

---

# A Bitmapper's Companion

---

epilys

December 1, 2021

an introduction  
to basic bitmap  
mathematics  
and algorithms  
with code  
samples in **Rust**





Table Of Contents	4	<b>toc</b>
Introduction	7	<b>intro</b>
Points And Lines	20	<b>lines</b>
Points and Line Segments	36	<b>segments</b>
Points, Lines and Circles	45	<b>circles</b>
Curves other than circles	57	<b>curves</b>
Points, Lines and Shapes	62	<b>shapes</b>
Vectors, matrices and transformations	73	<b>trans- forma- tions</b>
Addendum	98	<b>adden- dum</b>



Manos Pitsidianakis (epilys)

<https://nessuent.xyz>

<https://github.com/epilys>

[epilys@nessuent.xyz](mailto:epilys@nessuent.xyz)

All non-screenshot figures were generated by hand in Inkscape unless otherwise stated.

The skull in the cover is a transformed bitmap of the skull in the 1533 oil painting by Hans Holbein the Younger, *The Ambassadors*, which features a floating distorted skull rendered in anamorphic perspective.

*A Bitmapper's Companion*, 2021

**Special Topics ► Computer Graphics ► Programming**

006.6'6–dc20

Copyright © 2021 by Emmanouil Pitsidianakis

This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/3.0/> or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.

The source code for this work is available under the GNU GENERAL PUBLIC LICENSE version 3 or later. You can view it, study it, modify it for your purposes as long as you respect the license if you choose to distribute your modifications.

The source code is available here

<https://github.com/epilys/bitmappers-companion>

# Contents

<b>I</b>	<b>Introduction</b>	<b>9</b>
1	Data representation	11
2	Displaying pixels to your screen	13
3	Bits to byte pixels	15
4	Real pixels to byte pixels	17
5	Loading xbm files in <b>Rust</b>	19
<b>II</b>	<b>Points And Lines</b>	<b>21</b>
6	Distance between two points	23
7	Equations of a line	25
7.1	Line through a point $P = (x_p, y_p)$ and a slope $m$	25
7.2	Line through two points	26
8	Distance from a point to a line	27
8.1	Using the implicit equation form	27
8.2	Using an $L$ defined by two points $P_1, P_2$	28
8.3	Using an $L$ defined by a point $P_l$ and angle $\theta$	28
9	Angle between two lines	29
10	Intersection of two lines	31
11	Line equidistant from two points	33
12	Normal to a line through a point	35



<b>III</b>	<b>Points And Line Segments</b>	<b>37</b>
13	Drawing a line segment from its two endpoints	39
14	Drawing line segments with width	41
15	Intersection of two line segments	43
15.1	<i>Fast</i> intersection of two line segments	43
<b>IV</b>	<b>Points, Lines and Circles</b>	<b>47</b>
16	Equations of a circle	51
17	Bounding circle	53
<b>V</b>	<b>Curves other than circles</b>	<b>59</b>
18	Parametric elliptical arcs	61
<b>VI</b>	<b>Points, Lines and Shapes</b>	<b>63</b>
19	Union, intersection and difference of polygons	65
20	Centroid of polygon	67
21	Flood filling	69
<b>VII</b>	<b>Vectors, matrices and transformations</b>	<b>75</b>
22	Rotation of a bitmap	77
22.1	Fast 2D Rotation	81
23	90° Rotation of a bitmap by parallel recursive subdivision	83
24	Magnification/Scaling	85
24.1	Smoothing enlarged bitmaps	87
24.2	Stretching lines of bitmaps	88
25	Mirroring	91
26	Shearing	93
27	Projections	97

<i>CONTENTS</i>	7
<b>VIII Addendum</b>	<b>99</b>
27.1 Faster Drawing a line segment from its two endpoints using Symmetry	101
28 Joining the ends of two wide line segments together	103
29 Composing monochrome bitmaps with separate alpha channel data	105
30 Orthogonal connection of two points	107
31 Join segments with round corners	109
32 Faster line clipping	111
33 Space-filling Curves	113
33.1 Hilbert curves	114
33.2 Peano curves	116
33.3 Z-order curves	117
Index	119



intro



# Part I

## Introduction

intro

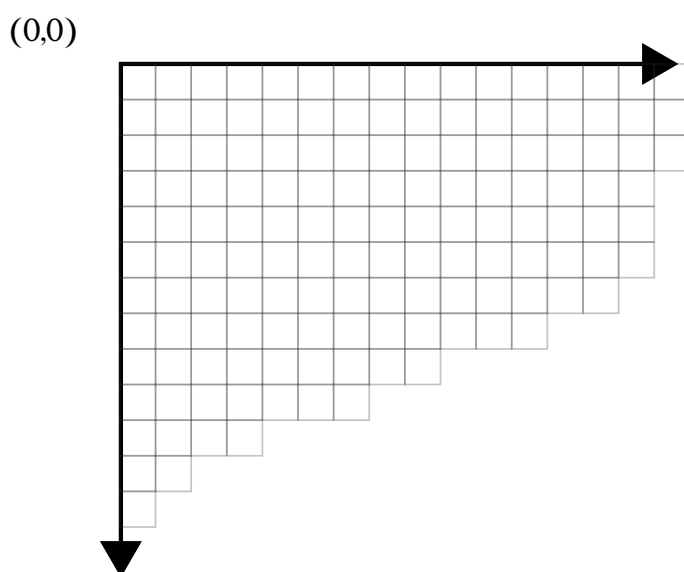
# Chapter 1

intro

## Data representation

The data structures we're going to use is *Point* and *Image*. *Image* represents a bitmap, although we will use full RGB colors for our points therefore the size of a pixel in memory will be u8 instead of l bit.

We will work on the cartesian grid representing the framebuffer that will show us the pixels. The *origin* of this grid (i.e. the center) is at  $(0,0)$ .



