
A Bitmapper's Companion

epilys

November 29, 2021

an introduction
to basic bitmap
mathematics
and algorithms
with code
samples in **Rust**



Table Of Contents	4	toc
Introduction	7	intro
Points And Lines	20	lines
Points and Line Segments	38	segments
Points, Lines and Circles	47	circles
Curves other than circles	56	curves
Points, Lines and Shapes	60	shapes
Vectors, matrices and transformations	71	trans- forma- tions
Advanced	96	ad- vanced



Manos Pitsidianakis (epilys)

<https://nessuent.xyz>

<https://github.com/epilys>

epilys@nessuent.xyz

All non-screenshot figures were generated by hand in Inkscape unless otherwise stated.

The skull in the cover is a transformed bitmap of the skull in the 1533 oil painting by Hans Holbein the Younger, *The Ambassadors*, which features a floating distorted skull rendered in anamorphic perspective.

A Bitmapper's Companion, 2021

Special Topics ► Computer Graphics ► Programming

006.6'6–dc20

Copyright © 2021 by Emmanouil Pitsidianakis

This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/3.0/> or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.

The source code for this work is available under the GNU GENERAL PUBLIC LICENSE version 3 or later. You can view it, study it, modify it for your purposes as long as you respect the license if you choose to distribute your modifications.

The source code is available here

<https://github.com/epilys/bitmappers-companion>

Contents

I	Introduction	9
1	Data representation	11
2	Displaying pixels to your screen	13
3	Bits to byte pixels	15
4	Real pixels to byte pixels	17
5	Loading xbm files in Rust	19
II	Points And Lines	21
6	Distance between two points	23
7	Equations of a line	25
7.1	Line through a point $P = (x_p, y_p)$ and a slope m	25
7.2	Line through two points	26
8	Distance from a point to a line	29
8.1	Using the implicit equation form	29
8.2	Using an L defined by two points P_1, P_2	29
8.3	Using an L defined by a point P_l and angle θ	30
9	Angle between two lines	31
10	Intersection of two lines	33
11	Line equidistant from two points	35
12	Normal to a line through a point	37



III	Points And Line Segments	39
13	Drawing a line segment from its two endpoints	41
14	Drawing line segments with width	43
15	Intersection of two line segments	45
15.1	<i>Fast</i> intersection of two line segments	45
IV	Points, Lines and Circles	49
16	Equations of a circle	53
17	Bounding circle	55
V	Curves other than circles	57
18	Parametric elliptical arcs	59
VI	Points, Lines and Shapes	61
19	Union, intersection and difference of polygons	63
20	Centroid of polygon	65
21	Flood filling	67
VII	Vectors, matrices and transformations	73
22	Rotation of a bitmap	75
22.1	Fast 2D Rotation	79
23	90° Rotation of a bitmap by parallel recursive subdivision	81
24	Magnification/Scaling	83
24.1	Smoothing enlarged bitmaps	85
24.2	Stretching lines of bitmaps	86
25	Mirroring	89
26	Shearing	91
27	Projections	95

<i>CONTENTS</i>	7
VIII Advanced	97
27.1 Faster Drawing a line segment from its two endpoints using Symmetry	99
28 Joining the ends of two wide line segments together	101
29 Composing monochrome bitmaps with separate alpha channel data	103
30 Orthogonal connection of two points	105
31 Join segments with round corners	107
32 Faster line clipping	109
33 Space-filling Curves	111
33.1 Hilbert curves	112
33.2 Peano curves	114
33.3 Z-order curves	115
Index	117



intro

Part I

Introduction

intro

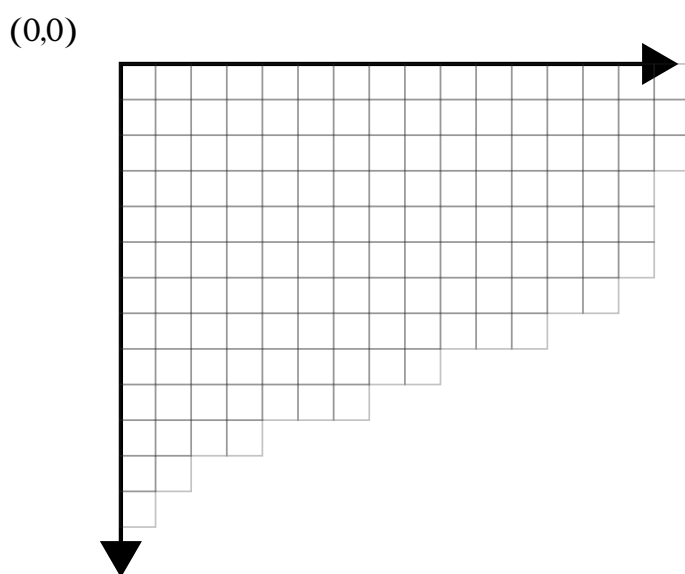
Chapter 1

intro

Data representation

The data structures we're going to use is *Point* and *Image*. *Image* represents a bitmap, although we will use full RGB colors for our points therefore the size of a pixel in memory will be u8 instead of 1 bit.

We will work on the cartesian grid representing the framebuffer that will show us the pixels. The *origin* of this grid (i.e. the center) is at $(0,0)$.



We will represent points as pairs of signed integers. When actually drawing them though, negative values and values outside the window's geometry will be ignored (clipped).

src/lib.rs:



This code file is a PDF attachment

```

pub type Point = (i64, i64);

pub const fn from_u8_rgb(r: u8, g: u8, b: u8) -> u32 {
    let (r, g, b) = (r as u32, g as u32, b as u32);
    (r << 16) | (g << 8) | b
}

pub const AZURE_BLUE: u32 = from_u8_rgb(0, 127, 255);
pub const RED: u32 = from_u8_rgb(157, 37, 10);
pub const WHITE: u32 = from_u8_rgb(255, 255, 255);
pub const BLACK: u32 = 0;

pub struct Image {
    pub bytes: Vec<u32>,
    pub width: usize,
    pub height: usize,
    pub x_offset: usize,
    pub y_offset: usize,
}

impl Image {
    pub fn new(width: usize, height: usize, x_offset: usize, y_offset: usize) -> Self;
    pub fn draw(&self, buffer: &mut Vec<u32>, fg: u32, bg: Option<u32>, window_width:
↳  usize);
    pub fn draw_outline(&mut self);
    pub fn clear(&mut self);
    pub fn plot(&mut self, x: i64, y: i64);
    pub fn get(&mut self, x: i64, y: i64) -> u32;
    pub fn plot_ellipse(
        &mut self,
        (xm, ym): (i64, i64),
        (a, b): (i64, i64),
        quadrants: [bool; 4],
        _wd: f64,
    );
    pub fn plot_line_width(&mut self, point_a: Point, point_b: Point, wd: f64);
    pub fn flood_fill(&mut self, mut x: i64, y: i64);
}

```

Chapter 2

Displaying pixels to your screen

A way to display an *Image* is to use the `minifb` crate which allows you to create a window and draw pixels directly on it. Here's how you could set it up:

