
A Bitmapper's Companion

epilys

November 29, 2021

an introduction
to basic bitmap
mathematics
and algorithms
with code
samples in **Rust**



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All non-screenshot figures were generated by hand in Inkscape unless otherwise stated.

The skull in the cover is a transformed bitmap of the skull in the 1533 oil painting by Hans Holbein the Younger, *The Ambassadors*, which features a floating distorted skull rendered in anamorphic perspective.

A Bitmapper's Companion, 2021

Special Topics ► Computer Graphics ► Programming

006.6'6–dc20

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The source code for this work is available under the GNU GENERAL PUBLIC LICENSE version 3 or later. You can view it, study it, modify it for your purposes as long as you respect the license if you choose to distribute your modifications.

The source code is available here

<https://github.com/epilys/bitmappers-companion>

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intro

Part I

Introduction

intro

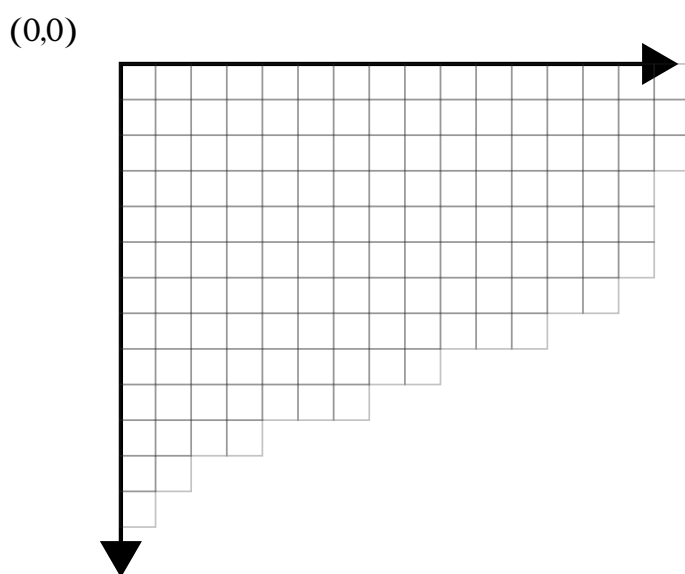
Chapter 1

intro

Data representation

The data structures we're going to use is *Point* and *Image*. *Image* represents a bitmap, although we will use full RGB colors for our points therefore the size of a pixel in memory will be u8 instead of 1 bit.

We will work on the cartesian grid representing the framebuffer that will show us the pixels. The *origin* of this grid (i.e. the center) is at $(0,0)$.



We will represent points as pairs of signed integers. When actually drawing them though, negative values and values outside the window's geometry will be ignored (clipped).

src/lib.rs:



This code file is a PDF attachment

```

pub type Point = (i64, i64);

pub const fn from_u8_rgb(r: u8, g: u8, b: u8) -> u32 {
    let (r, g, b) = (r as u32, g as u32, b as u32);
    (r << 16) | (g << 8) | b
}

pub const AZURE_BLUE: u32 = from_u8_rgb(0, 127, 255);
pub const RED: u32 = from_u8_rgb(157, 37, 10);
pub const WHITE: u32 = from_u8_rgb(255, 255, 255);
pub const BLACK: u32 = 0;

pub struct Image {
    pub bytes: Vec<u32>,
    pub width: usize,
    pub height: usize,
    pub x_offset: usize,
    pub y_offset: usize,
}

impl Image {
    pub fn new(width: usize, height: usize, x_offset: usize, y_offset: usize) -> Self;
    pub fn draw(&self, buffer: &mut Vec<u32>, fg: u32, bg: Option<u32>, window_width:
↳  usize);
    pub fn draw_outline(&mut self);
    pub fn clear(&mut self);
    pub fn plot(&mut self, x: i64, y: i64);
    pub fn get(&mut self, x: i64, y: i64) -> u32;
    pub fn plot_ellipse(
        &mut self,
        (xm, ym): (i64, i64),
        (a, b): (i64, i64),
        quadrants: [bool; 4],
        _wd: f64,
    );
    pub fn plot_line_width(&mut self, point_a: Point, point_b: Point, wd: f64);
    pub fn flood_fill(&mut self, mut x: i64, y: i64);
}

```

Chapter 2

Displaying pixels to your screen

A way to display an *Image* is to use the `minifb` crate which allows you to create a window and draw pixels directly on it. Here's how you could set it up:

`src/bin/introduction.rs`



This code file is a PDF attachment

```
use bitmappers_companion::*;
use minifb::{Key, Window, WindowOptions};

const WINDOW_WIDTH: usize = 400;
const WINDOW_HEIGHT: usize = 400;

fn main() {
    let mut buffer: Vec<u32> = vec![WHITE; WINDOW_WIDTH * WINDOW_HEIGHT];
    let mut window = Window::new(
        "Test - ESC to exit",
        WINDOW_WIDTH,
        WINDOW_HEIGHT,
        WindowOptions {
            title: true,
            //borderless: true,
            //resize: false,
            //transparency: true,
            ..WindowOptions::default()
        },
    )
    .unwrap();

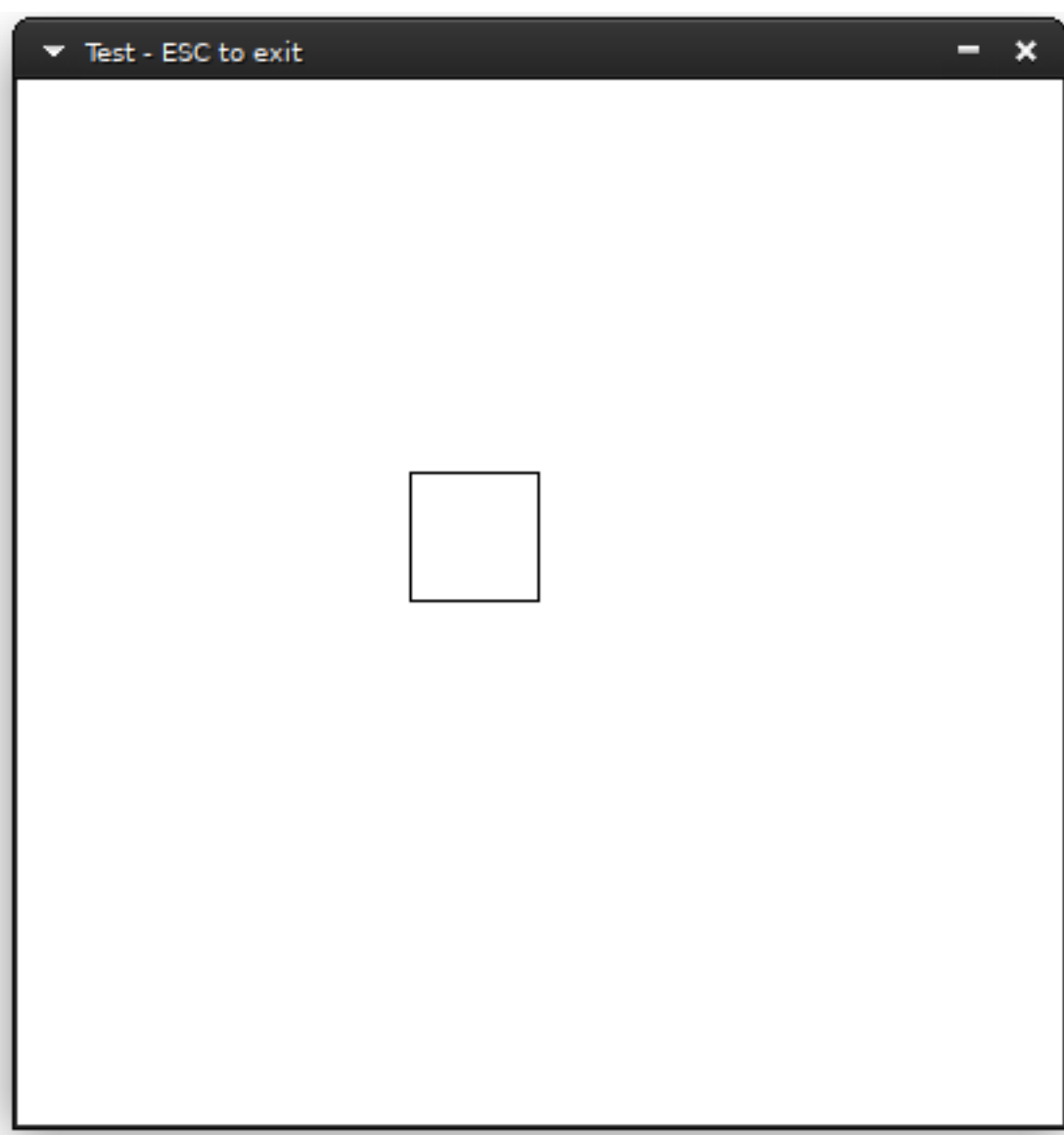
    // Limit to max ~60 fps update rate
    window.limit_update_rate(Some(std::time::Duration::from_micros(16600)));

    let mut image = Image::new(50, 50, 150, 150);
    image.draw_outline();
    image.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

    while window.is_open()
        && !window.is_key_down(Key::Escape)
        && !window.is_key_down(Key::Q) {
        window
            .update_with_buffer(&buffer, WINDOW_WIDTH, WINDOW_HEIGHT)
            .unwrap();
        let millis = std::time::Duration::from_millis(100);
        std::thread::sleep(millis);
    }
}
```

Running this will show you something like this:

intro



Chapter 3

Bits to byte pixels

Let's define a way to convert bit information to a byte vector:

```
pub fn bits_to_bytes(bits: &[u8], width: usize) -> Vec<u32> {  
    let mut ret = Vec::with_capacity(bits.len() * 8);  
    let mut current_row_count = 0;  
    for byte in bits {  
        for n in 0..8 {  
            if byte.rotate_right(n) & 0x01 > 0 {  
                ret.push(BLACK);  
            } else {  
                ret.push(WHITE);  
            }  
            current_row_count += 1;  
            if current_row_count == width {  
                current_row_count = 0;  
                break;  
            }  
        }  
    }  
    ret  
}
```

intro

Chapter 4

intro

Real pixels to byte pixels

1

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Chapter 5

Loading xbm files in Rust

xbm files are C source code files that contain the pixel information for an image as macro definitions for the dimensions and a static char array for the pixels, with each bit column representing a pixel. If the width dimension doesn't have 8 as a factor, the remaining bit columns are left blank/ignored.

They used to be a popular way to share user avatars in the old internet and are also good material for us to work with, since they are small and numerous. The following is such an image:



Then, we can convert the xbm file from C to **Rust** with the following transformations:

```
#define news_width 48  
#define news_height 48  
static char news_bits[] = {
```

to

```
const NEWS_WIDTH: usize = 48;  
const NEWS_HEIGHT: usize = 48;  
const NEWS_BITS: &[u8] = &[
```

And replace the closing `}` with `]`.

