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# A Bitmapper's Companion

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epilys

2021

an introduction  
to basic bitmap  
mathematics  
and algorithms  
with code  
samples in **Rust**





Table Of Contents	4	<b>toc</b>
Introduction	8	<b>intro</b>
Points And Lines	18	<b>lines</b>
Points and Line Segments	38	<b>segments</b>
Points, Lines and Circles	45	<b>circles</b>
Points, Lines and Shapes	53	<b>shapes</b>
Curves other than circles	62	<b>curves</b>
Vectors, matrices and transformations	71	<b>trans- forma- tions</b>
Addendum	86	<b>adden- dum</b>



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All non-screenshot figures were generated by hand in Inkscape unless otherwise stated.

The skull in the cover is a transformed bitmap of the skull in the 1533 oil painting by Hans Holbein the Younger, *The Ambassadors*, which features a floating distorted skull rendered in anamorphic perspective.

*A Bitmapper's Companion*, 2021

**Special Topics ► Computer Graphics ► Programming**

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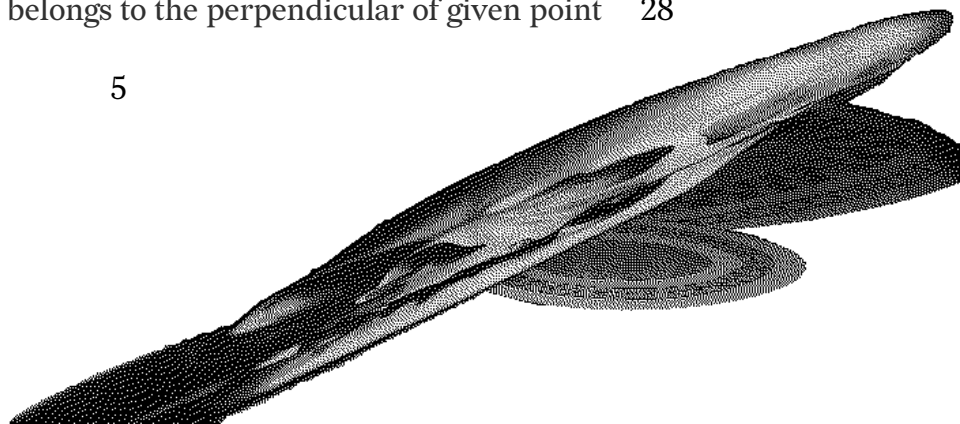
The source code for this work is available under the GNU GENERAL PUBLIC LICENSE version 3 or later. You can view it, study it, modify it for your purposes as long as you respect the license if you choose to distribute your modifications.

The source code is available here

<https://github.com/epilys/bitmappers-companion>

# Contents

<b>I</b>	<b>Introduction</b>	<b>9</b>
1	Data representation	10
2	Displaying pixels to your screen	12
3	Bits to byte pixels	14
4	Loading graphics files in <b>Rust</b>	15
5	Including xbm files in <b>Rust</b>	16
<b>II</b>	<b>Points And Lines</b>	<b>19</b>
6	Distance between two points	20
7	Moving a point to a distance at an angle	21
8	Equations of a line	22
8.1	Line through a point $P = (x_p, y_p)$ and a slope $m$	22
8.2	Line through two points	23
9	Drawing a line	25
10	Distance from a point to a line	26
10.1	Using the implicit equation form	26
10.2	Using an $L$ defined by two points $P_1, P_2$	27
10.3	Using an $L$ defined by a point $P_l$ and angle $\hat{\theta}$	27
11	Perpendicular lines	28
11.1	Find perpendicular to line that passes through given point	28
11.2	Find point in line that belongs to the perpendicular of given point	28



12	Angle between two lines	30
13	Intersection of two lines	32
14	Line equidistant from two points	34
15	Normal to a line through a point	36
16	Angle sectioning	37
16.1	Bisection	37
16.2	Trisection	37
<b>III</b>	<b>Points And Line Segments</b>	<b>39</b>
17	Drawing a line segment from its two endpoints	40
18	Drawing line segments with width	42
19	Intersection of two line segments	44
19.1	<i>Fast</i> intersection of two line segments	44
<b>IV</b>	<b>Points, Lines and Circles</b>	<b>46</b>
20	Equations of a circle	48
21	Bounding circle	49
<b>V</b>	<b>Points, Lines and Shapes</b>	<b>54</b>
22	Rectangles and parallelograms	55
22.1	From a center point	55
22.2	From a corner point	55
23	Triangles	56
23.1	Making a triangle from a point and given angles	56
24	Union, intersection and difference of polygons	57
25	Centroid of polygon	58
26	Polygon clipping	59
27	Triangle filling	60
28	Flood filling	62

<b>VI</b>	<b>Curves other than circles</b>	<b>63</b>
29	Parametric elliptical arcs	64
30	Bézier curves	66
30.1	Quadratic Bézier curves	67
30.1.1	Drawing the quadratic	67
30.2	Cubic Bézier curves	71
30.3	Weighted Béziers	71
<b>VII</b>	<b>Vectors, matrices and transformations</b>	<b>72</b>
31	Rotation of a bitmap	73
31.1	Fast 2D Rotation	77
32	90° Rotation of a bitmap by parallel recursive subdivision	78
33	Magnification/Scaling	79
33.1	Smoothing enlarged bitmaps	80
33.2	Stretching lines of bitmaps	80
34	Mirroring	82
35	Shearing	83
35.1	The relationship between shearing factor and angle	85
36	Projections	86
<b>VIII</b>	<b>Addendum</b>	<b>87</b>
37	Faster drawing a line segment from its two endpoints using symmetry	88
38	Joining the ends of two wide line segments together	89
39	Composing monochrome bitmaps with separate alpha channel data	90
40	Orthogonal connection of two points	91
41	Join segments with round corners	92
42	Squircle	96
43	Faster line clipping	99
44	Tilings	100

44.1	Hexagon Tiling	100
45	Space-filling Curves	101
45.1	Hilbert curve	102
45.2	Sierpiński curve	104
45.3	Peano curve	104
45.4	Z-order curve	105
45.5	Flowsnake curve	108
46	Dithering	110
46.1	Floyd-Steinberg	111
46.2	Atkinson dithering	113
47	Marching squares	115
	Index	117





# Part I

## Introduction

## Chapter 1

# Data representation

The data structures we're going to use is *Point* and *Image*. *Image* represents a bitmap, although we will use full RGB colors for our points therefore the size of a pixel in memory will be u8 instead of 1 bit.

We will work on the cartesian grid representing the framebuffer that will show us the pixels. The *origin* of this grid (i.e. the center) is at  $(0, 0)$ .



We will represent points as pairs of signed integers. When actually drawing them though, negative values and values outside the window's geometry will be

ignored (clipped).

src/lib.rs:



This code file is a PDF attachment

```
pub type Point = (i64, i64);

pub const fn from_u8_rgb(r: u8, g: u8, b: u8) -> u32 {
    let (r, g, b) = (r as u32, g as u32, b as u32);
    (r << 16) | (g << 8) | b
}

pub const AZURE_BLUE: u32 = from_u8_rgb(0, 127, 255);
pub const RED: u32 = from_u8_rgb(157, 37, 10);
pub const WHITE: u32 = from_u8_rgb(255, 255, 255);
pub const BLACK: u32 = 0;

pub struct Image {
    pub bytes: Vec<u32>,
    pub width: usize,
    pub height: usize,
    pub x_offset: usize,
    pub y_offset: usize,
}

impl Image {
    pub fn new(width: usize, height: usize, x_offset: usize, y_offset: usize) -> Self;
    pub fn magick_open(path: &str, x_offset: usize, y_offset: usize) -> Result<Self,
↳ Box<dyn Error>>;
    pub fn from_xbm(path: &str, x_offset: usize, y_offset: usize) -> Result<Self, Box<dyn
↳ Error>>;
    pub fn draw(&self, buffer: &mut Vec<u32>, fg: u32, bg: Option<u32>, window_width:
↳ usize);
    pub fn draw_outline(&mut self);
    pub fn clear(&mut self);
    pub fn plot(&mut self, x: i64, y: i64);
    pub fn get(&mut self, x: i64, y: i64) -> u32;
    pub fn plot_ellipse(
        &mut self,
        (xm, ym): (i64, i64),
        (a, b): (i64, i64),
        quadrants: [bool; 4],
        _wd: f64,
    );
    pub fn plot_line_width(&mut self, point_a: Point, point_b: Point, wd: f64);
    pub fn flood_fill(&mut self, mut x: i64, y: i64);
}
```

intro

## Chapter 2

# Displaying pixels to your screen

A way to display an *Image* is to use the `minifb` crate which allows you to create a window and draw pixels directly on it. Here's how you could set it up:

src/bin/introduction.rs:



This code file is a PDF attachment

```
use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

fn main() {
    let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
    let mut window = Window::new(
        "Test - ESC to exit",
        WINDOW_WIDTH,
        WINDOW_HEIGHT,
        WindowOptions {
            title: true,
            //borderless: true,
            //resize: false,
            //transparency: true,
            ..WindowOptions::default()
        },
    )
    .unwrap();

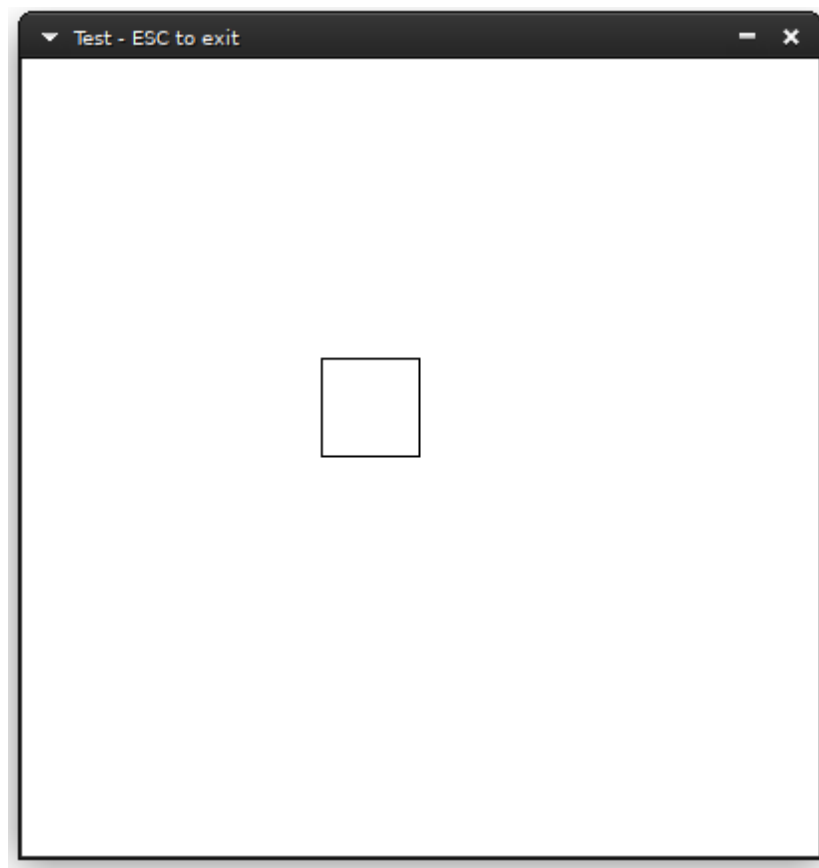
    // Limit to max ~60 fps update rate
    window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

    let mut image = Image::new(50, 50, 150, 150);
    image.draw_outline();
    image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

    while window.is_open()
        && !window.is_key_down(Key::Escape)
        && !window.is_key_down(Key::Q) {
        window
            .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
            .unwrap();

        let millis = std::time::Duration::from_millis(100);
        std::thread::sleep(millis);
    }
}
```

Running this will show you something like this:



intro

## Chapter 3

# Bits to byte pixels

Let's define a way to convert bit information to a byte vector:

```
pub fn bits_to_bytes(bits: &[u8], width: usize) -> Vec<u32> {  
    let mut ret = Vec::with_capacity(bits.len() * 8);  
    let mut current_row_count = 0;  
    for byte in bits {  
        for n in 0..8 {  
            if byte.rotate_right(n) & 0x01 > 0 {  
                ret.push(BLACK);  
            } else {  
                ret.push(WHITE);  
            }  
            current_row_count += 1;  
            if current_row_count == width {  
                current_row_count = 0;  
                break;  
            }  
        }  
    }  
    ret  
}
```

## Chapter 4

# Loading graphics files in Rust

The book's library includes a method to load xbm files on runtime (see *Including xbm files in **Rust*** for including them in your binary at compile time). If your system has ImageMagick installed and the commands `identify` and `magick` are in your `PATH` environment variable, you can use the `Image::magick_open` method:

```
impl Image {  
    ...  
    pub fn magick_open(path: &str, x_offset: usize, y_offset: usize) -> Result<Self,  
↳   Box<dyn Error>>;  
    ...  
}
```

It simply converts the image file you pass to it to raw bytes using the invocation `magick convert path RGB:-` which prints raw RGB content to `stdout`.

If you have another way to load pictures such as your own code or a picture format library crate, all you have to do is convert the pixel information to an `Image` whose definition we repeat here:

```
pub struct Image {  
    pub bytes: Vec<u32>,  
    pub width: usize,  
    pub height: usize,  
    pub x_offset: usize,  
    pub y_offset: usize,  
}
```

## Chapter 5

# Including xbm files in Rust

*The end of this chapter includes a short **Rust** program to automatically convert **xbm** files to equivalent **Rust** code.*

xbm files are C source code files that contain the pixel information for an image as macro definitions for the dimensions and a static char array for the pixels, with each bit column representing a pixel. If the width dimension doesn't have 8 as a factor, the remaining bit columns are left blank/ignored.

They used to be a popular way to share user avatars in the old internet and are also good material for us to work with, since they are small and numerous. The following is such an image:



Then, we can convert the xbm file from C to **Rust** with the following transformations:

```
#define news_width 48
#define news_height 48
static char news_bits[] = {
```

to

```
const NEWS_WIDTH: usize = 48;
const NEWS_HEIGHT: usize = 48;
const NEWS_BITS: &[u8] = &[
```

And replace the closing `}` with `]`.

We can then include the new file in our source code:

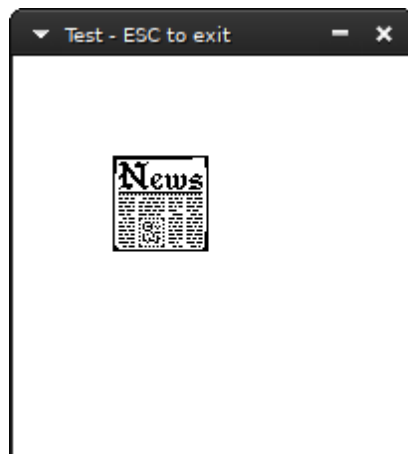


```
include!("news.xbm.rs");
```

load the image:

```
let mut image = Image::new(NEWS_WIDTH, NEWS_HEIGHT, 25, 25);  
image.bytes = bits_to_bytes(NEWS_BITS, NEWS_WIDTH);
```

and finally run it:



intro

The following short program uses the regex crate to match on these simple rules and print the equivalent code in stdout. You can use it like so:

```
cargo run --bin xbmtools -- file.xbm > file.xbm.rs
```

```
use regex;  
use regex::Regex;  
use std::fs::File;  
use std::io::prelude::*;  
  
fn main() {  
    let args = std::env::args().skip(1).collect::<Vec<String>>();  
    if args.len() != 1 {  
        println!("one argument expected, the xbm file path to convert.");  
        return;  
    }  
    let mut file = match File::open(&args[0]) {  
        Err(err) => panic!("couldn't open {}: {}", args[0], err),  
        Ok(file) => file,  
    };  
  
    let mut s = String::new();  
    if let Err(err) = file.read_to_string(&mut s) {  
        panic!("couldn't read {}: {}", args[0], err);  
    }  
  
    let re = Regex::new(  
        r"(?imax)  
        ^\s*\x23\s*define\s+(?P<i>.+?)_width\s+(?P<w>\d\d*)$  
    )".unwrap().unwrap();  
}
```

src/bin/xbmtors.rs:



This code file is a PDF attachment

```

    |s*
    ^|s*|x23|s*define\s+.+?_height\s+(?P<h>\d\d*)$
    |s*
    ^|s*static(\s+unsigned){0,1}\s+char\s+.+?_bits.. \s*=\s*\{(?P<b>[~}]+)\};
",
    )
    .unwrap();
    let caps = re
        .captures(&s)
        .expect("Could not convert file, regex doesn't match :(");
    let ident = caps.name("i").unwrap().as_str().to_uppercase();
    let out = re.replace_all(&s, format!("const {i}_WIDTH: usize = $w;\nconst {i}_HEIGHT:
↪   usize = $h;\nconst {i}_BITS: &[u8] = &[$b];", i = &ident));
    println!("{}", out.trim());
}

```

# **Part II**

## **Points And Lines**

## Chapter 6

# Distance between two points

lines



Given two points,  $K$  and  $L$ , an elementary application of Pythagoras' Theorem gives the distance between them as

$$r = \sqrt{(x_L - x_K)^2 + (y_L - y_K)^2} \quad (6.1)$$

which is simply coded:

```
pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {  
    let (x_k, y_k) = p_k;  
    let (x_l, y_l) = p_l;  
    let xlk = x_l - x_k;  
    let ylk = y_l - y_k;  
    f64::sqrt((xlk*xlk + ylk*ylk) as f64)  
}
```

## Chapter 7

# Moving a point to a distance at an angle

lines

Moving a point  $P = (x, y)$  at distance  $d$  at an angle of  $r$  radians is solved with simple trigonometry:

$$P' = (x + d \times \cos r, y + d \times \sin r)$$

Why? The problem is equivalent to calculating the point of a circle with  $P$  as the center,  $d$  the radius at angle  $r$  and as we will later\* see this is how the points of a circle are calculated.

```
pub fn move_point(p: Point, d: f64, r: f64) -> Point {  
    let (x, y) = p;  
    (x + (d * f64::cos(r)).round() as i64, y + (d * f64::sin(r)).round() as i64)  
}
```

---

\**Equations of a circle* page 48

## Chapter 8

# Equations of a line

lines

There are several ways to describe a line mathematically. We'll list the convenient ones for drawing pixels.