We will represent points as pairs of signed integers. When actually drawing them though, negative values and values outside the window's geometry will be ignored (clipped).

src/lib.rs:



This code file is a PDF attachment

```

pub type Point = (i64, i64);

pub const fn from_u8_rgb(r: u8, g: u8, b: u8) -> u32 {
    let (r, g, b) = (r as u32, g as u32, b as u32);
    (r << 16) | (g << 8) | b
}

pub const AZURE_BLUE: u32 = from_u8_rgb(0, 127, 255);
pub const RED: u32 = from_u8_rgb(157, 37, 10);
pub const WHITE: u32 = from_u8_rgb(255, 255, 255);
pub const BLACK: u32 = 0;

pub struct Image {
    pub bytes: Vec<u32>,
    pub width: usize,
    pub height: usize,
    pub x_offset: usize,
    pub y_offset: usize,
}

impl Image {
    pub fn new(width: usize, height: usize, x_offset: usize, y_offset: usize) -> Self;
    pub fn draw(&self, buffer: &mut Vec<u32>, fg: u32, bg: Option<u32>, window_width:
↳  usize);
    pub fn draw_outline(&mut self);
    pub fn clear(&mut self);
    pub fn plot(&mut self, x: i64, y: i64);
    pub fn get(&mut self, x: i64, y: i64) -> u32;
    pub fn plot_ellipse(
        &mut self,
        (xm, ym): (i64, i64),
        (a, b): (i64, i64),
        quadrants: [bool; 4],
        _wd: f64,
    );
    pub fn plot_line_width(&mut self, point_a: Point, point_b: Point, wd: f64);
    pub fn flood_fill(&mut self, mut x: i64, y: i64);
}

```

## Chapter 2

# Displaying pixels to your screen

A way to display an *Image* is to use the `minifb` crate which allows you to create a window and draw pixels directly on it. Here's how you could set it up:

`src/bin/introduction.rs`



This code file is a PDF attachment

```
use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

fn main() {
    let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
    let mut window = Window::new(
        "Test - ESC to exit",
        WINDOW_WIDTH,
        WINDOW_HEIGHT,
        WindowOptions {
            title: true,
            //borderless: true,
            //resize: false,
            //transparency: true,
            ..WindowOptions::default()
        },
    )
    .unwrap();

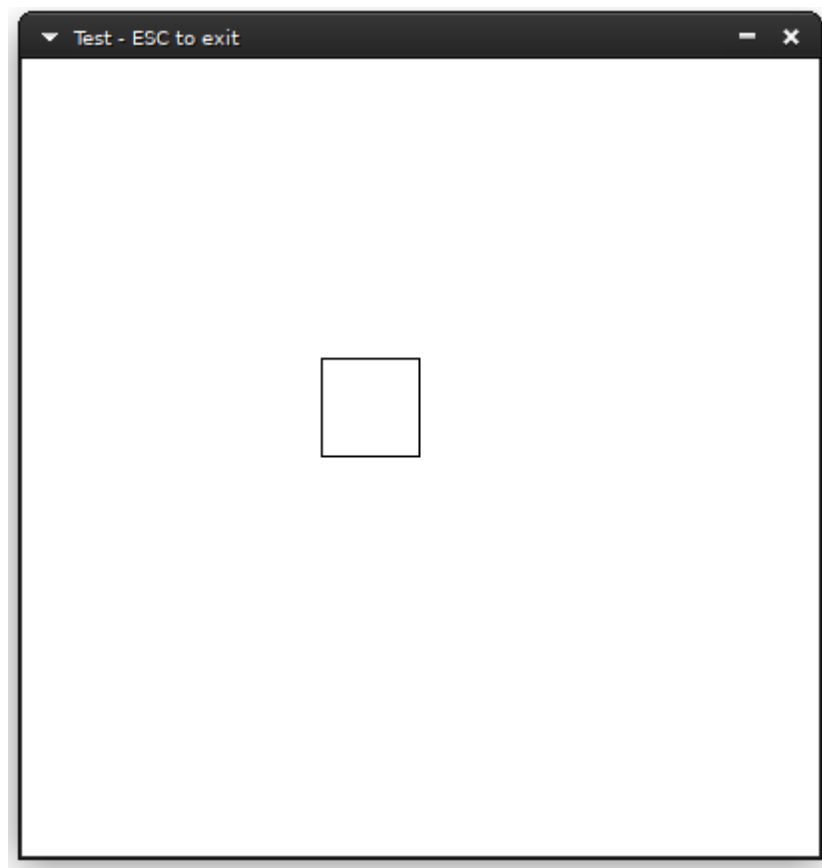
    // Limit to max ~60 fps update rate
    window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

    let mut image = Image::new(50, 50, 150, 150);
    image.draw_outline();
    image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

    while window.is_open()
        && !window.is_key_down(Key::Escape)
        && !window.is_key_down(Key::Q) {
        window
            .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
            .unwrap();
        let millis = std::time::Duration::from_millis(100);
        std::thread::sleep(millis);
    }
}
```

Running this will show you something like this:

intro



## Chapter 3

# Bits to byte pixels

Let's define a way to convert bit information to a byte vector:

```
pub fn bits_to_bytes(bits: &[u8], width: usize) -> Vec<u32> {  
    let mut ret = Vec::with_capacity(bits.len() * 8);  
    let mut current_row_count = 0;  
    for byte in bits {  
        for n in 0..8 {  
            if byte.rotate_right(n) & 0x01 > 0 {  
                ret.push(BLACK);  
            } else {  
                ret.push(WHITE);  
            }  
            current_row_count += 1;  
            if current_row_count == width {  
                current_row_count = 0;  
                break;  
            }  
        }  
    }  
    ret  
}
```

intro





[Redacted text block]

[Redacted text block]

[Redacted text block]

## Chapter 5

# Loading xbm files in Rust

xbm files are C source code files that contain the pixel information for an image as macro definitions for the dimensions and a static char array for the pixels, with each bit column representing a pixel. If the width dimension doesn't have 8 as a factor, the remaining bit columns are left blank/ignored.

They used to be a popular way to share user avatars in the old internet and are also good material for us to work with, since they are small and numerous. The following is such an image:



Then, we can convert the xbm file from C to **Rust** with the following transformations:

```
#define news_width 48  
#define news_height 48  
static char news_bits[] = {
```

to

```
const NEWS_WIDTH: usize = 48;  
const NEWS_HEIGHT: usize = 48;  
const NEWS_BITS: &[u8] = &[
```

And replace the closing `}` with `]`.

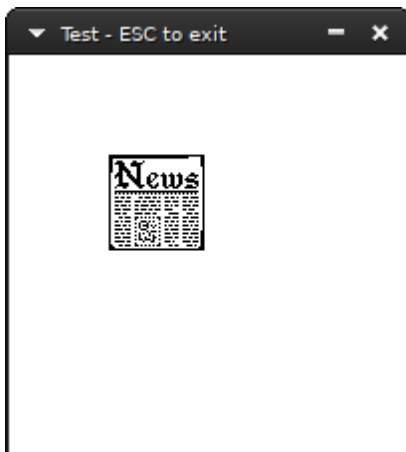
We can then include the new file in our source code:

```
include!("news.xbm.rs");
```

load the image:

```
let mut image = Image::new(NEWS_WIDTH, NEWS_HEIGHT, 25, 25);  
image.bytes = bits_to_bytes(NEWS_BITS, NEWS_WIDTH);
```

and finally run it:



# **Part II**

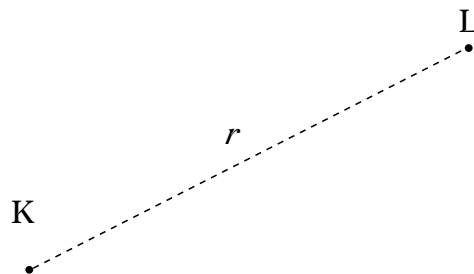
## **Points And Lines**

lines

## Chapter 6

# Distance between two points

lines



Given two points,  $K$  and  $L$ , an elementary application of Pythagoras' Theorem gives the distance between them as

$$r = \sqrt{(x_L - x_K)^2 + (y_L - y_K)^2} \quad (6.1)$$

which is simply coded:

```
pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {  
    let (x_k, y_k) = p_k;  
    let (x_l, y_l) = p_l;  
    let x_lk = x_l - x_k;  
    let y_lk = y_l - y_k;  
    f64::sqrt((x_lk*x_lk + y_lk*y_lk) as f64)  
}
```

lines



## Chapter 7

# Equations of a line

lines

There are several ways to describe a line mathematically. We'll list the convenient ones for drawing pixels.

