#define news_height 48
static char news_bits[] = {
```

to

```
const NEWS_WIDTH: usize = 48;
const NEWS_HEIGHT: usize = 48;
const NEWS_BITS: &[u8] = &[
```

And replace the closing `}` with `]`.

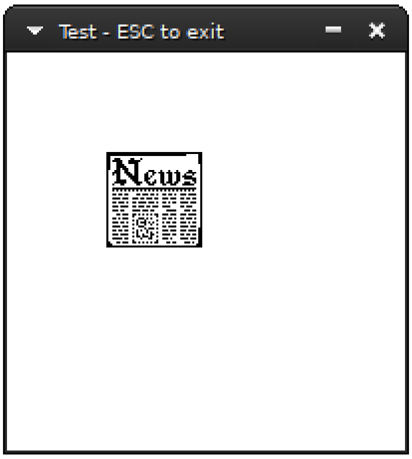
We can then include the new file in our source code:

```
include!("news.xbm.rs");
```

load the image:

```
let mut image = Image::new(NEWS_WIDTH, NEWS_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(NEWS_BITS, NEWS_WIDTH);
```

and finally run it:



The following short program uses the `regex` crate to match on these simple rules and print the equivalent code in `stdout`. You can use it like so:

```
cargo run --bin xbmtors -- file.xbm > file.xbm.rs
```

src/bin/xbmtors.rs:

This code file is a PDF attachment

```
use regex;
use regex::Regex;
use std::fs::File;
use std::io::prelude::*;

fn main() {
    let args = std::env::args().skip(1).collect::<Vec<String>>();
    if args.len() != 1 {
        println!("one argument expected, the xbm file path to convert.");
        return;
    }
    let mut file = match File::open(&args[0]) {
        Err(err) => panic!("couldn't open {}: {}", args[0], err),
        Ok(file) => file,
    };

    let mut s = String::new();
    if let Err(err) = file.read_to_string(&mut s) {
        panic!("couldn't read {}: {}", args[0], err);
    }

    let re = Regex::new(
        r"(?imx)
  ^\s*\x23\s*define\s+(?P<i>.+?)_width\s+(?P<w>\d\d*)$
  \s*
  ^\s*\x23\s*define\s+.+?_height\s+(?P<h>\d\d*)$
  \s*
  ^\s*static(\s+unsigned){0,1}\s+char\s+.+?_bits..\s*=\s*\{(?P<b>[^}]+)\};
",
    )
    .unwrap();
```

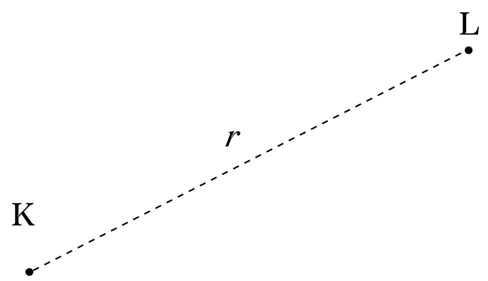
```
    let caps = re
        .captures(&s)
        .expect("Could not convert file, regex doesn't match :(");
    let ident = caps.name("i").unwrap().as_str().to_uppercase();
    let out = re.replace_all(&s, format!("const {i}_WIDTH: usize = $w;\nconst {i}_HEIGHT:
↪   usize = $h;\nconst {i}_BITS: &[u8] = &[$b];", i = &ident));
    println!("{}", out.trim());
}
```

intro

Part II

Points And Lines

Chapter 6
Distance between two points



Given two points, K and L, an elementary application of Pythagoras' Theorem gives the distance between them as

$$r = \sqrt{(x_L - x_K)^2 + (y_L - y_K)^2} \tag{6.1}$$

which is simply coded:

```
pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {
    let (x_k, y_k) = p_k;
    let (x_l, y_l) = p_l;
    let xlk = x_l - x_k;
    let ylk = y_l - y_k;
    f64::sqrt((xlk*xlk + ylk*ylk) as f64)
}
```

Chapter 7

Moving a point to a distance at an angle

Moving a point $P = (x, y)$ at distance d at an angle of r radians is solved with simple trigonometry:

$$P' = (x + d \times \cos r, y + d \times \sin r)$$

Why? The problem is equivalent to calculating the point of a circle with P as the center, d the radius at angle r and as we will later* see this is how the points of a circle are calculated.

```
pub fn move_point(p: Point, d: f64, r: f64) -> Point {
  let (x, y) = p;
  (x + (d * f64::cos(r)).round() as i64, y + (d * f64::sin(r)).round() as i64)
}
```

**Equations of a circle* page 47

Chapter 8

Equations of a line

lines

There are several ways to describe a line mathematically. We'll list the convenient ones for drawing pixels.

The equation that describes every possible line on a two dimensional grid is the *implicit* form $ax + by = c, (a, b) \neq (0, 0)$. We can generate equivalent equations by adding the equation to itself, i.e. $ax + by = c \equiv 2ax + 2by = 2c \equiv a'x+b'y = c', a' = 2a, b' = 2b, c' = 2c$ as many times as we want. To "minimize" the constants a, b, c we want to satisfy the relationship $a^2 + b^2 = 1$, and thus can convert the equivalent equations into one representative equation by multiplying the two sides with $\frac{1}{\sqrt{a^2+b^2}}$; this is called the normalized equation.

The *slope intercept form* describes any line that intercepts the y axis at $b \in \mathbb{R}$ with a specific slope a:

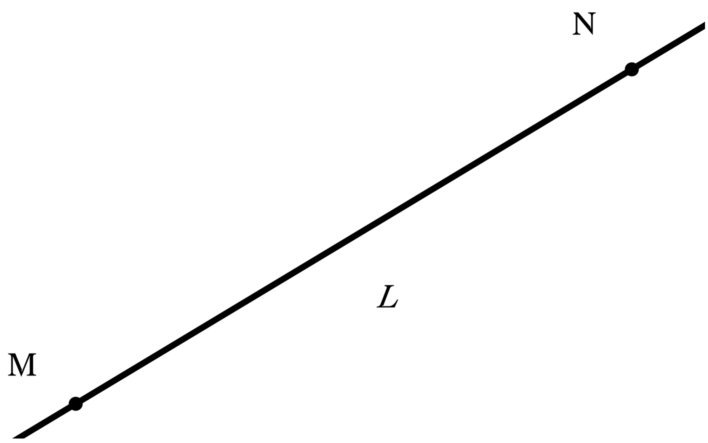
$$y = ax + b$$

The *parametric* form...

8.1 Line through a point $P = (x_p, y_p)$ and a slope m

$$y - y_p = m(x - x_p)$$

8.2 Line through two points



It seems sufficient, given the coordinates of two points M, N, to calculate a, b and c to form a line equation:

$$ax + by + c = 0$$

If the two points are not the same, they necessarily form such a line. To get there, we start from expressing the line as parametric over t: at $t = 0$ it's at point M and at $t = 1$ it's at point N:

$$c = c_M + (c_N - c_M)t, t \in R, c \in \{x, y\}$$
$$c = c_M, t \in R, c \in \{x, y\}$$

Substituting t in one of the equations we get:

$$(y_M - y_N)x + (x_N - x_M)y + (x_M y_N - x_N y_M) = 0$$

Which is what we were after. We should finish by normalising what we found with $\frac{1}{\sqrt{a^2+b^2}}$, but our coordinates are integers and have no decimal or floating point accuracy.

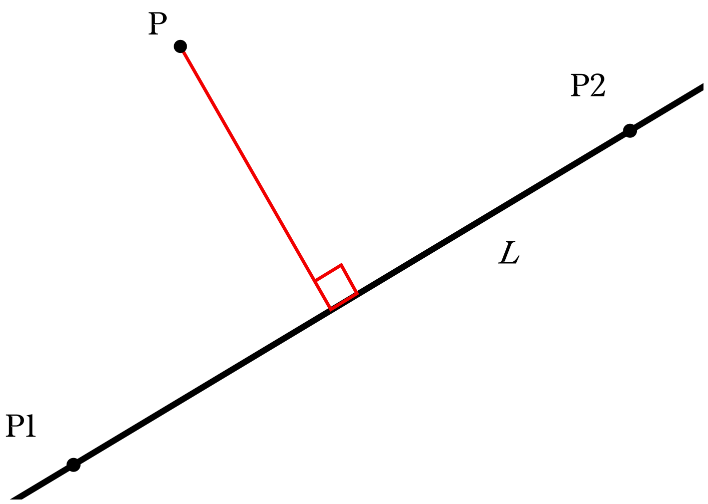
```
fn find_line(point_a: Point, point_b: Point) -> (i64, i64, i64) {
    let (xa, ya) = point_a;
    let (xb, yb) = point_b;
    let a = yb - ya;
    let b = xa - xb;
    let c = xb * ya - xa * yb;

    (a, b, c)
}
```

Chapter 9
Drawing a line

```
fn plot_line(image: &mut Image, (a, b, c): (i64, i64, i64)) {
    let x = if a != 0 { -1 * (c) / a } else { 0 };
    let mut prev_point = (x, 0);
    for y in 0..(WINDOW_HEIGHT as i64) {
        // ax+by+c =0 =>
        // x=(-c-by)/a
        let x = if a != 0 { -1 * (c + b * y) / a } else { 0 };
        let new_point = (x, y);
        image.plot_line_width(prev_point, new_point, 1.0);
        prev_point = new_point;
    }
}
```

Chapter 10

Distance from a point to a line



10.1 Using the implicit equation form

Let's find the distance from a given point P and a given line L. Let d be the distance between them. Bring L to the implicit form $ax + by = c$.

$$d = \frac{|ax_p + by_p + c|}{\sqrt{a^2 + b^2}}$$

10.2 Using an L defined by two points P_1, P_2

With $P = (x_0, y_0)$, $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$.

$$d = \frac{|(x_2 - x_1)(y_1 - y_0) - (x_1 - x_0)(y_2 - y_1)|}{\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

10.3 Using an L defined by a point P_l and angle $\hat{\theta}$

$$d = \left|\cos(\hat{\theta})(P_{ly} - y_p) - \sin(\hat{\theta})(P_{lx} - P_x)\right|$$

The code

This function uses the implicit form.

This code is included in the distributed library file in the *Data representation* chapter.

```
type Line = (i64, i64, i64);
pub fn distance_line_to_point((x, y): Point, (a, b, c): Line) -> f64 {
    let d = f64::sqrt((a * a + b * b) as f64);
    if d == 0.0 {
        0.
    } else {
        (a * x + b * y + c) as f64 / d
    }
}
```

Chapter 11
Perpendicular lines

11.1 Find perpendicular to line that passes through given point

lines

Now, we wish to find the equation of the line that passes through P and is perpendicular to L. Let's call it $L_\perp$. L in implicit form is $ax + by + c = 0$. The perpendicular will be:

$$L_\perp : bx - ay + (aP_y - bP_x) = 0$$

The code

This code is included in the distributed library file in the *Data representation* chapter.

```
type Line = (i64, i64, i64);
fn perpendicular((a, b, c): Line, p: Point) -> Line {
    (b, -1 * a, a * p.1 - b * p.0)
}
```

11.2 Find point in line that belongs to the perpendicular of given point

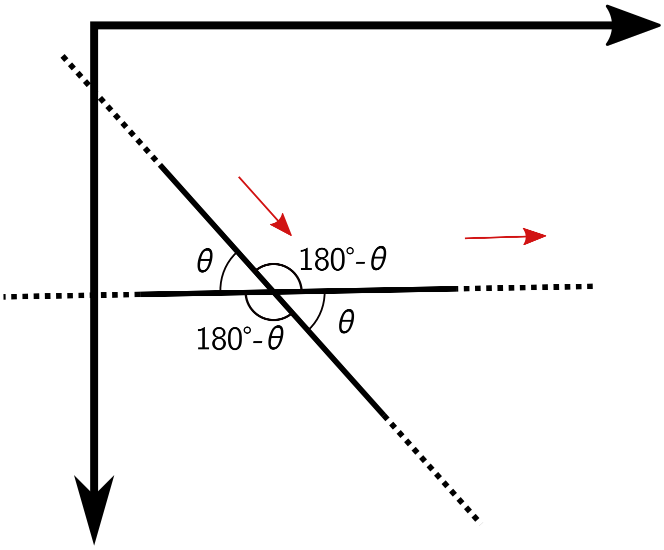
The code

This code is included in the distributed library file in the *Data representation* chapter.

```
fn point_perpendicular((a, b, c): Line, p: Point) -> Point {
    let d = (a * a + b * b) as f64;
    if d == 0. {
        return (0, 0);
    }
    let cp = a * p.1 - b * p.0;
    (
        ((-a * c - b * cp) as f64 / d) as i64,
        ((a * cp - b * c) as f64 / d) as i64,
    )
}
```

Chapter 12

Angle between two lines



By angle we mean the angle formed by the two directions of the lines; and direction vectors start from the origin (in the figure, they are the red arrows). So if we want any of the other three angles, we already know them from basic geometry as shown in the figure above.

If you prefer using the implicit equation, bring the two lines L_1 and L_2 to that form ($a_1x + b_1y + c = 0$ and $a_2x + b_2y + c_2 = 0$) and you can directly find $\hat{\theta}$ with the formula:

$$\hat{\theta} = \arccos \frac{a_1a_2 + b_1b_2}{\sqrt{\left(a_1^2 + b_1^2\right)\left(a_2^2 + b_2^2\right)}}$$

For the following parametric equations of L_1, L_2:

$$L_1 = (\{x = x_1 + f_1t\}, \{y = y_1 + g_1t\})$$
$$L_2 = (\{x = x_2 + f_2s\}, \{y = y_2 + g_2s\})$$

the formula is:

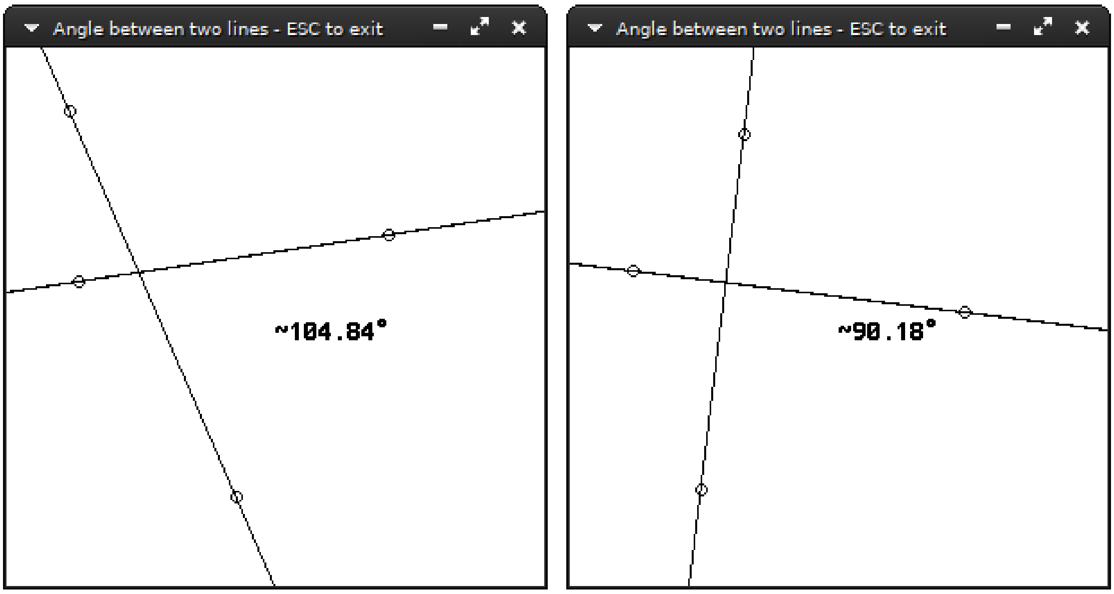
$$\hat{\theta} = \arccos \frac{f_1 f_2 + g_1 g_2}{\sqrt{\left(f_1^2 + g_1^2\right)\left(f_2^2 + g_2^2\right)}}$$

The code:

src/bin/anglebetweenlines.r

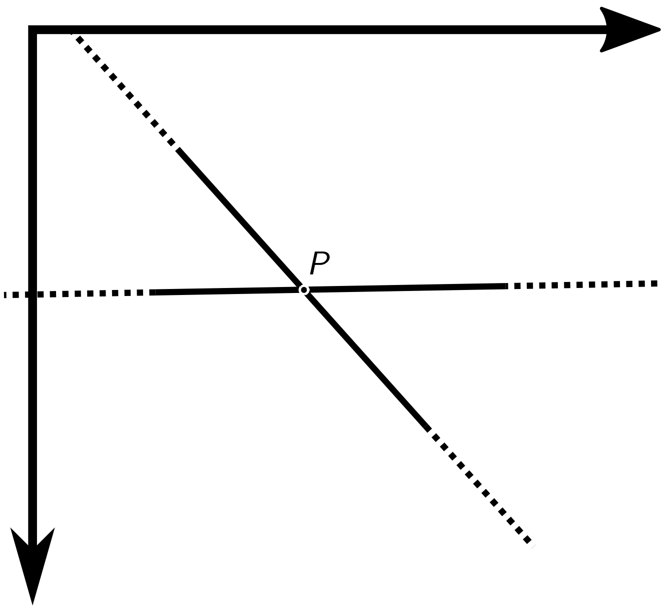
This code file is a PDF attachment