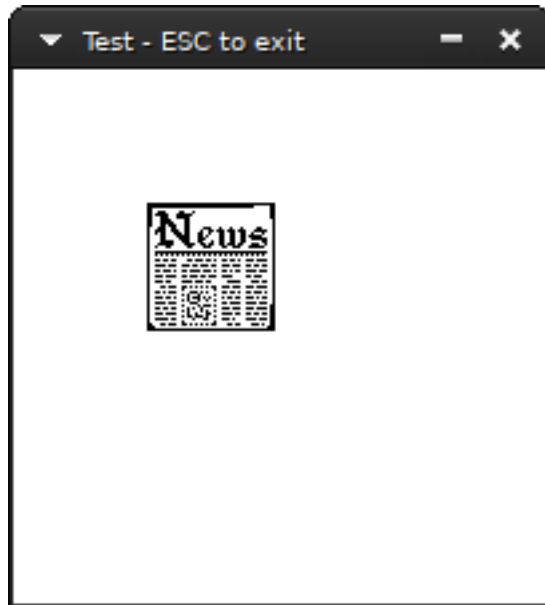
We can then include the new file in our source code:

```
include!("news.xbm.rs");
```

load the image:

```
let mut image = Image::new(NEWS_WIDTH, NEWS_HEIGHT, 25, 25);  
image.bytes = bits_to_bytes(NEWS_BITS, NEWS_WIDTH);
```

and finally run it:



Part II

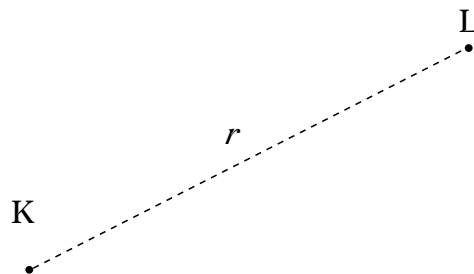
Points And Lines

lines

Chapter 6

Distance between two points

lines



Given two points, K and L , an elementary application of Pythagoras' Theorem gives the distance between them as

$$r = \sqrt{(x_L - x_K)^2 + (y_L - y_K)^2} \quad (6.1)$$

which is simply coded:

```
pub fn distance_between_two_points(p_k: Point, p_l: Point) -> f64 {  
  let (x_k, y_k) = p_k;  
  let (x_l, y_l) = p_l;  
  let x_lk = x_l - x_k;  
  let y_lk = y_l - y_k;  
  f64::sqrt((x_lk*x_lk + y_lk*y_lk) as f64)  
}
```

lines

Chapter 7

Equations of a line

lines

There are several ways to describe a line mathematically. We'll list the convenient ones for drawing pixels.

The equation that describes every possible line on a two dimensional grid is the *implicit* form $ax + by = c$, $(a, b) \neq (0, 0)$. We can generate equivalent equations by adding the equation to itself, i.e. $ax + by = c \equiv 2ax + 2by = 2c \equiv a'x + b'y = c'$, $a' = 2a, b' = 2b, c' = 2c$ as many times as we want. To "minimize" the constants a, b, c we want to satisfy the relationship $a^2 + b^2 = 1$, and thus can convert the equivalent equations into one representative equation by multiplying the two sides with $\frac{1}{\sqrt{a^2 + b^2}}$; this is called the normalized equation.

The *slope intercept form* describes any line that intercepts the y axis at $b \in \mathbb{R}$ with a specific slope a :

$$y = ax + b$$

The *parametric* form...

7.1 Line through a point $P = (x_p, y_p)$ and a slope m

$$y - y_p = m(x - x_p)$$

7.2 Line through two points

lines

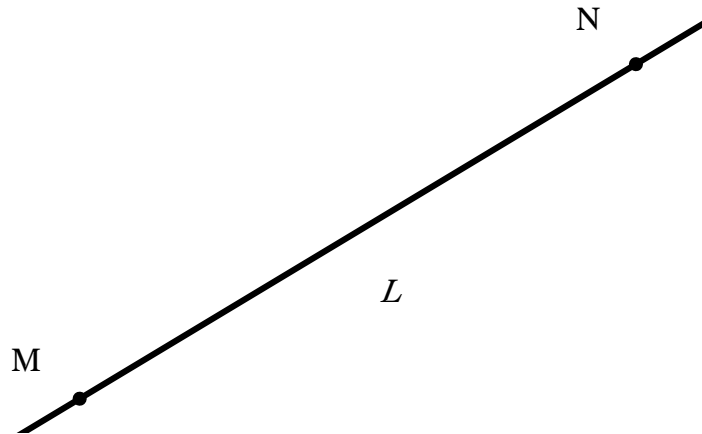


Figure 7.1:

It seems sufficient, given the coordinates of two points M, N , to calculate a, b and c to form a line equation:

$$ax + by + c = 0$$

If the two points are not the same, they necessarily form such a line. To get there, we start from expressing the line as parametric over t : at $t = 0$ it's at point M and at $t = 1$ it's at point N :

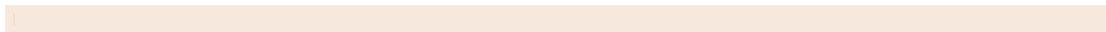
$$c = c_M + (c_N - c_M)t, t \in R, c \in \{x, y\}$$

$$c = c_M, t \in R, c \in \{x, y\}$$

Substituting t in one of the equations we get:

$$(y_M - y_N)x + (x_N - x_M)y + (x_M y_N - x_N y_M) = 0$$

Which is what we were after. We finish by normalising what we found with $\frac{1}{\sqrt{a^2 + b^2}}$:

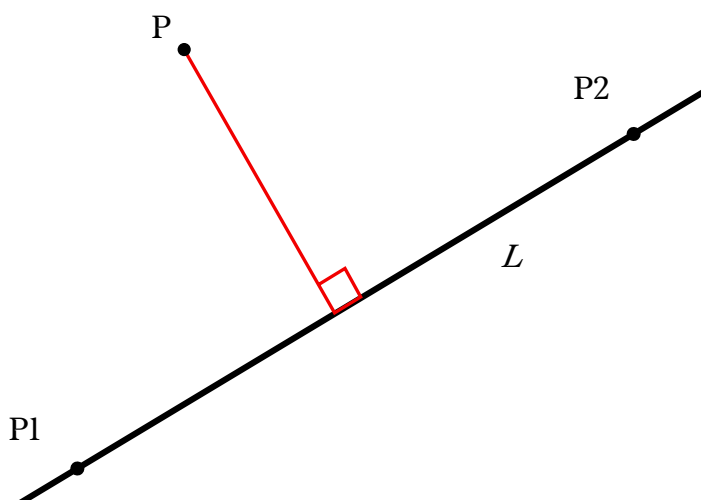


lines

Chapter 8

Distance from a point to a line

lines



8.1 Using the implicit equation form

Let's find the distance from a given point P and a given line L . Let d be the distance between them. Bring L to the implicit form $ax + by = c$.