`src/bin/introduction.rs`



This code file is a PDF attachment

```
use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

fn main() {
    let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
    let mut window = Window::new(
        "Test - ESC to exit",
        WINDOW_WIDTH,
        WINDOW_HEIGHT,
        WindowOptions {
            title: true,
            //borderless: true,
            //resize: false,
            //transparency: true,
            ..WindowOptions::default()
        },
    )
    .unwrap();

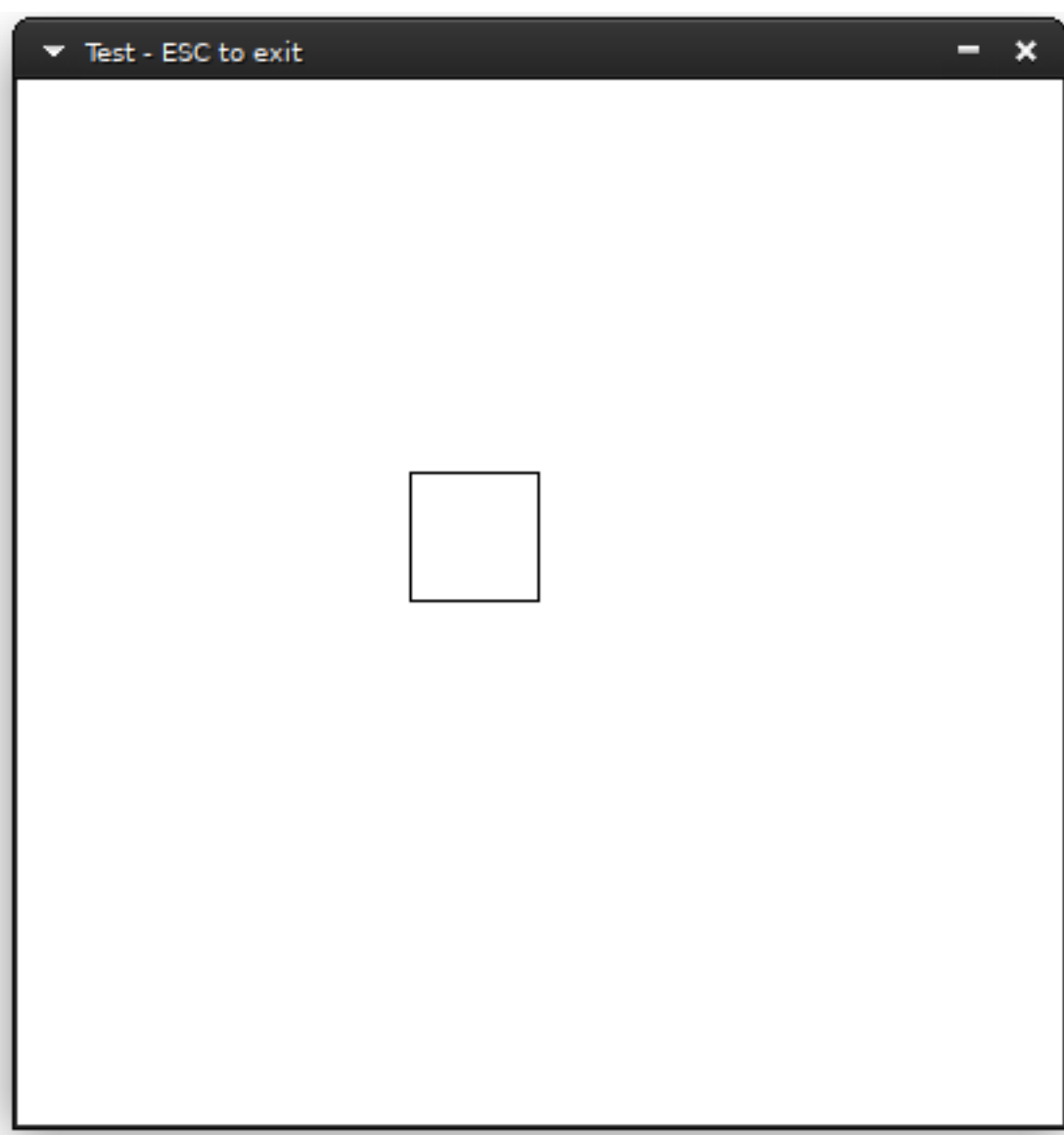
    // Limit to max ~60 fps update rate
    window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

    let mut image = Image::new(50, 50, 150, 150);
    image.draw_outline();
    image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

    while window.is_open()
        && !window.is_key_down(Key::Escape)
        && !window.is_key_down(Key::Q) {
        window
            .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
            .unwrap();
        let millis = std::time::Duration::from_millis(100);
        std::thread::sleep(millis);
    }
}
```

Running this will show you something like this:

intro



Chapter 3

Bits to byte pixels

Let's define a way to convert bit information to a byte vector:

```
pub fn bits_to_bytes(bits: &[u8], width: usize) -> Vec<u32> {  
    let mut ret = Vec::with_capacity(bits.len() * 8);  
    let mut current_row_count = 0;  
    for byte in bits {  
        for n in 0..8 {  
            if byte.rotate_right(n) & 0x01 > 0 {  
                ret.push(BLACK);  
            } else {  
                ret.push(WHITE);  
            }  
            current_row_count += 1;  
            if current_row_count == width {  
                current_row_count = 0;  
                break;  
            }  
        }  
    }  
    ret  
}
```

intro

Chapter 4

Real pixels to byte pixels

intro

1

[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 5

Loading xbm files in Rust

xbm files are C source code files that contain the pixel information for an image as macro definitions for the dimensions and a static char array for the pixels, with each bit column representing a pixel. If the width dimension doesn't have 8 as a factor, the remaining bit columns are left blank/ignored.

They used to be a popular way to share user avatars in the old internet and are also good material for us to work with, since they are small and numerous. The following is such an image:



Then, we can convert the xbm file from C to **Rust** with the following transformations:

```
#define news_width 48  
#define news_height 48  
static char news_bits[] = {
```

to

```
const NEWS_WIDTH: usize = 48;  
const NEWS_HEIGHT: usize = 48;  
const NEWS_BITS: &[u8] = &[
```

And replace the closing `}` with `]`.

