In[1]:=
```
dSdteqn[t_] = c - β S[t] II[t] - μS S[t] - D[S[t], t]
dEEdteqn[t_] = (1 - ϵ) β S[t] II[t] - δ EE[t] - μE EE[t] - D[EE[t], t]
dIIdteqn[t_] = ϵ β S[t] II[t] + δ EE[t] - μI II[t] - D[II[t], t]
```

Out[1]= $c - \mu S\, S[t] - \beta\, II[t]\, S[t] - S'[t]$

Out[2]= $-\delta\, EE[t] - \mu E\, EE[t] + \beta\, (1 - \epsilon)\, II[t]\, S[t] - EE'[t]$

Out[3]= $\delta\, EE[t] - \mu I\, II[t] + \beta \epsilon\, II[t]\, S[t] - II'[t]$

In[4]:=
```
yEeqn[t_] = kE EE[t] - yE[t]
yIeqn[t_] = kI II[t] - yI[t]
```

Out[4]= $kE\, EE[t] - yE[t]$

Out[5]= $kI\, II[t] - yI[t]$

In[6]:=
```
yEmap[t_] = Solve[yEeqn[t] == 0, EE[t]][[1]]
yImap[t_] = Solve[yIeqn[t] == 0, II[t]][[1]]
```

Out[6]= $\left\{EE[t] \to \frac{yE[t]}{kE}\right\}$

Out[7]= $\left\{II[t] \to \frac{yI[t]}{kI}\right\}$

In[8]:=
```
dSdteqn2[t_] = dSdteqn[t] /. yEmap[t] /. D[yEmap[t], t] /. yImap[t] /. D[yImap[t], t]
dEEdteqn2[t_] =
 dEEdteqn[t] /. yEmap[t] /. D[yEmap[t], t] /. yImap[t] /. D[yImap[t], t]
dIIdteqn2[t_] = dIIdteqn[t] /. yEmap[t] /. D[yEmap[t], t] /. yImap[t] /. D[yImap[t], t]
```

Out[8]= $c - \mu S\, S[t] - \frac{\beta\, S[t]\, yI[t]}{kI} - S'[t]$

Out[9]= $-\frac{\delta\, yE[t]}{kE} - \frac{\mu E\, yE[t]}{kE} + \frac{\beta\, (1 - \epsilon)\, S[t]\, yI[t]}{kI} - \frac{yE'[t]}{kE}$

Out[10]= $\frac{\delta\, yE[t]}{kE} - \frac{\mu I\, yI[t]}{kI} + \frac{\beta \epsilon\, S[t]\, yI[t]}{kI} - \frac{yI'[t]}{kI}$

In[11]:=
```
Smap[t_] = Solve[dIIdteqn2[t] == 0, S[t]][[1]]
```

Out[11]= $\left\{S[t] \to \frac{-kI\, \delta\, yE[t] + kE\, \mu I\, yI[t] + kE\, yI'[t]}{kE\, \beta \epsilon\, yI[t]}\right\}$

In[12]:=
```
TeXForm[Simplify[(-kI δ yE[t] + kE μI yI[t] + kE yI'[t]) / (kE β ϵ yI[t])]]
```

Out[12]//TeXForm=
```
\frac{\text{kE} \left(\text{yI}'(t)+\text{$\mu $I} \text{yI}(t)\right)-\delta  \text{kI}
   \text{yE}(t)}{\beta  \text{kE} \epsilon  \text{yI}(t)}
```

In[13]:=
```
dSdteqn3[t_] = dSdteqn2[t] /. Smap[t] /. D[Smap[t], t]
dEEdteqn3[t_] = dEEdteqn2[t] /. Smap[t] /. D[Smap[t], t]
```

Out[13]=
$$\mathbf{c} - \frac{-\mathrm{kI}\,\delta\,\mathrm{yE}[t] + \mathrm{kE}\,\mu\mathrm{I}\,\mathrm{yI}[t] + \mathrm{kE}\,\mathrm{yI}'[t]}{\mathrm{kE}\,\mathrm{kI}\,\epsilon} - \frac{\mu\mathbf{S}\,(-\mathrm{kI}\,\delta\,\mathrm{yE}[t] + \mathrm{kE}\,\mu\mathrm{I}\,\mathrm{yI}[t] + \mathrm{kE}\,\mathrm{yI}'[t])}{\mathrm{kE}\,\beta\,\epsilon\,\mathrm{yI}[t]} + \frac{\mathrm{yI}'[t]\,(-\mathrm{kI}\,\delta\,\mathrm{yE}[t] + \mathrm{kE}\,\mu\mathrm{I}\,\mathrm{yI}[t] + \mathrm{kE}\,\mathrm{yI}'[t])}{\mathrm{kE}\,\beta\,\epsilon\,\mathrm{yI}[t]^2} - \frac{-\mathrm{kI}\,\delta\,\mathrm{yE}'[t] + \mathrm{kE}\,\mu\mathrm{I}\,\mathrm{yI}'[t] + \mathrm{kE}\,\mathrm{yI}''[t]}{\mathrm{kE}\,\beta\,\epsilon\,\mathrm{yI}[t]}$$