The equation that describes every possible line on a two dimensional grid is the *implicit* form  $ax + by = c$ ,  $(a, b) \neq (0, 0)$ . We can generate equivalent equations by adding the equation to itself, i.e.  $ax + by = c \equiv 2ax + 2by = 2c \equiv a'x + b'y = c'$ ,  $a' = 2a, b' = 2b, c' = 2c$  as many times as we want. To "minimize" the constants  $a, b, c$  we want to satisfy the relationship  $a^2 + b^2 = 1$ , and thus can convert the equivalent equations into one representative equation by multiplying the two sides with  $\frac{1}{\sqrt{a^2 + b^2}}$ ; this is called the normalized equation.

The *slope intercept form* describes any line that intercepts the  $y$  axis at  $b \in \mathbb{R}$  with a specific slope  $a$ :

$$y = ax + b$$

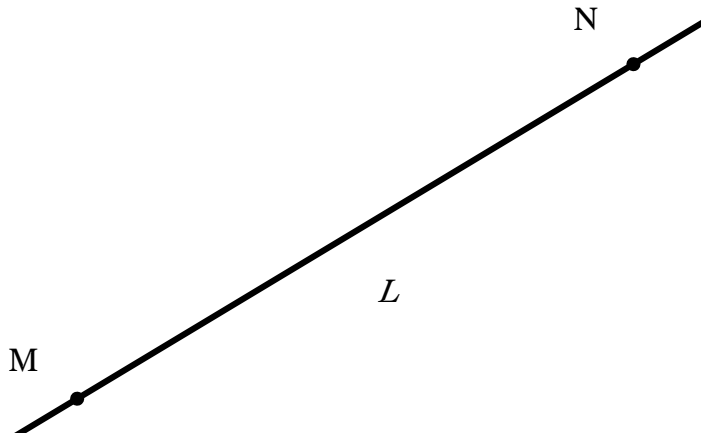
The *parametric* form...

### 7.1 Line through a point $P = (x_p, y_p)$ and a slope $m$

$$y - y_p = m(x - x_p)$$

## 7.2 Line through two points

lines



It seems sufficient, given the coordinates of two points  $M, N$ , to calculate  $a, b$  and  $c$  to form a line equation:

$$ax + by + c = 0$$

If the two points are not the same, they necessarily form such a line. To get there, we start from expressing the line as parametric over  $t$ : at  $t = 0$  it's at point  $M$  and at  $t = 1$  it's at point  $N$ :

$$x = x_M + (x_N - x_M)t, t \in R, c \in \{x, y\}$$

$$y = y_M + (y_N - y_M)t, t \in R, c \in \{x, y\}$$

Substituting  $t$  in one of the equations we get:

$$(y_M - y_N)x + (x_N - x_M)y + (x_M y_N - x_N y_M) = 0$$

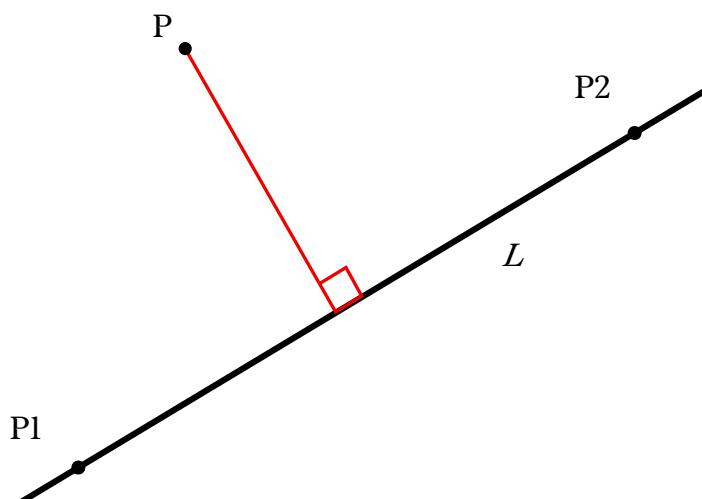
Which is what we were after. We finish by normalising what we found with  $\frac{1}{\sqrt{a^2 + b^2}}$ :

## Chapter 8

# Distance from a point to a line

lines

code samples



### 8.1 Using the implicit equation form

Let's find the distance from a given point  $P$  and a given line  $L$ . Let  $d$  be the distance between them. Bring  $L$  to the implicit form  $ax + by = c$ .

$$d = \frac{|ax_p + by_p + c|}{\sqrt{a^2 + b^2}}$$

## 8.2 Using an $L$ defined by two points $P_1, P_2$

With  $P = (x_0, y_0)$ ,  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$ .

$$d = \frac{|(x_2 - x_1)(y_1 - y_0) - (x_1 - x_0)(y_2 - y_1)|}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

## 8.3 Using an $L$ defined by a point $P_l$ and angle $\theta$

$$d = |\cos(\theta)(P_{ly} - y_p) - \sin(\theta)(P_{lx} - P_x)|$$

## Chapter 9

# Angle between two lines

[Redacted text block]

2

[Redacted text block]

[Redacted text block]

lines

lines

## Chapter 10

# Intersection of two lines

[Redacted content]

3

lines

lines

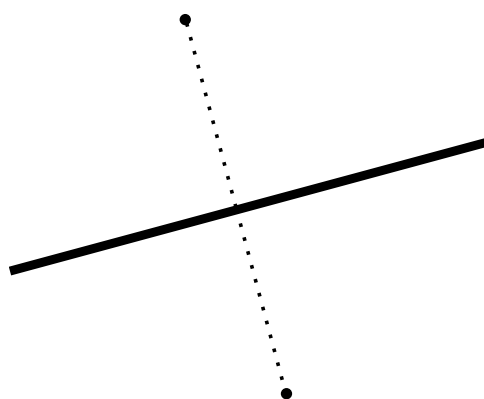




## Chapter 11

# Line equidistant from two points

lines



Let's name this line  $L$ . From the previous chapter we know how to get the line that's created by the two points  $M$  and  $N$ . If only we knew how to get a perpendicular line over the midpoint of a line segment!