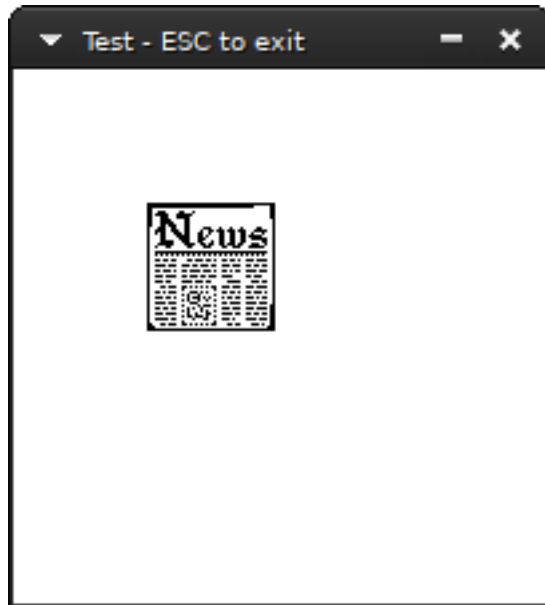
We can then include the new file in our source code:

```
include!("news.xbm.rs");
```

load the image:

```
let mut image = Image::new(NEWS_WIDTH, NEWS_HEIGHT, 25, 25);  
image.bytes = bits_to_bytes(NEWS_BITS, NEWS_WIDTH);
```

and finally run it:



Part II

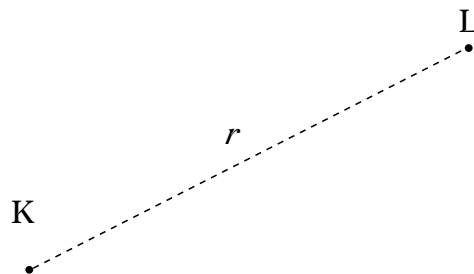
Points And Lines

lines

Chapter 6

Distance between two points

lines



Given two points, K and L , an elementary application of Pythagoras' Theorem gives the distance between them as

$$r = \sqrt{(x_L - x_K)^2 + (y_L - y_K)^2} \quad (6.1)$$

which is simply coded:

```
pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {  
    let (x_k, y_k) = p_k;  
    let (x_l, y_l) = p_l;  
    let x_lk = x_l - x_k;  
    let y_lk = y_l - y_k;  
    f64::sqrt((x_lk*x_lk + y_lk*y_lk) as f64)  
}
```

lines

Chapter 7

Equations of a line

lines

There are several ways to describe a line mathematically. We'll list the convenient ones for drawing pixels.

The equation that describes every possible line on a two dimensional grid is the *implicit* form $ax + by = c$, $(a, b) \neq (0, 0)$. We can generate equivalent equations by adding the equation to itself, i.e. $ax + by = c \equiv 2ax + 2by = 2c \equiv a'x + b'y = c'$, $a' = 2a, b' = 2b, c' = 2c$ as many times as we want. To "minimize" the constants a, b, c we want to satisfy the relationship $a^2 + b^2 = 1$, and thus can convert the equivalent equations into one representative equation by multiplying the two sides with $\frac{1}{\sqrt{a^2 + b^2}}$; this is called the normalized equation.

The *slope intercept form* describes any line that intercepts the y axis at $b \in \mathbb{R}$ with a specific slope a :

$$y = ax + b$$

The *parametric* form...

7.1 Line through a point $P = (x_p, y_p)$ and a slope m

$$y - y_p = m(x - x_p)$$

7.2 Line through two points

lines

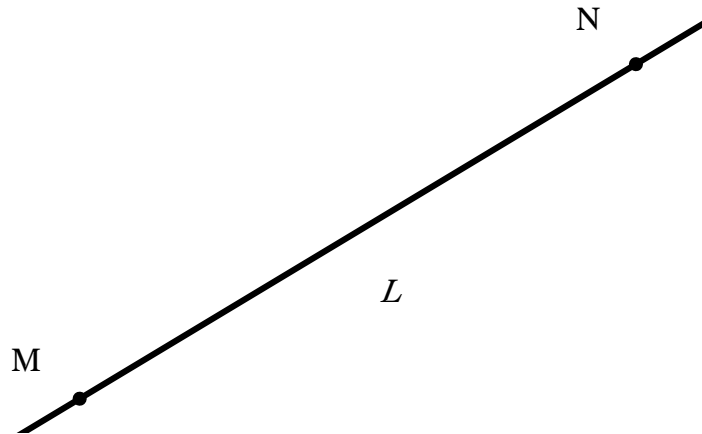


Figure 7.1:

It seems sufficient, given the coordinates of two points M, N , to calculate a, b and c to form a line equation:

$$ax + by + c = 0$$

If the two points are not the same, they necessarily form such a line. To get there, we start from expressing the line as parametric over t : at $t = 0$ it's at point M and at $t = 1$ it's at point N :

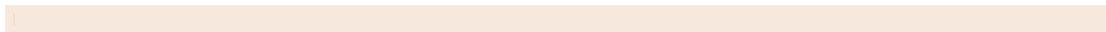
$$c = c_M + (c_N - c_M)t, t \in R, c \in \{x, y\}$$

$$c = c_M, t \in R, c \in \{x, y\}$$

Substituting t in one of the equations we get:

$$(y_M - y_N)x + (x_N - x_M)y + (x_M y_N - x_N y_M) = 0$$

Which is what we were after. We finish by normalising what we found with $\frac{1}{\sqrt{a^2 + b^2}}$:

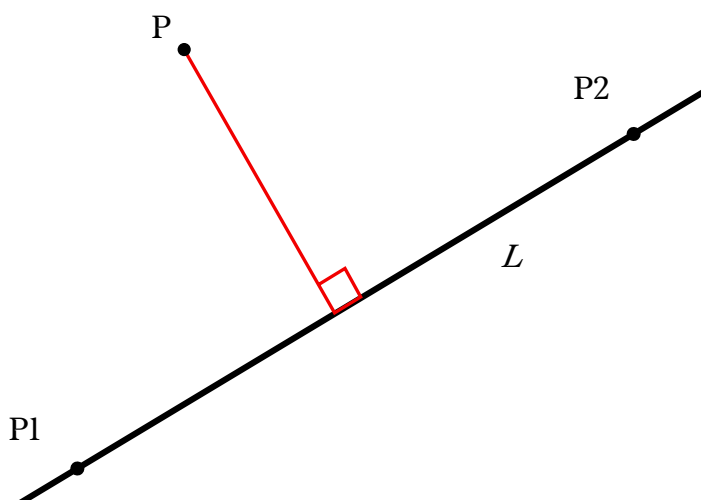


lines

Chapter 8

Distance from a point to a line

lines



8.1 Using the implicit equation form

Let's find the distance from a given point P and a given line L . Let d be the distance between them. Bring L to the implicit form $ax + by = c$.

$$d = \frac{|ax_p + by_p + c|}{\sqrt{a^2 + b^2}}$$

8.2 Using an L defined by two points P_1, P_2

With $P = (x_0, y_0)$, $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$.

$$d = \frac{|(x_2 - x_1)(y_1 - y_0) - (x_1 - x_0)(y_2 - y_1)|}{\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}}$$

8.3 Using an L defined by a point P_l and angle θ

$$d = |\cos(\theta)(P_{ly} - y_p) - \sin(\theta)(P_{lx} - P_x)|$$

Chapter 9

Angle between two lines

lines

2

lines

Chapter 10

Intersection of two lines

[Redacted content]

3

lines

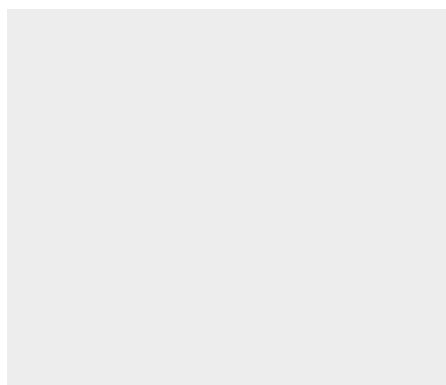
lines



Chapter 11

Line equidistant from two points

lines



4

Figure 11.1:

5

Let's name this line L . From the previous chapter we know how to get the line that's created by the two points M and N . If only we knew how to get a perpendicular line over the midpoint of a line segment!