The equation that describes every possible line on a two dimensional grid is the *implicit* form  $ax + by = c$ ,  $(a, b) \neq (0, 0)$ . We can generate equivalent equations by adding the equation to itself, i.e.  $ax + by = c \equiv 2ax + 2by = 2c \equiv a'x + b'y = c'$ ,  $a' = 2a, b' = 2b, c' = 2c$  as many times as we want. To "minimize" the constants  $a, b, c$  we want to satisfy the relationship  $a^2 + b^2 = 1$ , and thus can convert the equivalent equations into one representative equation by multiplying the two sides with  $\frac{1}{\sqrt{a^2 + b^2}}$ ; this is called the normalized equation.

The *slope intercept form* describes any line that intercepts the  $y$  axis at  $b \in \mathbb{R}$  with a specific slope  $a$ :

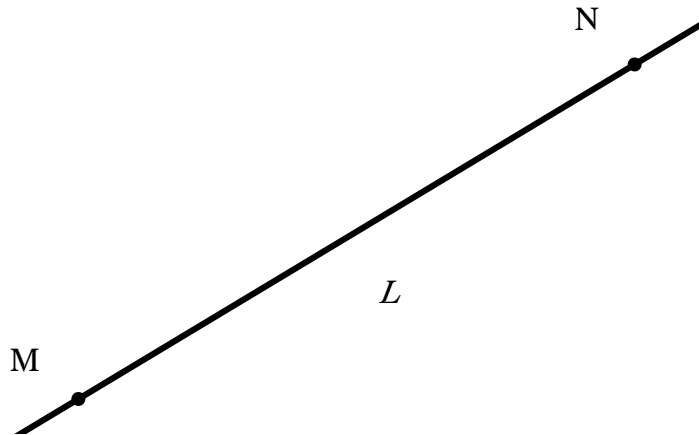
$$y = ax + b$$

The *parametric* form...

### 8.1 Line through a point $P = (x_p, y_p)$ and a slope $m$

$$y - y_p = m(x - x_p)$$

## 8.2 Line through two points



lines

It seems sufficient, given the coordinates of two points  $M, N$ , to calculate  $a, b$  and  $c$  to form a line equation:

$$ax + by + c = 0$$

If the two points are not the same, they necessarily form such a line. To get there, we start from expressing the line as parametric over  $t$ : at  $t = 0$  it's at point  $M$  and at  $t = 1$  it's at point  $N$ :

$$c = c_M + (c_N - c_M)t, t \in R, c \in \{x, y\}$$

$$c = c_M, t \in R, c \in \{x, y\}$$

Substituting  $t$  in one of the equations we get:

$$(y_M - y_N)x + (x_N - x_M)y + (x_M y_N - x_N y_M) = 0$$

Which is what we were after. We should finish by normalising what we found with  $\frac{1}{\sqrt{a^2+b^2}}$ , but our coordinates are integers and have no decimal or floating point accuracy.

```
fn find_line(point_a: Point, point_b: Point) -> (i64, i64, i64) {  
    let (xa, ya) = point_a;  
    let (xb, yb) = point_b;  
    let a = yb - ya;  
    let b = xa - xb;  
    let c = xb * ya - xa * yb;  
    (a, b, c)  
}
```

lines



## Chapter 9

# Drawing a line

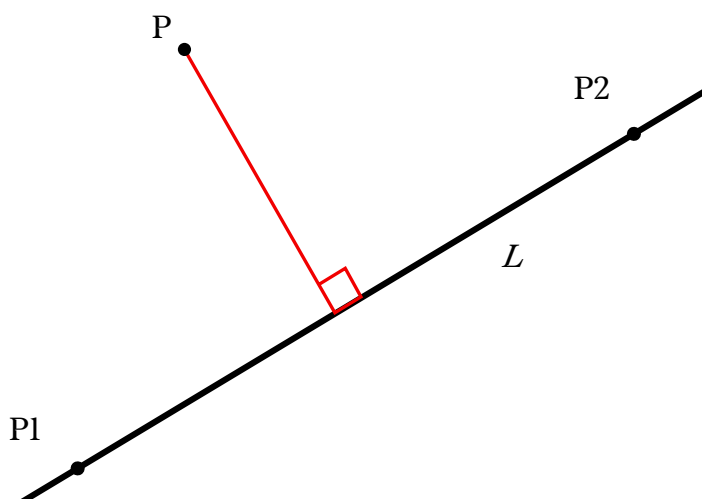
lines

```
fn plot_line(image: &mut Image, (a, b, c): (i64, i64, i64)) {
    let x = if a != 0 { -1 * (c) / a } else { 0 };
    let mut prev_point = (x, 0);
    for y in 0..(WINDOW_HEIGHT as i64) {
        //  $ax+by+c=0 \Rightarrow$ 
        //  $x=(-c-by)/a$ 
        let x = if a != 0 { -1 * (c + b * y) / a } else { 0 };
        let new_point = (x, y);
        image.plot_line_width(prev_point, new_point, 1.0);
        prev_point = new_point;
    }
}
```

## Chapter 10

# Distance from a point to a line

lines



### 10.1 Using the implicit equation form

Let's find the distance from a given point  $P$  and a given line  $L$ . Let  $d$  be the distance between them. Bring  $L$  to the implicit form  $ax + by = c$ .

$$d = \frac{|ax_p + by_p + c|}{\sqrt{a^2 + b^2}}$$

## 10.2 Using an $L$ defined by two points $P_1, P_2$

With  $P = (x_0, y_0)$ ,  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$ .

$$d = \frac{|(x_2 - x_1)(y_1 - y_0) - (x_1 - x_0)(y_2 - y_1)|}{\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}}$$

## 10.3 Using an $L$ defined by a point $P_l$ and angle $\hat{\theta}$

$$d = \left| \cos(\hat{\theta})(P_{ly} - y_p) - \sin(\hat{\theta})(P_{lx} - P_x) \right|$$

### The code

This function uses the implicit form.

```
type Line = (i64, i64, i64);
pub fn distance_line_to_point((x, y): Point, (a, b, c): Line) -> f64 {
  let d = f64::sqrt((a * a + b * b) as f64);
  if d == 0.0 {
    0.
  } else {
    (a * x + b * y + c) as f64 / d
  }
}
```

This code is included in the distributed library file in the *Data representation* chapter.

lines

## Chapter 11

# Perpendicular lines

## 11.1 Find perpendicular to line that passes through given point

Now, we wish to find the equation of the line that passes through  $P$  and is perpendicular to  $L$ . Let's call it  $L_{\perp}$ .  $L$  in implicit form is  $ax + by + c = 0$ . The perpendicular will be:

$$L_{\perp} : bx - ay + (aP_y - bP_x) = 0$$

### The code

This code is included in the distributed library file in the *Data representation* chapter.

```
type Line = (i64, i64, i64);
fn perpendicular((a, b, c): Line, p: Point) -> Line {
    (b, -1 * a, a * p.1 - b * p.0)
}
```

## 11.2 Find point in line that belongs to the perpendicular of given point

### The code

This code is included in the distributed library file in the *Data representation* chapter.

```
fn point_perpendicular((a, b, c): Line, p: Point) -> Point {
    let d = (a * a + b * b) as f64;
    if d == 0. {
        return (0, 0);
    }
    let cp = a * p.1 - b * p.0;
```

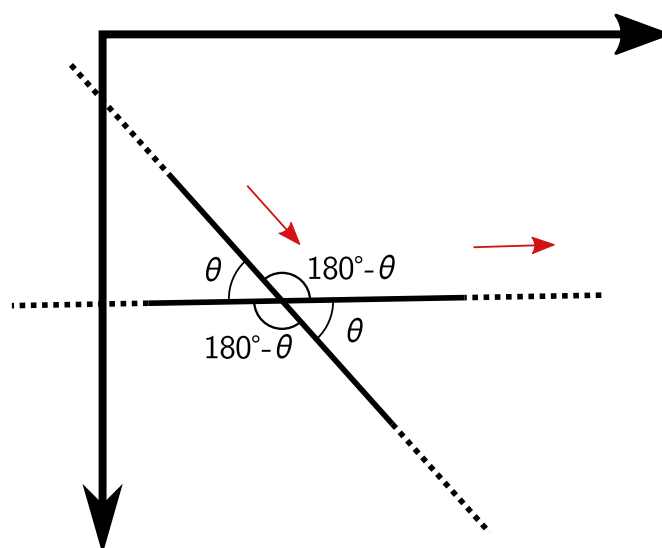
```
(  
    ((-a * c - b * cp) as f64 / d) as i64,  
    ((a * cp - b * c) as f64 / d) as i64,  
)  
}
```

lines

## Chapter 12

### Angle between two lines

lines



By angle we mean the angle formed by the two directions of the lines; and direction vectors start from the origin (in the figure, they are the **red arrows**). So if we want any of the other three angles, we already know them from basic geometry as shown in the figure above.

If you prefer using the implicit equation, bring the two lines  $L_1$  and  $L_2$  to that form  $(a_1x + b_1y + c = 0$  and  $a_2x + b_2y + c_2 = 0)$  and you can directly find  $\hat{\theta}$  with the formula:

$$\hat{\theta} = \arccos \frac{a_1a_2 + b_1b_2}{\sqrt{(a_1^2 + b_1^2) (a_2^2 + b_2^2)}}$$

For the following parametric equations of  $L_1, L_2$ :

$$L_1 = (\{x = x_1 + f_1 t\}, \{y = y_1 + g_1 t\})$$

$$L_2 = (\{x = x_2 + f_2 s\}, \{y = y_2 + g_2 s\})$$

the formula is:

$$\hat{\theta} = \arccos \frac{f_1 f_2 + g_1 g_2}{\sqrt{(f_1^2 + g_1^2)(f_2^2 + g_2^2)}}$$

The code:

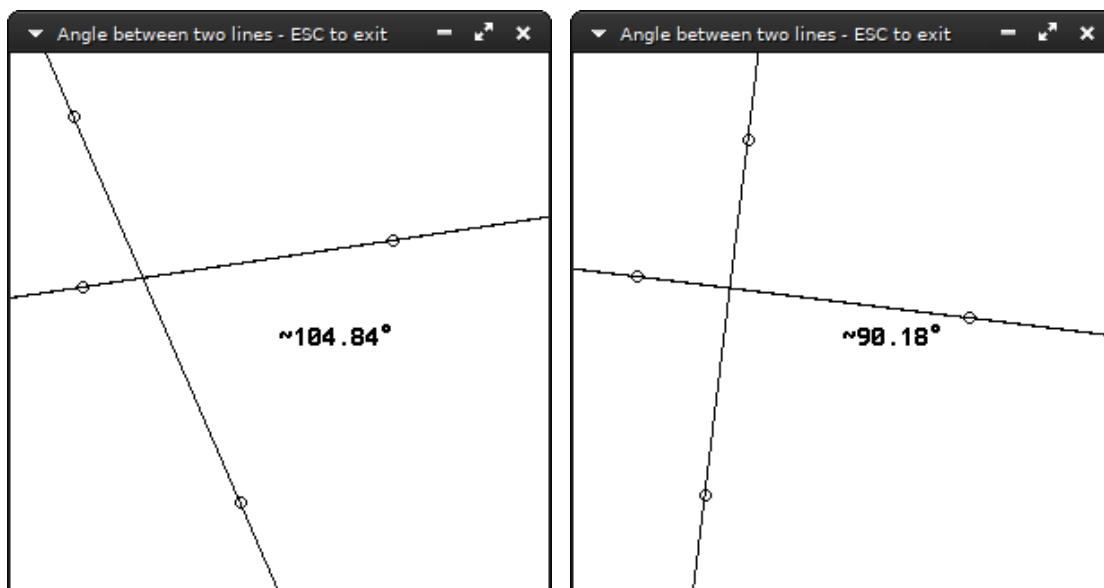
```
fn find_angle((a1, b1, c1): (i64, i64, i64), (a2, b2, c2): (i64, i64, i64)) -> f64 {
    let nom = (a1 * a2 + b1 * b2) as f64;
    let denom = ((a1 * a1 + b1 * b1) * (a2 * a2 + b2 * b2)) as f64;
    f64::acos(nom / f64::sqrt(denom))
}
```

src/bin/anglebe



lines

This code file is a PDF attachment

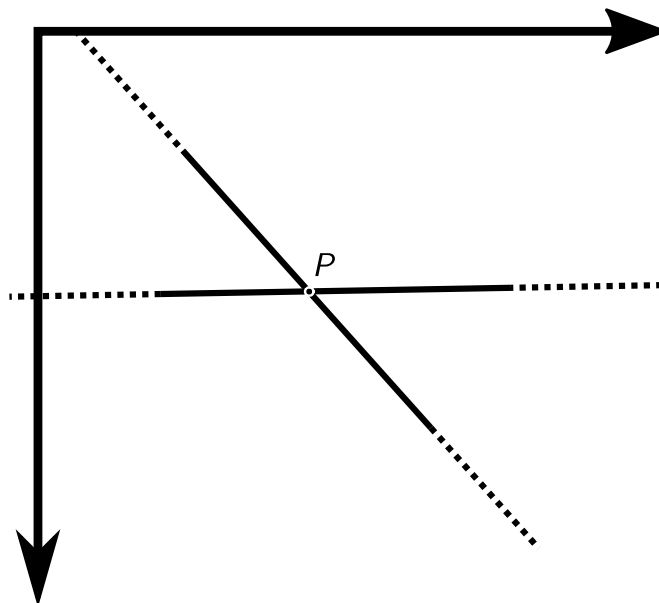


The `src/bin/anglebetweenlines.rs` example has two interactive lines and computes their angle with 64bit floating point accuracy.