$$d = \frac{|ax_p + by_p + c|}{\sqrt{a^2 + b^2}}$$

8.2 Using an L defined by two points P_1, P_2

With $P = (x_0, y_0)$, $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$.

$$d = \frac{|(x_2 - x_1)(y_1 - y_0) - (x_1 - x_0)(y_2 - y_1)|}{\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}}$$

8.3 Using an L defined by a point P_l and angle θ

$$d = |\cos(\theta)(P_{ly} - y_p) - \sin(\theta)(P_{lx} - P_x)|$$

Chapter 9

Angle between two lines

lines

2

lines

Chapter 10

Intersection of two lines

[Redacted content]

3

lines

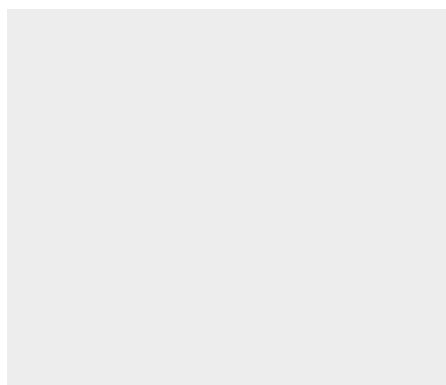
lines



Chapter 11

Line equidistant from two points

lines



4

Figure 11.1:

5

Let's name this line L . From the previous chapter we know how to get the line that's created by the two points M and N . If only we knew how to get a perpendicular line over the midpoint of a line segment!

Thankfully that midpoint also satisfies L 's equation, $ax + by + c$. The midpoint's coordinates are intuitively:

$$\left(\frac{x_M + x_N}{2}, \frac{y_M + y_N}{2} \right)$$

Putting them into the equation we can generate a triple of (a', b', c') and then normalize it to get L .

lines

Chapter 12

Normal to a line through a point

lines

6

lines

Part III

Points And Line Segments

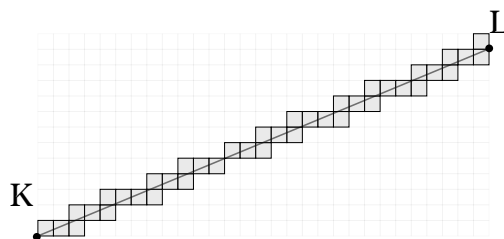
segments

segments

Chapter 13

Drawing a line segment from its two endpoints

For any line segment with any slope, pixels must be matched with the infinite amount of points contained in the segment. As shown in the following figure, a segment *touches* some pixels; we could fill them using an algorithm and get a bitmap of the line segment.



The algorithm presented here was first derived by Bresenham. In the *Image* implementation, it is used in the `plot_line_width` method.

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64)) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();
```

segments

```

sx = if dx > 0 { 1 } else { -1 };
dy = y2 - y1;
ay = (dy * 2).abs();
sy = if dy > 0 { 1 } else { -1 };
x = x1;
y = y1;

let b = dx / dy;
let a = 1;
let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;
let delta = double_d / 2;

if ax > ay {
  d = ay - ax / 2;
  loop {
    self.plot(x, y);
    if x == x2 {
      return;
    }
    if d >= 0 {
      y = y + sy;
      d = d - ax;
    }
    x = x + sx;
    d = d + ay;
  }
} else {
  d = ax - ay / 2;
  let delta = double_d / 3;
  loop {
    self.plot(x, y);
    if y == y2 {
      return;
    }
    if d >= 0 {
      x = x + sx;
      d = d - ay;
    }
    y = y + sy;
    d = d + ax;
  }
}
}
```

Chapter 14

Drawing line segments with width

```
pub fn plot_line_width(&mut self, (x1, y1): (i64, i64), (x2, y2): (i64, i64), _wd: f64) {  
    /* Bresenham's line algorithm */  
    let mut d;  
    let mut x: i64;  
    let mut y: i64;  
    let ax: i64;  
    let ay: i64;  
    let sx: i64;  
    let sy: i64;  
    let dx: i64;  
    let dy: i64;  
  
    dx = x2 - x1;  
    ax = (dx * 2).abs();  
    sx = if dx > 0 { 1 } else { -1 };  
  
    dy = y2 - y1;  
    ay = (dy * 2).abs();  
    sy = if dy > 0 { 1 } else { -1 };  
  
    x = x1;  
    y = y1;  
  
    let b = dx / dy;  
    let a = 1;  
    let double_d = (_wd * f64::sqrt((a * a + b * b) as f64)) as i64;  
    let delta = double_d / 2;  
  
    if ax > ay {  
        d = ay - ax / 2;  
        loop {  
            self.plot(x, y);  
            {  
                let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;  
                let mut _x = x;  
                loop {  
                    let t = total(_x);  
                    if t < -1 * delta || t > delta {  
                        break;  
                    }  
                    _x += 1;  
                    self.plot(_x, y);  
                }  
                let mut _x = x;  
                loop {  
                    let t = total(_x);  
                    if t < -1 * delta || t > delta {  
                        break;  
                    }  
                    _x -= 1;  
                    self.plot(_x, y);  
                }  
            }  
        }  
    }  
}
```

segments

segments