Thankfully that midpoint also satisfies L 's equation, $ax + by + c$. The midpoint's coordinates are intuitively:

$$\left(\frac{x_M + x_N}{2}, \frac{y_M + y_N}{2} \right)$$

Putting them into the equation we can generate a triple of (a', b', c') and then normalize it to get L .

lines

Chapter 12

Normal to a line through a point

lines

6

lines

Part III

Points And Line Segments

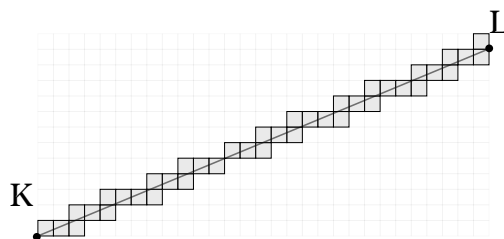
segments

segments

Chapter 13

Drawing a line segment from its two endpoints

For any line segment with any slope, pixels must be matched with the infinite amount of points contained in the segment. As shown in the following figure, a segment *touches* some pixels; we could fill them using an algorithm and get a bitmap of the line segment.



The algorithm presented here was first derived by Bresenham. In the *Image* implementation, it is used in the `plot_line_width` method.

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64)) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();
```

segments

```

sx = if dx > 0 { 1 } else { -1 };
dy = y2 - y1;
ay = (dy * 2).abs();
sy = if dy > 0 { 1 } else { -1 };
x = x1;
y = y1;

let b = dx / dy;
let a = 1;
let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;
let delta = double_d / 2;

if ax > ay {
  d = ay - ax / 2;
  loop {
    self.plot(x, y);
    if x == x2 {
      return;
    }
    if d >= 0 {
      y = y + sy;
      d = d - ax;
    }
    x = x + sx;
    d = d + ay;
  }
} else {
  d = ax - ay / 2;
  let delta = double_d / 3;
  loop {
    self.plot(x, y);
    if y == y2 {
      return;
    }
    if d >= 0 {
      x = x + sx;
      d = d - ay;
    }
    y = y + sy;
    d = d + ax;
  }
}
}
```

Chapter 14

Drawing line segments with width

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64), _wd: f64) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();  
    sx = if dx > 0 { 1 } else { -1 };  
  
    dy = y2 - y1;  
    ay = (dy * 2).abs();  
    sy = if dy > 0 { 1 } else { -1 };  
  
    x = x1;  
    y = y1;  
  
    let b = dx / dy;  
    let a = 1;  
    let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;  
    let delta = double_d / 2;  
  
    if ax > ay {  
        d = ay - ax / 2;  
        loop {  
            self.plot(x, y);  
            {  
                let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;  
                let mut _x = x;  
                loop {  
                    let t = total(_x);  
                    if t < -1 * delta || t > delta {  
                        break;  
                    }  
                    _x += 1;  
                    self.plot(_x, y);  
                }  
                let mut _x = x;  
                loop {  
                    let t = total(_x);  
                    if t < -1 * delta || t > delta {  
                        break;  
                    }  
                    _x -= 1;  
                    self.plot(_x, y);  
                }  
            }  
        }  
    }  
}
```

segments

segments

```
        if x == x2 {
            return;
        }
        if d >= 0 {
            y = y + sy;
            d = d - ax;
        }
        x = x + sx;
        d = d + ay;
    }
} else {
    d = ax - ay / 2;
    let delta = double_d / 3;
    loop {
        self.plot(x, y);
        {
            let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x += 1;
                self.plot(_x, y);
            }
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x -= 1;
                self.plot(_x, y);
            }
        }
    }
    if y == y2 {
        return;
    }
    if d >= 0 {
        x = x + sx;
        d = d - ay;
    }
    y = y + sy;
    d = d + ax;
}
}
```

Chapter 15

Intersection of two line segments

Let points **1** = (x_1, y_1) , **2** = (x_2, y_2) , **3** = (x_3, y_3) and **4** = (x_4, y_4) and **1,2**, **3,4** two line segments they form. We wish to find their intersection:

First, get the equation of line L_{12} and line L_{34} from chapter *Equations of a line*.

Substitute points **3** and **4** in equation L_{12} to compute $r_3 = L_{12}(\mathbf{3})$ and $r_4 = L_{12}(\mathbf{4})$ respectively.

If $r_3 \neq 0$, $r_4 \neq 0$ and $\text{sgn}(r_3) == \text{sign}(r_4)$ the line segments don't intersect, so stop.

In L_{34} substitute point **1** to compute r_1 , and do the same for point **2**.

If $r_1 \neq 0$, $r_2 \neq 0$ and $\text{sgn}(r_1) == \text{sign}(r_2)$ the line segments don't intersect, so stop.

At this point, L_{12} and L_{34} either intersect or are equivalent. Find their intersection point. (Refer to *Intersection of two lines*.)

15.1 Fast intersection of two line segments

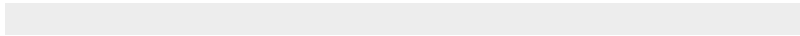


[Redacted text block 1]

[Redacted text block 2]

[Redacted text block 3]

[Redacted text block 4]



Part IV

Points, Lines and Circles

circles

[Redacted text block]

[Redacted text block]