Out[14]=
$$-\frac{\delta\,\mathrm{yE}[t]}{\mathrm{kE}} - \frac{\mu\mathrm{E}\,\mathrm{yE}[t]}{\mathrm{kE}} - \frac{\mathrm{yE}'[t]}{\mathrm{kE}} + \frac{(1-\epsilon)\,(-\mathrm{kI}\,\delta\,\mathrm{yE}[t] + \mathrm{kE}\,\mu\mathrm{I}\,\mathrm{yI}[t] + \mathrm{kE}\,\mathrm{yI}'[t])}{\mathrm{kE}\,\mathrm{kI}\,\epsilon}$$

In[15]:=
```
IPOPish1[t_] = Denominator[Together[dSdteqn3[t]]] Together[dSdteqn3[t]]
IPOPish2[t_] = Denominator[Together[dEEdteqn3[t]]] Together[dEEdteqn3[t]]
```

Out[15]=
$$\mathrm{kI}^2\,\delta\,\mu\mathbf{S}\,\mathrm{yE}[t]\,\mathrm{yI}[t] + \mathbf{c}\,\mathrm{kE}\,\mathrm{kI}\,\beta\,\epsilon\,\mathrm{yI}[t]^2 - \mathrm{kE}\,\mathrm{kI}\,\mu\mathrm{I}\,\mu\mathbf{S}\,\mathrm{yI}[t]^2 + \mathrm{kI}\,\beta\,\delta\,\mathrm{yE}[t]\,\mathrm{yI}[t]^2 - \mathrm{kE}\,\beta\,\mu\mathrm{I}\,\mathrm{yI}[t]^3 + \mathrm{kI}^2\,\delta\,\mathrm{yI}[t]\,\mathrm{yE}'[t] - \mathrm{kI}^2\,\delta\,\mathrm{yE}[t]\,\mathrm{yI}'[t] - \mathrm{kE}\,\mathrm{kI}\,\mu\mathbf{S}\,\mathrm{yI}[t]\,\mathrm{yI}'[t] - \mathrm{kE}\,\beta\,\mathrm{yI}[t]^2\,\mathrm{yI}'[t] + \mathrm{kE}\,\mathrm{kI}\,\mathrm{yI}'[t]^2 - \mathrm{kE}\,\mathrm{kI}\,\mathrm{yI}[t]\,\mathrm{yI}''[t]$$

Out[16]=
$$-\mathrm{kI}\,\delta\,\mathrm{yE}[t] - \mathrm{kI}\,\epsilon\,\mu\mathrm{E}\,\mathrm{yE}[t] + \mathrm{kE}\,\mu\mathrm{I}\,\mathrm{yI}[t] - \mathrm{kE}\,\epsilon\,\mu\mathrm{I}\,\mathrm{yI}[t] - \mathrm{kI}\,\epsilon\,\mathrm{yE}'[t] + \mathrm{kE}\,\mathrm{yI}'[t] - \mathrm{kE}\,\epsilon\,\mathrm{yI}'[t]$$

In[47]:=
```
TeXForm[IPOPish2[t]]
```

Out[47]//TeXForm=
```
-\text{kE} \epsilon  \text{yI}'(t)+\text{kE} \text{yI}'(t)+\text{kE} \text{$\mu $I} \text{
   \text{yE}'(t)+\delta  (-\text{kI}) \text{yE}(t)-\text{kI} \text{$\mu $E} \epsilon  \tex
```

In[17]:=
```
Monos1 = Sort[
  MonomialList[IPOPish1[t], {yI[t], yI'[t], yI''[t], yE[t], yE'[t], yE''[t]}]]
Monos2 = Sort[MonomialList[IPOPish2[t],
   {yI[t], yI'[t], yI''[t], yE[t], yE'[t], yE''[t]}]]
```