```
fn find_angle((a1, b1, c1): (i64, i64, i64), (a2, b2, c2): (i64, i64, i64)) -> f64 {
    let nom = (a1 * a2 + b1 * b2) as f64;
    let denom = ((a1 * a1 + b1 * b1) * (a2 * a2 + b2 * b2)) as f64;
    f64::acos(nom / f64::sqrt(denom))
}
```

The `src/bin/anglebetweenlines.rs` example has two interactive lines and computes their angle with 64bit floating point accuracy.

Chapter 13
Intersection of two lines



If the lines L_1, L_2 are in implicit form ($a_1x + b_1y + c = 0$ and $a_2x + b_2y + c_2 = 0$), the result comes after checking if the lines are parallel (in which case there's no single point of intersection):

$$a_1b_2 - a_2b_1 \neq 0$$

If they are not parallel, P is:

$$P = \left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \right)$$

The code:

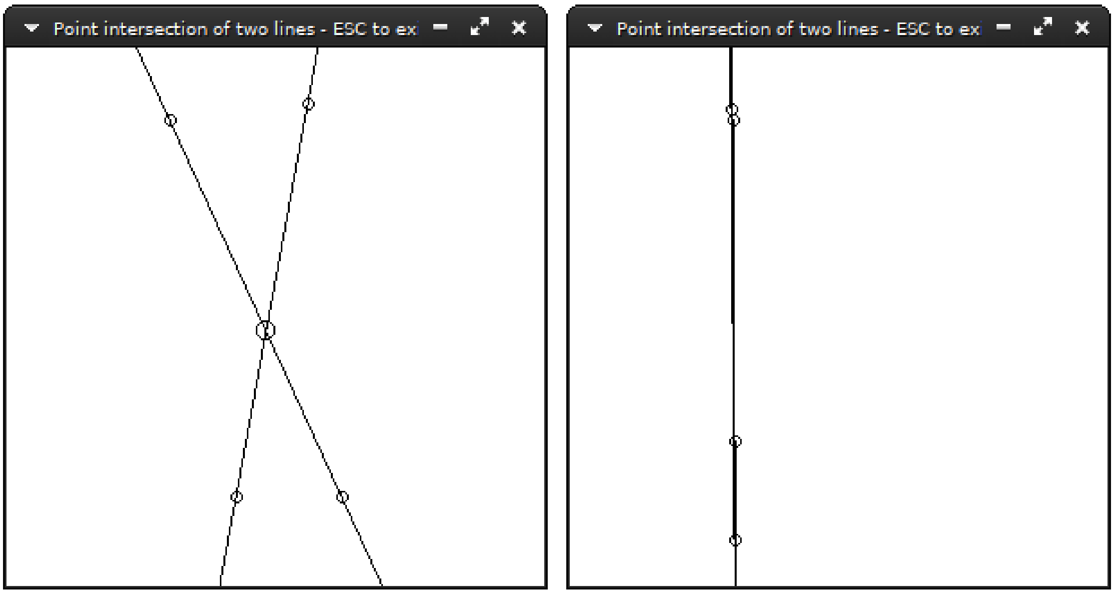
```
fn find_intersection((a1, b1, c1): (i64, i64, i64), (a2, b2, c2): (i64, i64, i64)) ->
↪  Option<Point> {
    let denom = a1 * b2 - a2 * b1;

    if denom == 0 {
        return None;
    }
```

src/bin/lineintersection.rs:

This code file is a PDF attachment

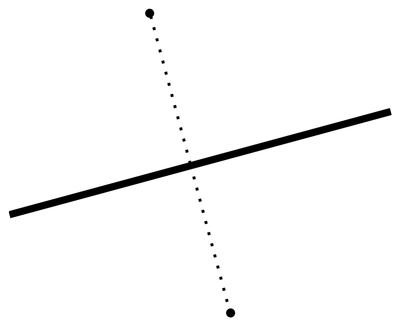
lines

```
    Some(((b1 * c2 - b2 * c1) / denom, (a2 * c1 - a1 * c2) / denom))
}
```



The `src/bin/lineintersection.rs` example has two interactive lines and computes their point of intersection.

Chapter 14
Line equidistant from two points



Let's name this line L. From previous chapter[*] we know how to get the line L that's created by the two points M and N:

$$L : (y_M - y_N)x + (x_N - x_M)y + (x_M y_N - x_N y_M) = 0$$

We need the perpendicular line over the midpoint of L.[†] The midpoint also satisfies L's equation. The midpoint's coordinates are intuitively:

$$P_{mid} = \left(\frac{x_M + x_N}{2}, \frac{y_M + y_N}{2}\right)$$

The perpendicular's $L_\perp$ equation is

$$L_{EQ} = L_\perp : yx - ay + \left(aP_{mid_y} - bP_{mid_x}\right) = 0$$

The code:

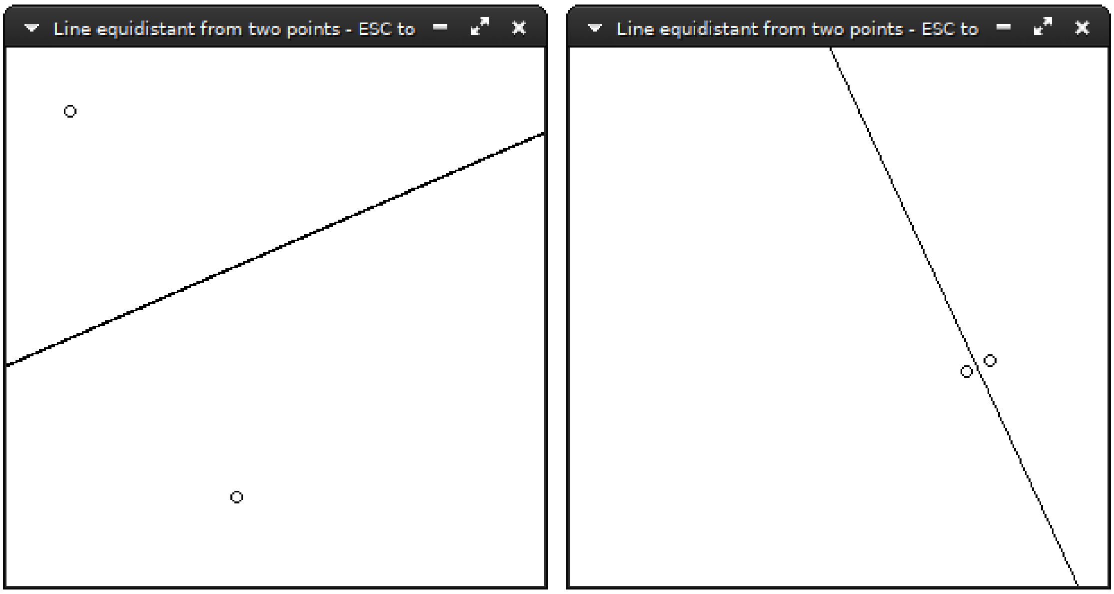
src/bin/equidistant.rs:

This code file is a PDF attachment

```
fn find_equidistant(point_a: Point, point_b: Point) -> (i64, i64, i64) {
    let (xa, ya) = point_a;
    let (xb, yb) = point_b;
    let midpoint = ((xa + xb) / 2, (ya + yb) / 2);

    let al = ya - yb;
    let bl = xb - xa;

    // If we had subpixel accuracy, we could do:
    //assert_eq!(al*midpoint.0+bl*midpoint.1, -cl);
```

[*] See *Line through two points*, page 23
[†] See *Perpendicular lines*, page 28

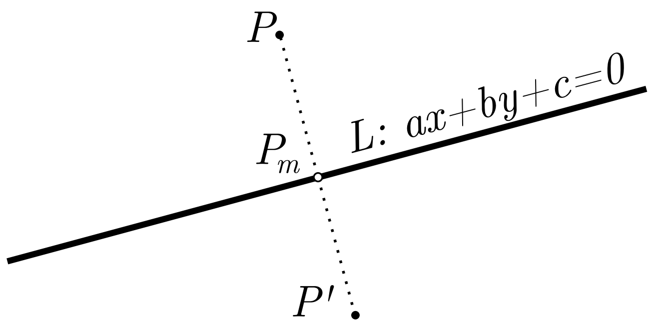
```
    let a = bl;
    let b = -1 * al;
    let c = (al * midpoint.1) - (bl * midpoint.0);

    (a, b, c)
}
```



The `src/bin/equidistant.rs` example has two interactive points and computes their L_{EQ}.

Chapter 15

Reflection of point on line



Line PP' will be perpendicular to $L : ax + by + c = 0$, meaning they will satisfy the equation $L_\perp : bx - ay + (aP_y - bP_x) = 0$.* We will find the middlepoint P_m. L and $L_\perp$ intercept at P_m, so substituting $L_\perp$'s y to L gives:

$$
\begin{aligned}
& a\mathbf{x} + b\left(\frac{b\mathbf{x} + (aP_y - bP_x)}{a}\right) + c = 0 \\
\implies & a\mathbf{x} + \frac{b^2}{a}\mathbf{x} + bP_y - \frac{b^2}{a}P_x + c = 0 \\
\implies & (a + \frac{b^2}{a})\mathbf{x} = \frac{b^2}{a}P_x - c - bP_y \\
\implies & \mathbf{x} = \left(\frac{\frac{b^2}{a}P_x - c - bP_y}{a + \frac{b^2}{a}}\right)
\end{aligned}
$$

P_{m_y} is found by substituting P_{m_x} to L. Now, knowing length of PP_m = length of P_mP', we can find P'_x and P'_y:

*See *Perpendicular lines*, page 28

lines

$$
\begin{aligned}
P_{m_x} - P_x &= P'_x - P_{m_x} \\
P_{m_y} - P_y &= P'_y - P_{m_y} \\
\implies P'_x &= 2P_{m_x} - P_x \\
P'_y &= 2P_{m_y} - P_y
\end{aligned}
$$

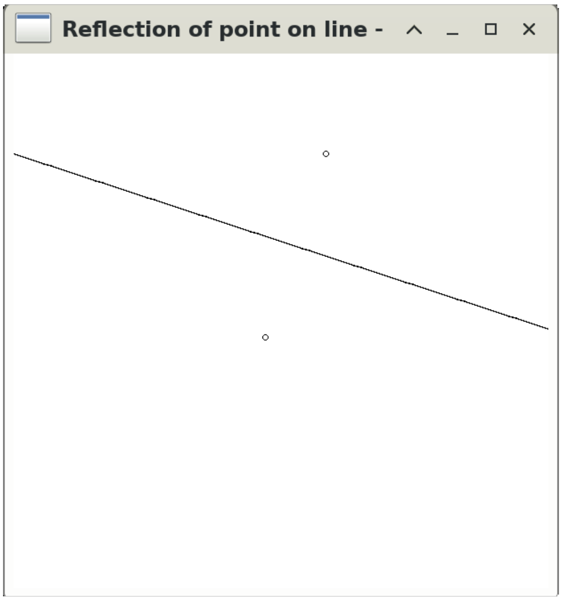
The code

lines

c/bin/mirror.rs:

de file is a PDF attachment

```
fn find_mirror(point: Point, l: Line) -> Point {
    let (x, y) = point;
    let (a, b, c) = l;
    let (a, b, c) = (a as f64, b as f64, c as f64);

    let b2a = (b * b) / a;
    let mx = (b2a * x as f64 - c - b * y as f64) / (a + b2a);
    let my = (-a * mx - c) / b;
    let (mx, my) = (mx as i64, my as i64);

    (2 * mx - x, 2 * my - y)
}
```



The `src/bin/mirror.rs` example lets you drag a point and draws its reflection across a line.

Chapter 16

Normal to a line through a point

Add *Normal to a line through a point*

lines

Chapter 17
Angle sectioning

17.1 Bisection

17.2 Trisection

If the title startled you, be assured it's not a joke. It's totally possible to trisect an angle... with a ruler. The adage that angle trisection is impossible refers to using only a compass and unmarked straightedge.

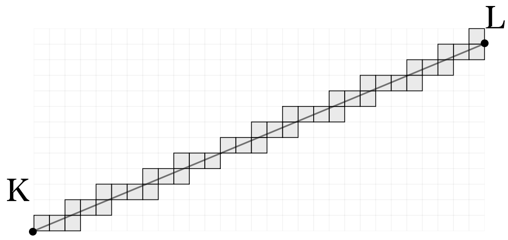
Part III

Points And Line Segments

Chapter 18

Drawing a line segment from its two endpoints

For any line segment with any slope, pixels must be matched with the infinite amount of points contained in the segment. As shown in the following figure, a segment *touches* some pixels; we could fill them using an algorithm and get a bitmap of the line segment.



The algorithm presented here was first derived by Bresenham. In the *Image* implementation, it is used in the `plot_line_width` method.

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64)) {
    /* Bresenham's line algorithm */
    let mut d;
    let mut x: i64;
    let mut y: i64;
    let ax: i64;
    let ay: i64;
    let sx: i64;
    let sy: i64;
    let dx: i64;
    let dy: i64;

    dx = x2 - x1;
    ax = (dx * 2).abs();
    sx = if dx > 0 { 1 } else { -1 };

    dy = y2 - y1;
    ay = (dy * 2).abs();
    sy = if dy > 0 { 1 } else { -1 };

    x = x1;
    y = y1;

    let b = dx / dy;
    let a = 1;
    let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;
    let delta = double_d / 2;

    if ax > ay {
        d = ay - ax / 2;
```

```
        loop {
            self.plot(x, y);
            if x == x2 {
                return;
            }
            if d >= 0 {
                y = y + sy;
                d = d - ax;
            }
            x = x + sx;
            d = d + ay;
        }
    } else {
        d = ax - ay / 2;
        let delta = double_d / 3;
        loop {
            self.plot(x, y);
            if y == y2 {
                return;
            }
            if d >= 0 {
                x = x + sx;
                d = d - ay;
            }
            y = y + sy;
            d = d + ax;
        }
    }
}
```


Add some explanation behind the algorithm in *Drawing a line segment from its two endpoints*

Chapter 19

Drawing line segments with width

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64), _wd: f64) {
    /* Bresenham's line algorithm */
    let mut d;
    let mut x: i64;
    let mut y: i64;
    let ax: i64;
    let ay: i64;
    let sx: i64;
    let sy: i64;
    let dx: i64;
    let dy: i64;

    dx = x2 - x1;
    ax = (dx * 2).abs();
    sx = if dx > 0 { 1 } else { -1 };

    dy = y2 - y1;
    ay = (dy * 2).abs();
    sy = if dy > 0 { 1 } else { -1 };

    x = x1;
    y = y1;

    let b = dx / dy;
    let a = 1;
    let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;
    let delta = double_d / 2;

    if ax > ay {
        d = ay - ax / 2;
        loop {
            self.plot(x, y);
            {
                let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;
                let mut _x = x;
                loop {
                    let t = total(_x);
                    if t < -1 * delta || t > delta {
                        break;
                    }
                    _x += 1;
                    self.plot(_x, y);
                }
                let mut _x = x;
                loop {
                    let t = total(_x);
                    if t < -1 * delta || t > delta {
                        break;
                    }
                    _x -= 1;
                    self.plot(_x, y);
                }
            }
            if x == x2 {
                return;
            }
            if d >= 0 {
                y = y + sy;
                d = d - ax;
            }
            x = x + sx;
            d = d + ay;
        }
    } else {
        d = ax - ay / 2;
        let delta = double_d / 3;
        loop {
```

```
            self.plot(x, y);
            {
                let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;
                let mut _x = x;
                loop {
                    let t = total(_x);
                    if t < -1 * delta || t > delta {
                        break;
                    }
                    _x += 1;
                    self.plot(_x, y);
                }
                let mut _x = x;
                loop {
                    let t = total(_x);
                    if t < -1 * delta || t > delta {
                        break;
                    }
                    _x -= 1;
                    self.plot(_x, y);
                }
            }
            if y == y2 {
                return;
            }
            if d >= 0 {
                x = x + sx;
                d = d - ay;
            }
            y = y + sy;
            d = d + ax;
        }
    }
}
```

Chapter 20

Intersection of two line segments

Let points $\mathbf{1} = (x_1, y_1)$, $\mathbf{2} = (x_2, y_2)$, $\mathbf{3} = (x_3, y_3)$ and $\mathbf{4} = (x_4, y_4)$ and **1,2**, **3,4** two line segments they form. We wish to find their intersection:

First, get the equation of line L_{12} and line L_{34} from chapter *Equations of a line*.

Substitute points **3** and **4** in equation L_{12} to compute $r_3 = L_{12}(\mathbf{3})$ and $r_4 = L_{12}(\mathbf{4})$ respectively.

If $r_3 \neq 0$, $r_4 \neq 0$ and $sgn(r_3) == sign(r_4)$ the line segments don't intersect, so stop.

In L_{34} substitute point **1** to compute r_1, and do the same for point **2**.

If $r_1 \neq 0$, $r_2 \neq 0$ and $sgn(r_1) == sign(r_2)$ the line segments don't intersect, so stop.

At this point, L_{12} and L_{34} either intersect or are equivalent. Find their intersection point. (See *Intersection of two lines* page 31)

20.1 *Fast* intersection of two line segments

Part IV

Points, Lines and Circles

Chapter 21

Equations of a circle

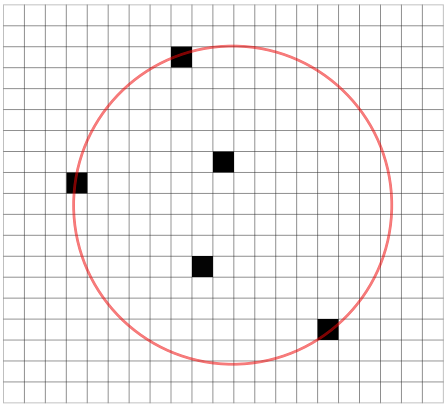
Add *Equations of a circle*

circles

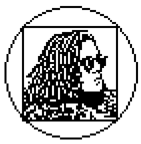
Chapter 22
Bounding circle

src/bin/boundingcircle.rs:

This code file is a PDF attachment

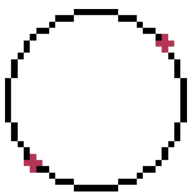


A bounding circle is a circle that includes all the points in a given set. Usually we're interested in one of the smallest ones possible.



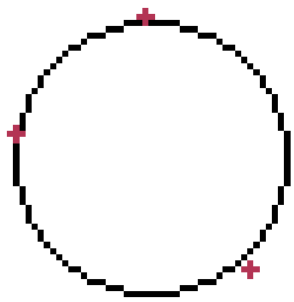
We can use the following methodology to find the bounding circle: start from two points and the circle they make up, and for each of the rest of the points check if the circle includes them. If not, make a bounding circle that includes every point up to the current one. To do this, we need some primitive operations.

We will need a way to construct a circle out of two points:



```
let p1 = points[0];
let p2 = points[1];
//The circle is determined by two points, P and Q. The center of the circle
↪ is
//at (P + Q)/2.0 and the radius is |(P - Q)/2.0|
let d_2 = (
(((p1.0 + p2.0) / 2), (p1.1 + p2.1) / 2),
(distance_between_two_points(p1, p2) / 2.0),
);
```

And a way to make a circle out of three points:



```
fn min_circle_w_3_points(q1: Point, q2: Point, q3: Point) -> Circle {
    let (ax, ay) = (q1.0 as f64, q1.1 as f64);
    let (bx, by) = (q2.0 as f64, q2.1 as f64);
    let (cx, cy) = (q3.0 as f64, q3.1 as f64);

    let mut d = 2. * (ax * (by - cy) + bx * (cy - ay) + cx * (ay - by));
    if d == 0.0 {
        d = std::cmp::max(
            std::cmp::max(
                distance_between_two_points(q1, q2) as i64,
                distance_between_two_points(q2, q3) as i64,
            ),
            distance_between_two_points(q1, q3) as i64,
        ) as f64
            / 2.;
    }
    let ux = ((ax * ax + ay * ay) * (by - cy)
        + (bx * bx + by * by) * (cy - ay)
        + (cx * cx + cy * cy) * (ay - by))
        / d;
    let uy = ((ax * ax + ay * ay) * (cx - bx)
        + (bx * bx + by * by) * (ax - cx)
        + (cx * cx + cy * cy) * (bx - ax))
        / d;
    let mut center = (ux as i64, uy as i64);

    if center.0 < 0 {
        center.0 = 0;
    }
    if center.1 < 0 {
        center.1 = 0;
    }
    let d = distance_between_two_points(center, q1);
    (center, d)
}
```

The algorithm:

circles

```rust
use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};
use rand::seq::SliceRandom;
use rand::thread_rng;
use std::f64::consts::{FRAC_PI_2, PI};

include!("../me.xbm.rs");

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {
    let (x_k, y_k) = p_k;
    let (x_l, y_l) = p_l;
    let xlk = x_l - x_k;
    let ylk = y_l - y_k;
    f64::sqrt((xlk * xlk + ylk * ylk) as f64)
}

fn image_to_points(image: &Image) -> Vec<Point> {
    let mut ret = Vec::with_capacity(image.bytes.len());
    for y in 0..(image.height as i64) {
        for x in 0..(image.width as i64) {
            if image.get(x, y) == Some(BLACK) {
                ret.push((x, y));
            }
        }
    }
    ret
}

type Circle = (Point, f64);

fn bc(image: &Image) -> Circle {
    let mut points = image_to_points(image);
    points.shuffle(&mut thread_rng());
    min_circle(&points)
}
fn min_circle(points: &[Point]) -> Circle {
    let mut points = points.to_vec();
    points.shuffle(&mut thread_rng());

    let p1 = points[0];
    let p2 = points[1];
    //The circle is determined by two points, P and Q. The center of the
↪   circle is
    //at (P + Q)/2.0 and the radius is |(P - Q)/2.0|
    let d_2 = (
        (((p1.0 + p2.0) / 2), (p1.1 + p2.1) / 2),
        (distance_between_two_points(p1, p2) / 2.0),
    );

    let mut d_prev = d_2;

    for i in 2..points.len() {
        let p_i = points[i];
        if distance_between_two_points(p_i, d_prev.0) <= (d_prev.1) {
            // then d_i = d_(i-1)
        } else {
            let new = min_circle_w_point(&points[..i], p_i);
            if distance_between_two_points(p_i, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }

    d_prev
}

fn min_circle_w_point(points: &[Point], q: Point) -> Circle {
    let mut points = points.to_vec();

    points.shuffle(&mut thread_rng());
    let p1 = points[0];
    //The circle is determined by two points, P_1 and Q. The center of the
↪   circle is
    //at (P_1 + Q)/2.0 and the radius is |(P_1 - Q)/2.0|
    let d_1 = (
        (((p1.0 + q.0) / 2), (p1.1 + q.1) / 2),
```

```
        (distance_between_two_points(p1, q) / 2.0),
    );

    let mut d_prev = d_1;

    for j in 1..points.len() {
        let p_j = points[j];
        if distance_between_two_points(p_j, d_prev.0) <= (d_prev.1) {
            //d_prev = d_prev;
        } else {
            let new = min_circle_w_points(&points[..j], p_j, q);
            if distance_between_two_points(p_j, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_points(points: &[Point], q1: Point, q2: Point) -> Circle {
    let mut points = points.to_vec();

    let d_0 = (
        (((q1.0 + q2.0) / 2), (q1.1 + q2.1) / 2),
        (distance_between_two_points(q1, q2) / 2.0),
    );

    let mut d_prev = d_0;
    for k in 0..points.len() {
        let p_k = points[k];
        if distance_between_two_points(p_k, d_prev.0) <= (d_prev.1) {
        } else {
            let new = min_circle_w_3_points(q1, q2, p_k);
            if distance_between_two_points(p_k, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_3_points(q1: Point, q2: Point, q3: Point) -> Circle {
    let (ax, ay) = (q1.0 as f64, q1.1 as f64);
    let (bx, by) = (q2.0 as f64, q2.1 as f64);
    let (cx, cy) = (q3.0 as f64, q3.1 as f64);

    let mut d = 2. * (ax * (by - cy) + bx * (cy - ay) + cx * (ay - by));
    if d == 0.0 {
        d = std::cmp::max(
            std::cmp::max(
                distance_between_two_points(q1, q2) as i64,
                distance_between_two_points(q2, q3) as i64,
            ),
            distance_between_two_points(q1, q3) as i64,
        ) as f64
            / 2.;
    }
    let ux = ((ax * ax + ay * ay) * (by - cy)
        + (bx * bx + by * by) * (cy - ay)
        + (cx * cx + cy * cy) * (ay - by))
        / d;
    let uy = ((ax * ax + ay * ay) * (cx - bx)
        + (bx * bx + by * by) * (ax - cx)
        + (cx * cx + cy * cy) * (bx - ax))
        / d;
    let mut center = (ux as i64, uy as i64);

    if center.0 < 0 {
        center.0 = 0;
    }
    if center.1 < 0 {
        center.1 = 0;
    }
    let d = distance_between_two_points(center, q1);
    (center, d)
}

fn main() {
```

```
    let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
    let mut window = Window::new(
        "Test - ESC to exit",
        WINDOW_WIDTH,
        WINDOW_HEIGHT,
        WindowOptions {
            title: true,
            //borderless: true,
            resize: true,
            //transparency: true,
            ..WindowOptions::default()
        },
    )
    .unwrap();

    // Limit to max ~60 fps update rate
    window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

    let mut full = Image::new(WINDOW_WIDTH, WINDOW_HEIGHT, 0, 0);
    let mut image = Image::new(ME_WIDTH, ME_HEIGHT, 45, 45);
    image.bytes = bits_to_bytes(ME_BITS, ME_WIDTH);
    let (center, r) = bc(&image);
    image.draw_outline();

    full.plot_circle((center.0 + 45, center.1 + 45), r as i64, 0.);
    while window.is_open() && !window.is_key_down(Key::Escape) && !window.is_key_down(Key::Q) {
        image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
        full.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

        window
            .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
            .unwrap();

        let millis = std::time::Duration::from_millis(100);

        std::thread::sleep(millis);
    }
}
```

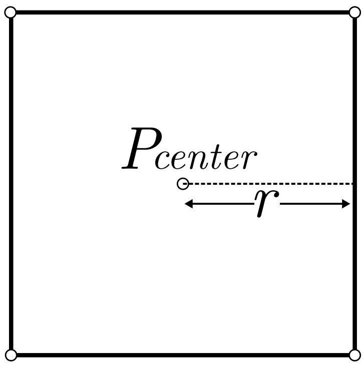
Part V

Points, Lines and Shapes

Chapter 23
Rectangles and parallelograms

23.1 Squares

23.1.1 From a center point



Square from given center point P_{center} and radius r

```
fn plot_square(image: &mut Image, center: Point, r: i64, wd: f64) {
  let (cx, cy) = center;
  let a = (cx - r, cy - r);
  let b = (cx + r, cy - r);
  let c = (cx + r, cy + r);
  let d = (cx - r, cy + r);
  image.plot_line_width(a, b, wd);
  image.plot_line_width(b, c, wd);
  image.plot_line_width(c, d, wd);
```

```
  image.plot_line_width(d, a, wd);
}
```

23.1.2 From a corner point

```
fn calc_center_point(p: Point, top: bool, right: bool, r: i64) -> Point {
    let (x, y) = p;
    match (top, right) {
        // Top right
        (true, true) => (x - r, y + r),
        // Top left
        (true, false) => (x + r, y + r),
        // Bottom right
        (false, true) => (x - r, y - r),
        // Bottom left
        (false, false) => (x + r, y - r),
    }
}
let r = 50;
let center_p = calc_center_point((155, 215), false, false, r);
//image.plot_circle(center_p, 3, 1.0);
plot_square(&mut image, center_p, r, 1.0);
```

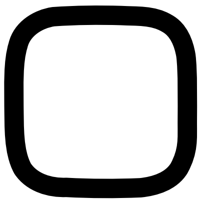
23.2 Rectangles

Chapter 24
Triangles

24.1 Making a triangle from a point and given angles

Chapter 25

Squircle



A *squircle* is a compromise between a square and a circle. It is purported to be more pleasing to the eye because the rounding corner is smoother than that of a circle arc (like the result of *Join segments with round corners*, page 109).

`src/bin/squircle.rs`:

This code file is a PDF attachment

A way to describe a squircle is as a superellipse, meaning a generalization of the ellipse equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by making the exponent parametric:

$$|x - a|^n + |y - b|^n = 1$$

The squircle as a superellipse is usually defined for $n = 4$.

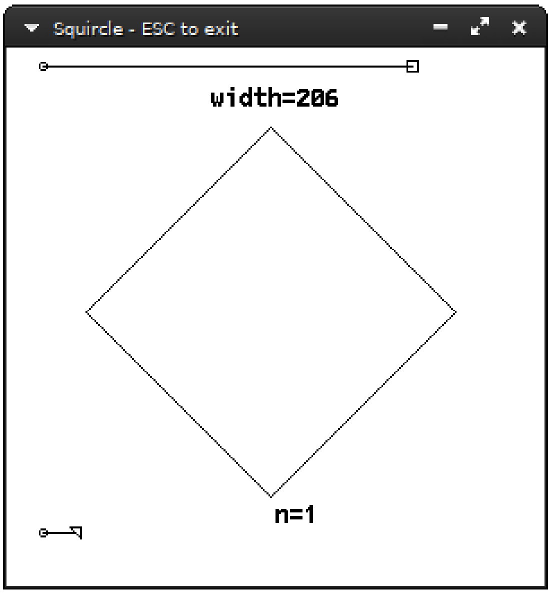
The code

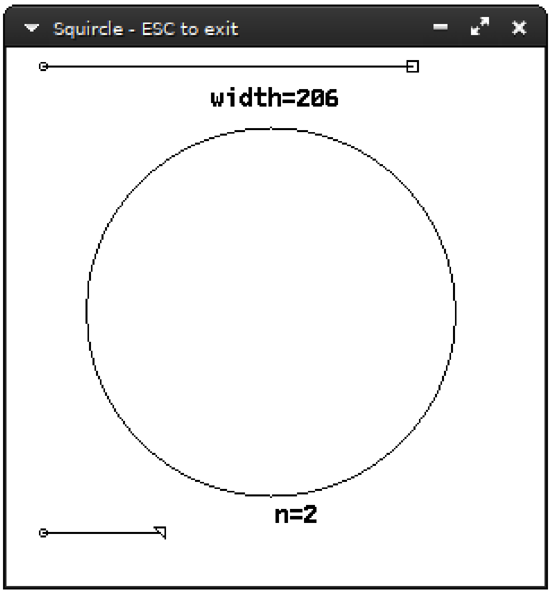
```
pub fn plot_squircle(
    image: &mut Image,
    (xm, ym): (i64, i64),
    width: i64,
    height: i64,
    n: i32,
    _wd: f64,
) {
    let r = width / 2;
    let w = width / 2;
    let h = height / 2;

    let mut prev_pos = (xm - w, xm - h);

    for i in 0..(2 * r + 1) {
        let x: i64 = (i - r) + w;
        let y: i64 = ((r as f64).powi(n) - (i as f64 - r as f64).abs().powi(n)).powf(1. /
 ↪  n as f64)
            as i64
            + h;
        if i != 0 {
            image.plot_line_width(prev_pos, (xm - x as i64, ym - y), _wd);
        }
        prev_pos = (xm - x as i64, ym - y);
    }
    for i in (2 * r)..(4 * r + 1) {
```

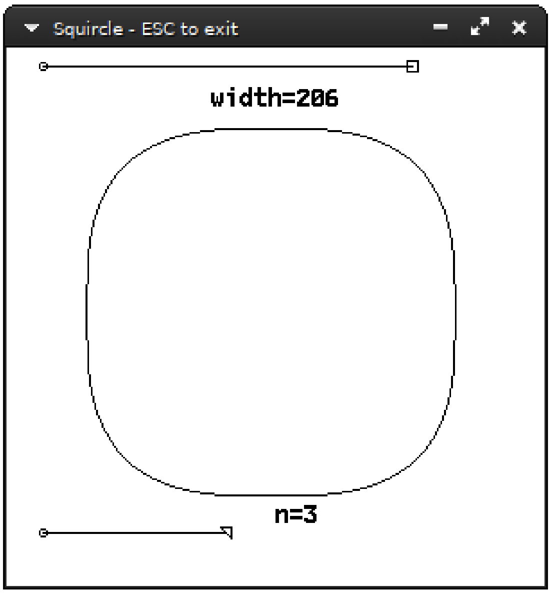
```
        let x: i64 = (3 * r - i) + w;
        let y = -1
            * (((r as f64).powi(n) - ((3 * r - i) as f64).abs().powi(n)).powf(1. / n as f64))
                as i64
            + h;
        image.plot_line_width(prev_pos, (xm - x as i64, ym - y), _wd);
        prev_pos = (xm - x as i64, ym - y);
    }
}
```

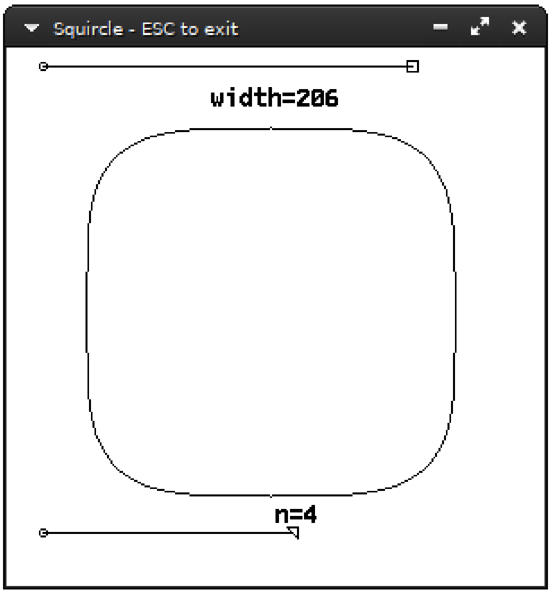
Different values of *n*

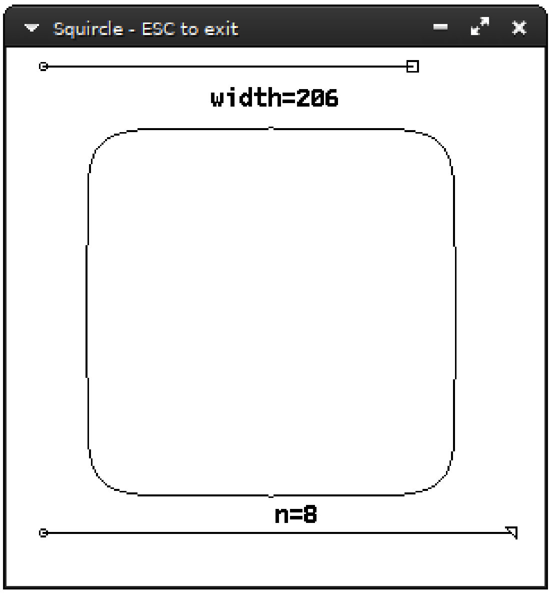
Increasing *n* in `src/bin/squircle.rs` makes the hyperellipse corners approach the square's.






Squircle - ESC to exit
width=206
n=3


Squircle - ESC to exit
width=206
n=4


Squircle - ESC to exit
width=206
n=8

Chapter 26

Union, intersection and difference of polygons

Add *Union, intersection and difference of polygons*

shapes

Chapter 27

Centroid of polygon

Add *Centroid of polygon*

shapes

Chapter 28
Polygon clipping

shapes

Chapter 29
Triangle filling

Add *Triangle filling* explanation

The book's library methods include a `fill_triangle` method:

This code is included in the distributed library file in the *Data representation* chapter.