[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 16

Equations of a circle

1. Find the equation of the circle with centre $(-2, 3)$ and radius 5.

2. Find the equation of the circle with centre $(4, -1)$ and radius 3.

3. Find the equation of the circle with centre $(-5, 2)$ and radius 4.

4. Find the equation of the circle with centre $(1, -4)$ and radius 2.

5. Find the equation of the circle with centre $(-3, 1)$ and radius 6.

6. Find the equation of the circle with centre $(2, -5)$ and radius 1.

7. Find the equation of the circle with centre $(-1, 4)$ and radius 3.

8. Find the equation of the circle with centre $(5, -2)$ and radius 4.

9. Find the equation of the circle with centre $(-4, 1)$ and radius 2.

10. Find the equation of the circle with centre $(3, -6)$ and radius 5.

9

11. Find the equation of the circle with centre $(-2, 3)$ and radius 5.

12. Find the equation of the circle with centre $(4, -1)$ and radius 3.

13. Find the equation of the circle with centre $(-5, 2)$ and radius 4.

14. Find the equation of the circle with centre $(1, -4)$ and radius 2.

15. Find the equation of the circle with centre $(-3, 1)$ and radius 6.

16. Find the equation of the circle with centre $(2, -5)$ and radius 1.

17. Find the equation of the circle with centre $(-1, 4)$ and radius 3.

18. Find the equation of the circle with centre $(5, -2)$ and radius 4.

19. Find the equation of the circle with centre $(-4, 1)$ and radius 2.

20. Find the equation of the circle with centre $(3, -6)$ and radius 5.

21. Find the equation of the circle with centre $(-2, 3)$ and radius 5.

22. Find the equation of the circle with centre $(4, -1)$ and radius 3.

23. Find the equation of the circle with centre $(-5, 2)$ and radius 4.

24. Find the equation of the circle with centre $(1, -4)$ and radius 2.

25. Find the equation of the circle with centre $(-3, 1)$ and radius 6.

26. Find the equation of the circle with centre $(2, -5)$ and radius 1.

27. Find the equation of the circle with centre $(-1, 4)$ and radius 3.

28. Find the equation of the circle with centre $(5, -2)$ and radius 4.

29. Find the equation of the circle with centre $(-4, 1)$ and radius 2.

30. Find the equation of the circle with centre $(3, -6)$ and radius 5.

circles

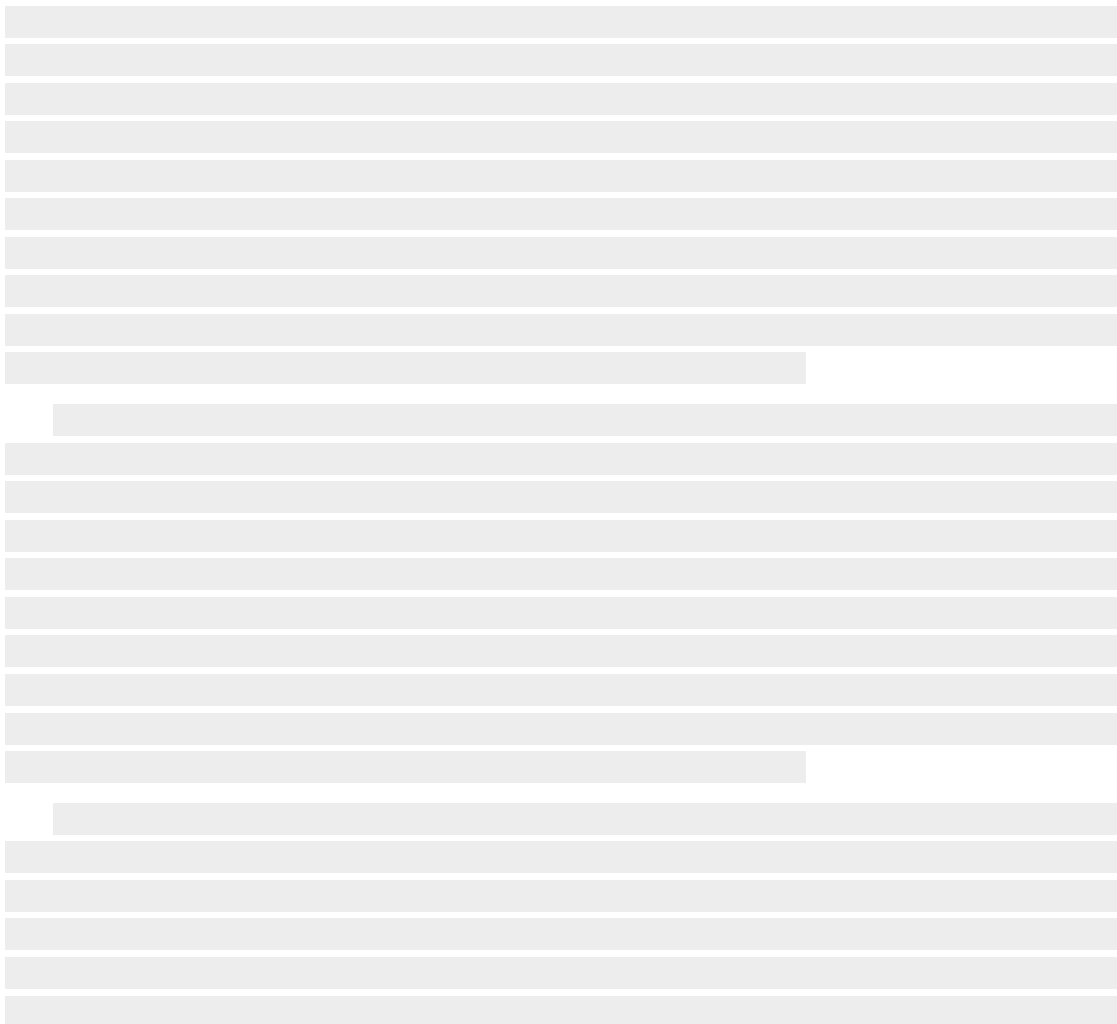
[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 17

Bounding circle



10

circles

[Redacted text block]

[Redacted text block]

[Redacted text block]

Part V

Curves other than circles

curves

Chapter 18

Parametric elliptical arcs

11

curves

[Redacted text block]

[Redacted text block]

[Redacted text block]

Part VI

Points, Lines and Shapes

shapes

Chapter 19

Union, intersection and difference of polygons

12

shapes

1. The first section of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes the need for transparency and accountability in financial reporting.

2. The second section outlines the various methods used to collect and analyze data. It includes a detailed description of the experimental procedures and the statistical techniques employed to interpret the results.

3. The third section presents the findings of the study, highlighting the key observations and conclusions. It discusses the implications of the results for future research and practical applications.

Chapter 20

Centroid of polygon



13

shapes

Chapter 21

Flood filling



14

shapes

[Redacted text block]

[Redacted text block]

[Redacted text block]

15

[Redacted text block]

[Redacted text block]



1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes the need for transparency and accountability in financial reporting.

2. The second part of the document focuses on the role of the board of directors in overseeing the company's financial health and ensuring compliance with applicable laws and regulations. It highlights the importance of regular communication and reporting between the board and management.

3. The third part of the document addresses the challenges faced by the company in managing its financial resources and maintaining a strong credit rating. It discusses the need for strategic financial planning and risk management.

4. The fourth part of the document provides a detailed analysis of the company's financial performance over the past year, including a breakdown of revenue, expenses, and profit. It also includes a comparison of the company's performance to industry benchmarks.

5. The fifth part of the document outlines the company's future financial goals and strategies for achieving them. It includes a discussion of the company's plans for expanding its operations and improving its financial performance.