Thankfully that midpoint also satisfies  $L$ 's equation,  $ax + by + c$ . The midpoint's coordinates are intuitively:

$$\left(\frac{x_M + x_N}{2}, \frac{y_M + y_N}{2}\right)$$

Putting them into the equation we can generate a triple of  $(a', b', c')$  and then normalize it to get  $L$ .

lines

## Chapter 12

# Normal to a line through a point

lines

4

lines

## Part III

# Points And Line Segments

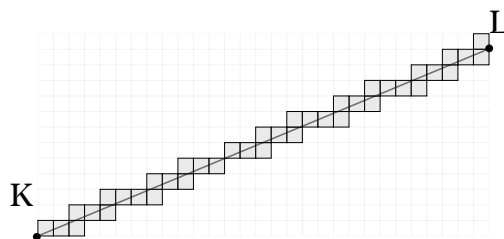
segments

segments

## Chapter 13

# Drawing a line segment from its two endpoints

For any line segment with any slope, pixels must be matched with the infinite amount of points contained in the segment. As shown in the following figure, a segment *touches* some pixels; we could fill them using an algorithm and get a bitmap of the line segment.



The algorithm presented here was first derived by Bresenham. In the *Image* implementation, it is used in the `plot_line_width` method.

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64)) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();
```

segments

```

sx = if dx > 0 { 1 } else { -1 };
dy = y2 - y1;
ay = (dy * 2).abs();
sy = if dy > 0 { 1 } else { -1 };
x = x1;
y = y1;

let b = dx / dy;
let a = 1;
let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;
let delta = double_d / 2;

if ax > ay {
  d = ay - ax / 2;
  loop {
    self.plot(x, y);
    if x == x2 {
      return;
    }
    if d >= 0 {
      y = y + sy;
      d = d - ax;
    }
    x = x + sx;
    d = d + ay;
  }
} else {
  d = ax - ay / 2;
  let delta = double_d / 3;
  loop {
    self.plot(x, y);
    if y == y2 {
      return;
    }
    if d >= 0 {
      x = x + sx;
      d = d - ay;
    }
    y = y + sy;
    d = d + ax;
  }
}
}
```

add some explanation behind the algorithm



## Chapter 14

# Drawing line segments with width

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64), _wd: f64) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();  
    sx = if dx > 0 { 1 } else { -1 };  
  
    dy = y2 - y1;  
    ay = (dy * 2).abs();  
    sy = if dy > 0 { 1 } else { -1 };  
  
    x = x1;  
    y = y1;  
  
    let b = dx / dy;  
    let a = 1;  
    let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;  
    let delta = double_d / 2;  
  
    if ax > ay {  
        d = ay - ax / 2;  
        loop {  
            self.plot(x, y);  
            {  
                let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;  
                let mut _x = x;  
                loop {  
                    let t = total(_x);  
                    if t < -1 * delta || t > delta {  
                        break;  
                    }  
                    _x += 1;  
                    self.plot(_x, y);  
                }  
                let mut _x = x;  
                loop {  
                    let t = total(_x);  
                    if t < -1 * delta || t > delta {  
                        break;  
                    }  
                    _x -= 1;  
                    self.plot(_x, y);  
                }  
            }  
        }  
    }  
}
```

segments

segments

```
        if x == x2 {
            return;
        }
        if d >= 0 {
            y = y + sy;
            d = d - ax;
        }
        x = x + sx;
        d = d + ay;
    }
} else {
    d = ax - ay / 2;
    let delta = double_d / 3;
    loop {
        self.plot(x, y);
        {
            let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x += 1;
                self.plot(_x, y);
            }
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x -= 1;
                self.plot(_x, y);
            }
        }
    }
}
if y == y2 {
    return;
}
if d >= 0 {
    x = x + sx;
    d = d - ay;
}
y = y + sy;
d = d + ax;
}
}
```

## Chapter 15

# Intersection of two line segments

Let points **1** =  $(x_1, y_1)$ , **2** =  $(x_2, y_2)$ , **3** =  $(x_3, y_3)$  and **4** =  $(x_4, y_4)$  and **1,2**, **3,4** two line segments they form. We wish to find their intersection:

First, get the equation of line  $L_{12}$  and line  $L_{34}$  from chapter *Equations of a line*.

Substitute points **3** and **4** in equation  $L_{12}$  to compute  $r_3 = L_{12}(\mathbf{3})$  and  $r_4 = L_{12}(\mathbf{4})$  respectively.

If  $r_3 \neq 0$ ,  $r_4 \neq 0$  and  $\text{sgn}(r_3) \neq \text{sgn}(r_4)$  the line segments don't intersect, so stop.

In  $L_{34}$  substitute point **1** to compute  $r_1$ , and do the same for point **2**.

If  $r_1 \neq 0$ ,  $r_2 \neq 0$  and  $\text{sgn}(r_1) \neq \text{sgn}(r_2)$  the line segments don't intersect, so stop.

At this point,  $L_{12}$  and  $L_{34}$  either intersect or are equivalent. Find their intersection point. (Refer to *Intersection of two lines*.)

[add code sample](#)

segments

## 15.1 Fast intersection of two line segments

5







## **Part IV**

# **Points, Lines and Circles**

**circles**





[Redacted text block 1]

[Redacted text block 2]

[Redacted text block 3]

[Redacted text block 4]

[Redacted text block]

## Chapter 16

# Equations of a circle

[Redacted text block]

7

[Redacted text block]

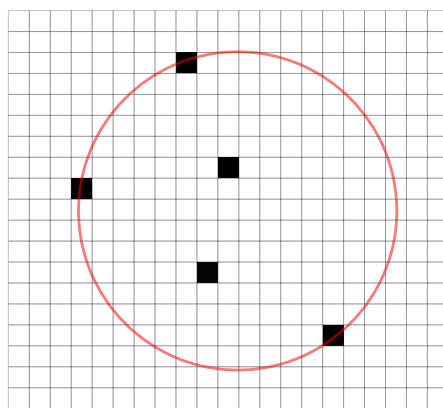
[Redacted text block]

circles



## Chapter 17

# Bounding circle



src/bin/boundingcircle.rs:



This code file is a PDF attachment

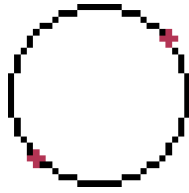
**circles**

A bounding circle is a circle that includes all the points in a given set. Usually we're interested in one of the smallest ones possible.



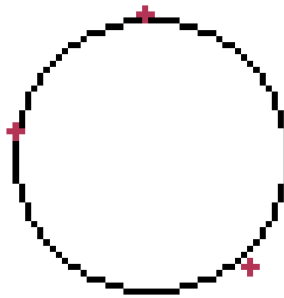
We can use the following methodology to find the bounding circle: start from two points and the circle they make up, and for each of the rest of the points check if the circle includes them. If not, make a bounding circle that includes every point up to the current one. To do this, we need some primitive operations.