Out[17]=
$$\{\mathrm{kI}^2\,\delta\,\mu\mathbf{S}\,\mathrm{yE}[t]\,\mathrm{yI}[t],\ (\mathbf{c}\,\mathrm{kE}\,\mathrm{kI}\,\beta\,\epsilon - \mathrm{kE}\,\mathrm{kI}\,\mu\mathrm{I}\,\mu\mathbf{S})\,\mathrm{yI}[t]^2,\ \mathrm{kI}\,\beta\,\delta\,\mathrm{yE}[t]\,\mathrm{yI}[t]^2,\ -\mathrm{kE}\,\beta\,\mu\mathrm{I}\,\mathrm{yI}[t]^3,\ \mathrm{kI}^2\,\delta\,\mathrm{yI}[t]\,\mathrm{yE}'[t],\ -\mathrm{kI}^2\,\delta\,\mathrm{yE}[t]\,\mathrm{yI}'[t],\ -\mathrm{kE}\,\mathrm{kI}\,\mu\mathbf{S}\,\mathrm{yI}[t]\,\mathrm{yI}'[t],\ -\mathrm{kE}\,\beta\,\mathrm{yI}[t]^2\,\mathrm{yI}'[t],\ \mathrm{kE}\,\mathrm{kI}\,\mathrm{yI}'[t]^2,\ -\mathrm{kE}\,\mathrm{kI}\,\mathrm{yI}[t]\,\mathrm{yI}''[t]\}$$

Out[18]=
$$\{(-\mathrm{kI}\,\delta - \mathrm{kI}\,\epsilon\,\mu\mathrm{E})\,\mathrm{yE}[t],\ (\mathrm{kE}\,\mu\mathrm{I} - \mathrm{kE}\,\epsilon\,\mu\mathrm{I})\,\mathrm{yI}[t],\ -\mathrm{kI}\,\epsilon\,\mathrm{yE}'[t],\ (\mathrm{kE} - \mathrm{kE}\,\epsilon)\,\mathrm{yI}'[t]\}$$

In[22]:=
```
Last[Monos2]
```

Out[22]=
$$(\mathrm{kE} - \mathrm{kE}\,\epsilon)\,\mathrm{yI}'[t]$$

In[20]:=
```
MonicMonos1 = Monos1 / (Last[Monos1] /.
    {yI[t] → 1, yI'[t] → 1, yI''[t] → 1, yE[t] → 1, yE'[t] → 1, yE''[t] → 1})
MonicMonos2 = Monos2 / (Last[Monos2] /. {yI[t] → 1, yI'[t] → 1,
     yI''[t] → 1, yE[t] → 1, yE'[t] → 1, yE''[t] → 1})
```

Out[20]=
$$\left\{-\frac{\mathrm{kI}\,\delta\,\mu\mathbf{S}\,\mathrm{yE}[t]\,\mathrm{yI}[t]}{\mathrm{kE}},\ -\frac{(\mathbf{c}\,\mathrm{kE}\,\mathrm{kI}\,\beta\,\epsilon - \mathrm{kE}\,\mathrm{kI}\,\mu\mathrm{I}\,\mu\mathbf{S})\,\mathrm{yI}[t]^2}{\mathrm{kE}\,\mathrm{kI}},\ -\frac{\beta\,\delta\,\mathrm{yE}[t]\,\mathrm{yI}[t]^2}{\mathrm{kE}},\ \frac{\beta\,\mu\mathrm{I}\,\mathrm{yI}[t]^3}{\mathrm{kI}},\ -\frac{\mathrm{kI}\,\delta\,\mathrm{yI}[t]\,\mathrm{yE}'[t]}{\mathrm{kE}},\ \frac{\mathrm{kI}\,\delta\,\mathrm{yE}[t]\,\mathrm{yI}'[t]}{\mathrm{kE}},\ \mu\mathbf{S}\,\mathrm{yI}[t]\,\mathrm{yI}'[t],\ \frac{\beta\,\mathrm{yI}[t]^2\,\mathrm{yI}'[t]}{\mathrm{kI}},\ -\mathrm{yI}'[t]^2,\ \mathrm{yI}[t]\,\mathrm{yI}''[t]\right\}$$

Out[21]=
$$\left\{\frac{(-\mathrm{kI}\,\delta - \mathrm{kI}\,\epsilon\,\mu\mathrm{E})\,\mathrm{yE}[t]}{\mathrm{kE} - \mathrm{kE}\,\epsilon},\ \frac{(\mathrm{kE}\,\mu\mathrm{I} - \mathrm{kE}\,\epsilon\,\mu\mathrm{I})\,\mathrm{yI}[t]}{\mathrm{kE} - \mathrm{kE}\,\epsilon},\ -\frac{\mathrm{kI}\,\epsilon\,\mathrm{yE}'[t]}{\mathrm{kE} - \mathrm{kE}\,\epsilon},\ \mathrm{yI}'[t]\right\}$$