Part VII

Vectors, matrices and transformations

trans-
forma-
tions

Chapter 22

Rotation of a bitmap

$$p' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

$$c = \cos\theta, s = \sin\theta, x_{p'} = x_p c - y_p s, y_{p'} = x_p s + y_p c.$$

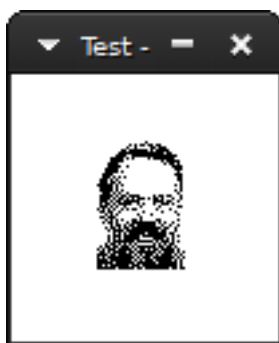
Let's load an xface. We will use `bits_to_bytes` (See Introduction).

```
include!("dmr.rs");
const WINDOW_WIDTH: usize = 100;
const WINDOW_HEIGHT: usize = 100;
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

src/bin/rotation.rs:



This code file is a PDF attachment



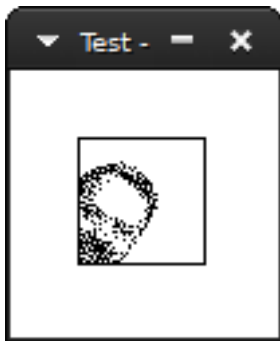
This is the xface of dmr. Instead of displaying the bitmap, this time we will rotate it 0.5 radians. Setup our image first:

```
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.draw_outline();
let dmr = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

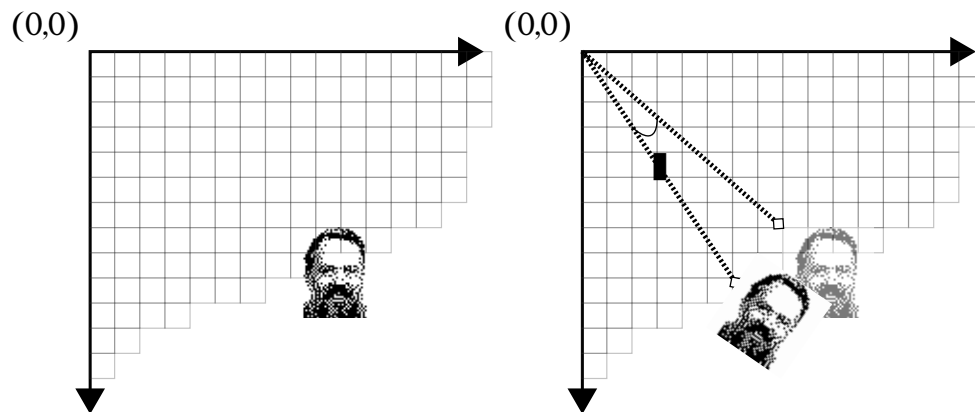
And then, loop for each byte in dmr's face and apply the rotation transformation.

```
let angle = 0.5;
let c = f64::cos(angle);
let s = f64::sin(angle);
for y in 0..DMR_HEIGHT {
    for x in 0..DMR_WIDTH {
        if dmr[y * DMR_WIDTH + x] == BLACK {
            let x = x as f64;
            let y = y as f64;
            let xr = x * c - y * s;
            let yr = x * s + y * c;
            image.plot(xr as i64, yr as i64);
        }
    }
}
```

The result:



We didn't mention in the beginning that the rotation has to be relative to a *point* and the given transformation is relative to the *origin*, in this case the upper left corner (0,0). So dmr was rotated relative to the origin:



(the distance to the origin (actually 0 pixels) has been exaggerated for the sake of the example)

Usually, we want to rotate something relative to itself. The right point to choose is the *centroid* of the object.

If we have a list of n points, the centroid is calculated as:

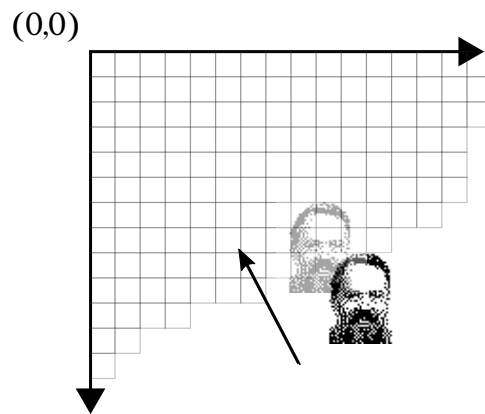
$$x_c = \frac{1}{n} \sum_{i=0}^n x_i$$

$$y_c = \frac{1}{n} \sum_{i=0}^n y_i$$

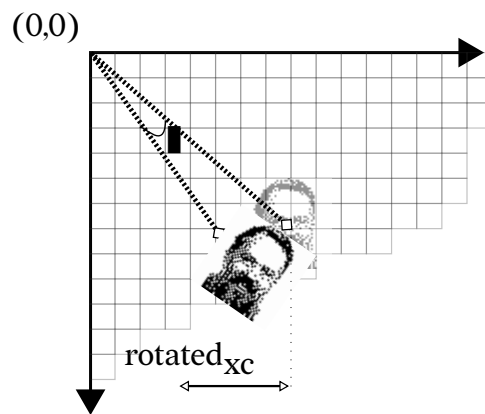
Since in this case we have a rectangle, the centroid has coordinates of half the width and half the height.

By subtracting the centroid from each point before we apply the transformation and then adding it back after we get what we want:

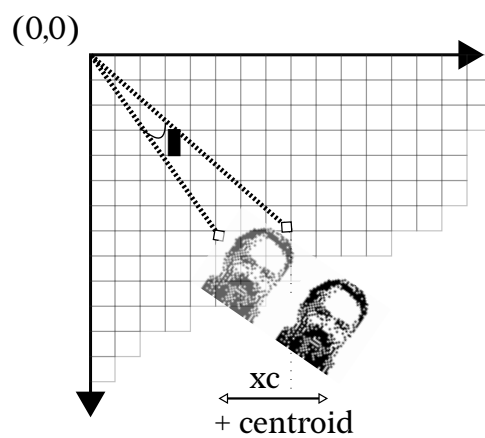
Here's it visually: First subtract the center point.



Then, rotate.

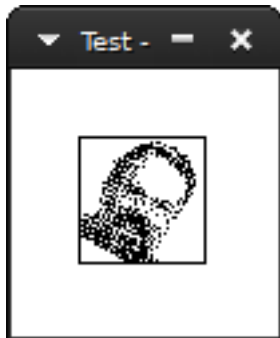


And subtract back to the original position.