```
        if x == x2 {
            return;
        }
        if d >= 0 {
            y = y + sy;
            d = d - ax;
        }
        x = x + sx;
        d = d + ay;
    }
} else {
    d = ax - ay / 2;
    let delta = double_d / 3;
    loop {
        self.plot(x, y);
        {
            let total = |_x| _x - (y * dx) / dy + (y1 * dx) / dy - x1;
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x += 1;
                self.plot(_x, y);
            }
            let mut _x = x;
            loop {
                let t = total(_x);
                if t < -1 * delta || t > delta {
                    break;
                }
                _x -= 1;
                self.plot(_x, y);
            }
        }
    }
}
if y == y2 {
    return;
}
if d >= 0 {
    x = x + sx;
    d = d - ay;
}
y = y + sy;
d = d + ax;
}
}
```

Chapter 15

Intersection of two line segments

Let points **1** = (x_1, y_1) , **2** = (x_2, y_2) , **3** = (x_3, y_3) and **4** = (x_4, y_4) and **1,2, 3,4** two line segments they form. We wish to find their intersection:

First, get the equation of line L_{12} and line L_{34} from chapter *Equations of a line*.

Substitute points **3** and **4** in equation L_{12} to compute $r_3 = L_{12}(\mathbf{3})$ and $r_4 = L_{12}(\mathbf{4})$ respectively.

If $r_3 \neq 0, r_4 \neq 0$ and $\text{sgn}(r_3) == \text{sign}(r_4)$ the line segments don't intersect, so stop.

In L_{34} substitute point **1** to compute r_1 , and do the same for point **2**.

If $r_1 \neq 0, r_2 \neq 0$ and $\text{sgn}(r_1) == \text{sign}(r_2)$ the line segments don't intersect, so stop.

At this point, L_{12} and L_{34} either intersect or are equivalent. Find their intersection point. (Refer to *Intersection of two lines*.)

15.1 Fast intersection of two line segments



[Redacted text block 1]

[Redacted text block 2]

[Redacted text block 3]

[Redacted text block 4]



Part IV

Points, Lines and Circles

circles

[Redacted text block 1]

[Redacted text block 2]

[Redacted text block 3]

[Redacted text block 4]

[Redacted text block]

Chapter 16

Equations of a circle

1. Find the equation of the circle with centre $(-2, 3)$ and radius 5.

2. Find the equation of the circle with centre $(4, -1)$ and radius 3.

3. Find the equation of the circle with centre $(-5, 2)$ and radius 4.

4. Find the equation of the circle with centre $(1, -4)$ and radius 2.

5. Find the equation of the circle with centre $(-3, 1)$ and radius 6.

6. Find the equation of the circle with centre $(2, -5)$ and radius 1.

7. Find the equation of the circle with centre $(-1, 0)$ and radius 3.

8. Find the equation of the circle with centre $(0, 4)$ and radius 2.

9. Find the equation of the circle with centre $(3, -2)$ and radius 4.

10. Find the equation of the circle with centre $(-4, 1)$ and radius 5.

9

11. Find the equation of the circle with centre $(-6, 3)$ and radius 2.

12. Find the equation of the circle with centre $(1, -3)$ and radius 4.

13. Find the equation of the circle with centre $(-2, 4)$ and radius 1.

14. Find the equation of the circle with centre $(5, -1)$ and radius 3.

15. Find the equation of the circle with centre $(-3, -2)$ and radius 5.

16. Find the equation of the circle with centre $(2, 1)$ and radius 2.

17. Find the equation of the circle with centre $(-1, -4)$ and radius 3.

18. Find the equation of the circle with centre $(4, 0)$ and radius 1.

19. Find the equation of the circle with centre $(-5, -3)$ and radius 4.

20. Find the equation of the circle with centre $(0, -2)$ and radius 5.

21. Find the equation of the circle with centre $(-7, 1)$ and radius 2.

22. Find the equation of the circle with centre $(3, -4)$ and radius 1.

23. Find the equation of the circle with centre $(-2, -5)$ and radius 3.

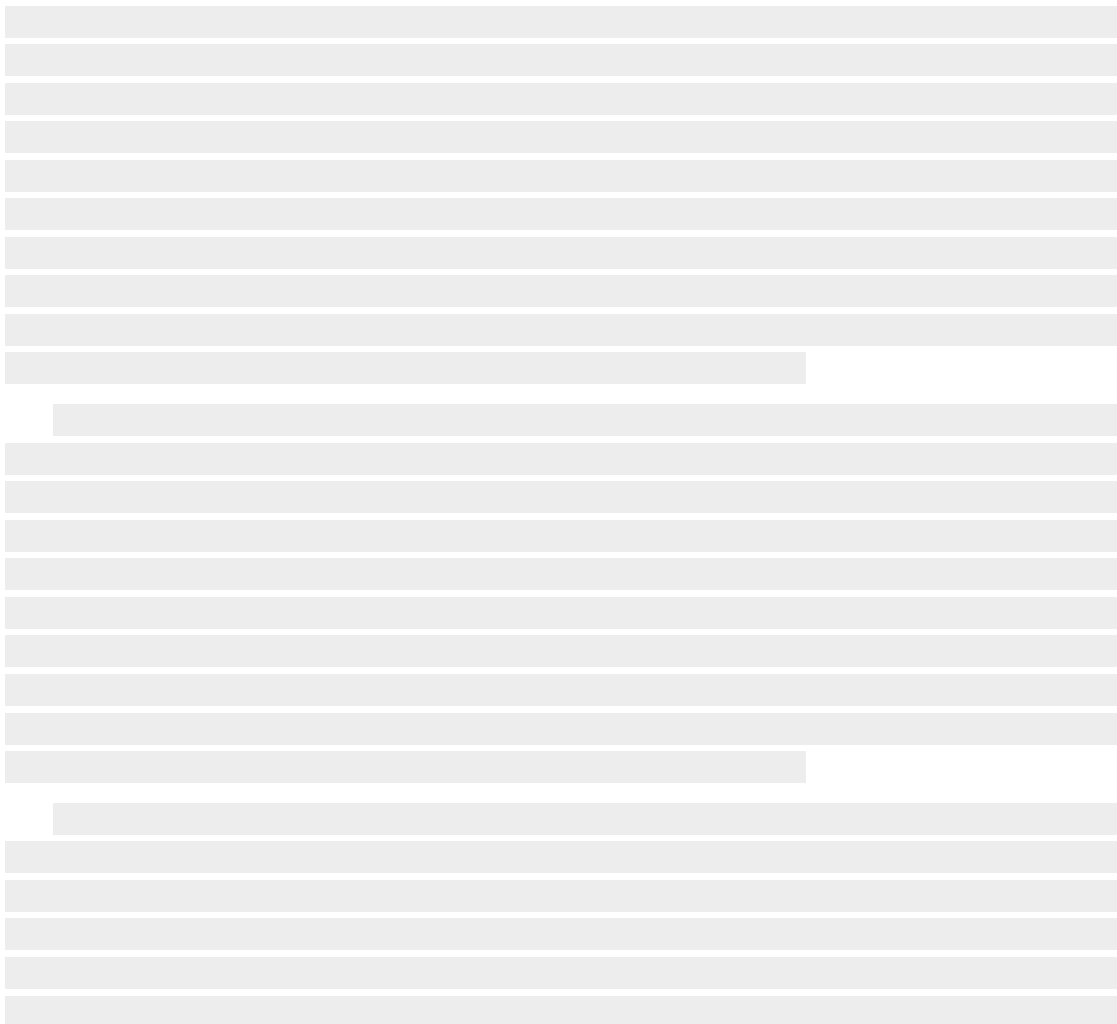
24. Find the equation of the circle with centre $(1, 2)$ and radius 4.

25. Find the equation of the circle with centre $(-4, -1)$ and radius 2.

circles

Chapter 17

Bounding circle



10

circles

circles

[Redacted text block]

[Redacted text block]

[Redacted text block]

Part V

Curves other than circles

curves

Chapter 18

Parametric elliptical arcs

11

curves

[Redacted text block]

[Redacted text block]

[Redacted text block]

Part VI

Points, Lines and Shapes

shapes

Chapter 19

Union, intersection and difference of polygons

12

shapes

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes the need for transparency and accountability in financial reporting.

2. The second part of the document outlines the various methods and techniques used to collect and analyze data. It includes a detailed description of the experimental procedures and the statistical analysis performed.

3. The third part of the document presents the results of the study and discusses the implications of the findings. It compares the results with previous research and provides a comprehensive analysis of the data.

Chapter 20

Centroid of polygon



13

shapes

Chapter 21

Flood filling



14

shapes

[Redacted text block]

[Redacted text block]

[Redacted text block]

15

[Redacted text block]

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1. The first section of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes the need for transparency and accountability in financial reporting.

2. The second section outlines the various methods used to collect and analyze data. It includes a detailed description of the experimental procedures and the statistical techniques employed to interpret the results.

3. The third section presents the findings of the study. It provides a comprehensive overview of the data collected and the conclusions drawn from the analysis. The results are presented in a clear and concise manner, supported by relevant evidence.

4. The fourth section discusses the implications of the findings and their potential applications. It explores the broader context of the research and its contribution to the field. The authors also address the limitations of the study and suggest areas for future research.

5. The final section provides a summary of the key points discussed throughout the document. It reiterates the main findings and the overall conclusions of the study. The authors express their gratitude to the funding agencies and the participants who made the research possible.



Part VII

Vectors, matrices and transformations

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forma-
tions

Chapter 22

Rotation of a bitmap

$$p' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

$$c = \cos\theta, s = \sin\theta, x_{p'} = x_p c - y_p s, y_{p'} = x_p s + y_p c.$$

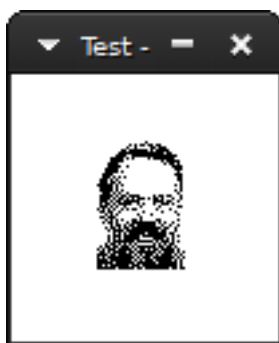
Let's load an xface. We will use `bits_to_bytes` (See Introduction).