We will need a way to construct a circle out of two points:



```
let p1 = points[0];
let p2 = points[1];
//The circle is determined by two points, P and Q. The center of the circle
↪ is
//at (P + Q)/2.0 and the radius is |(P - Q)/2.0|
let d_2 = (
  ((p1.0 + p2.0) / 2), (p1.1 + p2.1) / 2),
  (distance_between_two_points(p1, p2) / 2.0),
);
```

And a way to make a circle out of three points:



```
fn min_circle_w_3_points(q1: Point, q2: Point, q3: Point) -> Circle {
  let (ax, ay) = (q1.0 as f64, q1.1 as f64);
  let (bx, by) = (q2.0 as f64, q2.1 as f64);
  let (cx, cy) = (q3.0 as f64, q3.1 as f64);

  let mut d = 2. * (ax * (by - cy) + bx * (cy - ay) + cx * (ay - by));
  if d == 0.0 {
    d = std::cmp::max(
      std::cmp::max(
        distance_between_two_points(q1, q2) as i64,
        distance_between_two_points(q2, q3) as i64,
      ),
      distance_between_two_points(q1, q3) as i64,
    ) as f64
  }
  / 2.;
}
let ux = ((ax * ax + ay * ay) * (by - cy)
  + (bx * bx + by * by) * (cy - ay)
  + (cx * cx + cy * cy) * (ay - by))
  / d;
let uy = ((ax * ax + ay * ay) * (cx - bx)
```

```

    + (bx * bx + by * by) * (ax - cx)
    + (cx * cx + cy * cy) * (bx - ax))
    / d;
let mut center = (ux as i64, uy as i64);
if center.0 < 0 {
    center.0 = 0;
}
if center.1 < 0 {
    center.1 = 0;
}
let d = distance_between_two_points(center, q1);
(center, d)
}

```

The algorithm:

```

use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};
use rand::seq::SliceRandom;
use rand::thread_rng;
use std::f64::consts::{FRAC_PI_2, PI};

include!("../me.xbm.rs");

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {
    let (x_k, y_k) = p_k;
    let (x_l, y_l) = p_l;
    let xlk = x_l - x_k;
    let ylk = y_l - y_k;
    f64::sqrt((xlk * xlk + ylk * ylk) as f64)
}

fn image_to_points(image: &Image) -> Vec<Point> {
    let mut ret = Vec::with_capacity(image.bytes.len());
    for y in 0..(image.height as i64) {
        for x in 0..(image.width as i64) {
            if image.get(x, y) == Some(BLACK) {
                ret.push((x, y));
            }
        }
    }
    ret
}

type Circle = (Point, f64);

fn bc(image: &Image) -> Circle {
    let mut points = image_to_points(image);
    points.shuffle(&mut thread_rng());
    min_circle(&points)
}

fn min_circle(points: &[Point]) -> Circle {
    let mut points = points.to_vec();
    points.shuffle(&mut thread_rng());

    let p1 = points[0];
    let p2 = points[1];
    //The circle is determined by two points, P and Q. The center of the
    ↪ circle is
    //at (P + Q)/2.0 and the radius is |(P - Q)/2.0|
    let d_2 = (
        ((p1.0 + p2.0) / 2), (p1.1 + p2.1) / 2),
        (distance_between_two_points(p1, p2) / 2.0),
    );
    let mut d_prev = d_2;

    for i in 2..points.len() {
        let p_i = points[i];
        if distance_between_two_points(p_i, d_prev.0) <= (d_prev.1) {
            // then d_i = d_(i-1)

```

```

        } else {
            let new = min_circle_w_point(&points[..i], p_i);
            if distance_between_two_points(p_i, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_point(points: &[Point], q: Point) -> Circle {
    let mut points = points.to_vec();
    points.shuffle(&mut thread_rng());
    let p1 = points[0];
    //The circle is determined by two points, P_1 and Q. The center of the
    ↪ circle is
    //at (P_1 + Q)/2.0 and the radius is |(P_1 - Q)/2.0|
    let d_1 = (
        ((p1.0 + q.0) / 2), (p1.1 + q.1) / 2),
        (distance_between_two_points(p1, q) / 2.0),
    );
    let mut d_prev = d_1;
    for j in 1..points.len() {
        let p_j = points[j];
        if distance_between_two_points(p_j, d_prev.0) <= (d_prev.1) {
            //d_prev = d_prev;
        } else {
            let new = min_circle_w_points(&points[..j], p_j, q);
            if distance_between_two_points(p_j, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_points(points: &[Point], q1: Point, q2: Point) -> Circle {
    let mut points = points.to_vec();
    let d_0 = (
        ((q1.0 + q2.0) / 2), (q1.1 + q2.1) / 2),
        (distance_between_two_points(q1, q2) / 2.0),
    );
    let mut d_prev = d_0;
    for k in 0..points.len() {
        let p_k = points[k];
        if distance_between_two_points(p_k, d_prev.0) <= (d_prev.1) {
        } else {
            let new = min_circle_w_3_points(q1, q2, p_k);
            if distance_between_two_points(p_k, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_3_points(q1: Point, q2: Point, q3: Point) -> Circle {
    let (ax, ay) = (q1.0 as f64, q1.1 as f64);
    let (bx, by) = (q2.0 as f64, q2.1 as f64);
    let (cx, cy) = (q3.0 as f64, q3.1 as f64);
    let mut d = 2. * (ax * (by - cy) + bx * (cy - ay) + cx * (ay - by));
    if d == 0.0 {
        d = std::cmp::max(
            std::cmp::max(
                distance_between_two_points(q1, q2) as i64,
                distance_between_two_points(q2, q3) as i64,
            ),
            distance_between_two_points(q1, q3) as i64,
        ) as f64
        / 2.;
    }
}

```



```

let ux = ((ax * ax + ay * ay) * (by - cy)
  + (bx * bx + by * by) * (cy - ay)
  + (cx * cx + cy * cy) * (ay - by))
  / d;
let uy = ((ax * ax + ay * ay) * (cx - bx)
  + (bx * bx + by * by) * (ax - cx)
  + (cx * cx + cy * cy) * (bx - ax))
  / d;
let mut center = (ux as i64, uy as i64);
if center.0 < 0 {
  center.0 = 0;
}
if center.1 < 0 {
  center.1 = 0;
}
let d = distance_between_two_points(center, q1);
(center, d)
}

fn main() {
  let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
  let mut window = Window::new(
    "Test - ESC to exit",
    WINDOW_WIDTH,
    WINDOW_HEIGHT,
    WindowOptions {
      title: true,
      //borderless: true,
      resize: true,
      //transparency: true,
      ..WindowOptions::default()
    },
  )
  .unwrap();

  // Limit to max ~60 fps update rate
  window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

  let mut full = Image::new(WINDOW_WIDTH, WINDOW_HEIGHT, 0, 0);
  let mut image = Image::new(ME_WIDTH, ME_HEIGHT, 45, 45);
  image.bytes = bits_to_bytes(ME_BITS, ME_WIDTH);
  let (center, r) = bc(&image);
  image.draw_outline();

  full.plot_circle((center.0 + 45, center.1 + 45), r as i64, 0.);
  while window.is_open() && !window.is_key_down(Key::Escape) &&
  ↪ !window.is_key_down(Key::Q) {
    image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
    full.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

    window
      .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
      .unwrap();

    let millis = std::time::Duration::from_millis(100);
    std::thread::sleep(millis);
  }
}