In[23]:=
```
Coeffs1 = MonicMonos1 /.
  {yI[t] → 1, yI'[t] → 1, yI''[t] → 1, yE[t] → 1, yE'[t] → 1, yE''[t] → 1}
Coeffs2 = MonicMonos2 /. {yI[t] → 1, yI'[t] → 1,
   yI''[t] → 1, yE[t] → 1, yE'[t] → 1, yE''[t] → 1}
```

Out[23]=
$$\left\{-\frac{kI\,\delta\,\mu S}{kE},\ -\frac{c\,kE\,kI\,\beta\,\epsilon-kE\,kI\,\mu I\,\mu S}{kE\,kI},\ -\frac{\beta\,\delta}{kE},\ \frac{\beta\,\mu I}{kI},\ -\frac{kI\,\delta}{kE},\ \frac{kI\,\delta}{kE},\ \mu S,\ \frac{\beta}{kI},\ -1,\ 1\right\}$$

Out[24]=
$$\left\{\frac{-kI\,\delta-kI\,\epsilon\,\mu E}{kE-kE\,\epsilon},\ \frac{kE\,\mu I-kE\,\epsilon\,\mu I}{kE-kE\,\epsilon},\ -\frac{kI\,\epsilon}{kE-kE\,\epsilon},\ 1\right\}$$

In[25]:=
```
Coeffs = Simplify[Union[Coeffs1, Coeffs2]]
```

Out[25]=
$$\left\{-1,\ 1,\ \frac{\beta}{kI},\ -\frac{kI\,\delta}{kE},\ \frac{kI\,\delta}{kE},\ -\frac{\beta\,\delta}{kE},\ \frac{kI\,\epsilon}{kE\,(-1+\epsilon)},\right.$$
$$\left.\frac{kI\,(\delta+\epsilon\,\mu E)}{kE\,(-1+\epsilon)},\ \frac{\beta\,\mu I}{kI},\ \mu I,\ \mu S,\ -\frac{kI\,\delta\,\mu S}{kE},\ -c\,\beta\,\epsilon+\mu I\,\mu S\right\}$$

In[27]:=
```
TeXForm[Coeffs]
```

Out[27]//TeXForm=
```
\left\{-1,1,\frac{\beta }{\text{kI}},-\frac{\delta  \text{kI}}{\text{kE}},\frac{\delta  \t
   (\delta +\text{$\mu $E} \epsilon )}{\text{kE} (\epsilon -1)},\frac{\beta  \text{$\mu $I
   $S}-\beta  c \epsilon \right\}
```

In[28]:=
```
xCoeffs =
 Coeffs /. {β → a1, δ → a2, ϵ → a3, μS → a4, μE → a5, μI → a6, c → a7, kE → a8, kI → a9}
```

Out[28]=
$$\left\{-1,\ 1,\ \frac{a1}{a9},\ -\frac{a2\,a9}{a8},\ \frac{a2\,a9}{a8},\ -\frac{a1\,a2}{a8},\ \frac{a3\,a9}{(-1+a3)\,a8},\right.$$
$$\left.\frac{(a2+a3\,a5)\,a9}{(-1+a3)\,a8},\ \frac{a1\,a6}{a9},\ a6,\ a4,\ -\frac{a2\,a4\,a9}{a8},\ a4\,a6-a1\,a3\,a7\right\}$$

In[29]:=
```
Solve[Coeffs == xCoeffs, {β, δ, ϵ, μS, μE, μI, c, kE, kI}]
```

MessageTemplate[Solve, svars, Equations may not give solutions for all "solve" variables., 2, 29, 1, 33627853190062898696, Local]

Out[29]=
$$\left\{\left\{\beta\to\frac{a1\,kI}{a9},\ \delta\to\frac{a2\,(-1+a3)\,\epsilon}{a3\,(-1+\epsilon)},\ \mu S\to a4,\right.\right.$$
$$\left.\left.\mu E\to\frac{-a2\,a3-a3\,a5+a2\,\epsilon+a3\,a5\,\epsilon}{a3\,(-1+\epsilon)},\ \mu I\to a6,\ c\to\frac{a3\,a7\,a9}{kI\,\epsilon},\ kE\to\frac{(-1+a3)\,a8\,kI\,\epsilon}{a3\,a9\,(-1+\epsilon)}\right\}\right\}$$