```
include!("dmr.rs");
const WINDOW_WIDTH: usize = 100;
const WINDOW_HEIGHT: usize = 100;
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

src/bin/rotation.rs:



This code file is a PDF attachment



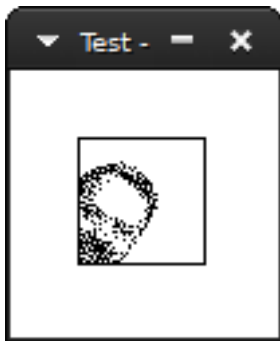
This is the xface of dmr. Instead of displaying the bitmap, this time we will rotate it 0.5 radians. Setup our image first:

```
let mut image = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
image.draw_outline();
let dmr = bits_to_bytes(DMR_BITS, DMR_WIDTH);
```

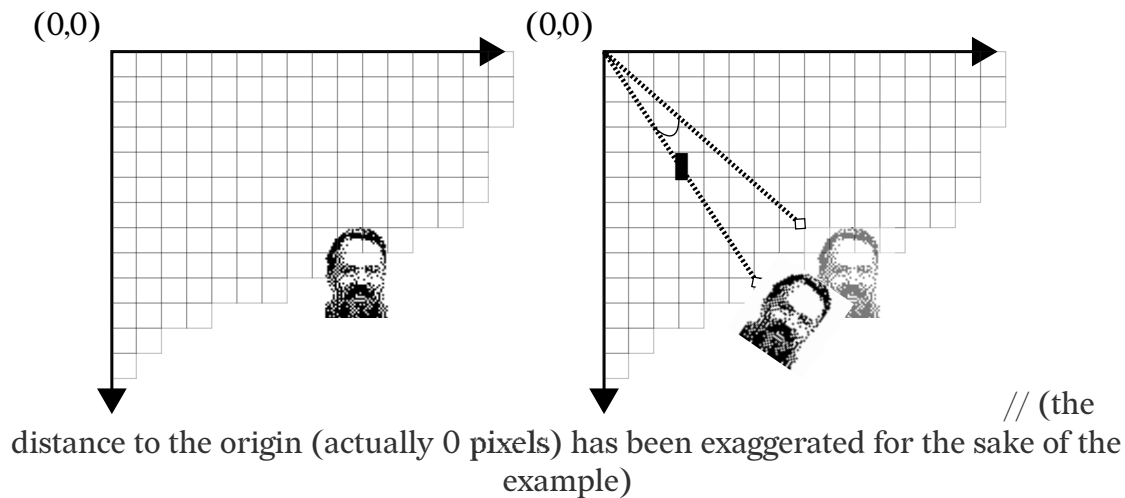
And then, loop for each byte in dmr's face and apply the rotation transformation.

```
let angle = 0.5;
let c = f64::cos(angle);
let s = f64::sin(angle);
for y in 0..DMR_HEIGHT {
    for x in 0..DMR_WIDTH {
        if dmr[y * DMR_WIDTH + x] == BLACK {
            let x = x as f64;
            let y = y as f64;
            let xr = x * c - y * s;
            let yr = x * s + y * c;
            image.plot(xr as i64, yr as i64);
        }
    }
}
```