```



## **Part V**

### **Curves other than circles**

curves



## Chapter 18

# Parametric elliptical arcs

8

## curves

[Redacted text block]

[Redacted text block]

[Redacted text block]

## **Part VI**

# **Points, Lines and Shapes**

shapes





## Chapter 19

# Union, intersection and difference of polygons



shapes

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes the need for transparency and accountability in financial reporting.

2. The second part of the document outlines the various methods and techniques used to collect and analyze data. It includes a detailed description of the experimental procedures and the statistical analysis performed.

3. The third part of the document presents the results of the study and discusses the implications of the findings. It compares the results with previous research and provides a comprehensive analysis of the data.

## Chapter 20

# Centroid of polygon



10

shapes

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

## Chapter 21

# Flood filling



11

shapes

[Redacted text block]

[Redacted text block]

[Redacted text block]

[Redacted text block]

[Redacted text block]



1. The first section of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes the need for transparency and accountability in financial reporting.

2. The second section outlines the various methods used to collect and analyze data. It includes a detailed description of the experimental procedures and the statistical techniques employed to interpret the results.

3. The third section presents the findings of the study. It provides a comprehensive overview of the data collected and the conclusions drawn from the analysis. The results are presented in a clear and concise manner, supported by relevant evidence.

4. The fourth section discusses the implications of the findings and their potential applications. It explores the broader context of the research and its contribution to the field. The authors also address the limitations of the study and suggest areas for future research.

5. The final section provides a summary of the key points discussed throughout the document. It reiterates the main findings and the overall conclusions of the study. The authors express their gratitude to the funding agencies and the participants who made the research possible.







## **Part VII**

# **Vectors, matrices and transformations**

trans-  
forma-  
tions



## Chapter 22

# Rotation of a bitmap

$$p' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

$$c = \cos\theta, s = \sin\theta, x_{p'} = x_p c - y_p s, y_{p'} = x_p s + y_p c.$$

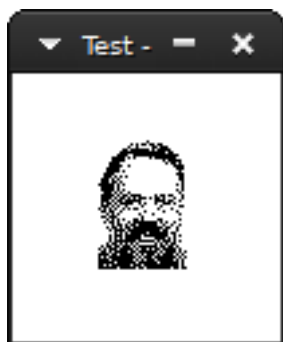
Let's load an xface. We will use `bits_to_bytes` (See Introduction).

```
include!("dmr.rs");
const WINDOW_WIDTH: usize = 100;
const WINDOW_HEIGHT: usize = 100;
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

src/bin/rotation.rs:



This code file is a PDF attachment



trans-  
forma-  
tions

This is the xface of dmr. Instead of displaying the bitmap, this time we will

rotate it 0.5 radians. Setup our image first:

```
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.draw_outline();
let dmr = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

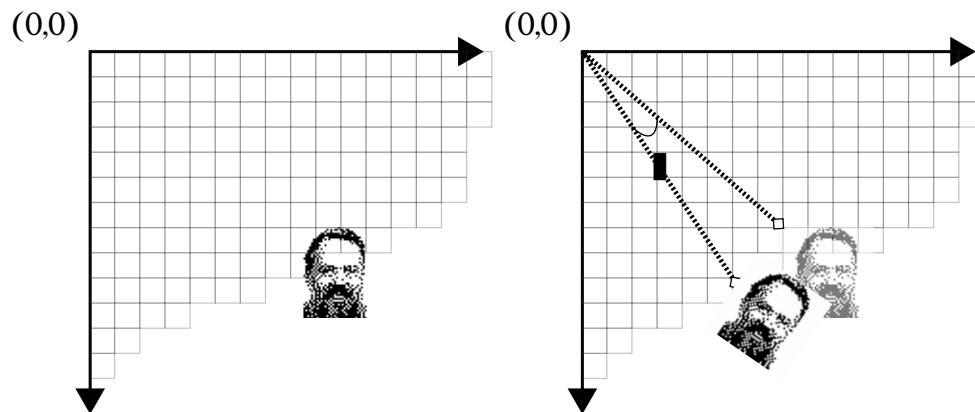
And then, loop for each byte in dmr's face and apply the rotation transformation.

```
let angle = 0.5;
let c = f64::cos(angle);
let s = f64::sin(angle);
for y in 0..DMR_HEIGHT {
    for x in 0..DMR_WIDTH {
        if dmr[y * DMR_WIDTH + x] == BLACK {
            let x = x as f64;
            let y = y as f64;
            let xr = x * c - y * s;
            let yr = x * s + y * c;
            image.plot(xr as i64, yr as i64);
        }
    }
}
```

The result:



We didn't mention in the beginning that the rotation has to be relative to a *point* and the given transformation is relative to the *origin*, in this case the upper left corner (0,0). So dmr was rotated relative to the origin:



(the distance to the origin (actually 0 pixels) has been exaggerated for the sake of the example)

Usually, we want to rotate something relative to itself. The right point to choose is the *centroid* of the object.

If we have a list of  $n$  points, the centroid is calculated as:

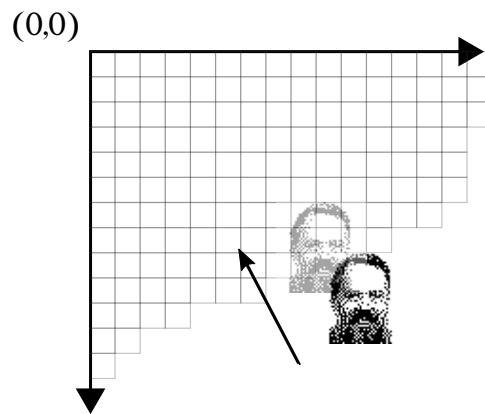
$$x_c = \frac{1}{n} \sum_{i=0}^n x_i$$

$$y_c = \frac{1}{n} \sum_{i=0}^n y_i$$

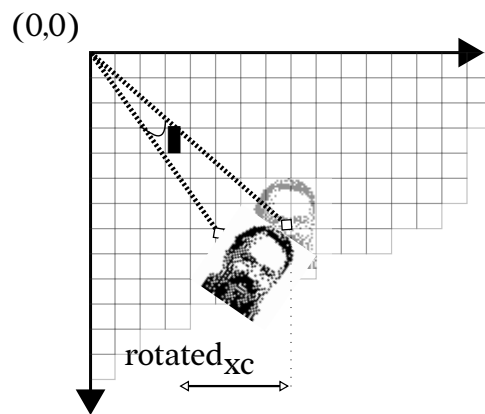
Since in this case we have a rectangle, the centroid has coordinates of half the width and half the height.