In code:

```
let center_point = ((DMR_WIDTH/2) as i64, (DMR_HEIGHT/2) as i64);
for y in 0..DMR_HEIGHT {
  for x in 0..DMR_WIDTH {
    if dmr[y * DMR_WIDTH + x] == BLACK {
      let x = (x as i64 - center_point.0) as f64;
      let y = (y as i64 - center_point.1) as f64;
      let xr = x * c - y * s;
      let yr = x * s + y * c;
      image.plot(xr as i64 + center_point.0,
                 yr as i64 + center_point.1);
    }
  }
}
```



The result:

22.1 Fast 2D Rotation

17

[Redacted text block]

[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 23

90° Rotation of a bitmap by parallel recursive subdivision



18

trans-
forma-
tions

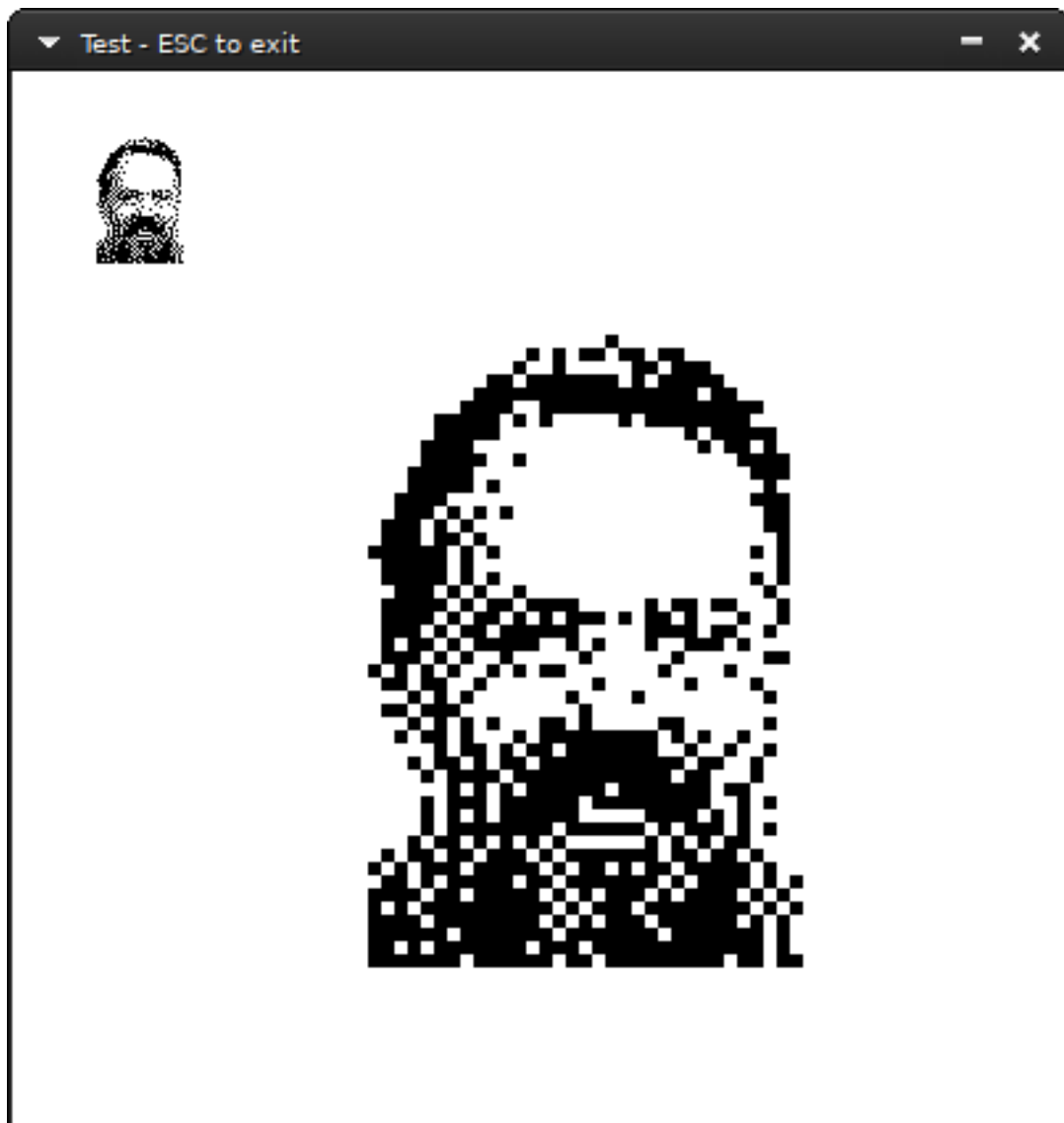
[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 24

Magnification/Scaling



```
let mut original = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
original.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
original.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

let mut scaled = Image::new(DMR_WIDTH * 5, DMR_HEIGHT * 5, 100, 100);
let mut sx: i64; //source
let mut sy: i64; //source
let mut dx: i64; //destination
let mut dy: i64 = 0; //destination

let og_height = original.height as i64;
let og_width = original.width as i64;
let scaled_height = scaled.height as i64;
let scaled_width = scaled.width as i64;

while dy < scaled_height {
    sy = (dy * og_height) / scaled_height;
    dx = 0;
    while dx < scaled_width {
        sx = (dx * og_width) / scaled_width;
        if original.get(sx, sy) == Some(BLACK) {
            scaled.plot(dx, dy);
        }
        dx += 1;
    }
    dy += 1;
}
scaled.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
```

src/bin/scale.rs:



This code file is a PDF attachment

24.1 Smoothing enlarged bitmaps

19

The first part of the chapter discusses the basic concepts of transformations and how they can be used to create complex shapes. It covers topics such as translation, rotation, scaling, and shearing. The second part of the chapter focuses on the application of these transformations in computer graphics, including how they are used to animate objects and create realistic simulations.

The third part of the chapter introduces the concept of projective transformations, which are used to map 3D objects onto a 2D plane. This is a fundamental technique in computer graphics, and it is used to create the illusion of depth and perspective. The fourth part of the chapter discusses the use of transformations in the design of user interfaces, where they are used to create interactive elements and animations.

The fifth part of the chapter covers the use of transformations in the design of fonts, where they are used to create custom typefaces and to adjust the spacing and alignment of characters. The sixth part of the chapter discusses the use of transformations in the design of web pages, where they are used to create responsive layouts and to optimize the performance of the site.

24.2 Stretching lines of bitmaps

20

This section discusses the process of stretching lines of bitmaps, which is a common technique used in computer graphics to create smooth transitions between different colors and shades. It covers the mathematical principles behind the stretching process and provides examples of how it can be applied in practice.

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]



Chapter 25

Mirroring

Mirroring to an axis is the transformation of one coordinate to its equidistant value across the axis:

To mirror a across the x axis, simply multiply its coordinates with the following matrix:

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

This results in the y coordinate's sign being flipped.