The result:



We didn't mention in the beginning that the rotation has to be relative to a *point* and the given transformation is relative to the *origin*, in this case the upper left corner (0,0). So dmr was rotated relative to the origin:



Usually, we want to rotate something relative to itself. The right point to choose is the *centroid* of the object.

If we have a list of n points, the centroid is calculated as:

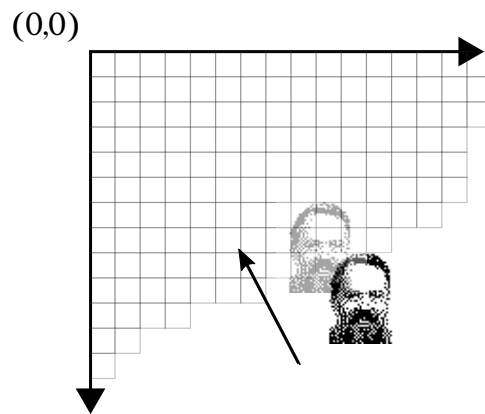
$$x_c = \frac{1}{n} \sum_{i=0}^n x_i$$

$$y_c = \frac{1}{n} \sum_{i=0}^n y_i$$

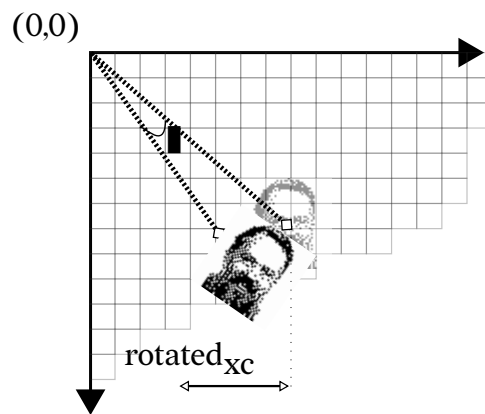
Since in this case we have a rectangle, the centroid has coordinates of half the width and half the height.

By subtracting the centroid from each point before we apply the transformation and then adding it back after we get what we want:

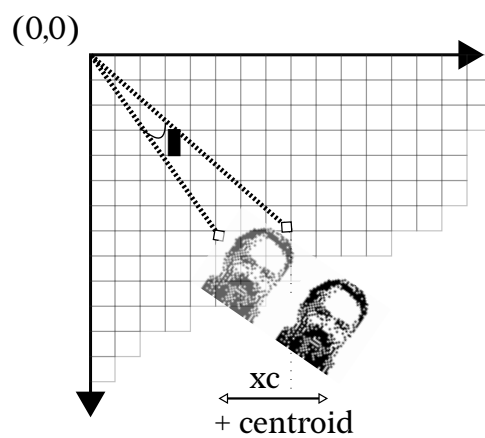
Here's it visually: First subtract the center point.



Then, rotate.

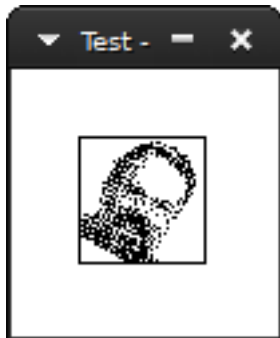


And subtract back to the original position.



In code:

```
let center_point = ((DMR_WIDTH/2) as i64, (DMR_HEIGHT/2) as i64);
for y in 0..DMR_HEIGHT {
  for x in 0..DMR_WIDTH {
    if dmr[y * DMR_WIDTH + x] == BLACK {
      let x = (x as i64 - center_point.0) as f64;
      let y = (y as i64 - center_point.1) as f64;
      let xr = x * c - y * s;
      let yr = x * s + y * c;
      image.plot(xr as i64 + center_point.0,
                 yr as i64 + center_point.1);
    }
  }
}
```



The result:

22.1 Fast 2D Rotation

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Chapter 23

90° Rotation of a bitmap by parallel recursive subdivision



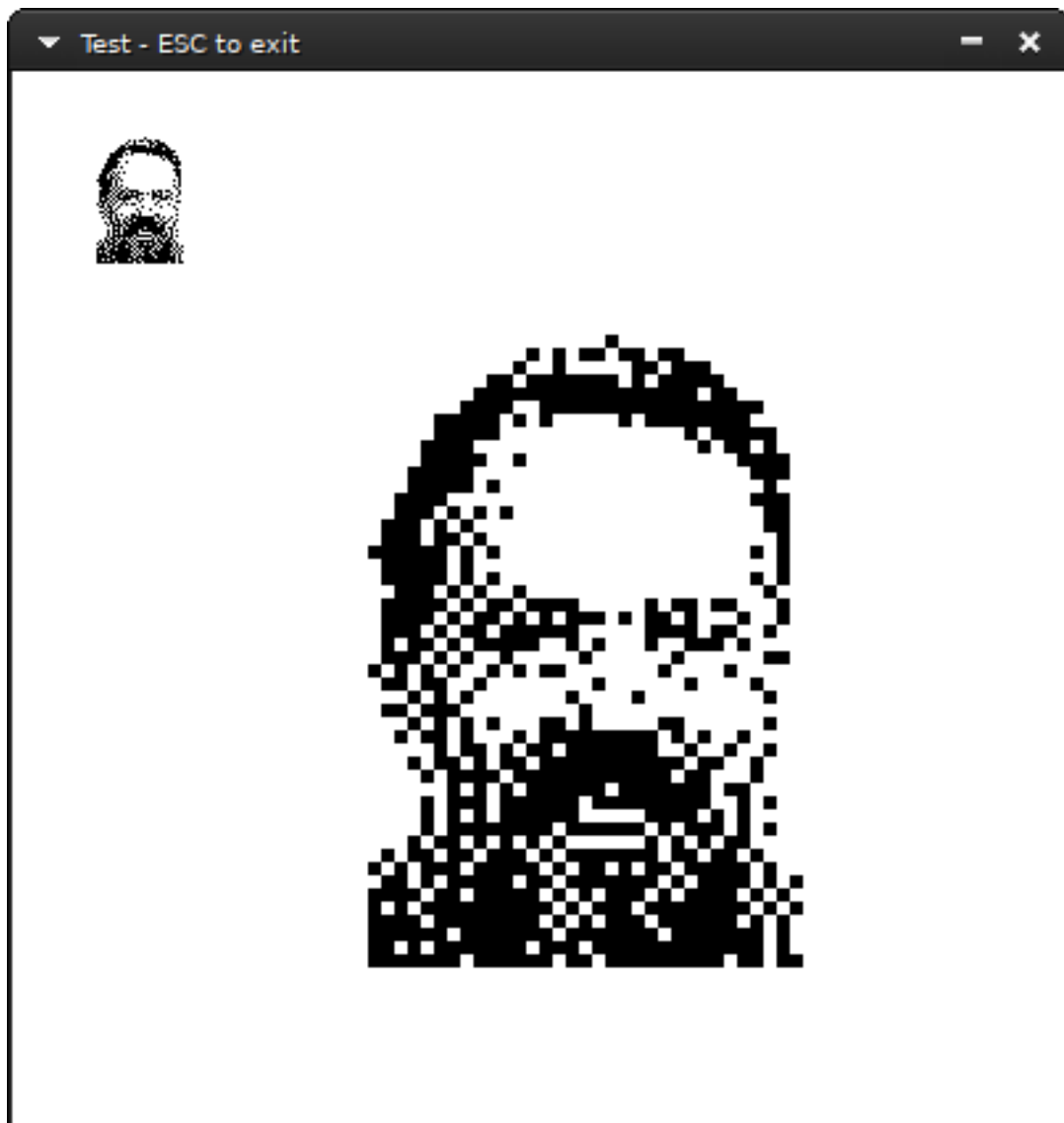
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Chapter 24

Magnification/Scaling



```
let mut original = Image::new(DMR_WIDTH, DMR_HEIGHT, 25, 25);
original.bytes = bits_to_bytes(DMR_BITS, DMR_WIDTH);
original.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);

let mut scaled = Image::new(DMR_WIDTH * 5, DMR_HEIGHT * 5, 100, 100);
let mut sx: i64; //source
let mut sy: i64; //source
let mut dx: i64; //destination
let mut dy: i64 = 0; //destination

let og_height = original.height as i64;
let og_width = original.width as i64;
let scaled_height = scaled.height as i64;
let scaled_width = scaled.width as i64;

while dy < scaled_height {
    sy = (dy * og_height) / scaled_height;
    dx = 0;
    while dx < scaled_width {
        sx = (dx * og_width) / scaled_width;
        if original.get(sx, sy) == Some(BLACK) {
            scaled.plot(dx, dy);
        }
        dx += 1;
    }
    dy += 1;
}
scaled.draw(&mut buffer, BLACK, None, WINDOW_WIDTH);
```

src/bin/scale.rs:



This code file is a PDF attachment

24.1 Smoothing enlarged bitmaps

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The first part of the chapter discusses the basic concepts of transformations and how they can be used to create complex shapes. It covers topics such as translation, rotation, scaling, and shearing. The second part of the chapter focuses on the application of these transformations in computer graphics, including how they are used to animate objects and create realistic simulations.

The third part of the chapter introduces the concept of affine transformations, which are a subset of linear transformations that preserve the straightness of lines. It discusses how affine transformations can be used to map points from one coordinate system to another and how they can be used to create perspective projections.

The final part of the chapter discusses the application of transformations in computer graphics, including how they are used to animate objects and create realistic simulations. It covers topics such as the use of transformations in 2D and 3D graphics, and how they can be used to create complex, multi-layered images.

24.2 Stretching lines of bitmaps

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This section discusses the process of stretching lines of bitmaps, which is a common technique used in computer graphics to create smooth, continuous images. It covers topics such as the use of bilinear interpolation to stretch lines and how this technique can be used to create high-quality, anti-aliased images.

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Chapter 25

Mirroring

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Chapter 26

Shearing

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Chapter 27

Projections

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Part VIII

Advanced

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27.1 Faster Drawing a line segment from its two endpoints using Symmetry

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Chapter 28

Joining the ends of two wide line segments together



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Chapter 29

Composing monochrome bitmaps with separate alpha channel data

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Chapter 30

Orthogonal connection of two points

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Chapter 31

Join segments with round corners



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Chapter 32

Faster line clipping



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Chapter 33

Space-filling Curves



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33.1 Hilbert curves

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33.2 Peano curves

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33.3 Z-order curves



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About this text

The text has been typeset in \LaTeX using the book class and:

- **Redaction** for the main text.
- **Fira Sans** for referring to the programming language **Rust**.
- **Redaction20** for referring to the words bitmap and pixels as a concept.