By subtracting the centroid from each point before we apply the transformation and then adding it back after we get what we want:

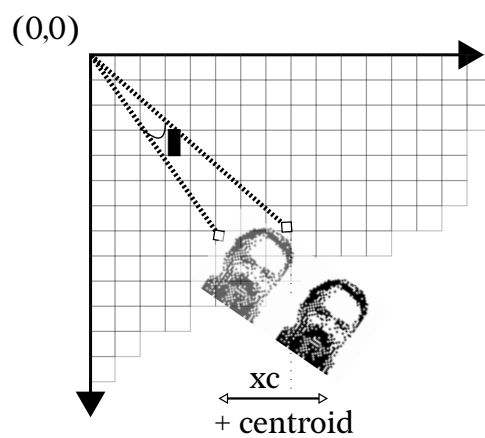
Here's it visually: First subtract the center point.



Then, rotate.



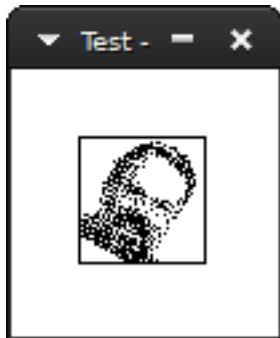
And subtract back to the original position.





In code:

```
let center_point = ((DMR_WIDTH/2) as i64, (DMR_HEIGHT/2) as i64);
for y in 0..DMR_HEIGHT {
  for x in 0..DMR_WIDTH {
    if dmr[y * DMR_WIDTH + x] == BLACK {
      let x = (x as i64 - center_point.0) as f64;
      let y = (y as i64 - center_point.1) as f64;
      let xr = x * c - y * s;
      let yr = x * s + y * c;
      image.plot(xr as i64 + center_point.0,
                 yr as i64 + center_point.1);
    }
  }
}
```



The result:

## 22.1 Fast 2D Rotation

14

[Redacted text block]

[Redacted text block]

[Redacted text block]

[Redacted text block]

## Chapter 23

# 90° Rotation of a bitmap by parallel recursive subdivision



15

[Redacted text block]

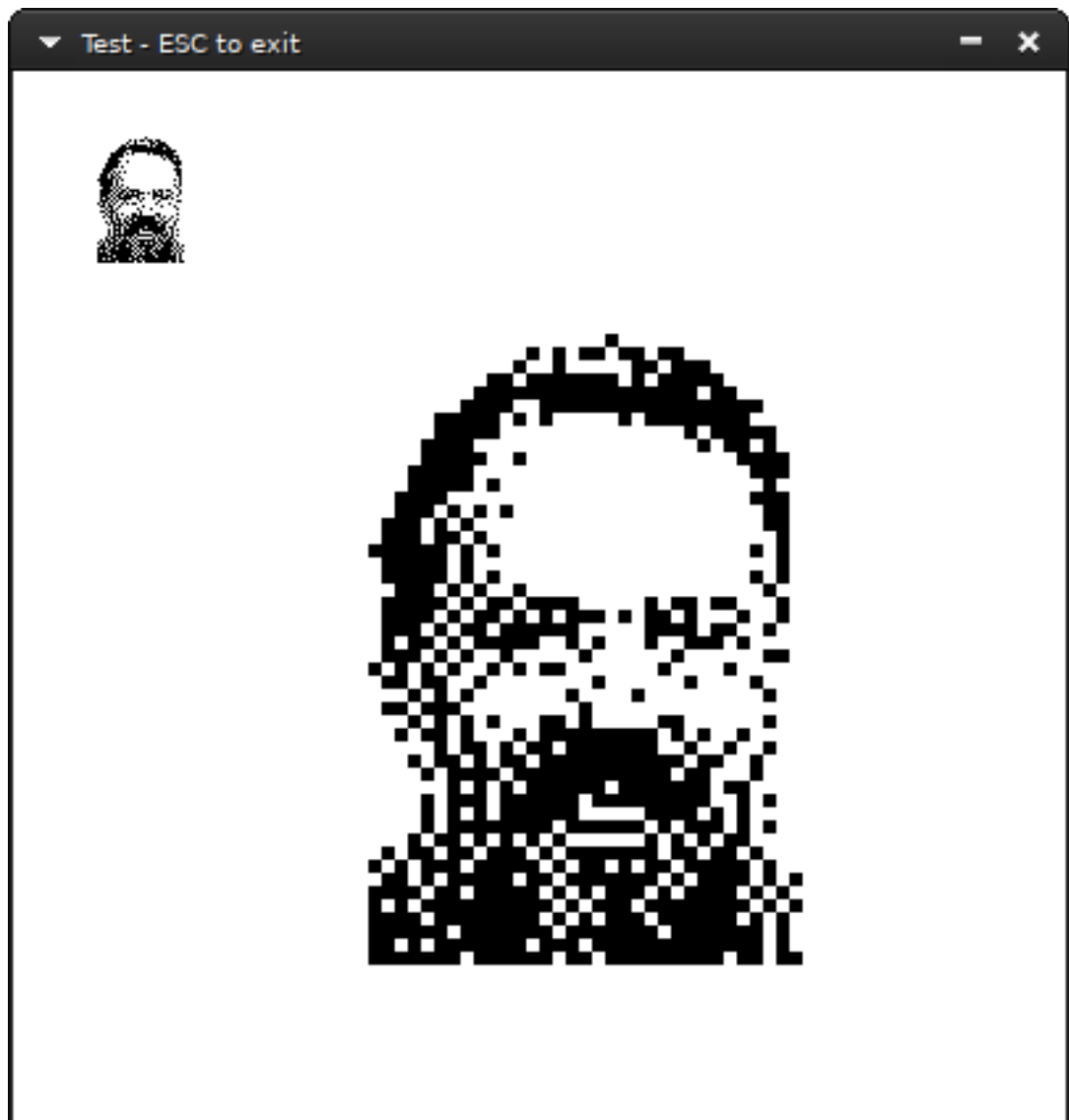
[Redacted text block]

[Redacted text block]



## Chapter 24

# Magnification/Scaling



```
let mut original = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
original.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
original.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

let mut scaled = Image::new(DMR_WIDTH * 5, DMR_HEIGHT * 5, 100, 100);
let mut sx: i64; //source
let mut sy: i64; //source
let mut dx: i64; //destination
let mut dy: i64 = 0; //destination

let og_height = original.height as i64;
let og_width = original.width as i64;
let scaled_height = scaled.height as i64;
let scaled_width = scaled.width as i64;

while dy < scaled_height {
    sy = (dy * og_height) / scaled_height;
    dx = 0;
    while dx < scaled_width {
        sx = (dx * og_width) / scaled_width;
        if original.get(sx, sy) == Some(BLACK) {
            scaled.plot(dx, dy);
        }
        dx += 1;
    }
    dy += 1;
}
scaled.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
```

src/bin/scale.rs:



This code file is a PDF attachment

## 24.1 Smoothing enlarged bitmaps

16

The first part of the chapter discusses the basic concepts of transformations and how they can be used to create complex shapes. It covers topics such as translation, rotation, scaling, and shearing. The second part of the chapter focuses on the application of these transformations in computer graphics, including how they are used to animate objects and create realistic simulations.