For y -mirroring, the transformation follows the same logic:

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Chapter 26

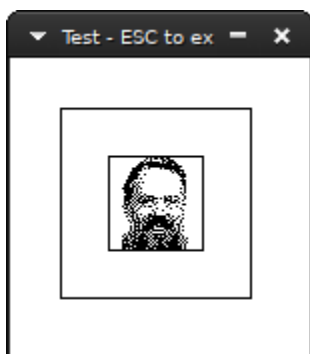
Shearing

Simple shearing is the transformation of one dimension by a distance proportional to the other dimension. In x -shearing (or horizontal shearing) only the x coordinate is affected, and likewise in y -shearing only y as well.

src/bin/shearing.rs



This code file is a PDF attachment



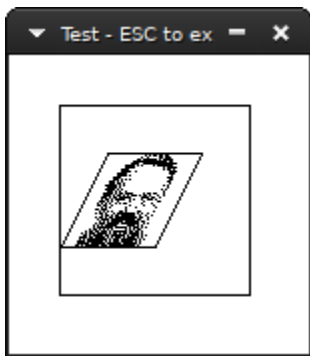
With l being equal to the desired tilt away from the y axis, the transformation is described by the following matrix:

$$S_x = \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}$$

Which is as simple as this function:

```
fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p + (l * (y_p as f64)) as i64, y_p)
}
```

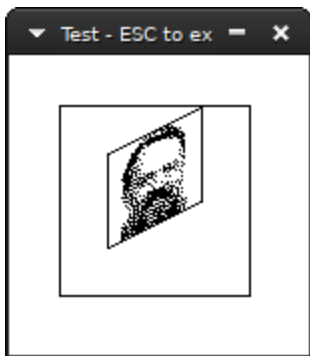
trans-
forma-
tions



For y -shearing, we have the following:

$$S_y = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix}$$

```
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}
```



A full example:

```
include!("../dmr.xbm.rs");
const WINDOW_WIDTH: usize = 200;
const WINDOW_HEIGHT: usize = 200;
fn shear_x((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p+(l*(y_p as f64)) as i64, y_p)
}
fn shear_y((x_p, y_p): (i64, i64), l: f64) -> (i64, i64) {
    (x_p, (l*(x_p as f64)) as i64 + y_p)
}

let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
image.draw_outline();

let l = -0.5;
let mut sheared = Image::new(DMR_WIDTH*2, DMR_HEIGHT*2, 25, 25);
for x in 0..DMR_WIDTH {
    for y in 0..DMR_HEIGHT {
        if image.bytes[y * DMR_WIDTH + x] == BLACK {
```

```
        let p = shear_x((x as i64 ,y as i64 ), l);
        sheared.plot(p.0+(DMR_WIDTH/2) as i64, p.1+(DMR_HEIGHT/2) as i64);
    }
}
sheared.draw_outline();
```


Chapter 27

Projections

21

trans-
forma-
tions

[Redacted text block]

[Redacted text block]

[Redacted text block]

Part VIII

Advanced

ad-
vanced

27.1 Faster Drawing a line segment from its two endpoints using Symmetry

22



[Redacted text block]

[Redacted text block]

Chapter 28

Joining the ends of two wide line segments together



ad-
vanced

[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 29

Composing monochrome bitmaps with separate alpha channel data



ad-
vanced

[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 30

Orthogonal connection of two points

[Redacted text block]

25

ad-
vanced

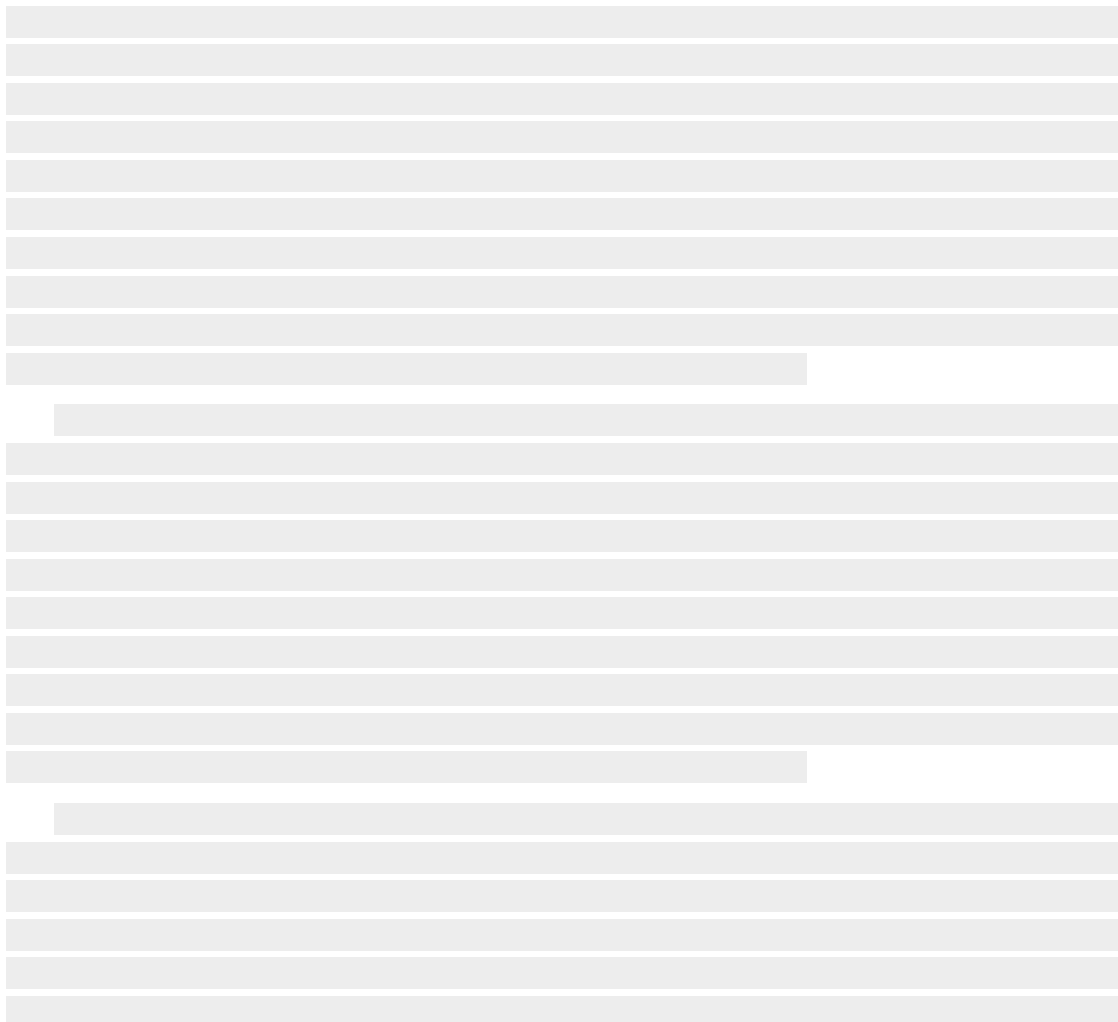
[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 31

Join segments with round corners



26

ad-
vanced

[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 32

Faster line clipping

[Redacted content]

27

[Redacted text block]

[Redacted text block]

[Redacted text block]

Chapter 33

Space-filling Curves

[Redacted content]

28

[Redacted text block]

[Redacted text block]

[Redacted text block]

33.1 Hilbert curves

29

[Redacted text block]

[Redacted text block]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

33.2 Peano curves

30

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

[REDACTED]

33.3 Z-order curves

[REDACTED]

31

[REDACTED]

[REDACTED]

[Redacted text block]

[Redacted text block]

[Redacted text block]

Index

centroid, 65, 77

shearing, 91

About this text

The text has been typeset in \LaTeX using the book class and:

- **Redaction** for the main text.
- **Fira Sans** for referring to the programming language **Rust** .
- **Redaction20** for referring to the words bitmap and pixels as a concept.