## Chapter 13

# Intersection of two lines

lines



If the lines  $L_1, L_2$  are in implicit form ( $a_1x + b_1y + c = 0$  and  $a_2x + b_2y + c_2 = 0$ ), the result comes after checking if the lines are parallel (in which case there's no single point of intersection):

$$a_1b_2 - a_2b_1 \neq 0$$

If they are not parallel,  $P$  is:

$$P = \left( \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \right)$$



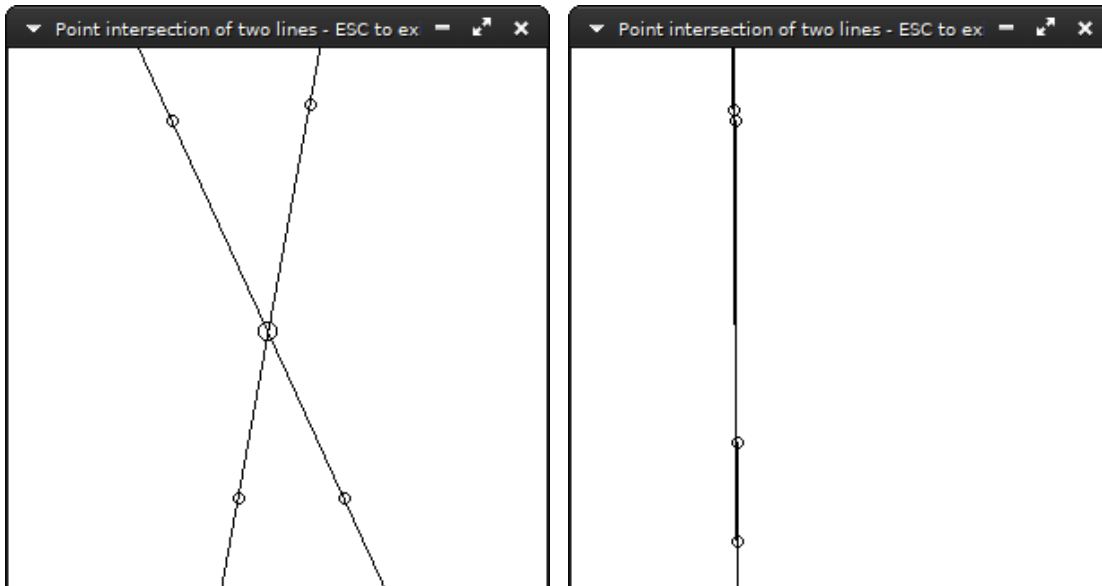
The code:

```
fn find_intersection((a1, b1, c1): (i64, i64, i64), (a2, b2, c2): (i64, i64, i64)) ->
  ↪ Option<Point> {
    let denom = a1 * b2 - a2 * b1;
    if denom == 0 {
      return None;
    }
    Some(((b1 * c2 - b2 * c1) / denom, (a2 * c1 - a1 * c2) / denom))
  }
```

src/bin/lineintersection.rs:



This code file is a PDF attachment



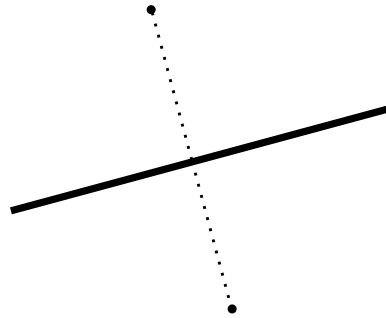
The src/bin/lineintersection.rs example has two interactive lines and computes their point of intersection.

lines

## Chapter 14

# Line equidistant from two points

lines



Let's name this line  $L$ . From previous chapter\* we know how to get the line  $L$  that's created by the two points  $M$  and  $N$ :

$$L : (y_M - y_N)x + (x_N - x_M)y + (x_M y_N - x_N y_M) = 0$$

We need the perpendicular line over the midpoint of  $L$ .<sup>†</sup> The midpoint also satisfies  $L$ 's equation. The midpoint's coordinates are intuitively:

$$P_{mid} = \left( \frac{x_M + x_N}{2}, \frac{y_M + y_N}{2} \right)$$

The perpendicular's  $L_{\perp}$  equation is

$$L_{EQ} = L_{\perp} : yx - ay + (aP_{mid_y} - bP_{mid_x}) = 0$$

---

\*See *Line through two points*, page 23

<sup>†</sup>See *Perpendicular lines*, page 28

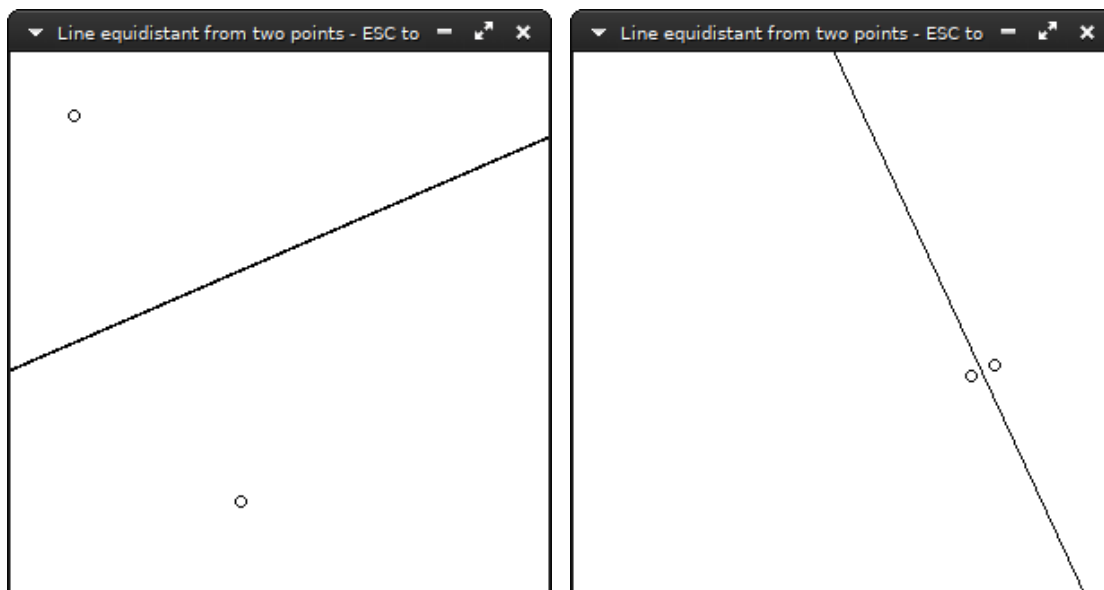
The code:

```
fn find_equidistant(point_a: Point, point_b: Point) -> (i64, i64, i64) {
    let (xa, ya) = point_a;
    let (xb, yb) = point_b;
    let midpoint = ((xa + xb) / 2, (ya + yb) / 2);
    let al = ya - yb;
    let bl = xb - xa;
    // If we had subpixel accuracy, we could do:
    //assert_eq!(al*midpoint.0+bl*midpoint.1, -cl);
    let a = bl;
    let b = -1 * al;
    let c = (al * midpoint.1) - (bl * midpoint.0);
    (a, b, c)
}
```

src/bin/equidistant.rs:



This code file is a PDF attachment



The `src/bin/equidistant.rs` example has two interactive points and computes their  $L_{EQ}$ .

lines

## Chapter 15

# Normal to a line through a point

lines

Add Normal to a line through a point

A series of horizontal gray bars, likely representing a list or table. The bars are arranged in a single column and are of varying lengths, suggesting a list of items or a table with multiple rows. The bars are light gray and have a thin black border.

## Chapter 16

# Angle sectioning

lines

### 16.1 Bisection



### 16.2 Trisection

If the title startled you, be assured it's not a joke. It's totally possible to trisect an angle... with a ruler. The adage that angle trisection is impossible refers to using only a compass and unmarked straightedge.





lines

## **Part III**

# **Points And Line Segments**

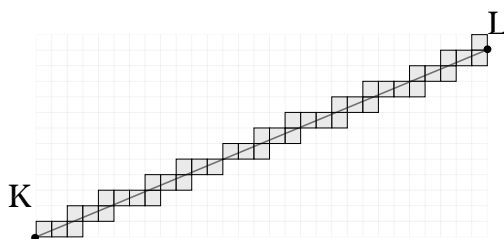
segments

## Chapter 17

# Drawing a line segment from its two endpoints

segments

For any line segment with any slope, pixels must be matched with the infinite amount of points contained in the segment. As shown in the following figure, a segment *touches* some pixels; we could fill them using an algorithm and get a bitmap of the line segment.



The algorithm presented here was first derived by Bresenham. In the *Image* implementation, it is used in the `plot_line_width` method.

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64)) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();  
    sx = if dx > 0 { 1 } else { -1 };  
}
```



```

dy = y2 - y1;
ay = (dy * 2).abs();
sy = if dy > 0 { 1 } else { -1 };
x = x1;
y = y1;

let b = dx / dy;
let a = 1;
let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;
let delta = double_d / 2;

if ax > ay {
  d = ay - ax / 2;
  loop {
    self.plot(x, y);
    if x == x2 {
      return;
    }
    if d >= 0 {
      y = y + sy;
      d = d - ax;
    }
    x = x + sx;
    d = d + ay;
  }
} else {
  d = ax - ay / 2;
  let delta = double_d / 3;
  loop {
    self.plot(x, y);
    if y == y2 {
      return;
    }
    if d >= 0 {
      x = x + sx;
      d = d - ay;
    }
    y = y + sy;
    d = d + ax;
  }
}
}

```

segments

Add some explanation behind the algorithm in *Drawing a line segment from its two endpoints*

## Chapter 18

# Drawing line segments with width

segments

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64), _wd: f64) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();  
    sx = if dx > 0 { 1 } else { -1 };  
    dy = y2 - y1;  
    ay = (dy * 2).abs();  
    sy = if dy > 0 { 1 } else { -1 };  
  
    x = x1;  
    y = y1;  
  
    let b = dx / dy;  
    let a = 1;  
    let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;  
    let delta = double_d / 2;  
  
    if ax > ay {  
        d = ay - ax / 2;  
        loop {  
            self.plot(x, y);  
            {  
                let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;  
                let mut _x = x;  
                loop {  
                    let t = total(_x);  
                    if t < -1 * delta || t > delta {  
                        break;  
                    }  
                    _x += 1;  
                    self.plot(_x, y);  
                }  
            }  
            let mut _x = x;  
            loop {  
                let t = total(_x);  
                if t < -1 * delta || t > delta {  
                    break;  
                }  
                _x -= 1;  
                self.plot(_x, y);  
            }  
        }  
    }  
}
```

```

        if x == x2 {
            return;
        }
        if d >= 0 {
            y = y + sy;
            d = d - ax;
        }
        x = x + sx;
        d = d + ay;
    }
} else {
    d = ax - ay / 2;
    let delta = double_d / 3;
    loop {
        self.plot(x, y);
        {
            let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x += 1;
                self.plot(_x, y);
            }
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x -= 1;
                self.plot(_x, y);
            }
        }
    }
    if y == y2 {
        return;
    }
    if d >= 0 {
        x = x + sx;
        d = d - ay;
    }
    y = y + sy;
    d = d + ax;
}
}
}
}

```

segments

## Chapter 19

# Intersection of two line segments

Let points  $\mathbf{1} = (x_1, y_1)$ ,  $\mathbf{2} = (x_2, y_2)$ ,  $\mathbf{3} = (x_3, y_3)$  and  $\mathbf{4} = (x_4, y_4)$  and  $\mathbf{1,2}$ ,  $\mathbf{3,4}$  two line segments they form. We wish to find their intersection:

First, get the equation of line  $L_{12}$  and line  $L_{34}$  from chapter *Equations of a line*.

Substitute points  $\mathbf{3}$  and  $\mathbf{4}$  in equation  $L_{12}$  to compute  $r_3 = L_{12}(\mathbf{3})$  and  $r_4 = L_{12}(\mathbf{4})$  respectively.

If  $r_3 \neq 0$ ,  $r_4 \neq 0$  and  $\text{sgn}(r_3) \neq \text{sgn}(r_4)$  the line segments don't intersect, so stop.

In  $L_{34}$  substitute point  $\mathbf{1}$  to compute  $r_1$ , and do the same for point  $\mathbf{2}$ .

If  $r_1 \neq 0$ ,  $r_2 \neq 0$  and  $\text{sgn}(r_1) \neq \text{sgn}(r_2)$  the line segments don't intersect, so stop.

At this point,  $L_{12}$  and  $L_{34}$  either intersect or are equivalent. Find their intersection point. (See *Intersection of two lines* page 32)

### 19.1 Fast intersection of two line segments




segments

## **Part IV**

# **Points, Lines and Circles**

**circles**

[Redacted text block]

## Chapter 20

# Equations of a circle

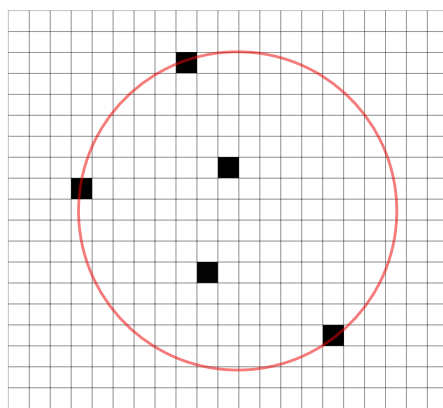
Add Equations of a circle

circles



## Chapter 21

# Bounding circle



src/bin/boundingcircle.rs:



This code file is a PDF attachment

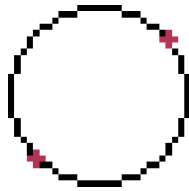
**circles**

A bounding circle is a circle that includes all the points in a given set. Usually we're interested in one of the smallest ones possible.



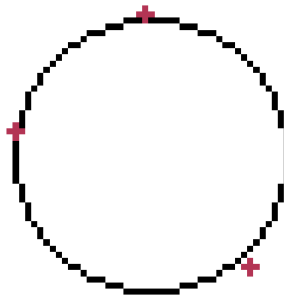
We can use the following methodology to find the bounding circle: start from two points and the circle they make up, and for each of the rest of the points check if the circle includes them. If not, make a bounding circle that includes every point up to the current one. To do this, we need some primitive operations.

We will need a way to construct a circle out of two points:



```
let p1 = points[0];
let p2 = points[1];
//The circle is determined by two points, P and Q. The center of the circle
↪ is
//at (P + Q)/2.0 and the radius is |(P - Q)/2.0|
let d_2 = (
  ((p1.0 + p2.0) / 2), (p1.1 + p2.1) / 2),
  (distance_between_two_points(p1, p2) / 2.0),
);
```

And a way to make a circle out of three points:



```
fn min_circle_w_3_points(q1: Point, q2: Point, q3: Point) -> Circle {
  let (ax, ay) = (q1.0 as f64, q1.1 as f64);
  let (bx, by) = (q2.0 as f64, q2.1 as f64);
  let (cx, cy) = (q3.0 as f64, q3.1 as f64);

  let mut d = 2. * (ax * (by - cy) + bx * (cy - ay) + cx * (ay - by));
  if d == 0.0 {
    d = std::cmp::max(
      std::cmp::max(
        distance_between_two_points(q1, q2) as i64,
        distance_between_two_points(q2, q3) as i64,
      ),
      distance_between_two_points(q1, q3) as i64,
    ) as f64
    / 2.;
  }
  let ux = ((ax * ax + ay * ay) * (by - cy)
    + (bx * bx + by * by) * (cy - ay)
    + (cx * cx + cy * cy) * (ay - by))
    / d;
  let uy = ((ax * ax + ay * ay) * (cx - bx)
```

```

    + (bx * bx + by * by) * (ax - cx)
    + (cx * cx + cy * cy) * (bx - ax))
    / d;
let mut center = (ux as i64, uy as i64);
if center.0 < 0 {
    center.0 = 0;
}
if center.1 < 0 {
    center.1 = 0;
}
let d = distance_between_two_points(center, q1);
(center, d)
}

```

## The algorithm:

```

use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};
use rand::seq::SliceRandom;
use rand::thread_rng;
use std::f64::consts::{FRAC_PI_2, PI};

include!("../me.xbm.rs");

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {
    let (x_k, y_k) = p_k;
    let (x_l, y_l) = p_l;
    let xlk = x_l - x_k;
    let ylk = y_l - y_k;
    f64::sqrt((xlk * xlk + ylk * ylk) as f64)
}

fn image_to_points(image: &Image) -> Vec<Point> {
    let mut ret = Vec::with_capacity(image.bytes.len());
    for y in 0..(image.height as i64) {
        for x in 0..(image.width as i64) {
            if image.get(x, y) == Some(BLACK) {
                ret.push((x, y));
            }
        }
    }
    ret
}

type Circle = (Point, f64);

fn bc(image: &Image) -> Circle {
    let mut points = image_to_points(image);
    points.shuffle(&mut thread_rng());
    min_circle(&points)
}

fn min_circle(points: &[Point]) -> Circle {
    let mut points = points.to_vec();
    points.shuffle(&mut thread_rng());

    let p1 = points[0];
    let p2 = points[1];

    //The circle is determined by two points, P and Q. The center of the
    circle is
    //at (P + Q)/2.0 and the radius is |(P - Q)/2.0|
    let d_2 = (
        ((p1.0 + p2.0) / 2), (p1.1 + p2.1) / 2),
        (distance_between_two_points(p1, p2) / 2.0),
    );

    let mut d_prev = d_2;

    for i in 2..points.len() {
        let p_i = points[i];
        if distance_between_two_points(p_i, d_prev.0) <= (d_prev.1) {
            // then d_i = d_{i-1}

```

circles

```

        } else {
            let new = min_circle_w_point(&points[..i], p_i);
            if distance_between_two_points(p_i, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_point(points: &[Point], q: Point) -> Circle {
    let mut points = points.to_vec();
    points.shuffle(&mut thread_rng());
    let p1 = points[0];
    //The circle is determined by two points, P_1 and Q. The center of the
    ↪ circle is
    //at (P_1 + Q)/2.0 and the radius is |(P_1 - Q)/2.0|
    let d_1 = (
        ((p1.0 + q.0) / 2), (p1.1 + q.1) / 2),
        (distance_between_two_points(p1, q) / 2.0),
    );
    let mut d_prev = d_1;
    for j in 1..points.len() {
        let p_j = points[j];
        if distance_between_two_points(p_j, d_prev.0) <= (d_prev.1) {
            //d_prev = d_prev;
        } else {
            let new = min_circle_w_points(&points[..j], p_j, q);
            if distance_between_two_points(p_j, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_points(points: &[Point], q1: Point, q2: Point) -> Circle {
    let mut points = points.to_vec();
    let d_0 = (
        ((q1.0 + q2.0) / 2), (q1.1 + q2.1) / 2),
        (distance_between_two_points(q1, q2) / 2.0),
    );
    let mut d_prev = d_0;
    for k in 0..points.len() {
        let p_k = points[k];
        if distance_between_two_points(p_k, d_prev.0) <= (d_prev.1) {
        } else {
            let new = min_circle_w_3_points(q1, q2, p_k);
            if distance_between_two_points(p_k, new.0) <= (new.1) {
                d_prev = new;
            }
        }
    }
    d_prev
}

fn min_circle_w_3_points(q1: Point, q2: Point, q3: Point) -> Circle {
    let (ax, ay) = (q1.0 as f64, q1.1 as f64);
    let (bx, by) = (q2.0 as f64, q2.1 as f64);
    let (cx, cy) = (q3.0 as f64, q3.1 as f64);
    let mut d = 2. * (ax * (by - cy) + bx * (cy - ay) + cx * (ay - by));
    if d == 0.0 {
        d = std::cmp::max(
            std::cmp::max(
                distance_between_two_points(q1, q2) as i64,
                distance_between_two_points(q2, q3) as i64,
            ),
            distance_between_two_points(q1, q3) as i64,
        ) as f64
        / 2.;
    }
}