```
pub fn fill_triangle(&mut self, q1: Point, q2: Point, q3: Point) {
    let make_equation =
        |p1: Point, p2: Point, p3: Point, a: &mut i64, b: &mut i64, c: &mut i64| {
            *a = p2.1 - p1.1;
            *b = p1.0 - p2.0;
            *c = p1.0 * p2.1 - p1.1 * p2.0;

            if *a * p3.0 + *b * p3.1 + *c < 0 {
                *a = -*a;
                *b = -*b;
                *c = -*c;
            }
        };
    let mut x_min = q1.0;
    let mut y_min = q1.1;
    let mut x_max = q1.0;
    let mut y_max = q1.1;
    let mut a = [0_i64; 3];
    let mut b = [0_i64; 3];
    let mut c = [0_i64; 3];

    // find bounding box
    for q in [q1, q2, q3] {
        x_min = std::cmp::min(x_min, q.0);
        x_max = std::cmp::max(x_max, q.0);

        y_min = std::cmp::min(y_min, q.1);
        y_max = std::cmp::max(y_max, q.1);
    }
    make_equation(q1, q2, q3, &mut a[0], &mut b[0], &mut c[0]);
    make_equation(q1, q3, q2, &mut a[1], &mut b[1], &mut c[1]);
    make_equation(q2, q3, q1, &mut a[2], &mut b[2], &mut c[2]);

    let mut d0 = a[0] * x_min + b[0] * y_min + c[0];
    let mut d1 = a[1] * x_min + b[1] * y_min + c[1];
    let mut d2 = a[2] * x_min + b[2] * y_min + c[2];

    for y in y_min..=y_max {
        let mut f0 = d0;
        let mut f1 = d1;
        let mut f2 = d2;

        d0 += b[0];
        d1 += b[1];
        d2 += b[2];

        for x in x_min..=x_max {
            if f0 >= 0 && f1 >= 0 && f2 >= 0 {
                self.plot(x, y);
            }
            f0 += a[0];
            f1 += a[1];
            f2 += a[2];
        }
    }
}
```

shapes

Chapter 30
Flood filling

Add *Flood filling*

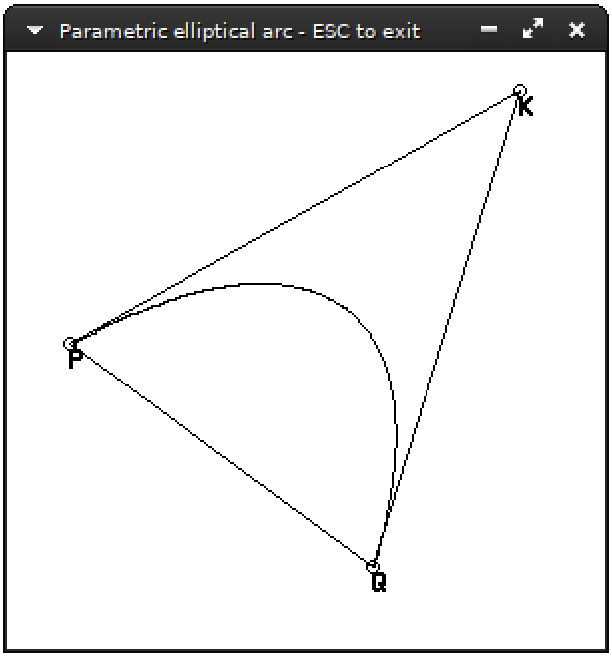
shapes

Part VI

Curves other than circles

curves

Chapter 31
Parametric elliptical arcs



P, Q and K are the arc's control points.

curves

This algorithm* draws an elliptical arc starting from point P and ending at Q. The control point K mirrors the ellipse's center J: drawing the quadrilateral $PKQJ$ would appear as a lozenge, or rhombus.

The parameter t defines the step angle in radians and is limited to $0 < t \leq 1$. For each point calculation, the point is t radians away from the previous one, so to increase the amount of points calculated keep t small.

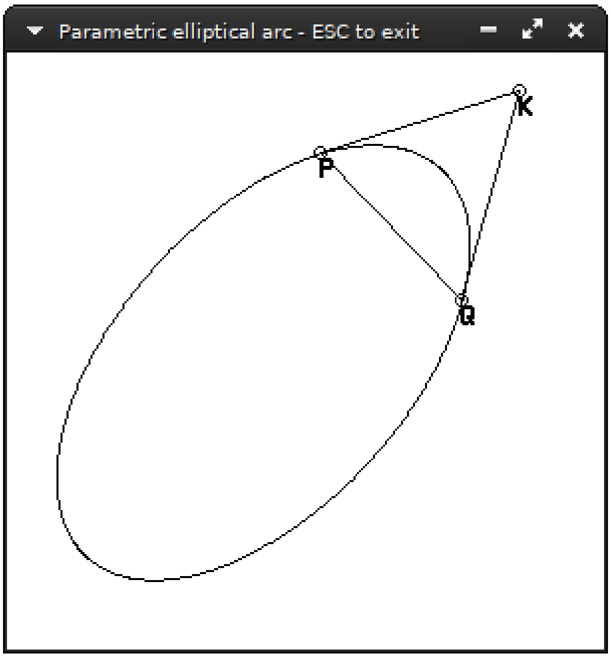
src/bin/parellarc.rs:

This code file is a PDF attachment

```
fn parellarc(image: &mut Image, p: Point, q: Point, k: Point, t: f64) {
    if t <= 0. || t > 1. {
        return;
    }
    let mut v = ((k.0 - q.0) as f64, (k.1 - q.1) as f64);
    let mut u = ((k.0 - p.0) as f64, (k.1 - p.1) as f64);
    let j = ((p.0 as f64 - v.0 + 0.5), (p.1 as f64 - v.1 + 0.5));
```

*_Graphics Gems III_ page 164

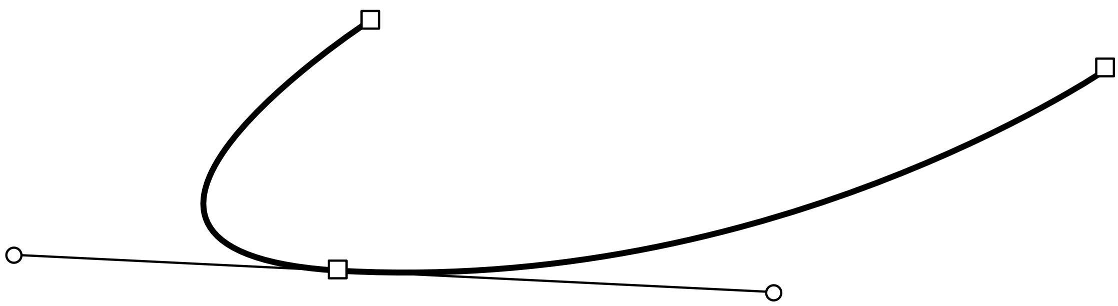
```
    u = (
        (u.0 * f64::sqrt(1. - t * t * 0.25) - v.0 * t * 0.5),
        (u.1 * f64::sqrt(1. - t * t * 0.25) - v.1 * t * 0.5),
    );

    let n = (std::f64::consts::FRAC_PI_2 / t).floor() as u64;

    let mut prev_pos = p;
    for _ in 0..n {
        let x = (v.0 + j.0).round() as i64;
        let y = (v.1 + j.1).round() as i64;
        let new_point = (x, y);
        image.plot_line_width(prev_pos, new_point, 1.);
        prev_pos = new_point;

        u.0 -= v.0 * t;
        v.0 += u.0 * t;
        u.1 -= v.1 * t;
        v.1 += u.1 * t;
    }
}
```



Changing n to $\frac{2\pi}{t}$ draws the entire ellipse.

curves

Chapter 32
Bézier curves

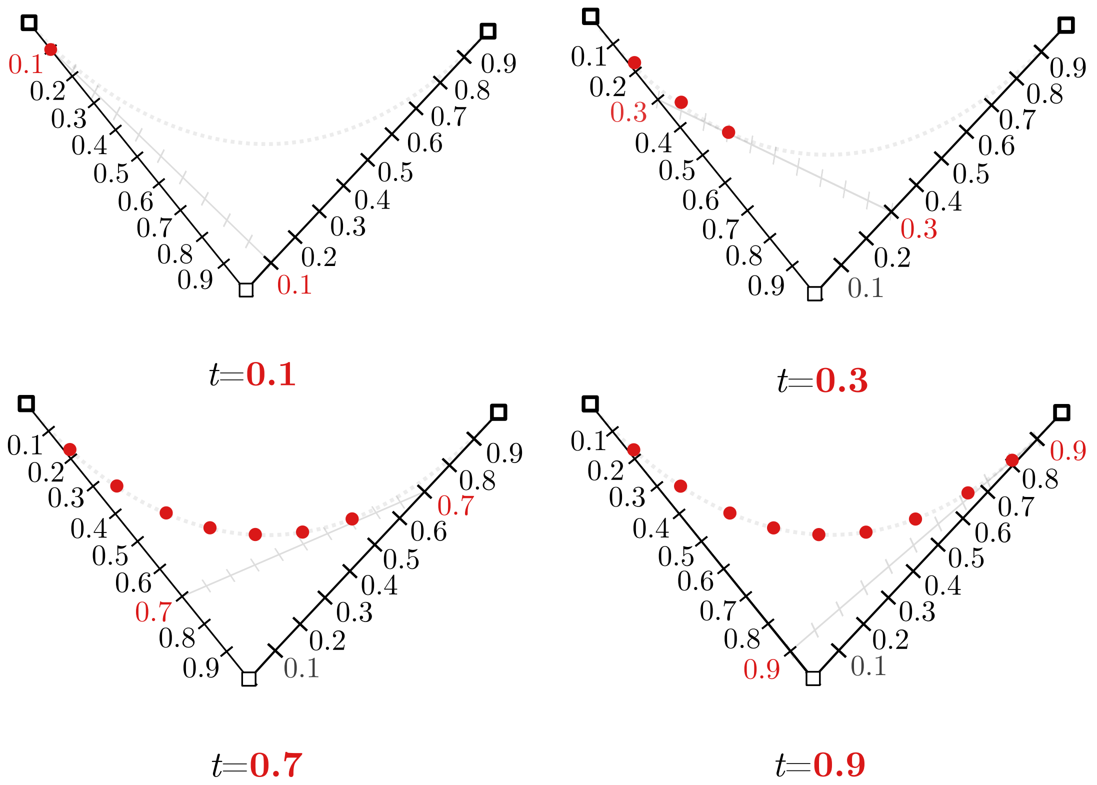


Two cubic *Bézier* curves joined together as displayed in graphics software.

32.1 Quadratic Bézier curves

32.1.1 Drawing the quadratic

To actually draw a curve, i.e. with points P_1, P_2, P_3 we will use *de Casteljau's algorithm*. The gist behind the algorithm is that the length of the curve is visited at specific percentages (e.g. 0%, 0.2%, 0.4% ... 99.8%, 100%), meaning we will have that many steps, and for each such percentage t we calculate a line starting at the t-nth point of P_1P_2 and ending at the t-nth point of P_2P_3. The t-eth point of that line also belongs to the curve, so we plot it.



Computing curve points for values of $t \in [0, 1]$ with de Casteljau's algorithm

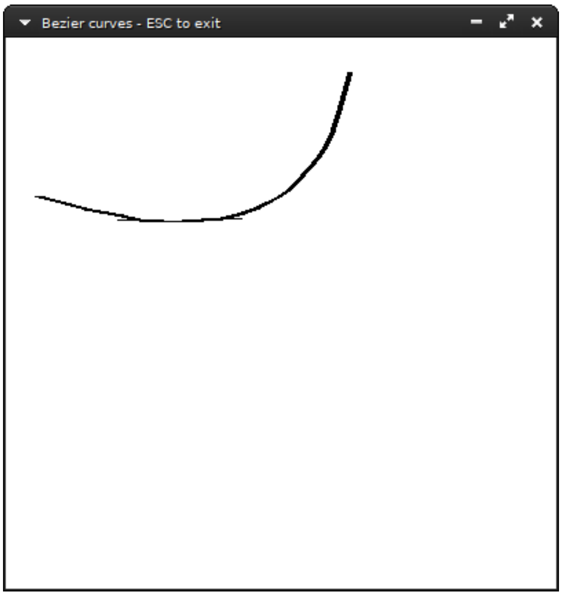
Let's draw the curve $P_1 = (25, 115), P_2 = (225, 180), P_3 = (250, 25)$

src/bin/bezier.rs:

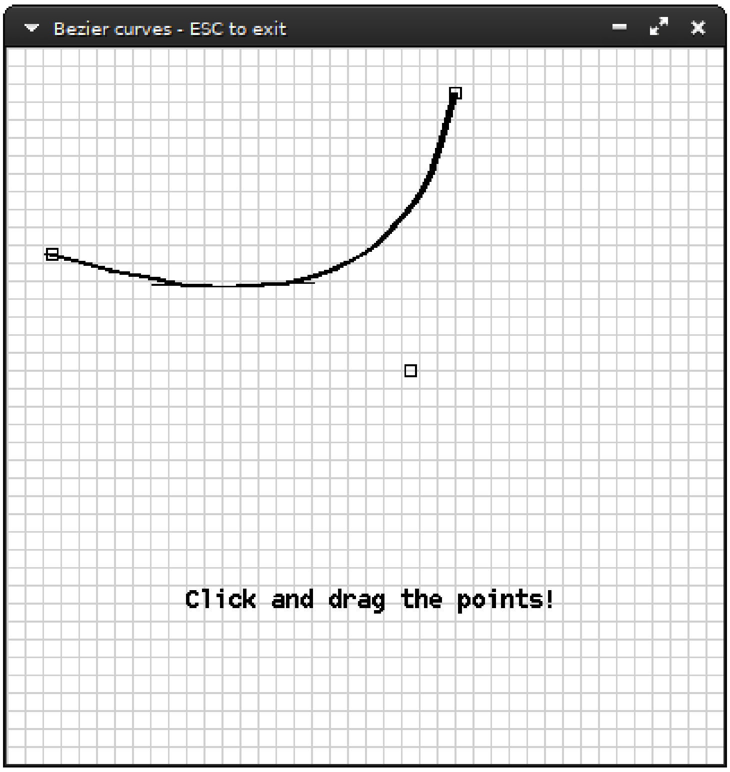
This code file is a PDF attachment

The result:

curves



The `minifb` library allows to track user input, so we detect user clicks and the mouse's position; thus we can interactively modify a curve with some modifications in the code:



Interactively modifying a curve with the `bezier.rs` tool.

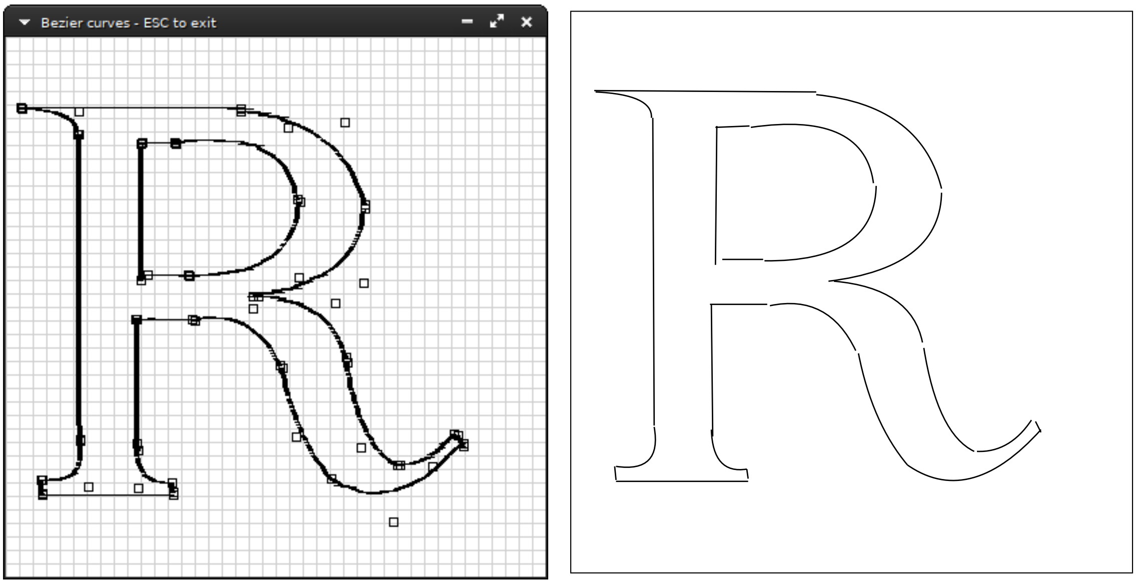
We can go one step further and insult type designers* and use the tool to make a font glyph.

src/bin/bezierg

This code file is a PDF attachment

Of course, it requires effort to match the beginning and end of each curve that makes up the glyph. That's why font designing tools have *point snapping* to ensure curve continuation. But for a quick font designer app prototype, it's good enough.

*who use cubic Béziers or other fancier curves (*splines*)



Left: A font glyph drawn with the interactive `bezierglyph.rs` tool. *Right*: the same glyph exported to `SVG`.

32.2 Cubic Bézier curves

32.3 Weighted Béziers

Part VII

Vectors, matrices and transformations

trans-
forma-
tions

Chapter 33
Rotation of a bitmap

$$p' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

$$c = \cos\theta, s = \sin\theta, x_{p'} = x_p c - y_p s, y_{p'} = x_p s + y_p c.$$

Let's load an `xface`. We will use `bits_to_bytes` (See *Bits to byte pixels*, page 14).

src/bin/rotation.rs:

This code file is a PDF attachment