The third part of the chapter introduces the concept of projective transformations, which are used to map 3D objects onto a 2D plane. This is a fundamental technique in computer graphics, and it is used to create the illusion of depth and perspective. The fourth part of the chapter discusses the use of transformations in the design of user interfaces, where they are used to create interactive elements and animations.

The fifth part of the chapter covers the use of transformations in the design of fonts, where they are used to create custom typefaces and ligatures. The sixth part of the chapter discusses the use of transformations in the design of web pages, where they are used to create responsive layouts and animations. The final part of the chapter discusses the use of transformations in the design of mobile applications, where they are used to create intuitive and engaging user experiences.

## 24.2 Stretching lines of bitmaps

17

This section discusses the process of stretching lines of bitmaps, which is a common technique used in computer graphics to create smooth transitions between different colors or patterns. It covers the mathematical principles behind this process and provides examples of how it can be applied in practice.



[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]



## Chapter 25

# Mirroring

add screenshots and figure and code

Mirroring to an axis is the transformation of one coordinate to its equidistant value across the axis:

To mirror a pixel across the  $x$  axis, simply multiply its coordinates with the following matrix:

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

This results in the  $y$  coordinate's sign being flipped.

For  $y$ -mirroring, the transformation follows the same logic:

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



## Chapter 26

# Shearing

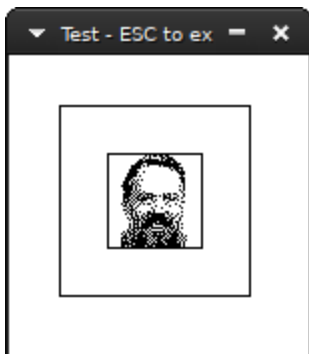
add figure

src/bin/shearing.rs:



This code file is a PDF attachment

Simple shearing is the transformation of one dimension by a distance proportional to the other dimension. In  $x$ -shearing (or horizontal shearing) only the  $x$  coordinate is affected, and likewise in  $y$ -shearing only  $y$  as well.



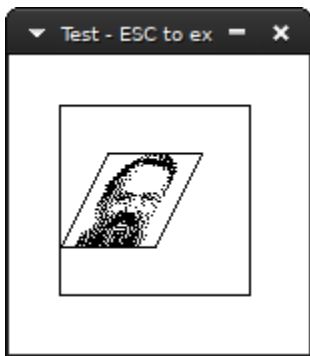
With  $l$  being equal to the desired tilt away from the  $y$  axis, the transformation is described by the following matrix:

$$S_x = \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}$$

Which is as simple as this function:

```
fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {  
    (x_p + (l * (y_p as f64)) as i64, y_p)  
}
```

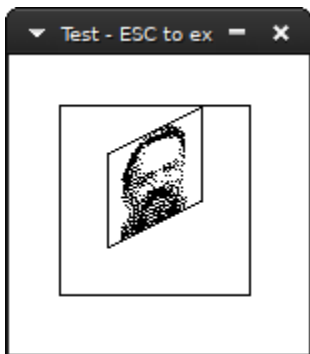
trans-  
forma-  
tions



For  $y$ -shearing, we have the following:

$$S_y = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix}$$

```
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}
```



A full example:

```
include!("../dmr.xbm.rs");
const WINDOW_WIDTH: usize = 200;
const WINDOW_HEIGHT: usize = 200;

fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p+(l*(y_p as f64)) as i64, y_p)
}
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}

let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
image.draw_outline();
```

```
let l = -0.5;
let mut sheared = Image::new(DMR_WIDTH*2, DMR_HEIGHT*2, 25, 25);
for x in 0..DMR_WIDTH {
  for y in 0..DMR_HEIGHT {
    if image.bytes[y * DMR_WIDTH + x] == BLACK {
      let p = shear_x((x as i64 ,y as i64 ), l);
      sheared.plot(p.0+(DMR_WIDTH/2) as i64, p.1+(DMR_HEIGHT/2) as i64);
    }
  }
}
sheared.draw_outline();
```





## Chapter 27

# Projections

18

trans-  
forma-  
tions

[Redacted text block]

[Redacted text block]

[Redacted text block]

# **Part VIII**

## **Addendum**

adden-  
dum



## 27.1 Faster Drawing a line segment from its two endpoints using Symmetry

19

[Redacted text block]

[Redacted text block]

## Chapter 28

# Joining the ends of two wide line segments together



addendum

[Redacted text block]

[Redacted text block]

[Redacted text block]



## Chapter 29

## Composing monochrome bitmaps with separate alpha channel data

[Redacted text block]

[Redacted text block]

[Redacted text block]

## Chapter 30

# Orthogonal connection of two points

[Redacted content]

22

addendum

[Redacted text block]

[Redacted text block]

[Redacted text block]

## Chapter 31

# Join segments with round corners



23

addendum

[Redacted text block]

[Redacted text block]

[Redacted text block]

## Chapter 32

# Faster line clipping

[Redacted content]

24

addendum

[Redacted text block]

[Redacted text block]

[Redacted text block]



## Chapter 33

# Space-filling Curves



25

addendum

[Redacted text block]

[Redacted text block]

[Redacted text block]

### 33.1 Hilbert curves

26

[Redacted text block]

[Redacted text block]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

## 33.2 Peano curves

27

[Redacted text block]

[Redacted text block]

[Redacted text block]

[Redacted text block]

[REDACTED]

### 33.3 Z-order curves

[REDACTED]

28

[REDACTED]

[REDACTED]

[Redacted text block 1]

[Redacted text block 2]

[Redacted text block 3]

# Index

centroid, 67, 79

circle out of three points, 54

circle out of two points, 54

midpoint, 33

shearing, 93





# About this text

The text has been typeset in  $\text{\LaTeX}$  using the book class and:

- **Redaction** for the main text.
- **Fira Sans** for referring to the programming language **Rust**.
- **Redaction20** for referring to the words bitmap and pixels as a concept.



# Todo list

code samples	27
add some explanation behind the algorithm	40
add code sample	43
add screenshots and figure and code	91
add figure	93