```

```

let ux = ((ax * ax + ay * ay) * (by - cy)
  + (bx * bx + by * by) * (cy - ay)
  + (cx * cx + cy * cy) * (ay - by))
  / d;
let uy = ((ax * ax + ay * ay) * (cx - bx)
  + (bx * bx + by * by) * (ax - cx)
  + (cx * cx + cy * cy) * (bx - ax))
  / d;
let mut center = (ux as i64, uy as i64);
if center.0 < 0 {
  center.0 = 0;
}
if center.1 < 0 {
  center.1 = 0;
}
let d = distance_between_two_points(center, q1);
(center, d)
}

fn main() {
  let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
  let mut window = Window::new(
    "Test - ESC to exit",
    WINDOW_WIDTH,
    WINDOW_HEIGHT,
    WindowOptions {
      title: true,
      //borderless: true,
      resize: true,
      //transparency: true,
      ..WindowOptions::default()
    },
  )
  .unwrap();

  // Limit to max ~60 fps update rate
  window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

  let mut full = Image::new(WINDOW_WIDTH, WINDOW_HEIGHT, 0, 0);
  let mut image = Image::new(ME_WIDTH, ME_HEIGHT, 45, 45);
  image.bytes = bits_to_bytes(ME_BITS, ME_WIDTH);
  let (center, r) = bc(&image);
  image.draw_outline();

  full.plot_circle((center.0 + 45, center.1 + 45), r as i64, 0.);
  while window.is_open() && !window.is_key_down(Key::Escape) &&
↪ !window.is_key_down(Key::Q) {
    image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
    full.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

    window
      .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
      .unwrap();

    let millis = std::time::Duration::from_millis(100);
    std::thread::sleep(millis);
  }
}

```

circles

## **Part V**

# **Points, Lines and Shapes**

shapes

## Chapter 22

# Rectangles and parallelograms



### 22.1 From a center point



### 22.2 From a corner point

## Chapter 23

# Triangles



### 23.1 Making a triangle from a point and given angles





## Chapter 24

# Union, intersection and difference of polygons

Add *Union, intersection and difference of polygons*



shapes

## Chapter 25

# Centroid of polygon

Add Centroid of polygon



shapes

## Chapter 26

# Polygon clipping



shapes

## Chapter 27

# Triangle filling

Add *Triangle filling* explanation

This code is included in the distributed library file in the *Data representation* chapter.

The book's library methods include a `fill_triangle` method:

```
pub fn fill_triangle(&mut self, q1: Point, q2: Point, q3: Point) {
    let make_equation =
        |p1: Point, p2: Point, p3: Point, a: &mut i64, b: &mut i64, c: &mut i64| {
            *a = p2.1 - p1.1;
            *b = p1.0 - p2.0;
            *c = p1.0 * p2.1 - p1.1 * p2.0;

            if *a * p3.0 + *b * p3.1 + *c < 0 {
                *a = -*a;
                *b = -*b;
                *c = -*c;
            }
        };

    let mut x_min = q1.0;
    let mut y_min = q1.1;
    let mut x_max = q1.0;
    let mut y_max = q1.1;
    let mut a = [0_i64; 3];
    let mut b = [0_i64; 3];
    let mut c = [0_i64; 3];

    // find bounding box
    for q in [q1, q2, q3] {
        x_min = std::cmp::min(x_min, q.0);
        x_max = std::cmp::max(x_max, q.0);
        y_min = std::cmp::min(y_min, q.1);
        y_max = std::cmp::max(y_max, q.1);
    }
    make_equation(q1, q2, q3, &mut a[0], &mut b[0], &mut c[0]);
    make_equation(q1, q3, q2, &mut a[1], &mut b[1], &mut c[1]);
    make_equation(q2, q3, q1, &mut a[2], &mut b[2], &mut c[2]);

    let mut d0 = a[0] * x_min + b[0] * y_min + c[0];
    let mut d1 = a[1] * x_min + b[1] * y_min + c[1];
    let mut d2 = a[2] * x_min + b[2] * y_min + c[2];

    for y in y_min..=y_max {
        let mut f0 = d0;
        let mut f1 = d1;
        let mut f2 = d2;

        d0 += b[0];
        d1 += b[1];
        d2 += b[2];

        for x in x_min..=x_max {
```

```
        if f0 >= 0 && f1 >= 0 && f2 >= 0 {  
            self.plot(x, y);  
        }  
        f0 += a[0];  
        f1 += a[1];  
        f2 += a[2];  
    }  
}
```

shapes

## Chapter 28

# Flood filling

Add Flood filling



shapes

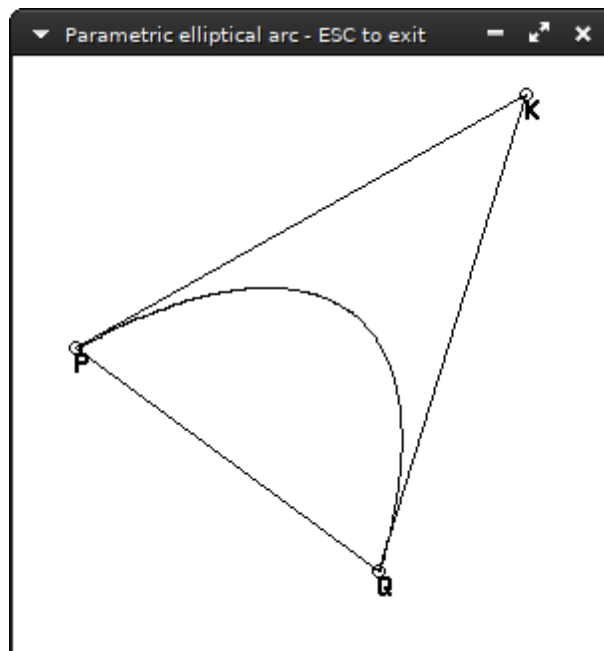
## **Part VI**

### **Curves other than circles**

curves

## Chapter 29

# Parametric elliptical arcs



$P$ ,  $Q$  and  $K$  are the arc's control points.

This algorithm\* draws an elliptical arc starting from point  $P$  and ending at  $Q$ . The control point  $K$  mirrors the ellipse's center  $J$ : drawing the quadrilateral  $PKQJ$  would appear as a lozenge, or rhombus.

The parameter  $t$  defines the step angle in radians and is limited to  $0 < t \leq 1$ .

---

\**Graphics Gems III* page 164



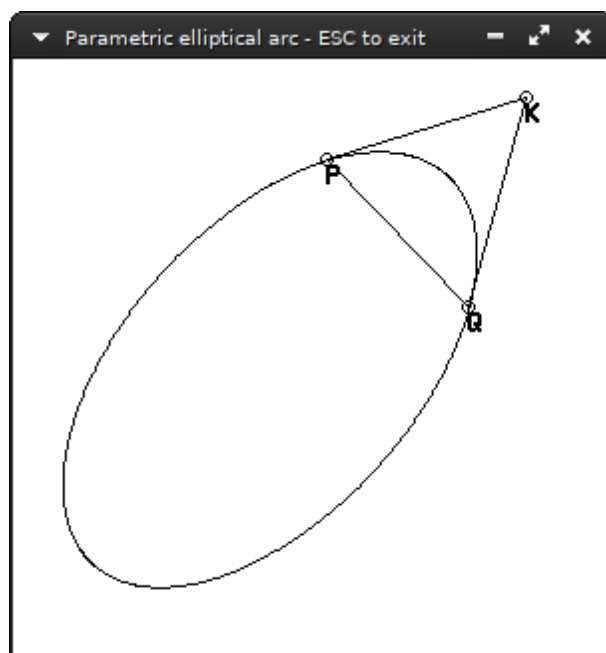
For each point calculation, the point is  $t$  radians away from the previous one, so to increase the amount of points calculated keep  $t$  small.

src/bin/parellarc.rs:



This code file is a PDF attachment

```
fn parellarc(image: &mut Image, p: Point, q: Point, k: Point, t: f64) {
    if t <= 0. || t > 1. {
        return;
    }
    let mut v = ((k.0 - q.0) as f64, (k.1 - q.1) as f64);
    let mut u = ((k.0 - p.0) as f64, (k.1 - p.1) as f64);
    let j = ((p.0 as f64 - v.0 + 0.5), (p.1 as f64 - v.1 + 0.5));
    u = (
        (u.0 * f64::sqrt(1. - t * t * 0.25) - v.0 * t * 0.5),
        (u.1 * f64::sqrt(1. - t * t * 0.25) - v.1 * t * 0.5),
    );
    let n = (std::f64::consts::FRAC_PI_2 / t).floor() as u64;
    let mut prev_pos = p;
    for _ in 0..n {
        let x = (v.0 + j.0).round() as i64;
        let y = (v.1 + j.1).round() as i64;
        let new_point = (x, y);
        image.plot_line_width(prev_pos, new_point, 1.);
        prev_pos = new_point;
        u.0 -= v.0 * t;
        v.0 += u.0 * t;
        u.1 -= v.1 * t;
        v.1 += u.1 * t;
    }
}
```

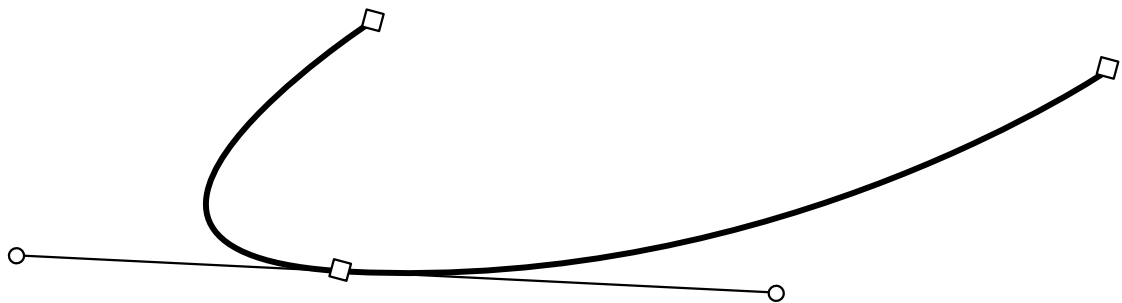


Changing  $n$  to  $\frac{2\pi}{t}$  draws the entire ellipse.

curves

## Chapter 30

# Bézier curves



Two cubic *Bézier* curves joined together as displayed in graphics software.

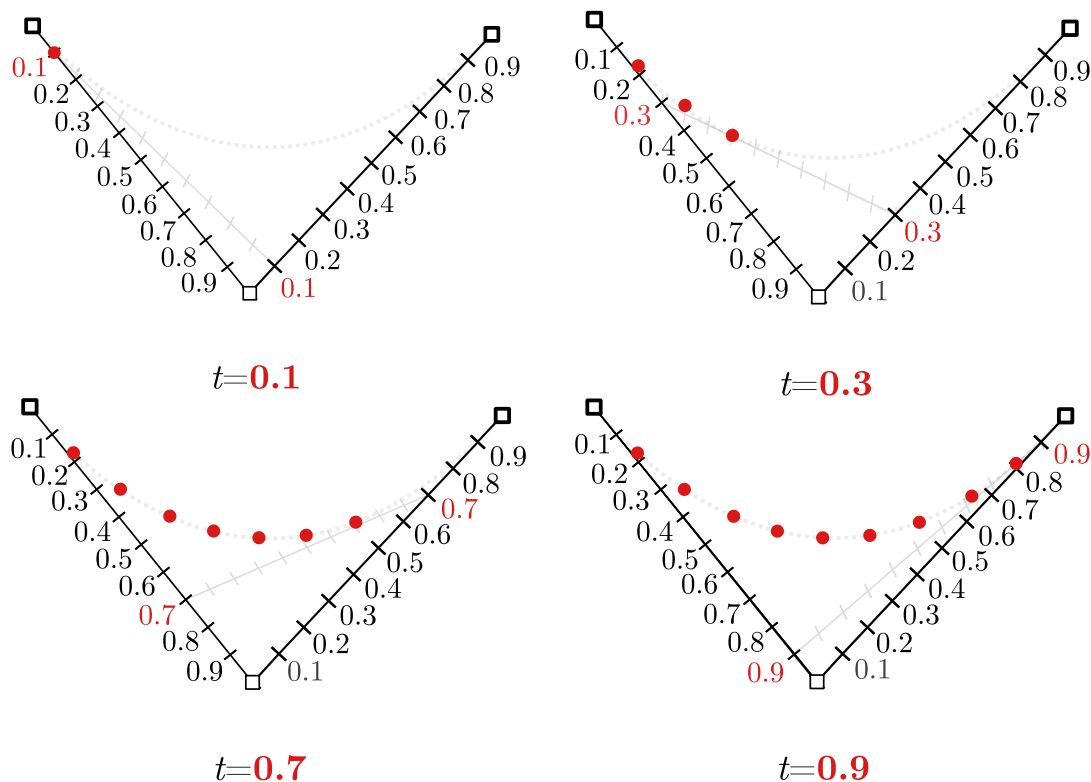


curves

## 30.1 Quadratic Bézier curves

### 30.1.1 Drawing the quadratic

To actually draw a curve, i.e. with points  $P_1, P_2, P_3$  we will use *de Casteljau's algorithm*. The gist behind the algorithm is that the length of the curve is visited at specific percentages (e.g. 0%, 0.2%, 0.4% ... 99.8%, 100%), meaning we will have that many steps, and for each such percentage  $t$  we calculate a line starting at the  $t$ -nth point of  $P_1P_2$  and ending at the  $t$ -nth point of  $P_2P_3$ . The  $t$ -eth point of that line also belongs to the curve, so we plot it.



Computing curve points for values of  $t \in [0, 1]$  with de Casteljau's algorithm

Let's draw the curve  $P_1 = (25, 115), P_2 = (225, 180), P_3 = (250, 25)$

src/bin/bezier.rs:

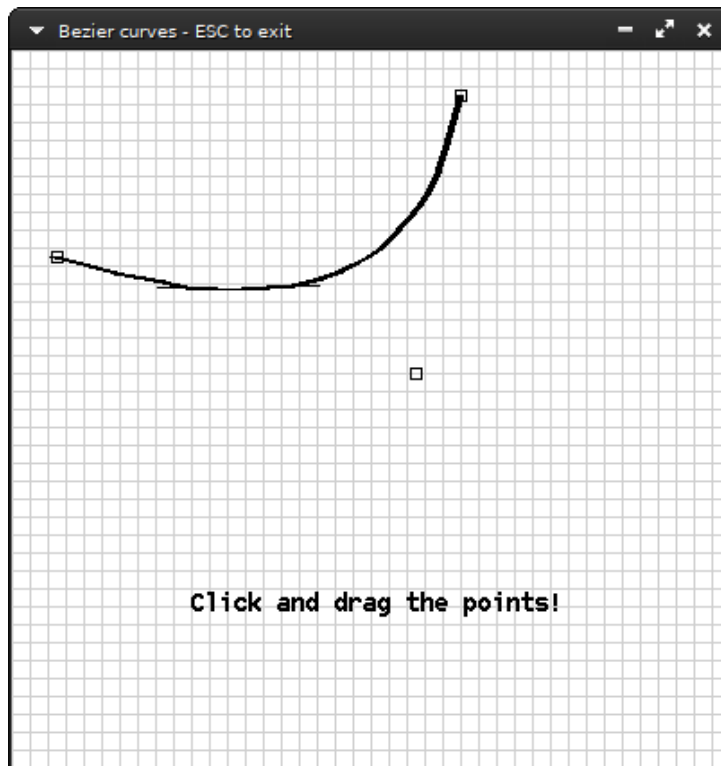


This code file is a PDF attachment

The result:



The `minifb` library allows to track user input, so we detect user clicks and the mouse's position; thus we can interactively modify a curve with some modifications in the code:



Interactively modifying a curve with the `bezier.rs` tool.

We can go one step further and insult type designers\* and use the tool to make a font glyph.