```
include!("dmr.rs");
const WINDOW_WIDTH: usize = 100;
const WINDOW_HEIGHT: usize = 100;
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

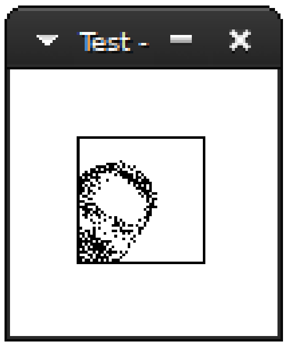
This is the `xface` of `dmr`. Instead of displaying the bitmap, this time we will rotate it 0.5 radians. Setup our image first:

```
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.draw_outline();
let dmr = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

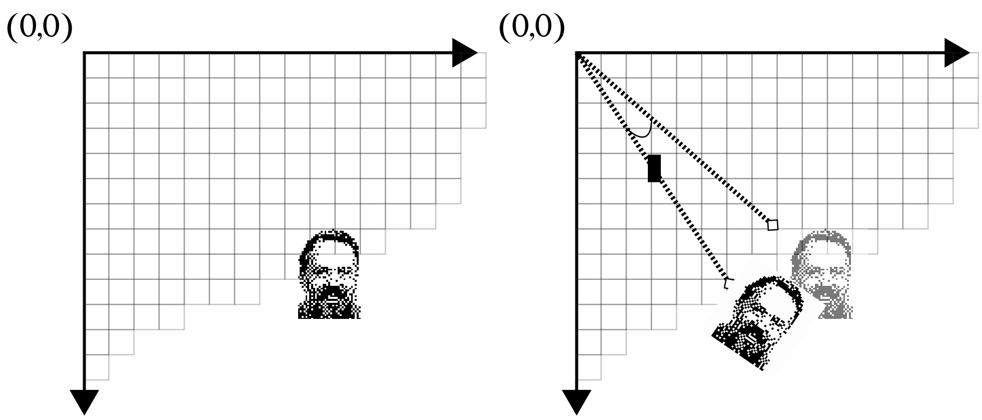
And then, loop for each byte in `dmr`'s face and apply the rotation transformation.

```
let angle = 0.5;

let c = f64::cos(angle);
let s = f64::sin(angle);

for y in 0..DMR_HEIGHT {
    for x in 0..DMR_WIDTH {
        if dmr[y * DMR_WIDTH + x] == BLACK {
            let x = x as f64;
            let y = y as f64;
            let xr = x * c - y * s;
            let yr = x * s + y * c;
            image.plot(xr as i64, yr as i64);
        }
    }
}
```

The result:



We didn't mention in the beginning that the rotation has to be relative to a *point* and the given transformation is relative to the *origin*, in this case the upper left corner $(0,0)$. So `dmr` was rotated relative to the origin :



(the distance to the origin (actually 0 pixels) has been exaggerated for the sake of the example)

Usually, we want to rotate something relative to itself. The right point to choose is the *centroid* of the object.

If we have a list of n points, the centroid is calculated as:

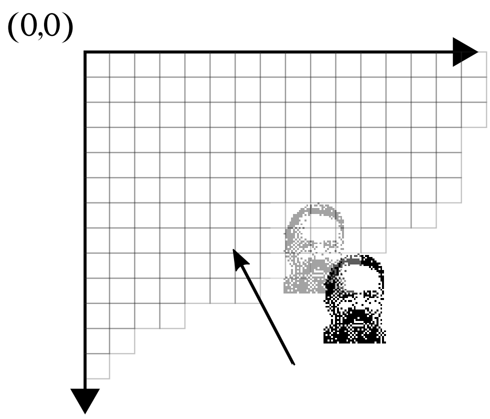
$$x_c = \frac{1}{n}\sum_{i=0}^{n} x_i$$

$$y_c = \frac{1}{n}\sum_{i=0}^{n} y_i$$

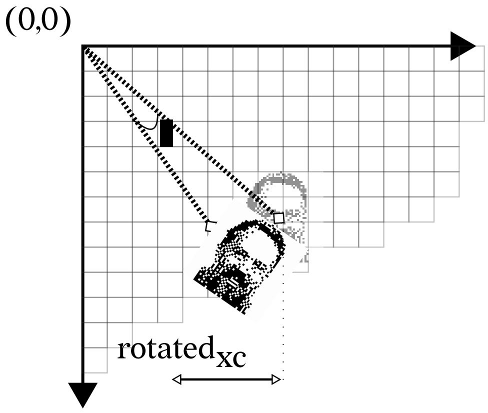
Since in this case we have a rectangle, the centroid has coordinates of half the width and half the height.

By subtracting the centroid from each point before we apply the transformation and then adding it back after we get what we want:

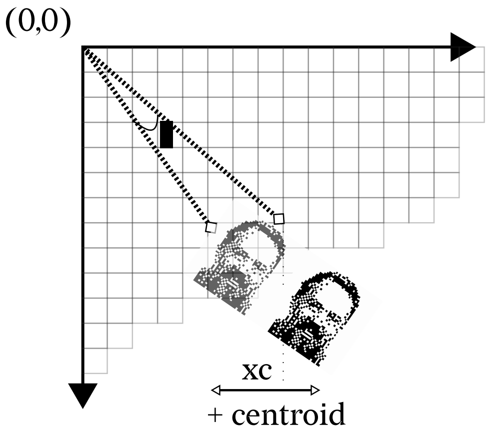
Here's it visually: First subtract the center point.



Then, rotate.



And subtract back to the original position.



In code:

```
let center_point = ((DMR_WIDTH/2) as i64, (DMR_HEIGHT/2) as i64);
for y in 0..DMR_HEIGHT {
    for x in 0..DMR_WIDTH {
        if dmr[y * DMR_WIDTH + x] == BLACK {
            let x = (x as i64 -center_point.0) as f64;
            let y = (y as i64 -center_point.1) as f64;
            let xr = x * c - y * s;
            let yr = x * s + y * c;
            image.plot(xr as i64+center_point.0,
                       yr as i64 + center_point.1);
        }
    }
}
```

The result: 

33.1 Fast 2D Rotation

Add *Fast 2D Rotation*

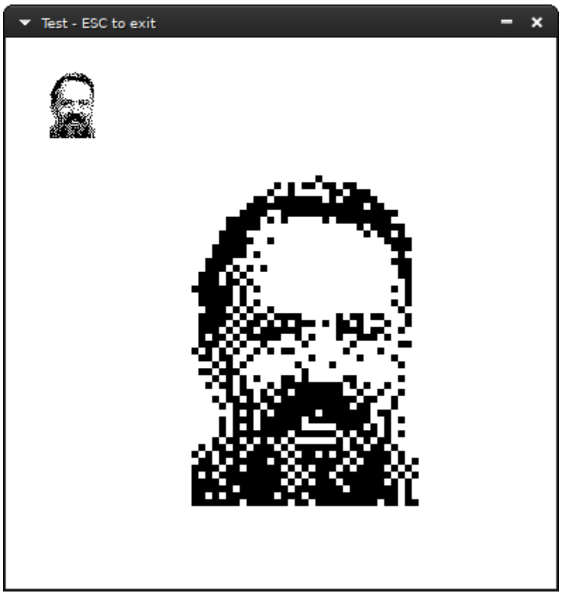
Chapter 34

90° Rotation of a bitmap by parallel recursive subdivision

Add *90° Rotation of a bitmap by parallel recursive subdivision*

trans-
forma-
tions

Chapter 35
Magnification/Scaling



We want to magnify a bitmap without any smoothing. We define an `Image` scaled to the dimensions we want, and loop for every pixel in the scaled `Image`. Then, for each pixel, calculate its source in the original bitmap: if the coordinates in the scaled bitmap are (x, y) then the source coordinates (sx, sy) are:

$$sx = \frac{x * original.width}{scaled.width}$$
$$sy = \frac{y * original.height}{scaled.height}$$

So, if (sx, sy) are painted, then (x, y) must be painted as well.

```
let mut original = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
original.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
original.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

let mut scaled = Image::new(DMR_WIDTH * 5, DMR_HEIGHT * 5, 100, 100);
let mut sx: i64; //source
let mut sy: i64; //source
let mut dx: i64; //destination
let mut dy: i64 = 0; //destination
```

`src/bin/scale.rs`:

This code file is a PDF attachment

trans-forma-tions

```
let og_height = original.height as i64;
let og_width = original.width as i64;
let scaled_height = scaled.height as i64;
let scaled_width = scaled.width as i64;

while dy < scaled_height {
    sy = (dy * og_height) / scaled_height;
    dx = 0;
    while dx < scaled_width {
        sx = (dx * og_width) / scaled_width;
        if original.get(sx, sy) == Some(BLACK) {
            scaled.plot(dx, dy);
        }
        dx += 1;
    }
    dy += 1;
}
scaled.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
```

35.1 Smoothing enlarged bitmaps

Add *Smoothing enlarged bitmaps*

35.2 Stretching lines of bitmaps

Add *Stretching lines of bitmaps*

trans-
forma-
tions

Chapter 36
Mirroring

Add screenshots and figure and code in *Mirroring*

Mirroring to an axis is the transformation of one coordinate to its equidistant value across the axis:

To mirror a pixelacross the x axis, simply multiply its coordinates with the following matrix:

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

This results in the y coordinate's sign being flipped.

For y-mirroring, the transformation follows the same logic:

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

trans-
forma-
tions

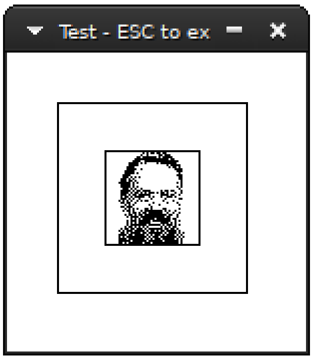
Chapter 37

Shearing

src/bin/shearing.rs:

This code file is a PDF attachment

Simple shearing is the transformation of one dimension by a distance proportional to the other dimension, In x-shearing (or horizontal shearing) only the x coordinate is affected, and likewise in y-shearing only y as well.



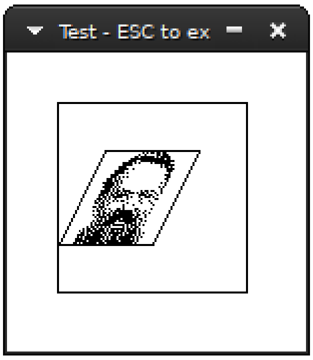
With l being equal to the desired tilt away from the y axis, the transformation is described by the following matrix:

$$S_x = \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}$$

Which is as simple as this function:

```
fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p+(l*(y_p as f64)) as i64, y_p)
}
```

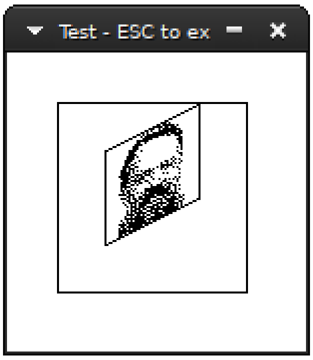

trans-
forma-
tions



For y-shearing, we have the following:

$$S_y = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix}$$

```
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}
```

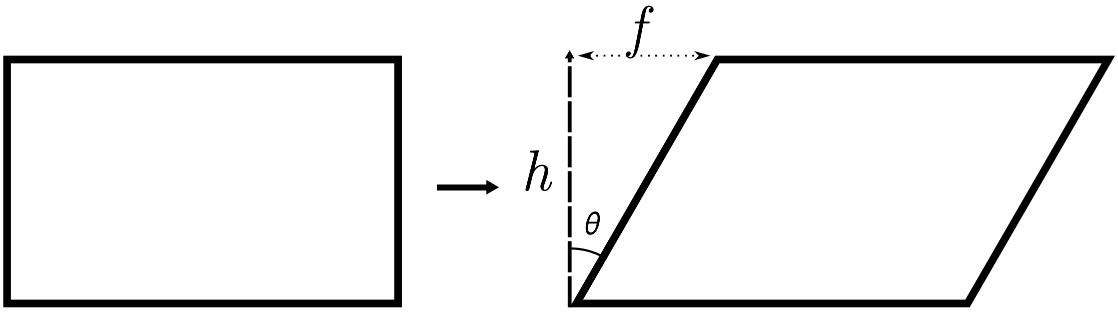


A full example:

```
include!("../dmr.xbm.rs");
const WINDOW_WIDTH: usize = 200;
const WINDOW_HEIGHT: usize = 200;
fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p+(l*(y_p as f64)) as i64, y_p)
}
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
image.draw_outline();
```

```
let l = -0.5;
let mut sheared = Image::new(DMR_WIDTH*2, DMR_HEIGHT*2, 25, 25);
for x in 0..DMR_WIDTH {
    for y in 0..DMR_HEIGHT  {
        if image.bytes[y * DMR_WIDTH + x] == BLACK {
          let p = shear_x((x as i64 ,y as i64 ), l);
          sheared.plot(p.0+(DMR_WIDTH/2) as i64, p.1+(DMR_HEIGHT/2) as i64);
        }
    }
}
sheared.draw_outline();
```

37.1 The relationship between shearing factor and angle



Shearing is a delta movement in one dimension, thus the point before moving and the point after form an angle with the x axis. To move a point $(x, 0)$ by 30° forward we will have the new point $(x + f, 0)$ where f is the shear factor. These two points and (x, h) where h is the height of the bitmap form a triangle, thus the following are true:

$$\cot \theta = \frac{h}{f}$$

Therefore to find your factor for any angle θ replace its cotangent in the following formula:

$$f = \frac{h}{\cot \theta}$$

For example to shear by $-30°$ (meaning the bitmap will move to the right, since rotations are always clockwise) we need $\cot(-30deg) = -\sqrt{3}$ and $f = -\frac{h}{\sqrt{3}}$.

trans-forma-tions

Chapter 38
Projections

Add *Projections*

Part VIII

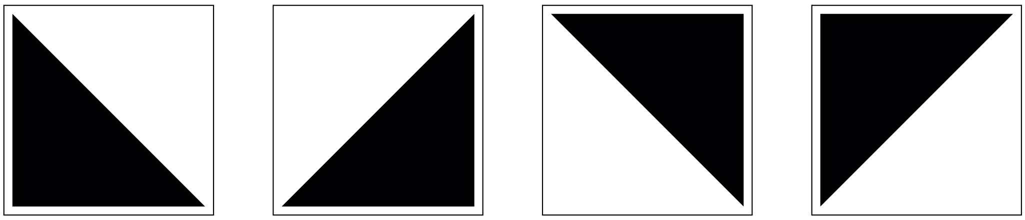
Patterns

Chapter 39
Tilings

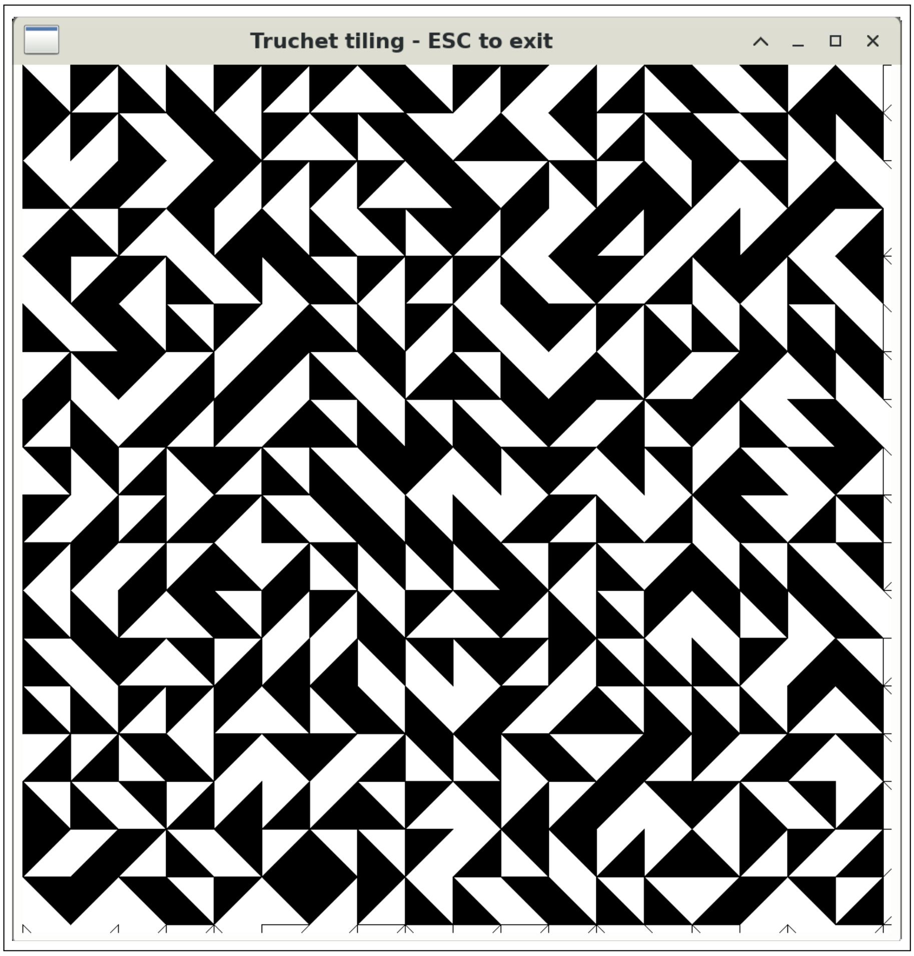
patterns

39.1 Truchet Tiling

Truchet tiling is a repetition of four specific tiles in any specific order. It can be random or deterministic.



The four tiles



Random arrangement of truchet tiles using `rand`.

The code

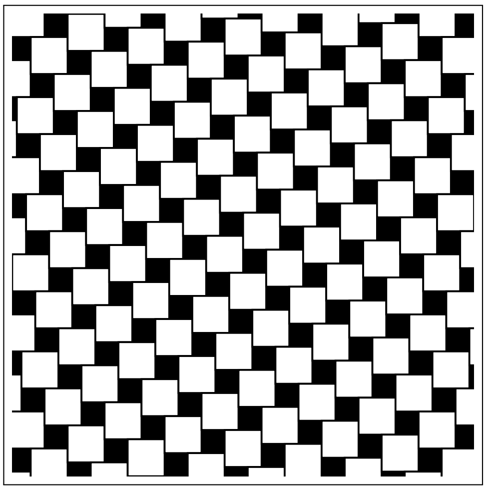
src/bin/floyddither.rs:

This code file is a PDF attachment