`src/bin/bezier`

curves

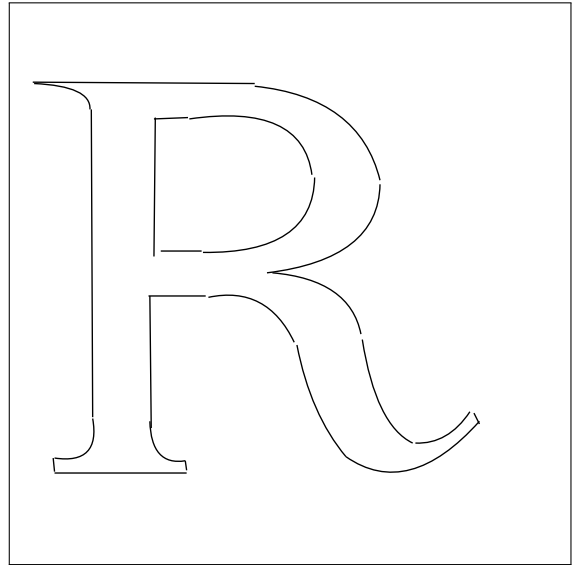
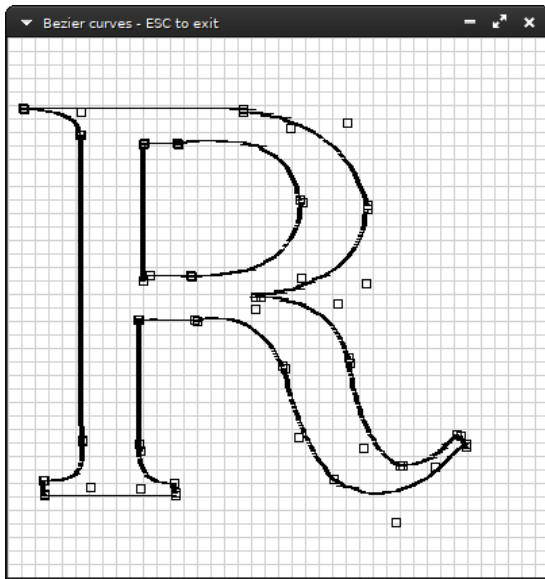


This code file is a PDF attachment

Of course, it requires effort to match the beginning and end of each curve that makes up the glyph. That's why font designing tools have *point snapping* to ensure curve continuation. But for a quick font designer app prototype, it's good enough.

---

\*who use cubic Béziers or other fancier curves (*splines*)



*Left:* A font glyph drawn with the interactive `bezieryglyph.rs` tool. *Right:* the same glyph exported to SVG.

## 30.2 Cubic Bézier curves



## 30.3 Weighted Béziers



## **Part VII**

# **Vectors, matrices and transformations**



## Chapter 31

# Rotation of a bitmap

$$p' = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

$$c = \cos \theta, s = \sin \theta, x_{p'} = x_p c - y_p s, y_{p'} = x_p s + y_p c.$$

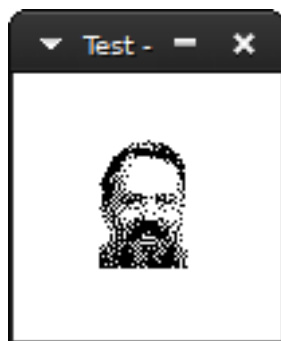
Let's load an xface. We will use `bits_to_bytes` (See *Bits to byte pixels*, page 14).

```
include!("dmr.rs");
const WINDOW_WIDTH: usize = 100;
const WINDOW_HEIGHT: usize = 100;
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

src/bin/rotation.rs:



This code file is a PDF attachment



This is the xface of dmr. Instead of displaying the bitmap, this time we will rotate it 0.5 radians. Setup our image first:

```
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.draw_outline();
let dmr = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

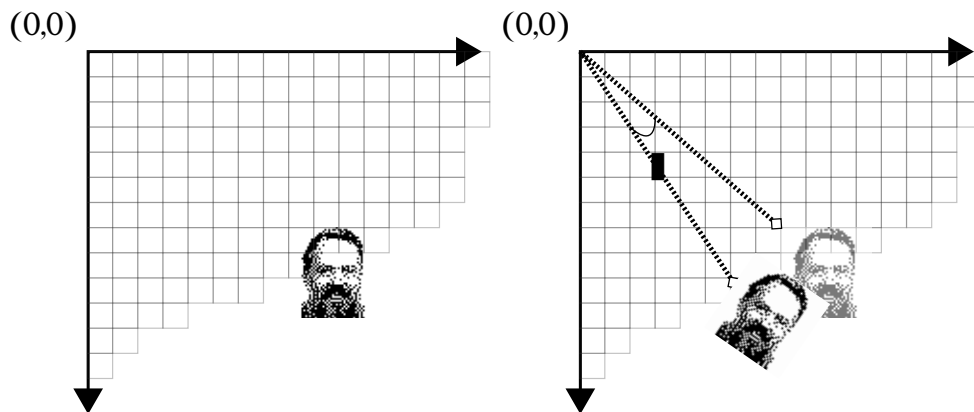
And then, loop for each byte in dmr's face and apply the rotation transformation.

```
let angle = 0.5;
let c = f64::cos(angle);
let s = f64::sin(angle);
for y in 0..DMR_HEIGHT {
    for x in 0..DMR_WIDTH {
        if dmr[y * DMR_WIDTH + x] == BLACK {
            let x = x as f64;
            let y = y as f64;
            let xr = x * c - y * s;
            let yr = x * s + y * c;
            image.plot(xr as i64, yr as i64);
        }
    }
}
```

The result:



We didn't mention in the beginning that the rotation has to be relative to a *point* and the given transformation is relative to the *origin*, in this case the upper left corner (0,0). So dmr was rotated relative to the origin:



(the distance to the origin (actually 0 pixels) has been exaggerated for the sake of the example)

Usually, we want to rotate something relative to itself. The right point to choose is the *centroid* of the object.

If we have a list of  $n$  points, the centroid is calculated as:

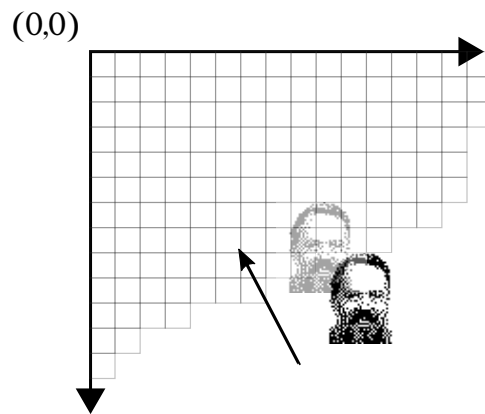
$$x_c = \frac{1}{n} \sum_{i=0}^n x_i$$

$$y_c = \frac{1}{n} \sum_{i=0}^n y_i$$

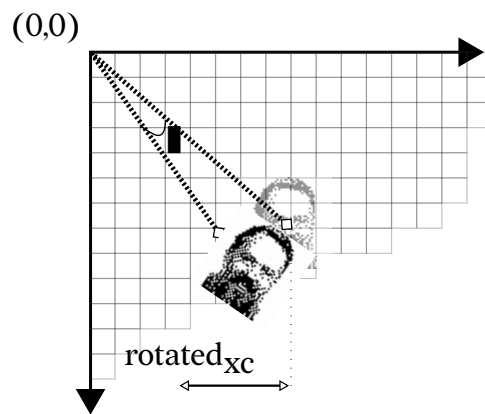
Since in this case we have a rectangle, the centroid has coordinates of half the width and half the height.

By subtracting the centroid from each point before we apply the transformation and then adding it back after we get what we want:

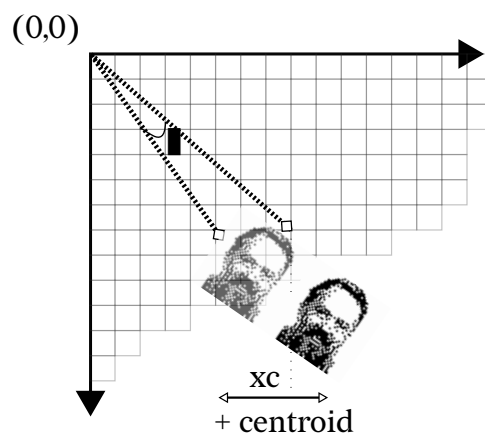
Here's it visually: First subtract the center point.



Then, rotate.



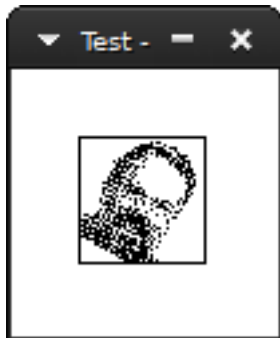
And subtract back to the original position.



trans-  
forma-  
tions

In code:

```
let center_point = ((DMR_WIDTH/2) as i64, (DMR_HEIGHT/2) as i64);
for y in 0..DMR_HEIGHT {
  for x in 0..DMR_WIDTH {
    if dmr[y * DMR_WIDTH + x] == BLACK {
      let x = (x as i64 - center_point.0) as f64;
      let y = (y as i64 - center_point.1) as f64;
      let xr = x * c - y * s;
      let yr = x * s + y * c;
      image.plot(xr as i64 + center_point.0,
                 yr as i64 + center_point.1);
    }
  }
}
```



The result:

## 31.1 Fast 2D Rotation

Add Fast 2D Rotation



trans-  
forma-  
tions

## Chapter 32

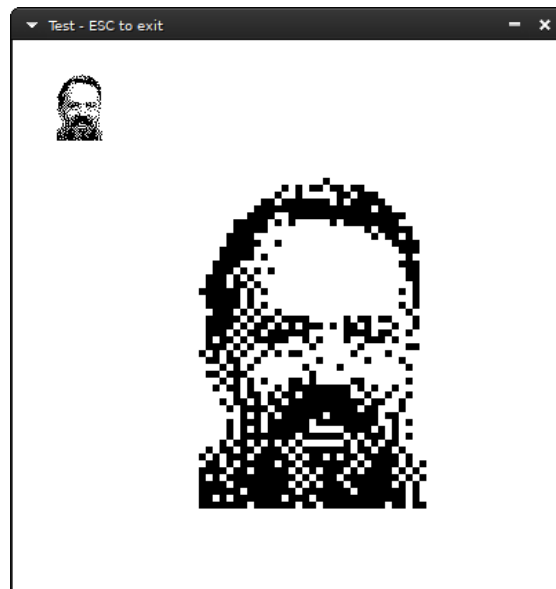
# 90° Rotation of a bitmap by parallel recursive subdivision

Add 90° Rotation of a bitmap by parallel recursive subdivision



## Chapter 33

# Magnification/Scaling



We want to magnify a bitmap without any smoothing. We define an Image scaled to the dimensions we want, and loop for every pixel in the scaled Image. Then, for each pixel, calculate its source in the original bitmap: if the coordinates in the scaled bitmap are  $(x, y)$  then the source coordinates  $(sx, sy)$  are:

$$sx = \frac{x * original.width}{scaled.width}$$
$$sy = \frac{y * original.height}{scaled.height}$$

So, if  $(sx, sy)$  are painted, then  $(x, y)$  must be painted as well.

src/bin/scale.rs:



This code file is a PDF attachment

```
let mut original = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
original.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
original.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

let mut scaled = Image::new(DMR_WIDTH * 5, DMR_HEIGHT * 5, 100, 100);
let mut sx: i64; //source
let mut sy: i64; //source
let mut dx: i64; //destination
let mut dy: i64 = 0; //destination

let og_height = original.height as i64;
let og_width = original.width as i64;
let scaled_height = scaled.height as i64;
let scaled_width = scaled.width as i64;

while dy < scaled_height {
    sy = (dy * og_height) / scaled_height;
    dx = 0;
    while dx < scaled_width {
        sx = (dx * og_width) / scaled_width;
        if original.get(sx, sy) == Some(BLACK) {
            scaled.plot(dx, dy);
        }
        dx += 1;
    }
    dy += 1;
}
scaled.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
```

## 33.1 Smoothing enlarged bitmaps

Add Smoothing enlarged bitmaps



## 33.2 Stretching lines of bitmaps

Add Stretching lines of bitmaps







## Chapter 34

# Mirroring

Add screenshots and figure and code in *Mirroring*

Mirroring to an axis is the transformation of one coordinate to its equidistant value across the axis:

To mirror a pixel across the  $x$  axis, simply multiply its coordinates with the following matrix:

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

This results in the  $y$  coordinate's sign being flipped.

For  $y$ -mirroring, the transformation follows the same logic:

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

## Chapter 35

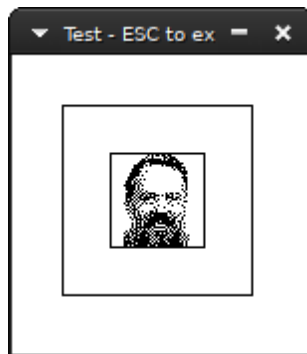
# Shearing

Simple shearing is the transformation of one dimension by a distance proportional to the other dimension. In  $x$ -shearing (or horizontal shearing) only the  $x$  coordinate is affected, and likewise in  $y$ -shearing only  $y$  as well.

src/bin/shearing.rs:



This code file is a PDF attachment

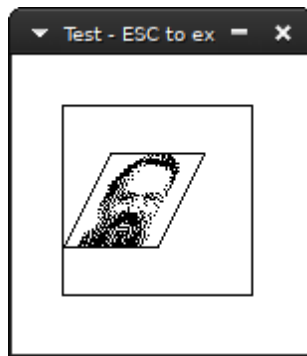


With  $l$  being equal to the desired tilt away from the  $y$  axis, the transformation is described by the following matrix:

$$S_x = \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}$$

Which is as simple as this function:

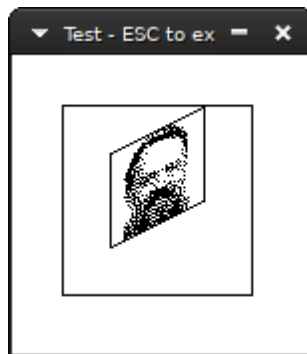
```
fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {  
    (x_p + (l * (y_p as f64)) as i64, y_p)  
}
```



For  $y$ -shearing, we have the following:

$$S_y = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix}$$

```
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}
```



A full example:

```
include!("../dmr.xbm.rs");
const WINDOW_WIDTH: usize = 200;
const WINDOW_HEIGHT: usize = 200;

fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p+(l*(y_p as f64)) as i64, y_p)
}
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}

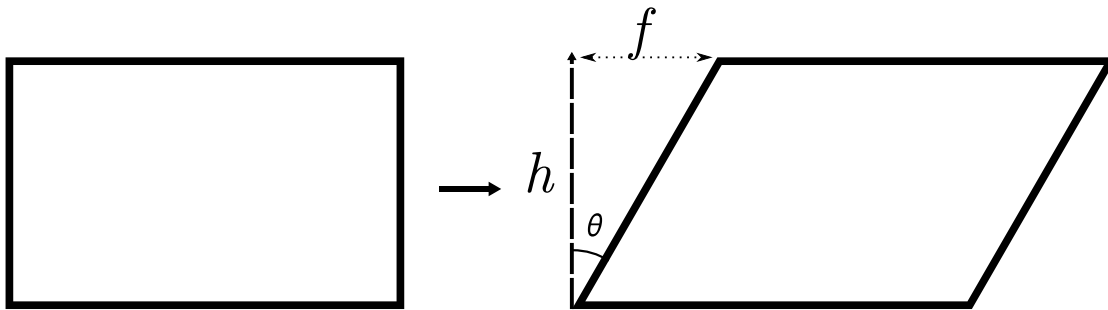
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
image.draw_outline();
```

```

let l = -0.5;
let mut sheared = Image::new(DMR_WIDTH*2, DMR_HEIGHT*2, 25, 25);
for x in 0..DMR_WIDTH {
    for y in 0..DMR_HEIGHT {
        if image.bytes[y * DMR_WIDTH + x] == BLACK {
            let p = shear_x((x as i64 ,y as i64 ), l);
            sheared.plot(p.0+(DMR_WIDTH/2) as i64, p.1+(DMR_HEIGHT/2) as i64);
        }
    }
}
sheared.draw_outline();

```

## 35.1 The relationship between shearing factor and angle



Shearing is a delta movement in one dimension, thus the point before moving and the point after form an angle with the  $x$  axis. To move a point  $(x, 0)$  by  $30^\circ$  forward we will have the new point  $(x + f, 0)$  where  $f$  is the shear factor. These two points and  $(x, h)$  where  $h$  is the height of the bitmap form a triangle, thus the following are true:

$$\cot \theta = \frac{h}{f}$$

Therefore to find your factor for any angle  $\theta$  replace its cotangent in the following formula:

$$f = \frac{h}{\cot \theta}$$

For example to shear by  $-30^\circ$  (meaning the bitmap will move to the right, since rotations are always clockwise) we need  $\cot(-30deg) = -\sqrt{3}$  and  $f = -\frac{h}{\sqrt{3}}$ .

## Chapter 36

# Projections

Add Projections


# **Part VIII**

## **Addendum**

adden-  
dum

## Chapter 37

# Faster drawing a line segment from its two endpoints using symmetry

Add *Faster drawing a line segment from its two endpoints using symmetry*

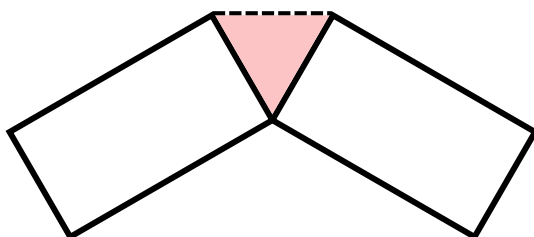




## Chapter 38

# Joining the ends of two wide line segments together

Add *Joining the ends of two wide line segments together*



A series of horizontal gray bars, likely representing a stack of paper or a timeline, used for additional content or notes.

## Chapter 39

# Composing monochrome bitmaps with separate alpha channel data

Add *Composing monochrome bitmaps with separate alpha channel data*



## Chapter 40

# Orthogonal connection of two points

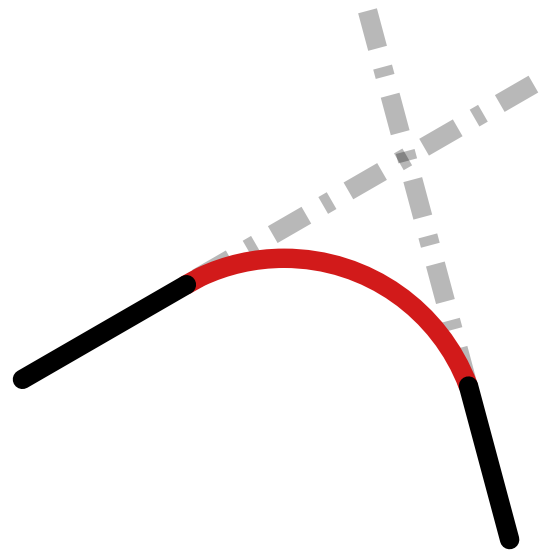
Add *Orthogonal connection of two points*



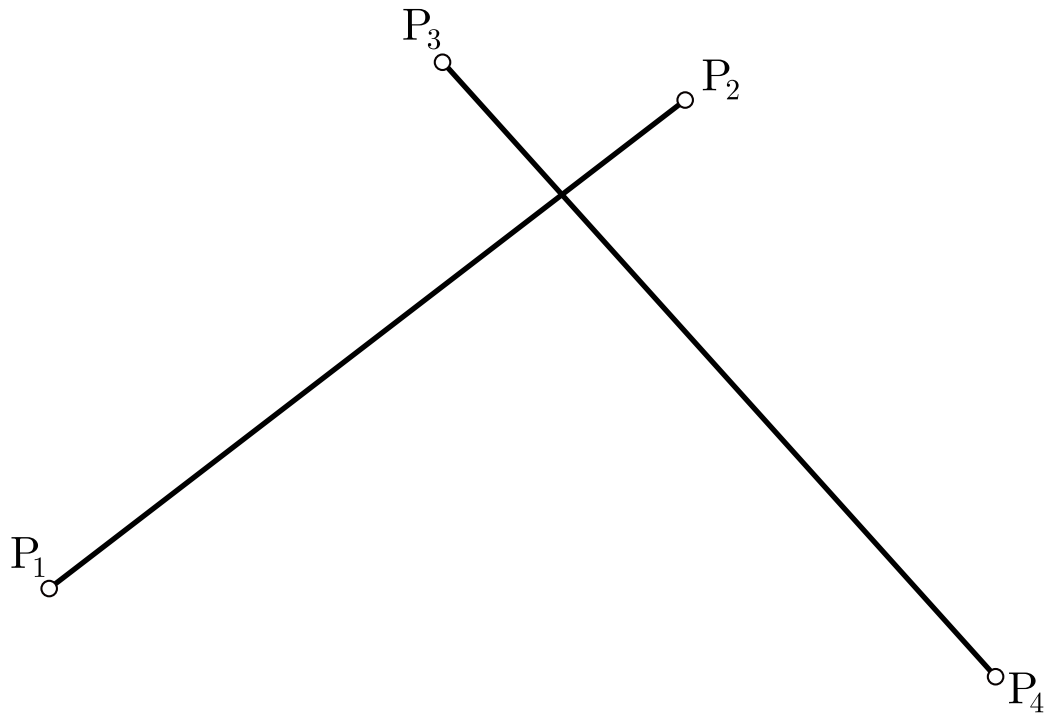
## Chapter 41

# Join segments with round corners

Round corners are everywhere around us. It is useful to know at least one method of construction. This specific method constructs a circle that has a common point with each given line segment, and calculates the arc that when added to the line segments they are smoothly joined. The excess length, since those common points will be before the end of the line segments, must be erased. Therefore, it's best to begin with just the points of the two segments before starting to draw anything.



Since the segments intercept, the round corner will end up beneath the intersection. We wish to find a circle that has a common point with each segment and the arc made up from those points and the circle is the round corner we are after.



We are given 4 points,  $P_1, P_2$  and  $P_3, P_4$  that make up segments  $S_1$  and  $S_2$ . Begin by finding the midpoints  $m_1$  and  $m_2$  of segments  $S_1$  and  $S_2$ . These will be:

$$m_1 = \frac{P_1 + P_2}{2}$$

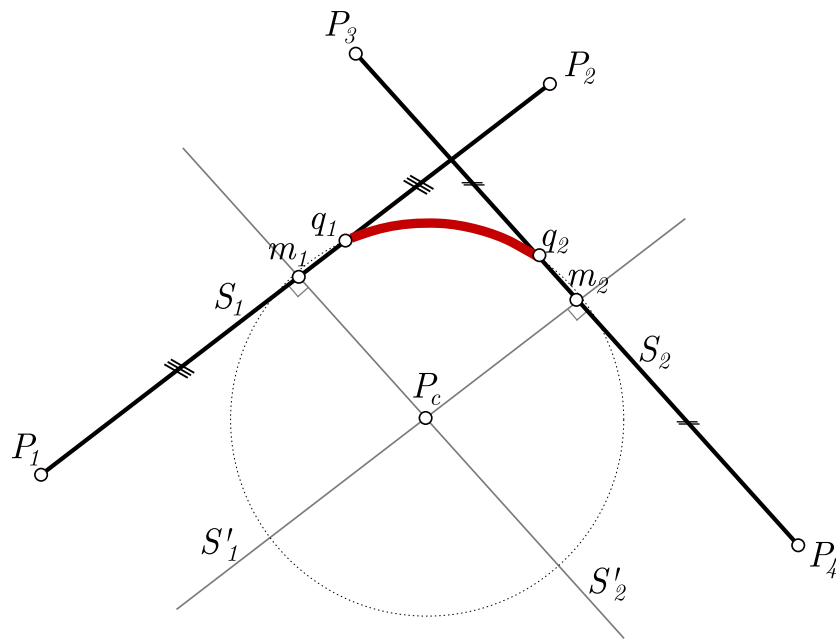
$$m_2 = \frac{P_3 + P_4}{2}$$

Then, find the signed distances (i.e. don't use the absolute value of distance)  $d_1$  of  $m_1$  from  $S_2$  and  $d_2$  of  $m_2$  from  $S_1$ .

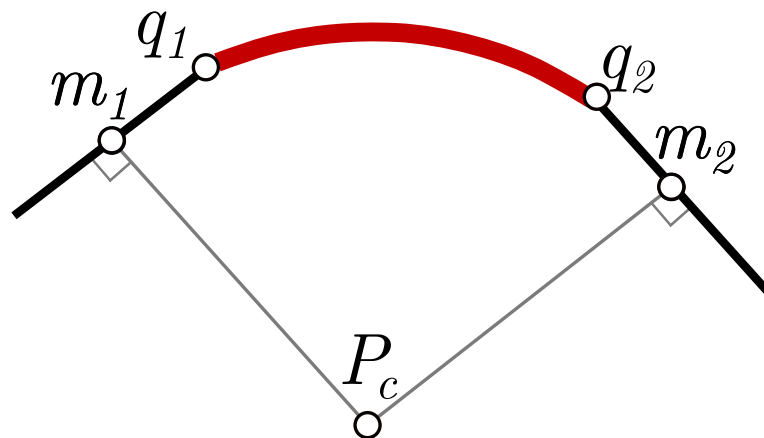
Construct parallel lines  $l_1$  to  $S_1$  that is  $d_1$  pixels away. Repeat with  $l_2$  for  $S_2$  and  $d_2$ .

Their intersection is the circle's center,  $P_c$ .

The intersection of  $l_1, l_2$  with the two segments are the points where we should clip or extend the segments:  $q_1$  and  $q_2$ .



The starting angle is found by calculating the angle of  $q_1P_c$  with the  $x$ -axis with the `atan2` math library procedure.



addendum

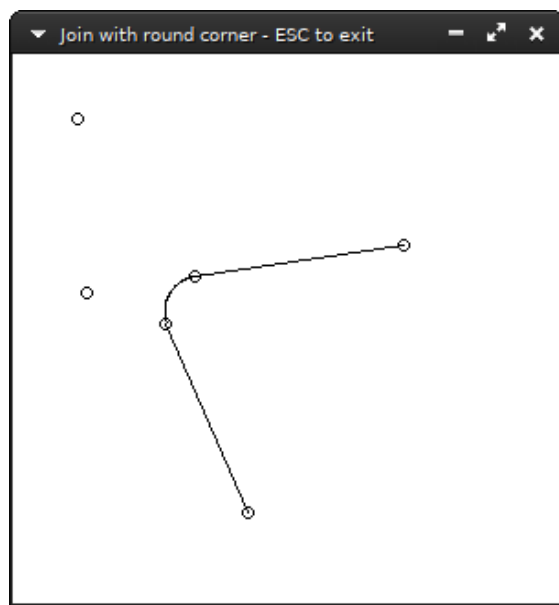
The *subtended* angle\* of the arc from the center  $P_c$  is found by calculating the dot product of  $q_1P_c$  and  $q_2P_c$ :

The code:

src/bin/roundcorner.rs:



This code file is a PDF attachment

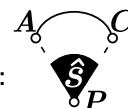


The src/bin/roundcorner.rs example has two interactive lines and computes the joining fillet.

addendum

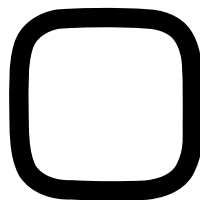
---

\*the *subtended* angle of an arc  $\widehat{AC}$  to a point  $P$  is the angle between  $PA$  and  $PC$ :



## Chapter 42

# Squircle



src/bin/squircle.rs:



This code file is a PDF  
attachment

A *squircle* is a compromise between a square and a circle. It is purported to be more pleasing to the eye because the rounding corner is smoother than that of a circle arc (like the result of *Join segments with round corners*, page 92).

A way to describe a squircle is as a superellipse, meaning a generalization of the ellipse equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  by making the exponent parametric:

$$|x - a|^n + |y - b|^n = 1$$

The squircle as a superellipse is usually defined for  $n = 4$ .

## The code

```
pub fn plot_squircle(
    image: &mut Image,
    (xm, ym): (i64, i64),
    width: i64,
    height: i64,
    n: i32,
    _wd: f64,
) {
    let r = width / 2;
    let w = width / 2;
    let h = height / 2;
```



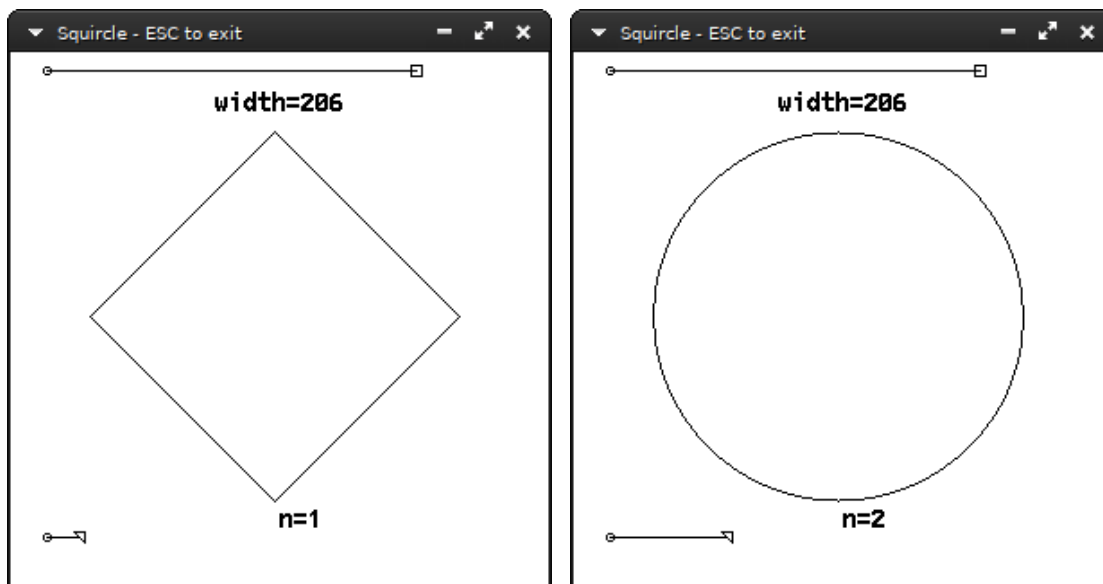
```

let mut prev_pos = (xm - w, xm - h);
for i in 0..(2 * r + 1) {
    let x: i64 = (i - r) + w;
    let y: i64 = ((r as f64).powi(n) - (i as f64 - r as f64).abs().powi(n)).powf(1. /
↪ n as f64)
        as i64
        + h;
    if i != 0 {
        image.plot_line_width(prev_pos, (xm - x as i64, ym - y), _wd);
    }
    prev_pos = (xm - x as i64, ym - y);
}
for i in (2 * r)..(4 * r + 1) {
    let x: i64 = (3 * r - i) + w;
    let y = -1
        * ((r as f64).powi(n) - ((3 * r - i) as f64).abs().powi(n)).powf(1. / n as
↪ f64))
        as i64
        + h;
    image.plot_line_width(prev_pos, (xm - x as i64, ym - y), _wd);
    prev_pos = (xm - x as i64, ym - y);
}
}

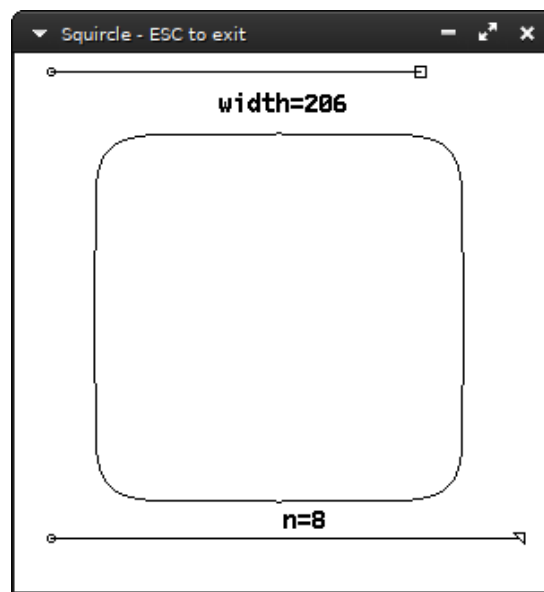
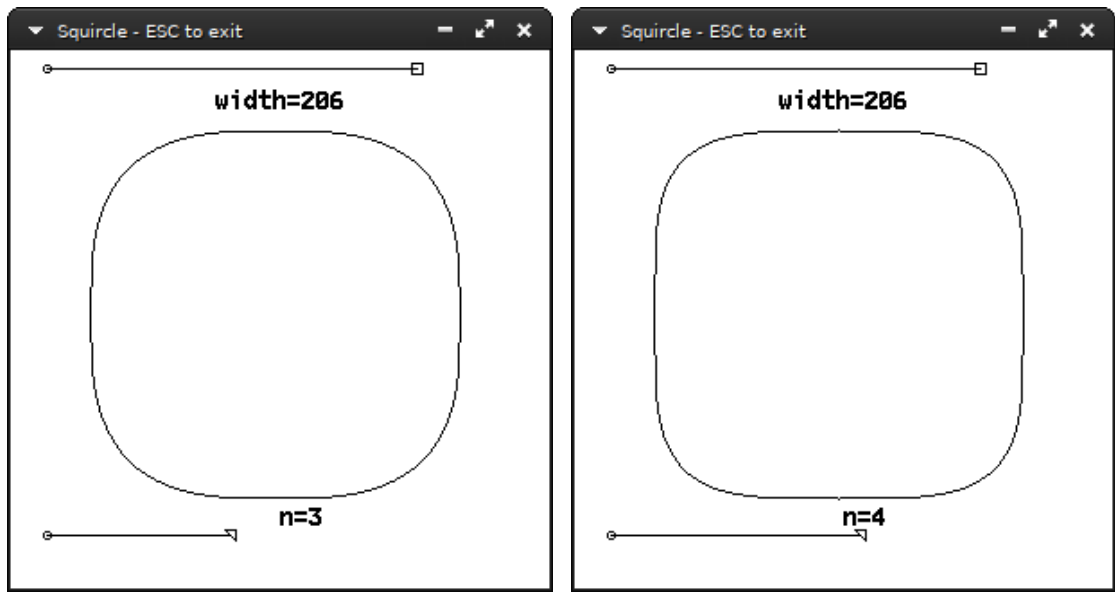
```