```
fn truchet(image: &mut Image, size: i64) {
    let mut x = 0;
    let mut y = 0;
    #[repr(u8)]
    enum Tile {
        A = 0,
        B,
        C,
        D,
    }
    let tiles = [Tile::A, Tile::B, Tile::C, Tile::D];
    let width = image.width as i64;
    let height = image.height as i64;
    let mut rng = thread_rng();
    while y < height {
        while x < width {
            let t = tiles.choose(&mut rng).unwrap();
            let (a, b, c) = match t {
                Tile::A => {
                    let a = (x, y + size);
                    let b = (x + size, y + size);
                    let c = (x + size, y);
                    (a, b, c)
                }
                Tile::B => {
                    let a = (x, y);
                    let b = (x, y + size);
                    let c = (x + size, y + size);
                    (a, b, c)
                }
                Tile::C => {
                    let a = (x, y);
                    let b = (x + size, y);
                    let c = (x, y + size);
                    (a, b, c)
                }
                Tile::D => {
                    let a = (x, y);
                    let b = (x + size, y);
                    let c = (x + size, y + size);
                    (a, b, c)
                }
            };
            image.plot_line_width(a, b, 1.);
            image.plot_line_width(b, c, 1.);
            image.plot_line_width(c, a, 1.);
            let c = ((a.0 + b.0 + c.0) / 3, (a.1 + b.1 + c.1) / 3);
            image.flood_fill(c.0, c.1);
            x += size;
        }
        x = 0;
        y += size;
    }
}
```

39.2 Pythagorean Tiling

Pythagorean tiling consists of two squares, one filled and one blank and is described by the ratio of their sizes.



Pythagorean tiling using the golden ratio $\phi \equiv \frac{1+\sqrt{5}}{2}$

The code

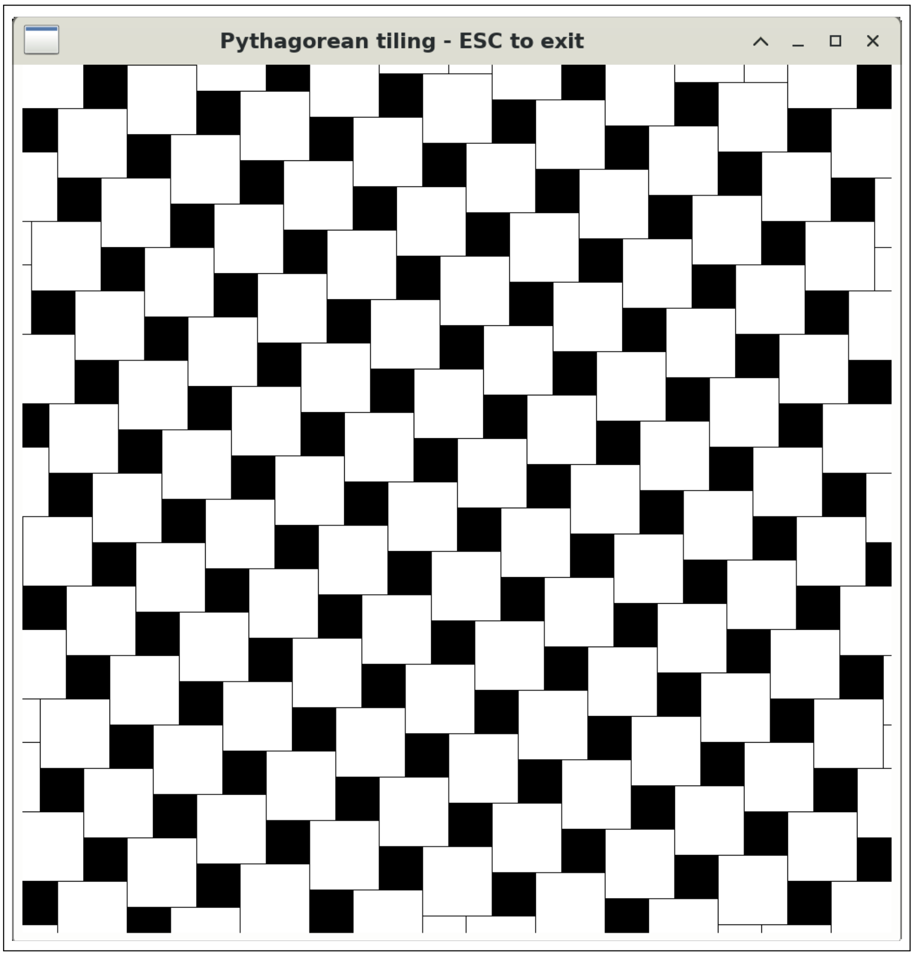
src/bin/pythagorean.rs:

This code file is a PDF attachment

```
fn pythagorean(image: &mut Image, size_a: i64, size_b: i64) {
    let width = image.width as i64;
    let height = image.height as i64;
    let times = 4 * width / (size_a + size_b);
    for i in -times..times {
        let mut x = -width + i * (size_b - size_a);
        let mut y = -height - i * (size_b + size_a);
        while y < 2 * height && x < 2 * width {
            // Draw the first smaller and filled rectangle
            let a = (x, y);
            let b = (x + size_a, y);
            let c = (x + size_a, y + size_a);
            let d = (x, y + size_a);
            image.plot_line_width(a, b, 0.);
            image.plot_line_width(b, c, 0.);
            image.plot_line_width(c, d, 0.);
            image.plot_line_width(d, a, 0.);
            // Calculate the center point of the rectangle in order to start flood
 ↪  filling from it
            let (cx, cy) = ((a.0 + b.0 + c.0 + d.0) / 4, (a.1 + b.1 + c.1 + d.1) / 4);
            image.flood_fill(cx, cy);
            x += size_a;
            // Draw the second bigger rectangle
            let a = b;
            let b = (a.0 + size_b, y);
            let c = (a.0 + size_b, y + size_b);
            let d = (a.0, y + size_b);
            image.plot_line_width(a, b, 1.);
            image.plot_line_width(b, c, 1.);
            image.plot_line_width(c, d, 1.);
```

patterns

```
            image.plot_line_width(d, a, 1.);
            y += size_b;
        }
    }
}
```



The output of `src/bin/pythagorean.rs`

patterns

39.3 Hexagon tiling

patterns

Chapter 40

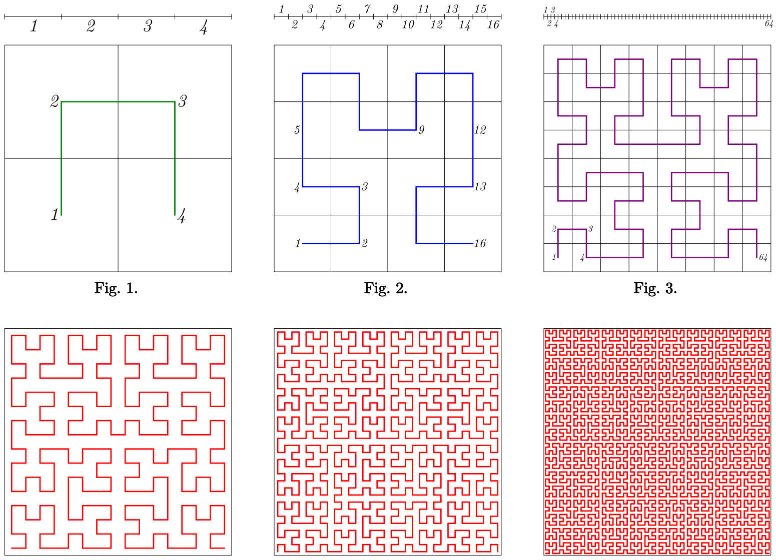
Space-filling Curves

Add *Space-filling Curves*

patterns

40.1 Hilbert curve

Add *Hilbert curve* explanation



The first six iterations of the Hilbert curve by Braindrain0000

src/bin/hilbert.rs:

This code file is a PDF attachment

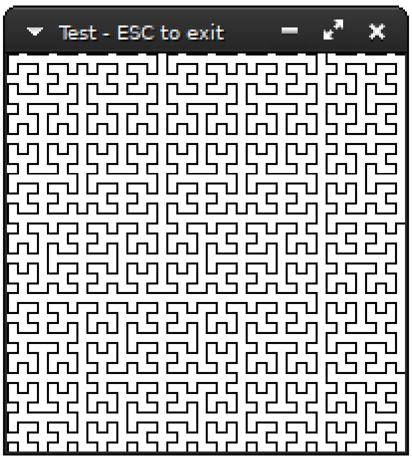
Here's a simple algorithm for drawing a Hilbert curve.*

```
const HILBERT: &[&[usize]] = &[
    &[22, 10, 16, 38],
    &[10, 22, 24, 48],
    &[44, 36, 30, 18],
    &[36, 44, 42, 28],
];
fn curve(img: &mut Image, k: usize, order: i64, mut x: i64, mut y: i64) -> (i64, i64) {
    const STEP_SIZE: i64 = 5;
    let mut row: usize;
    let mut direction: usize;
    if order > 0 {
        for j in 0..4 {
            let step = HILBERT[k][j];
            row = (step / 10) - 1;
            let (xn, yn) = curve(img, row, order - 1, x, y);
            x = xn;
            y = yn;
            direction = step % 10;
            let prev = (x, y);
            match direction {
                8 => {
                    // null op
                }
                2 => {
                    //N
                    y -= STEP_SIZE;
                }
                1 => {
```


*Griffiths, J. G. (1985). *Table-driven algorithms for generating space-filling curves*. Computer-Aided Design, 17(1), 37–41. doi:10.1016/0010-4485(85)90009-0

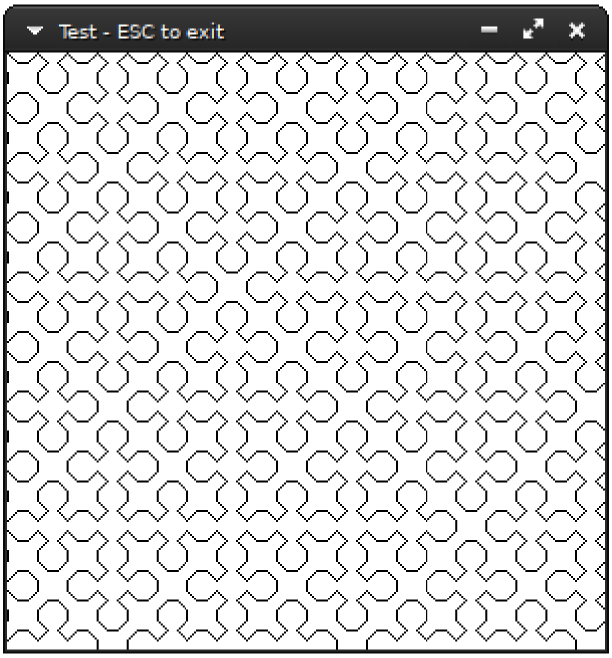
```
                    // NE
                    y -= STEP_SIZE;
                    x += STEP_SIZE;
                }
                0 => {
                    //E
                    x += STEP_SIZE;
                }
                7 => {
                    //SE
                    x += STEP_SIZE;
                    y += STEP_SIZE;
                }
                6 => {
                    //S
                    y += STEP_SIZE;
                }
                5 => {
                    //SW
                    y += STEP_SIZE;
                    x -= STEP_SIZE;
                }
                4 => {
                    //W
                    x -= STEP_SIZE;
                }
                3 => {
                    //NW
                    y -= STEP_SIZE;
                    x -= STEP_SIZE;
                }
                other => unreachable!("{}", other),
            }
            img.plot_line_width(prev, (x, y), 0.);
        }
    }
    (x, y)
}
```

```
let mut image = Image::new(WINDOW_WIDTH, WINDOW_WIDTH, 0, 0);
curve(&mut image, 0, 7, 0, WINDOW_WIDTH as i64);
```



patterns

40.2 Sierpiński curve



Switching the table from the Hilbert implementation to this:

```
const SIERP: &[&[usize]] = &[
    &[17, 25, 33, 41],
    &[17, 20, 41, 18],
    &[25, 36, 17, 28],
    &[33, 44, 25, 38],
    &[41, 12, 33, 48],
];
```

And switching two lines from the function to