## Different values of $n$

Increasing  $n$  in `src/bin/squircle.rs` makes the hyperellipse corners approach the square's.



addendum



addendum

## Chapter 43

# Faster line clipping

Add *Faster line clipping*



## Chapter 44

# Tilings

Add Tilings

### 44.1 Hexagon Tiling

## Chapter 45

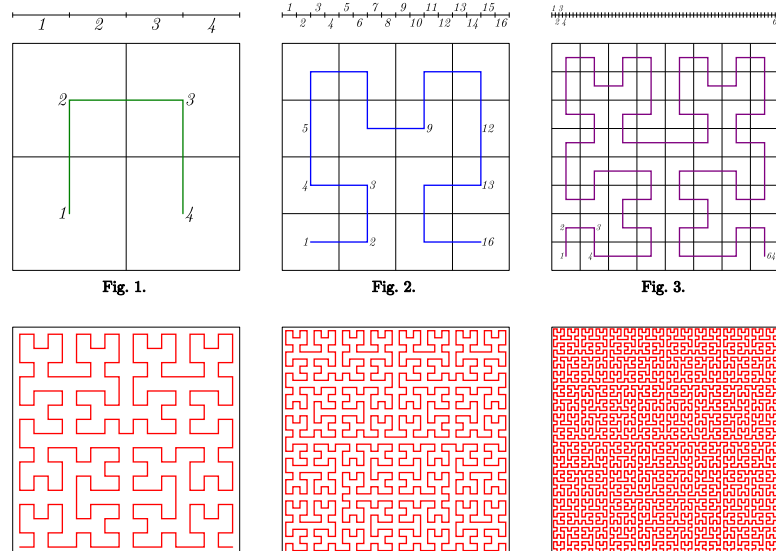
# Space-filling Curves

Add Space-filling Curves



## 45.1 Hilbert curve

Add Hilbert curve explanation



The first six iterations of the Hilbert curve by [Braindrain0000](#)

src/bin/hilbert.rs:



This code file is a PDF attachment

Here's a simple algorithm for drawing a Hilbert curve.\*

```
const HILBERT: &[[usize]] = &[
    &[22, 10, 16, 38],
    &[10, 22, 24, 48],
    &[44, 36, 30, 18],
    &[36, 44, 42, 28],
];

fn curve(img: &mut Image, k: usize, order: i64, mut x: i64, mut y: i64) -> (i64, i64) {
    const STEP_SIZE: i64 = 5;
    let mut row: usize;
    let mut direction: usize;
    if order > 0 {
        for j in 0..4 {
            let step = HILBERT[k][j];
            row = (step / 10) - 1;
            let (xn, yn) = curve(img, row, order - 1, x, y);
            x = xn;
            y = yn;
            direction = step % 10;
            let prev = (x, y);
            match direction {
                8 => {
                    // null op
                }
                2 => {
                    //N
                    y -= STEP_SIZE;
                }
                1 => {
```

\*Griffiths, J. G. (1985). *Table-driven algorithms for generating space-filling curves*. Computer-Aided Design, 17(1), 37-41. doi:10.1016/0010-4485(85)90009-0

```

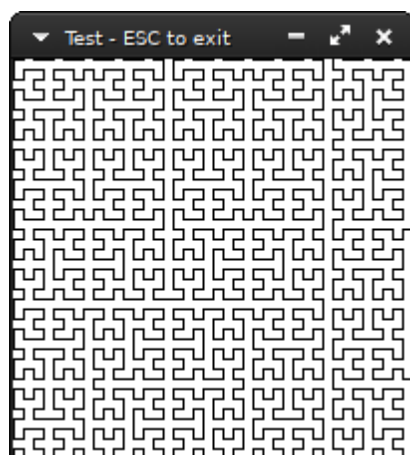
        // NE
        y -= STEP_SIZE;
        x += STEP_SIZE;
    }
    0 => {
        //E
        x += STEP_SIZE;
    }
    7 => {
        //SE
        x += STEP_SIZE;
        y += STEP_SIZE;
    }
    6 => {
        //S
        y += STEP_SIZE;
    }
    5 => {
        //SW
        y += STEP_SIZE;
        x -= STEP_SIZE;
    }
    4 => {
        //W
        x -= STEP_SIZE;
    }
    3 => {
        //NW
        y -= STEP_SIZE;
        x -= STEP_SIZE;
    }
    other => unreachable!("{}", other),
}
img.plot_line_width(prev, (x, y), 0.);
}
}
(x, y)
}

```

```

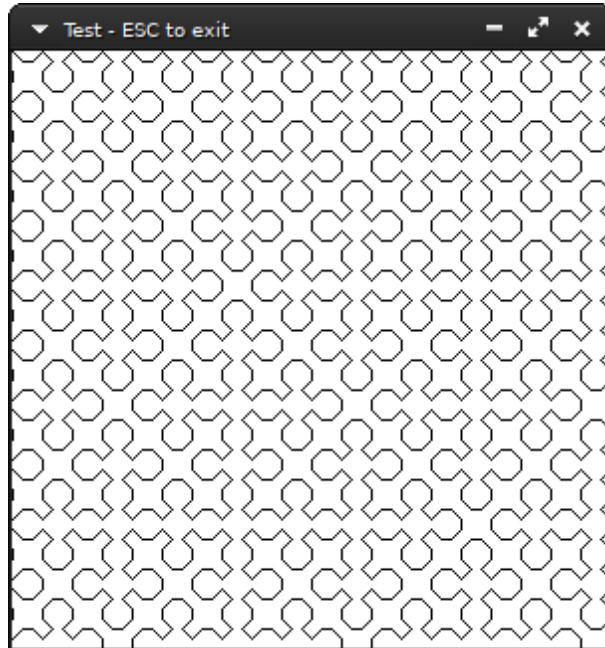
let mut image = Image::new(WINDOW_WIDTH, WINDOW_WIDTH, 0, 0);
curve(&mut image, 0, 7, 0, WINDOW_WIDTH as i64);

```



addendum

## 45.2 Sierpiński curve



Switching the table from the Hilbert implementation to this:

```
const SIERP: &[[usize]] = &[
  &[17, 25, 33, 41],
  &[17, 20, 41, 18],
  &[25, 36, 17, 28],
  &[33, 44, 25, 38],
  &[41, 12, 33, 48],
];
```

And switching two lines from the function to

```
- let step = HILBERT[k][j];
- row = (step / 10) - 1;
+ let step = SIERP[k][j];
+ row = (step / 10);
```

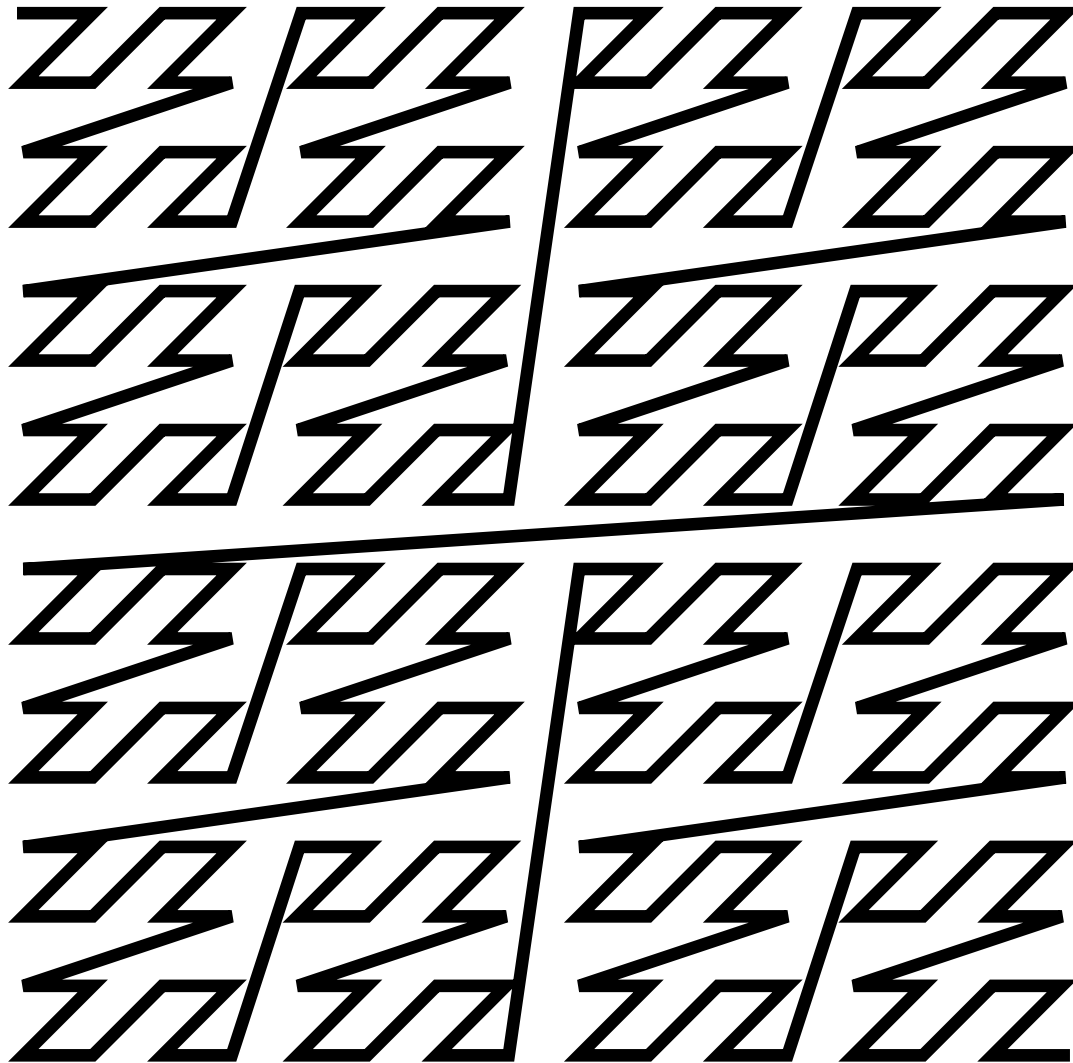
You can draw a Sierpinshi curve of order  $n$  by calling `curve(&mut image, 0, n+1, 0, 0)`.

## 45.3 Peano curve

Add Peano curve



## 45.4 Z-order curve



Drawing the Z-order curve is really simple: first, have a counter variable that starts from zero and is incremented by one at each step. Then, you extract the  $(x,y)$  coordinates the new step represents from its binary representation. The bits for the  $x$  coordinate are located at the odd bits, and for  $y$  at the even bits. I.e. the values are interleaved as bits in the value of the step:

$$x = 0b110011 = 51$$

110110101111

$$y = 0b101111 = 47$$

Knowing this, implementing the drawing process will consist of computing the next step, drawing a line segment from the current step and the next, set the current step as the next and continue;

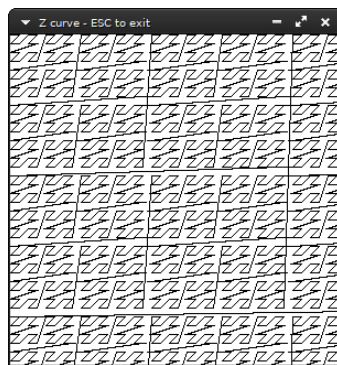
```
fn zcurve(img: &mut Image, x_offset: i64, y_offset: i64) {
    const STEP_SIZE: i64 = 8;
    let mut sx: i64 = 0;
    let mut sy: i64 = 0;
    let mut b: u64 = 0;
    let mut prev_pos = (sx + x_offset, sy + y_offset);
    loop {
        let next = b + 1;
        sx = 0;
        if (next & 1) as i64 > 0 {
            sx += STEP_SIZE;
        }
        if next & 0b100 > 0 {
            sx += 2 * STEP_SIZE;
        }
        if next & 0b10_000 > 0 {
            sx += 4 * STEP_SIZE;
        }
        if next & 0b1_000_000 > 0 {
            sx += 8 * STEP_SIZE;
        }
        if next & 0b100_000_000 > 0 {
            sx += 16 * STEP_SIZE;
        }
        if next & 0b10_000_000_000 > 0 {
            sx += 32 * STEP_SIZE;
        }
        if next & 0b1_000_000_000_000 > 0 {
            sx += 64 * STEP_SIZE;
        }
        if next & 0b100_000_000_000_000 > 0 {
            sx += 128 * STEP_SIZE;
        }
    }
}
```

addendum

```

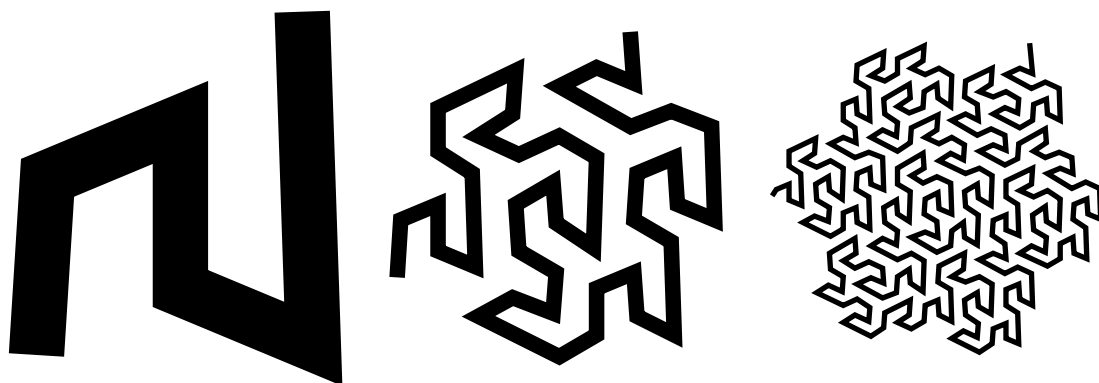
    }
    if next & 0b10_000_000_000_000_000 > 0 {
        sx += 256 * STEP_SIZE;
    }
    if next & 0b1_000_000_000_000_000_000 > 0 {
        sx += 512 * STEP_SIZE;
    }
    sy = 0;
    if (next & 0b10) as i64 > 0 {
        sy += STEP_SIZE;
    }
    if next & 0b1_000 > 0 {
        sy += 2 * STEP_SIZE;
    }
    if next & 0b100_000 > 0 {
        sy += 4 * STEP_SIZE;
    }
    if next & 0b10_000_000 > 0 {
        sy += 8 * STEP_SIZE;
    }
    if next & 0b1_000_000_000 > 0 {
        sy += 16 * STEP_SIZE;
    }
    if next & 0b100_000_000_000 > 0 {
        sy += 32 * STEP_SIZE;
    }
    if next & 0b10_000_000_000_000 > 0 {
        sy += 64 * STEP_SIZE;
    }
    if next & 0b1_000_000_000_000_000 > 0 {
        sy += 128 * STEP_SIZE;
    }
    if next & 0b100_000_000_000_000_000 > 0 {
        sy += 256 * STEP_SIZE;
    }
    if next & 0b10_000_000_000_000_000_000 > 0 {
        sy += 512 * STEP_SIZE;
    }
    img.plot_line_width(prev_pos, (sx + x_offset, sy + y_offset), 1.0);
    if next == 0b111_111_111_111_111_111_111 {
        break;
    }
    if sx as usize > img.width && sy as usize > img.height {
        break;
    }
    prev_pos = (sx + x_offset, sy + y_offset);
    b = next;
}
}

```



addendum

## 45.5 Flowsnake curve



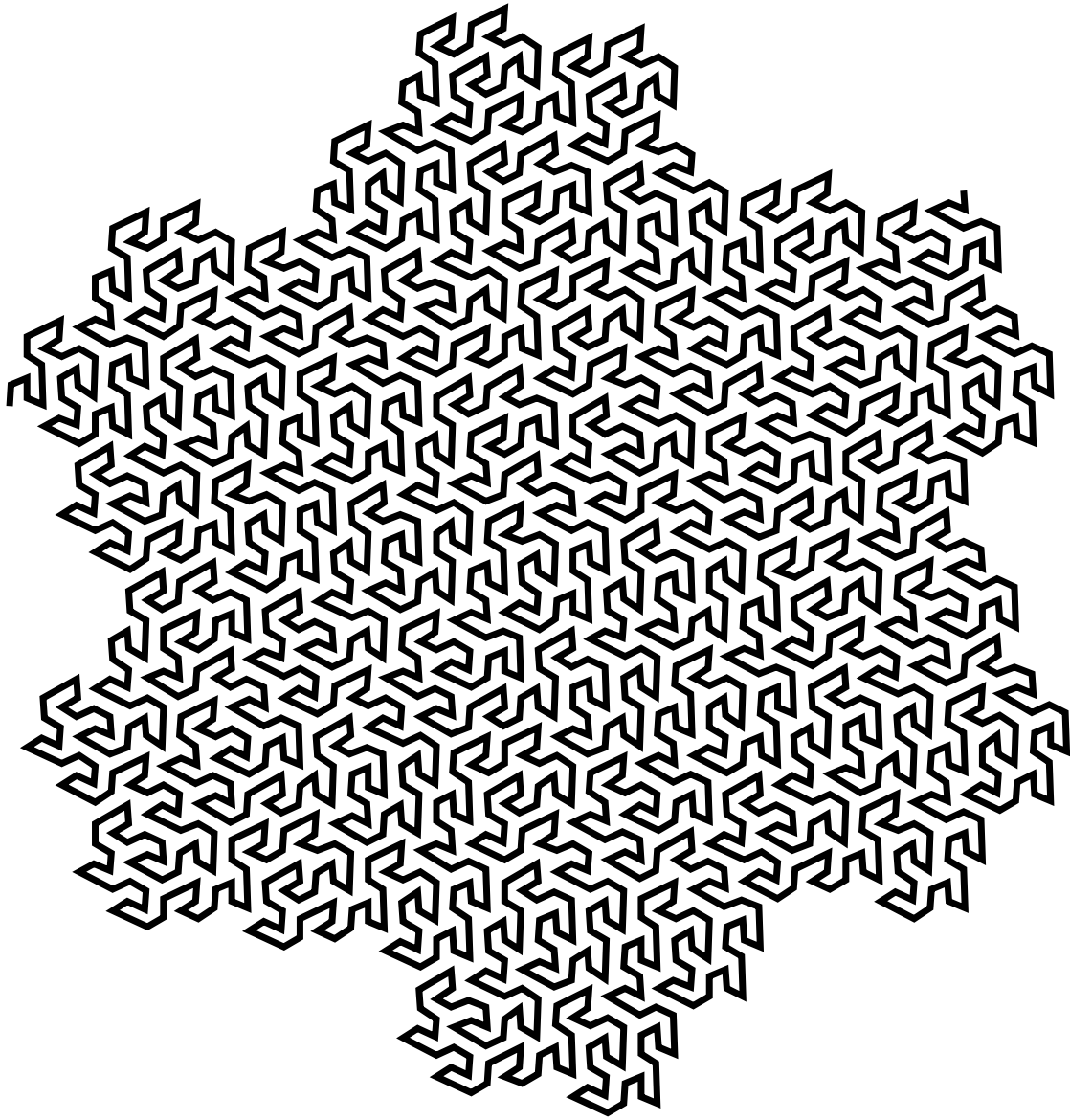
The first three orders of the Gosper curve.

As a fractal curve, the *flowsnake curve* or *Gosper curve* is defined by a set of recursive rules for drawing it. There are four kind of rules and two of them define rulesets (i.e. they are non-terminal steps).

$$A \mapsto A-B--B+A++AA+B-$$

$$B \mapsto +A-BB--B-A++A+B$$



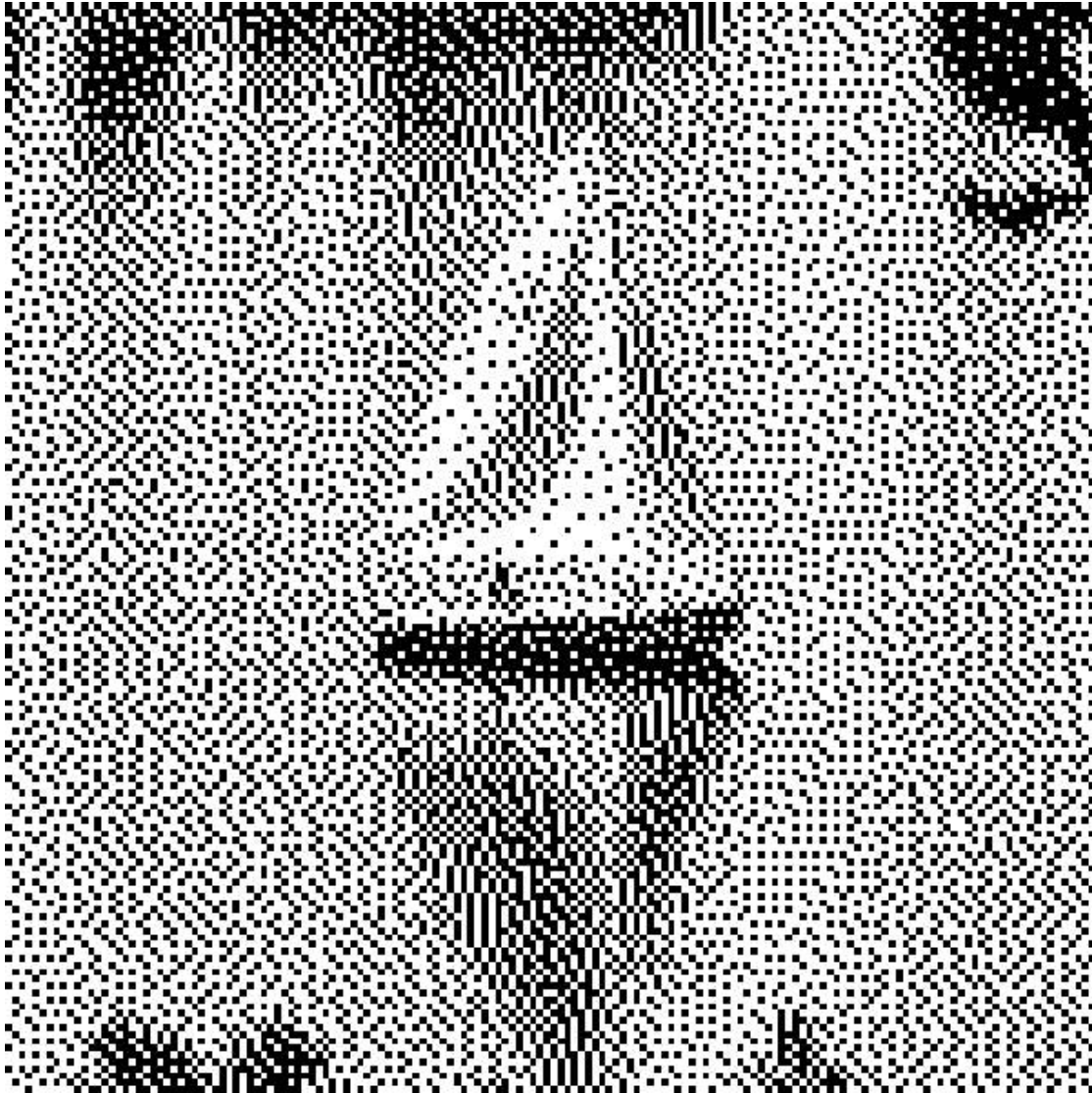


The fourth order Gosper curve consists of a minimum of 2057 distinct line segments (but our algorithm draws 36015)

## **Chapter 46**

# **Dithering**

## 46.1 Floyd-Steinberg



detail of a standard test image, [\*Sailboat on lake\*](#), with Floyd-Steinberg dithering

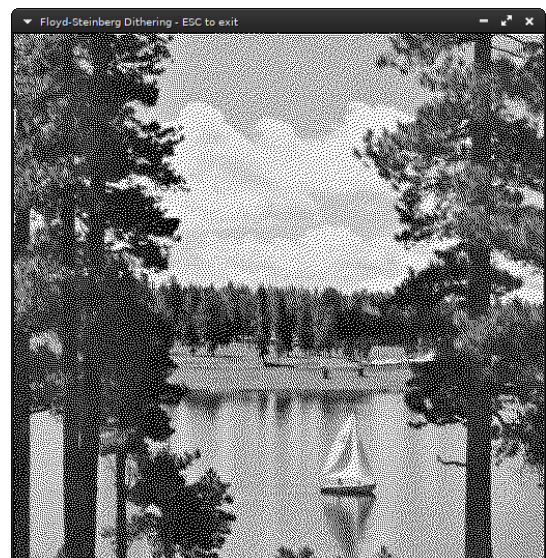
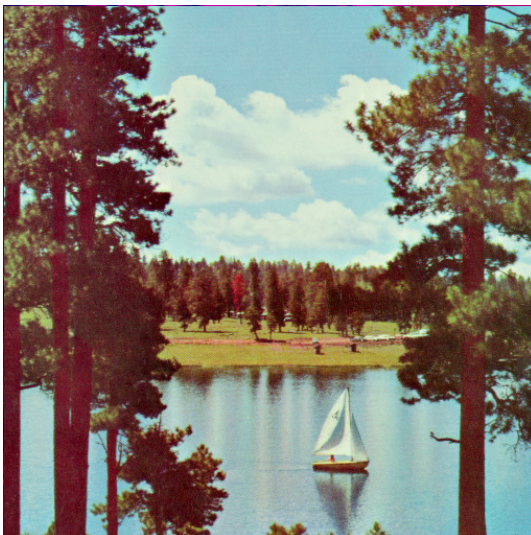


src/bin/floydither.rs:



This code file is a PDF attachment

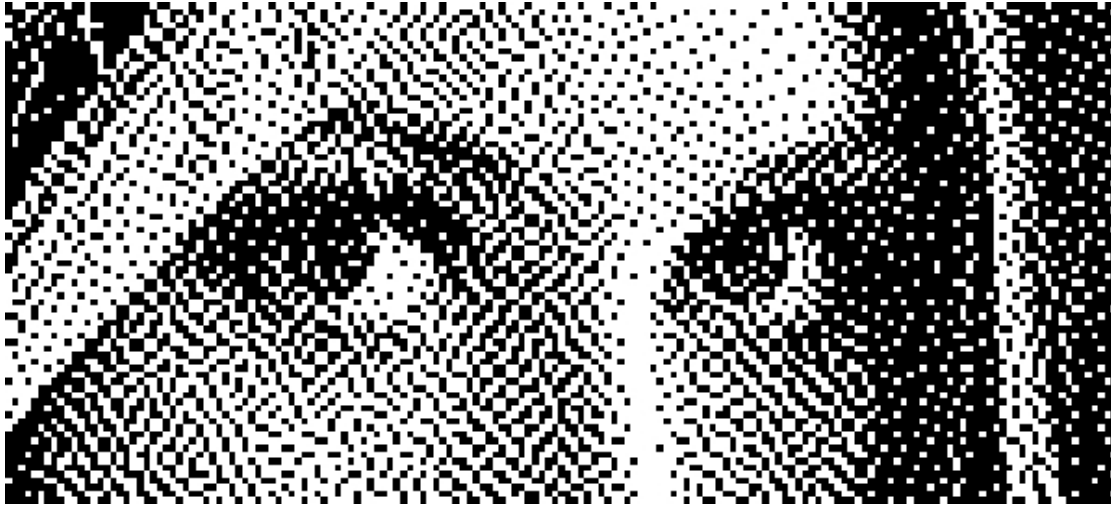
```
fn floyd(image: &mut Image) {
    let w = image.width;
    let m = [(0, 7), (w - 2, 3), (w - 1, 5), (w, 1)];
    let mut e = vec![0.0; w + 1];
    let bytes = image
        .bytes
        .iter()
        .map(|&byte| {
            let (r, g, b) = from_u32_rgb(byte);
            let g: f64 = (0.299 * (r as f64)) + (0.587 * (g as f64)) + (0.114 * (b as
↪ f64));
            let pix = g / 255.0 + {
                e.push(0.);
                e.remove(0)
            };
            let col = if pix > 0.5 { 1. } else { 0. };
            let err = (pix - col) / 16.;
            for (x, y) in m.iter() {
                e[*x] += err * (*y as f64);
            }
            if col.floor() as u32 == 1 {
                WHITE
            } else {
                BLACK
            }
        })
        .collect::<Vec<u32>>();
    image.bytes = bytes;
}
```



addendum



## 46.2 Atkinson dithering



detail of a standard test image, *Lenna*, with Atkinson dithering



The following code implements Atkinson dithering:\*

```
fn atkinson(image: &mut Image) {
    let w= image.width;
    let mut e = vec![0.0;2*w];
    let m = [0, 1, w-2, w-1, w, 2*w-1];
    for byte in image.bytes.iter_mut() {
        let (r,g,b) = from_u32_rgb(*byte);
        let g:f64 = ((0.299*(r as f64)) ) + ((0.587_f64*(g as f64)) ) + ((0.114*(b as
↪ f64)) );
        let pix = g/255.0 + { e.push(0.); e.remove(0)};
        let col = if pix > 0.5 { 1. } else { 0. };
        let err = (pix-col)/8.;
        for m in m.iter() {
            e[*m] += err;
        }
        *byte = if (col.floor() as u32 == 1) {
            WHITE
        }
```

src/bin/atkinsondither.rs:

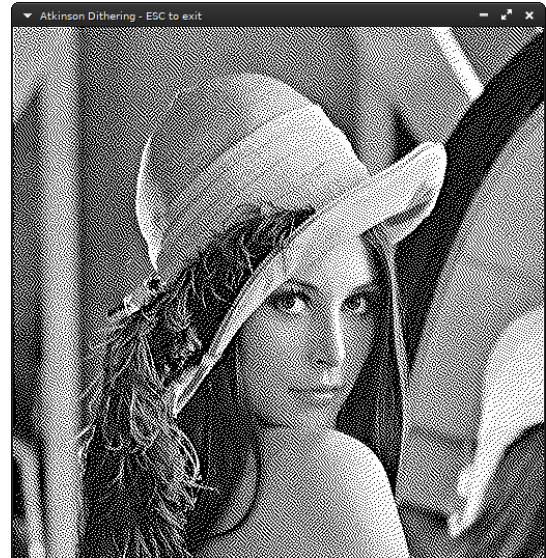


This code file is a PDF  
attachment

addendum

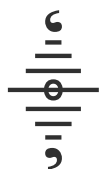
\*Algorithm taken from <https://beyondloom.com/blog/dither.html>

```
    } else {  
      BLACK  
    };  
  }  
}
```



## Chapter 47

# Marching squares





# Index

- alpha channel, 90
- angle
  - between two lines, 30
  - bisectioning, 37
  - trisectioning, 37
- area filling, *see* flood filling
- Atkinson dithering, 113
- bucket filling, *see* flood filling
- centroid
  - polygon, 58
  - rectangle, 75
- circle
  - bounding, 49
  - equations, 48
  - out of three points, 50
  - out of two points, 50
- contour, *see* marching squares
- curves
  - Bézier, 66
    - cubic, 71
    - quadratic, 67
    - weighted, 71
  - elliptical, 64
  - Flowsnake curve, 108
  - Hilbert curve, 102
  - Peano curve, 104
  - space-filling, 101
- de Casteljau's algorithm, 67
- distance
  - between two points, 20
  - moving a point, 21
  - point from a line, 26
- dithering, 110
  - Atkinson, 113
  - Floyd-Steinberg, 111
- equidistant line, 34
- flood filling, 62
  - triangle filling, 60
- Flowsnake curve, 108
- Floyd-Steinberg dithering, 111
- Gosper curve, *see* Flowsnake curve
- Hilbert curve, 102
- line
  - drawing, 25
  - equations, 22
  - equidistant, 34
  - intersection, 32
  - perpendicular, 28
  - through point and slope, 22
  - through two points, 23
- magnification, 79
- marching squares, 115
- midpoint, 34, 93
- Peano curve, 104
- perpendicular, 28
- polygon
  - boolean operations, 57

- centroid, 58
- clipping, 59
- rotation, 73
- scaling, 79
- shearing, 83
- skewing, *see* shearing
- smoothing, 80
- stretching, 80
- triangle, 56
  - filling, 60
  - from point and angles, 56

# About this text

The text has been typeset in  $\text{\LaTeX}$  using the book class and:

- **Redaction** for the main text.
- **Fira Sans** for referring to the programming language **Rust**.
- **Redaction20** for referring to the words bitmap and pixels as a concept.

# Todo list

Add <i>Normal to a line through a point</i>	36
Add some explanation behind the algorithm in <i>Drawing a line segment from its two endpoints</i>	41
Add <i>Equations of a circle</i>	48
Add <i>Union, intersection and difference of polygons</i>	57
Add <i>Centroid of polygon</i>	58
Add <i>Triangle filling</i> explanation	60
Add <i>Flood filling</i>	62
Add <i>Fast 2D Rotation</i>	77
Add <i>90° Rotation of a bitmap by parallel recursive subdivision</i>	78
Add <i>Smoothing enlarged bitmaps</i>	80
Add <i>Stretching lines of bitmaps</i>	80
Add screenshots and figure and code in <i>Mirroring</i>	82
Add <i>Projections</i>	86
Add <i>Faster drawing a line segment from its two endpoints using symmetry</i>	88
Add <i>Joining the ends of two wide line segments together</i>	89
Add <i>Composing monochrome bitmaps with separate alpha channel data</i>	90
Add <i>Orthogonal connection of two points</i>	91
Add <i>Faster line clipping</i>	99
Add <i>Tilings</i>	100
Add <i>Space-filling Curves</i>	101
Add <i>Hilbert curve</i> explanation	102
Add <i>Peano curve</i>	104