```
- let step = HILBERT[k][j];
- row = (step / 10) - 1;
+ let step = SIERP[k][j];
+ row = (step / 10);
```

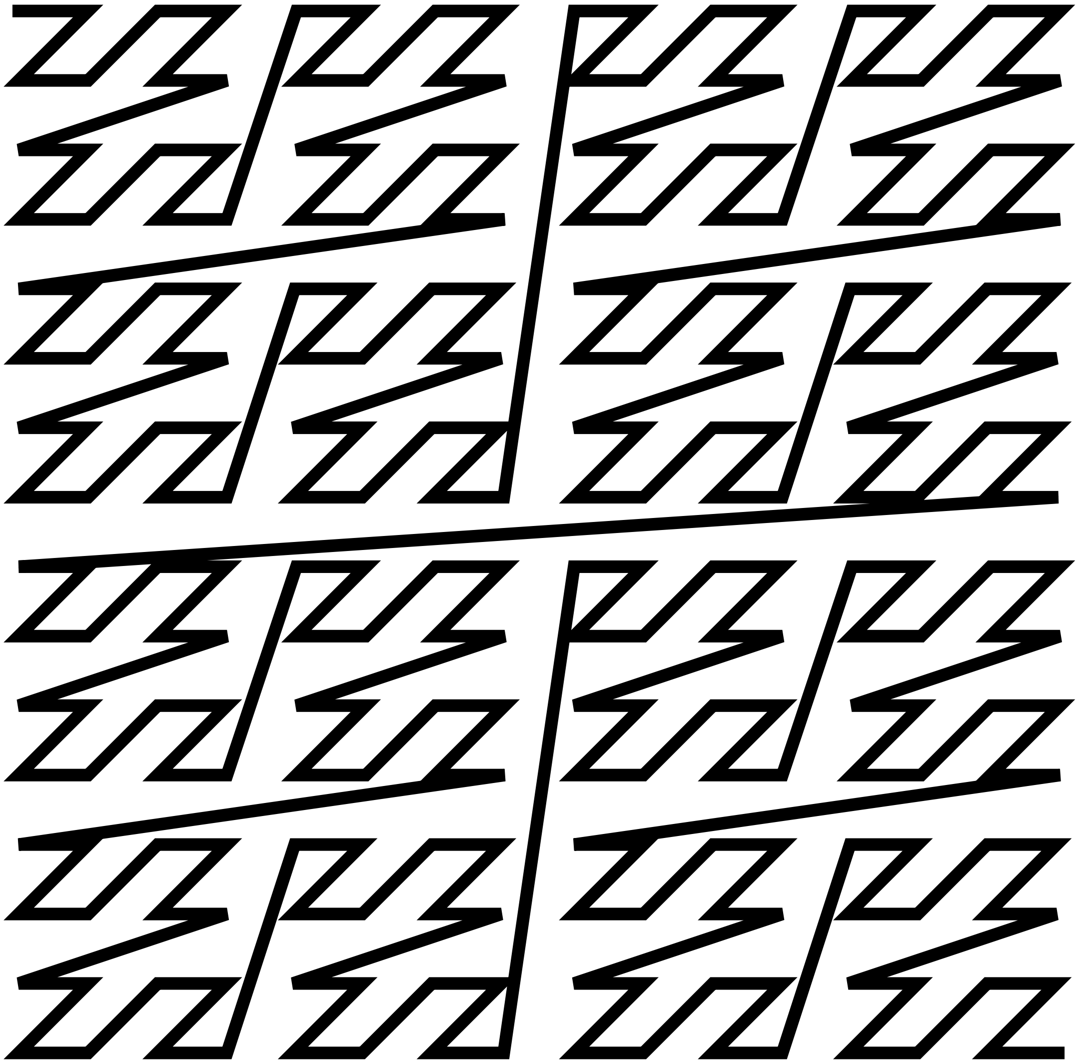
patterns

You can draw a Sierpinshi curve of order n by calling `curve(&mut image, 0,n+1, 0, 0)`.

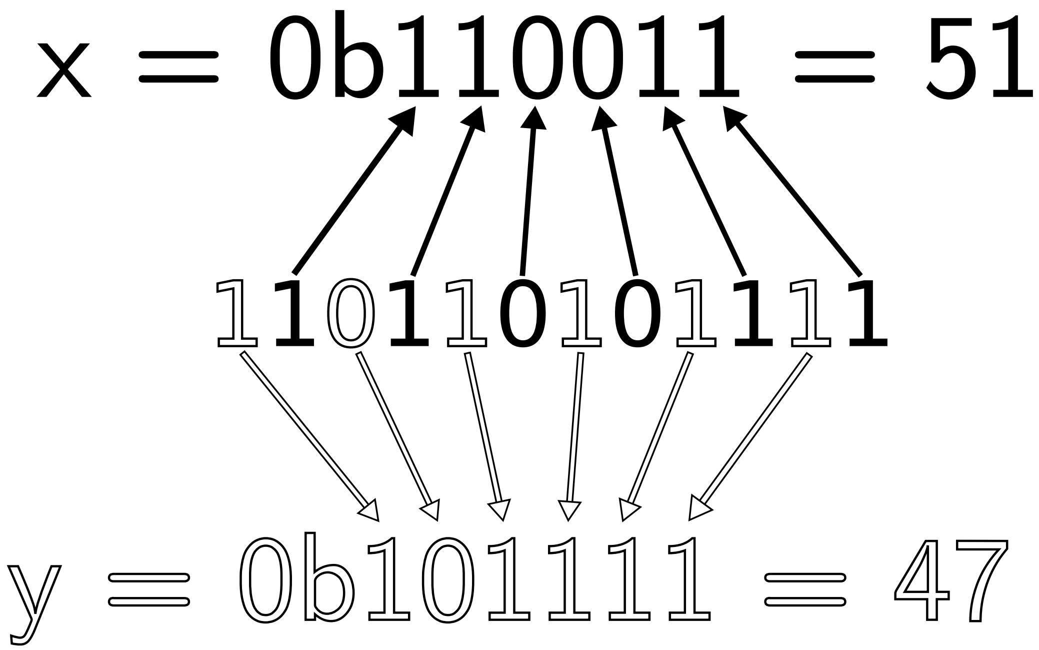
40.3 Peano curve

Add *Peano curve*

40.4 Z-order curve



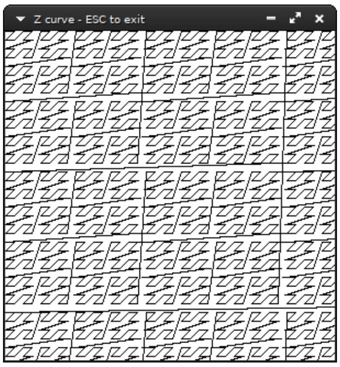
Drawing the Z-order curve is really simple: first, have a counter variable that starts from zero and is incremented by one at each step. Then, you extract the (x, y) coordinates the new step represents from its binary representation. The bits for the x coordinate are located at the odd bits, and for y at the even bits. I.e. the values are interleaved as bits in the value of the step:



Knowing this, implementing the drawing process will consist of computing the next step, drawing a line segment from the current step and the next, set the current step as the next and continue;

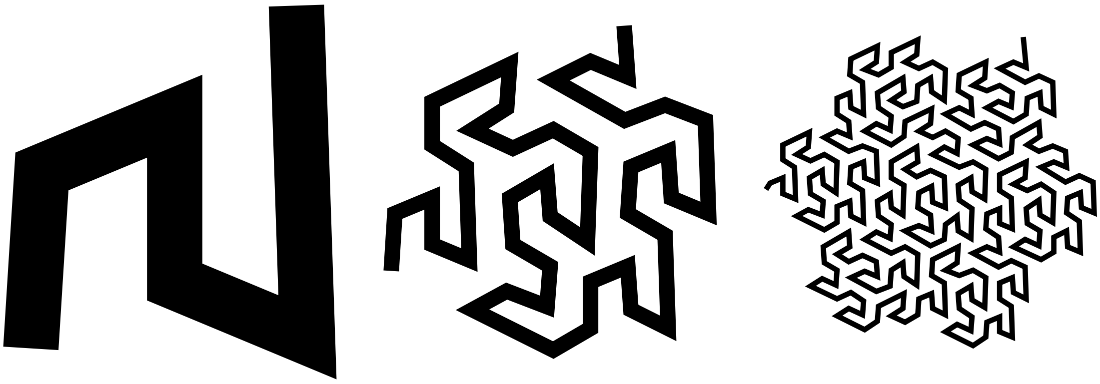
```
fn zcurve(img: &mut Image, x_offset: i64, y_offset: i64) {
    const STEP_SIZE: i64 = 8;
    let mut sx: i64 = 0;
    let mut sy: i64 = 0;
    let mut b: u64 = 0;

    let mut prev_pos = (sx + x_offset, sy + y_offset);
    loop {
        let next = b + 1;
        sx = 0;
        if (next & 1) as i64 > 0 {
            sx += STEP_SIZE;
        }
        if next & 0b100 > 0 {
            sx += 2 * STEP_SIZE;
        }
        if next & 0b10_000 > 0 {
            sx += 4 * STEP_SIZE;
        }
        if next & 0b1_000_000 > 0 {
            sx += 8 * STEP_SIZE;
        }
        if next & 0b100_000_000 > 0 {
            sx += 16 * STEP_SIZE;
        }
        if next & 0b10_000_000_000 > 0 {
            sx += 32 * STEP_SIZE;
        }
        if next & 0b1_000_000_000_000 > 0 {
            sx += 64 * STEP_SIZE;
        }
        if next & 0b100_000_000_000_000 > 0 {
            sx += 128 * STEP_SIZE;
```

```
        }
        if next & 0b10_000_000_000_000_000 > 0 {
            sx += 256 * STEP_SIZE;
        }
        if next & 0b1_000_000_000_000_000_000 > 0 {
            sx += 512 * STEP_SIZE;
        }
        sy = 0;
        if (next & 0b10) as i64 > 0 {
            sy += STEP_SIZE;
        }
        if next & 0b1_000 > 0 {
            sy += 2 * STEP_SIZE;
        }
        if next & 0b100_000 > 0 {
            sy += 4 * STEP_SIZE;
        }
        if next & 0b10_000_000 > 0 {
            sy += 8 * STEP_SIZE;
        }
        if next & 0b1_000_000_000 > 0 {
            sy += 16 * STEP_SIZE;
        }
        if next & 0b100_000_000_000 > 0 {
            sy += 32 * STEP_SIZE;
        }
        if next & 0b10_000_000_000_000 > 0 {
            sy += 64 * STEP_SIZE;
        }
        if next & 0b1_000_000_000_000_000 > 0 {
            sy += 128 * STEP_SIZE;
        }
        if next & 0b100_000_000_000_000_000 > 0 {
            sy += 256 * STEP_SIZE;
        }
        if next & 0b10_000_000_000_000_000_000 > 0 {
            sy += 512 * STEP_SIZE;
        }
        img.plot_line_width(prev_pos, (sx + x_offset, sy + y_offset), 1.0);

        if next == 0b111_111_111_111_111_111_111_111 {
            break;
        }
        if sx as usize > img.width && sy as usize > img.height {
            break;
        }

        prev_pos = (sx + x_offset, sy + y_offset);
        b = next;
    }
}
```



patterns

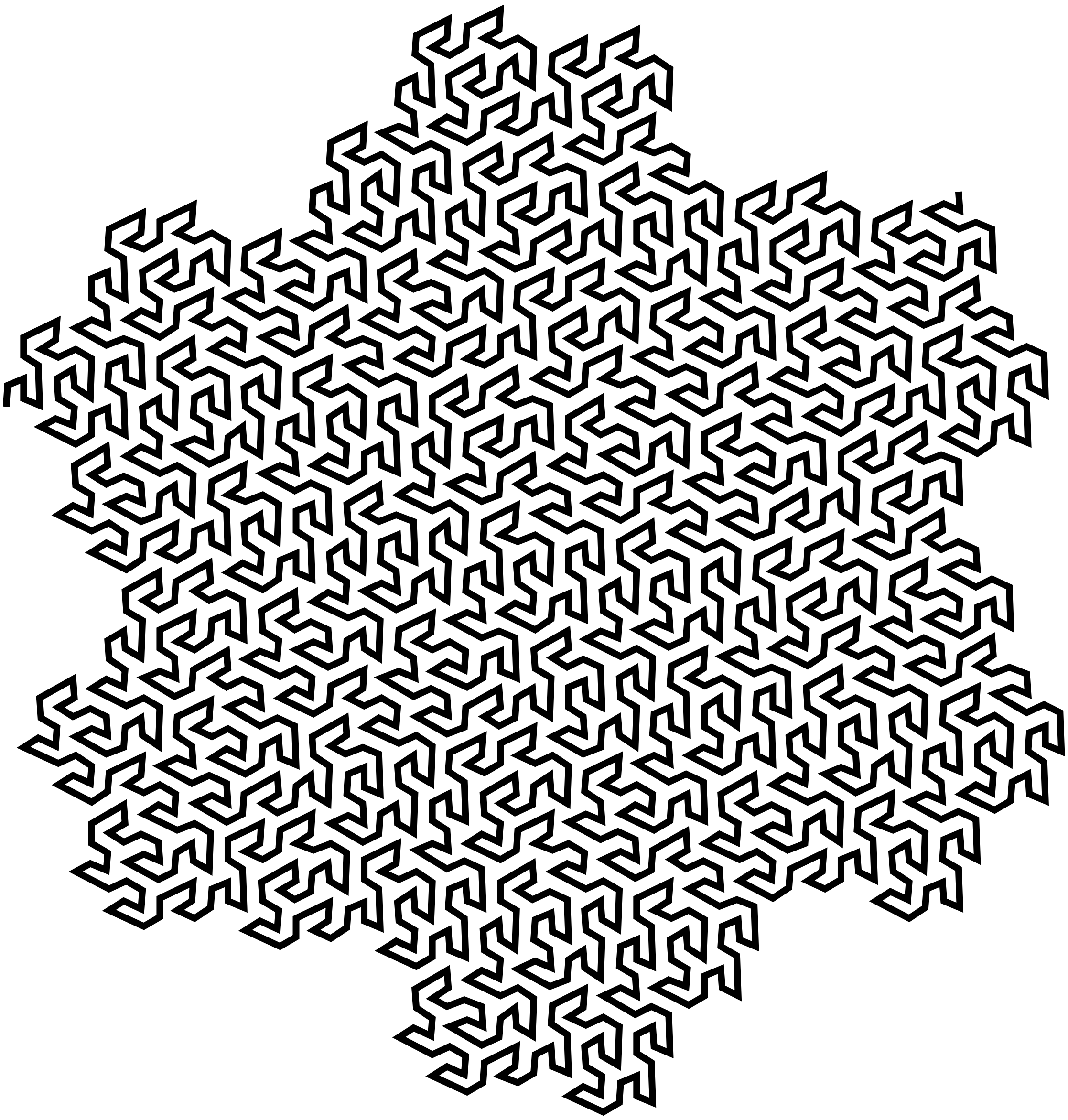
40.5 Flowsnake curve



The first three orders of the Gosper curve.

As a fractal curve, the *flowsnake curve* or *Gosper curve* is defined by a set of recursive rules for drawing it. There are four kind of rules and two of them define rulesets (i.e. they are non-terminal steps).

$$A \mapsto A{-}B{-}{-}B{+}A{+}{+}AA{+}B{-}$$
$$B \mapsto {+}A{-}BB{-}{-}B{-}A{+}{+}A{+}B$$



The fourth order Gosper curve consists of a minimum of 2057 distinct line segments (but our algorithm draws 36015)

patterns

Part IX

Addendum

addendum

Chapter 41

Faster drawing a line segment from its two endpoints using symmetry

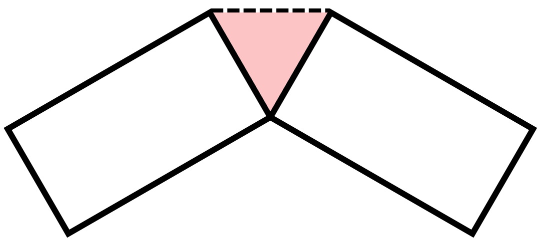
Add *Faster drawing a line segment from its two endpoints using symmetry*

addendum

Chapter 42

Joining the ends of two wide line segments together

Add *Joining the ends of two wide line segments together*



addendum

Chapter 43

Composing monochrome bitmaps with separate alpha channel data

Add *Composing monochrome bitmaps with separate alpha channel data*

addendum

Chapter 44

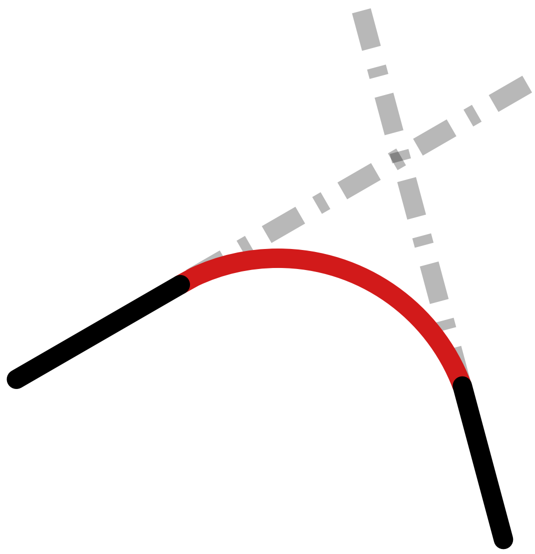
Orthogonal connection of two points

Add *Orthogonal connection of two points*

Chapter 45

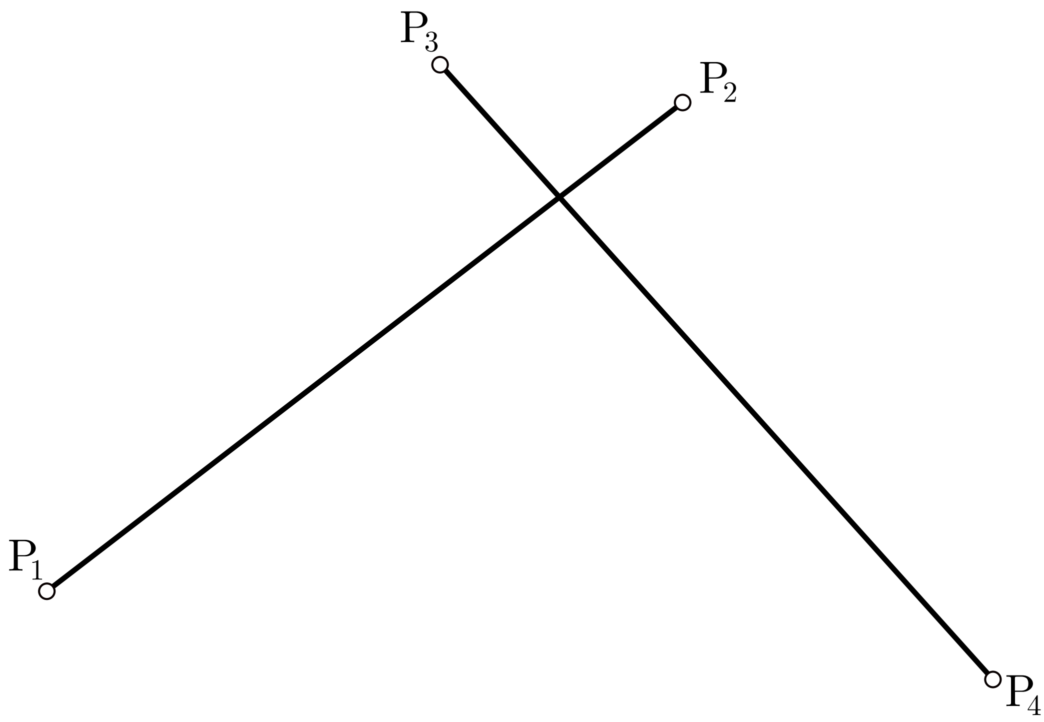
Join segments with round corners

Round corners are everywhere around us. It is useful to know at least one method of construction. This specific method constructs a circle that has a common point with each given line segment, and calculates the arc that when added to the line segments they are smoothly joined. The excess length, since those common points will be before the end of the line segments, must be erased. Therefore, it's best to begin with just the points of the two segments before starting to draw anything.



Since the segments intercept, the round corner will end up beneath the intersection. We wish to find a circle that has a common point with each segment and the arc made up from those points and the circle is the round corner we are after.

addendum



We are given 4 points, P_1, P_2 and P_3, P_4 that make up segments S_1 and S_2. Begin by finding the midpoints m_1 and m_2 of segments S_1 and S_2. These will be:

$$m_1 = \frac{P_1 + P_2}{2}$$
$$m_2 = \frac{P_3 + P_4}{2}$$

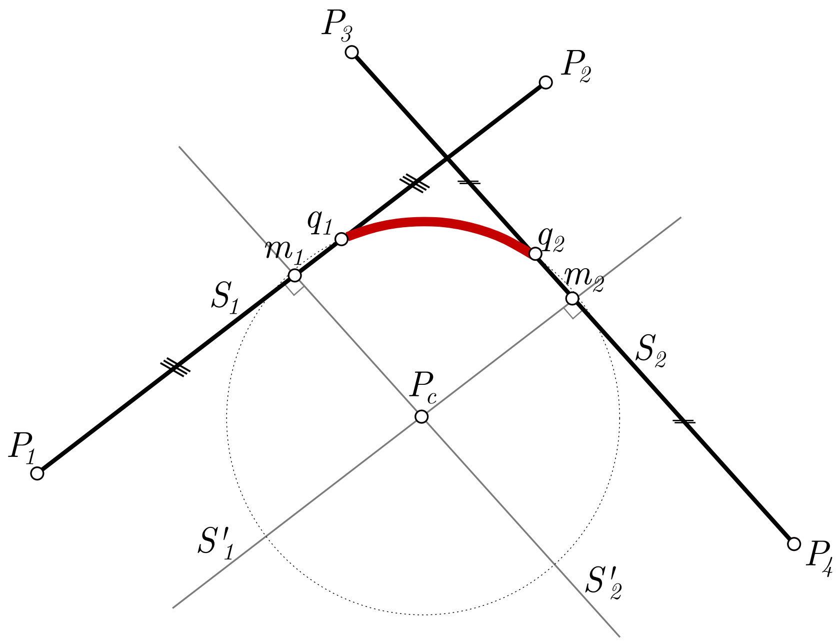
Then, find the signed distances (i.e. don't use the absolute value of distance) d_1 of m_1 from S_2 and d_2 of m_2 from S_1.

Construct parallel lines l_1 to S_1 that is d_1 pixels away. Repeat with l_2 for S_2 and d_2.

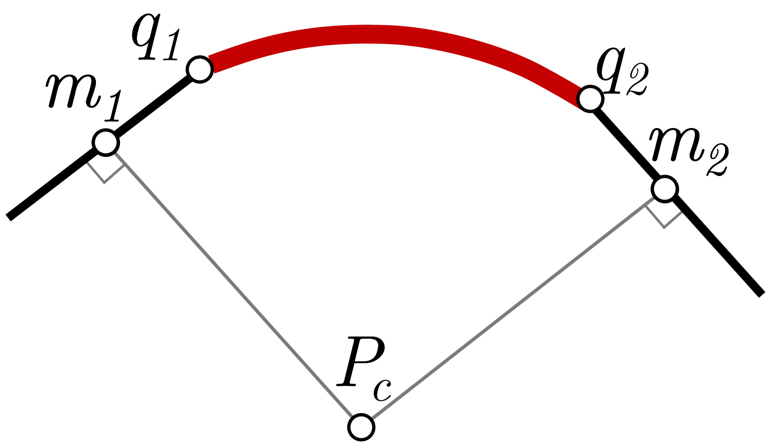
Their intersection is the circle's center, P_c.

The intersection of l_1, l_2 with the two segments are the points where we should clip or extend the segments: q_1 and q_2.

addenddum



The starting angle is found by calculating the angle of q_1P_c with the x-axis with the `atan2` math library procedure.



addendum

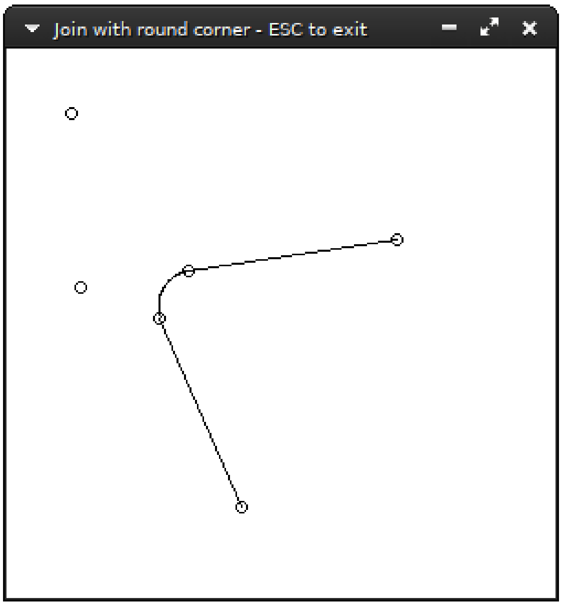
The *subtended* angle* of the arc from the center P_c is found by calculating the dot product of $q_1 P_c$ and $q_2 P_c$:

src/bin/roundcorner.rs:

This code file is a PDF attachment

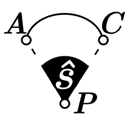
The code:

```
```



The `src/bin/roundcorner.rs` example has two interactive lines and computes the joining fillet.

*the *subtended* angle of an arc $\overset{\frown}{AC}$ to a point P is the angle between PA and PC:



Chapter 46
Faster line clipping

Add *Faster line clipping*

adden-
dum

Chapter 47
Archimedean spiral

The code

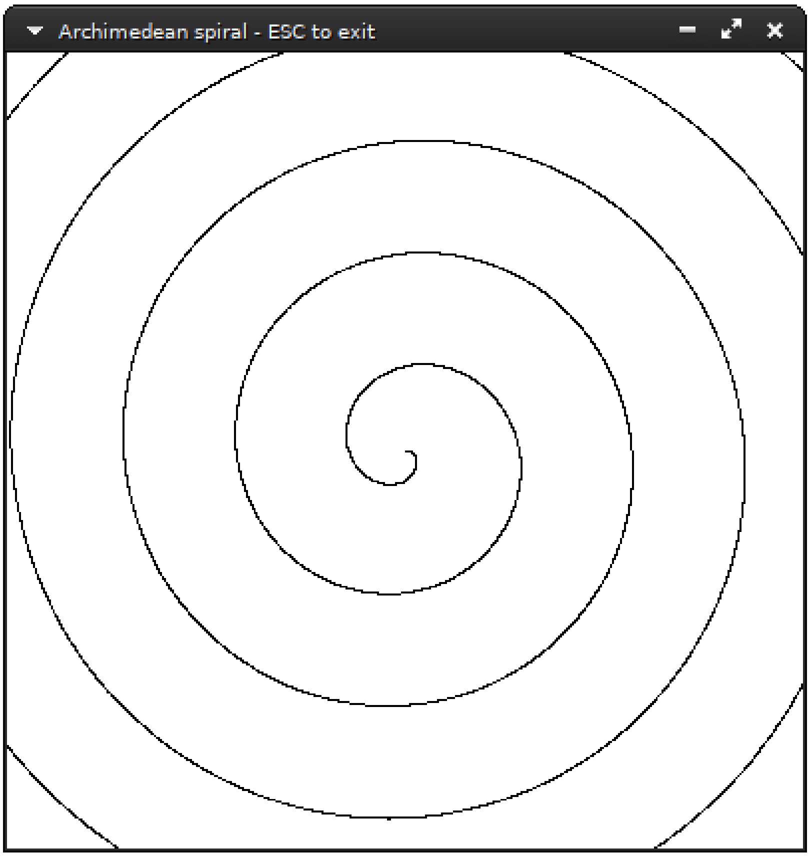
src/bin/archimedeanspiral.rs:

This code file is a PDF attachment

```
pub fn arch(image: &mut Image, center: Point) {
    let a = 1.0_f64;
    let b = 9.0_f64;

    // max_angle = number of spirals * 2pi.
    let max_angle = 5.0_f64 * 2.0_f64 * std::f64::consts::PI;

    let mut theta = 0.0_f64;
    let (dx, dy) = center;
    let mut prev_point = center;
    while theta < max_angle {
        theta = theta + 0.002_f64;

        let r = a + b * theta;
        let x = (r * theta.cos()) as i64 + dx;
```

addendum

```
        let y = (r * theta.sin()) as i64 + dy;
        image.plot_line_width(prev_point, (x, y), 1.0);
        prev_point = (x, y);
    }
}
```

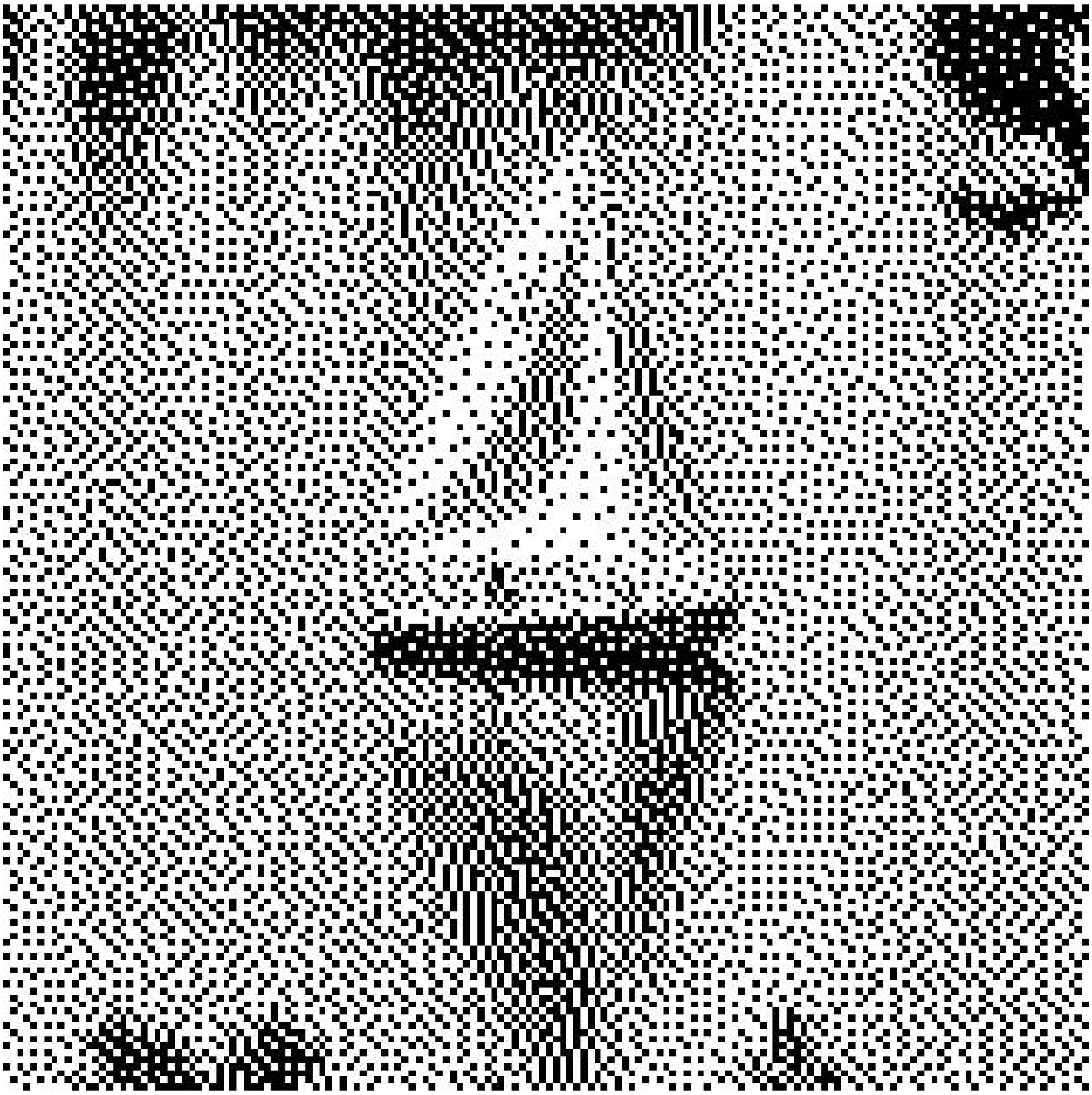


addendum

Chapter 48
Dithering

addendum

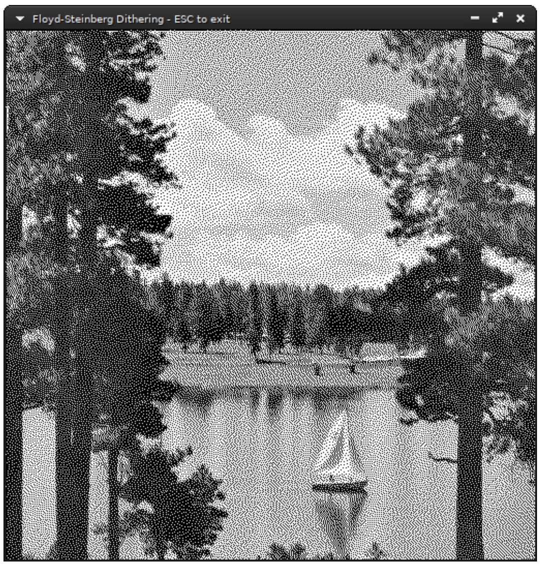
48.1 Floyd-Steinberg



detail of a standard test image, *Sailboat on lake*, with Floyd-Steinberg dithering src/bin/floyddither.rs:

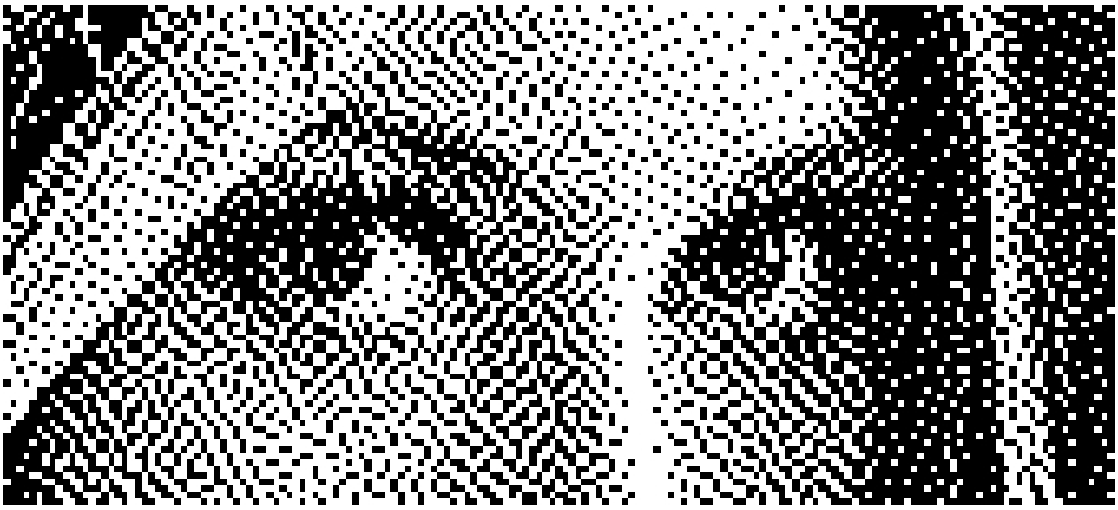
This code file is a PDF attachment

```
fn floyd(image: &mut Image) {
    let w = image.width;
    let m = [(0, 7), (w - 2, 3), (w - 1, 5), (w, 1)];
    let mut e = vec![0.0; w + 1];
    let bytes = image
        .bytes
        .iter()
        .map(|&byte| {
            let (r, g, b) = from_u32_rgb(byte);
            let g: f64 = (0.299 * (r as f64)) + (0.587_f64 * (g as f64)) + (0.114 * (b as f64));
            let pix = g / 255.0 + {
                e.push(0.);
                e.remove(0)
            };
            let col = if pix > 0.5 { 1. } else { 0. };
            let err = (pix - col) / 16.;
            for (x, y) in m.iter() {
                e[*x] += err * (*y as f64);
            }
            if col.floor() as u32 == 1 {
                WHITE
            } else {
                BLACK
            }
        })
        .collect::<Vec<u32>>();
    image.bytes = bytes;
}
```



adden-
dum

48.2 Atkinson dithering



detail of a standard test image, *Lenna*, with Atkinson dithering

The following code implements Atkinson dithering:*

src/bin/atkinsondither.rs:

This code file is a PDF attachment

```
fn atkinson(image: &mut Image) {
    let w= image.width;
    let mut e = vec![0.0;2*w];
    let m = [0, 1, w-2, w-1, w, 2*w-1];
    for byte in image.bytes.iter_mut() {
        let (r,g,b) = from_u32_rgb(*byte);
        let g:f64 = ((0.299*(r as f64)) ) + ((0.587_f64*(g as f64)) ) + ((0.114*(b as
↪   f64)) );
        let pix = g/255.0 + { e.push(0.); e.remove(0)};
        let col = if pix > 0.5 { 1. } else { 0. };
        let err = (pix-col)/8.;
        for m in m.iter() {
            e[*m] += err;
        }
        *byte = if (col.floor() as u32 == 1) {
            WHITE
```


*Algorithm taken from https://beyondloom.com/blog/dither.html

```
        } else {
            BLACK
        };
    }
}
```

addendum

Chapter 49

Marching squares

addendum

Index

About this text

The text has been typeset in XƎLaTeX using the `book` class and:

- **Redaction** for the main text.
- **Fira Sans** for referring to the programming language **Rust** .
- **Redaction20** for referring to the words bitmap and pixels as a concept.

Todo list