```
dSdteqn[a_] = - β S[0, a] II[0, a] / (S[0, a] + EE[0, a] + II[0, a]) -
  μS S[0, a] - (D[S[t, a], t] /. t → 0) - D[S[0, a], a]
dEEdteqn[a_] = (1 - ϵ) β S[0, a] II[0, a] / (S[0, a] + EE[0, a] + II[0, a]) -
  δ EE[0, a] - μE EE[0, a] - (D[EE[t, a], t] /. t → 0) - D[EE[0, a], a]
dIIdteqn[a_] = ϵ β S[0, a] II[0, a] / (S[0, a] + EE[0, a] + II[0, a]) +
  δ EE[0, a] - μI II[0, a] - (D[II[t, a], t] /. t → 0) - D[II[0, a], a]
```

$$-\mu S \, S[0, a] - \frac{\beta \, II[0, a] \, S[0, a]}{EE[0, a] + II[0, a] + S[0, a]} - S^{(0,1)}[0, a] - S^{(1,0)}[0, a]$$

$$-\delta \, EE[0, a] - \mu E \, EE[0, a] + \frac{\beta \, (1 - \epsilon) \, II[0, a] \, S[0, a]}{EE[0, a] + II[0, a] + S[0, a]} - EE^{(0,1)}[0, a] - EE^{(1,0)}[0, a]$$

$$\delta \, EE[0, a] - \mu I \, II[0, a] + \frac{\beta \, \epsilon \, II[0, a] \, S[0, a]}{EE[0, a] + II[0, a] + S[0, a]} - II^{(0,1)}[0, a] - II^{(1,0)}[0, a]$$

```
yEeqn[t_, a_] = kE EE[t, a] - yE[t, a]
yIeqn[t_, a_] = kI II[t, a] - yI[t, a]
Seqn[t_, a_] = S[t, a] + EE[t, a] + II[t, a] - Ntot[t, a]
```

kE EE[t, a] - yE[t, a]

kI II[t, a] - yI[t, a]

EE[t, a] + II[t, a] - Ntot[t, a] + S[t, a]

```
yEmap[t_, a_] = Solve[yEeqn[t, a] == 0, EE[t, a]][[1]]
yImap[t_, a_] = Solve[yIeqn[t, a] == 0, II[t, a]][[1]]
Smap[t_, a_] = Solve[Seqn[t, a] == 0, S[t, a]][[1]]
```

$$\left\{ EE[t, a] \to \frac{yE[t, a]}{kE} \right\}$$

$$\left\{ II[t, a] \to \frac{yI[t, a]}{kI} \right\}$$

{S[t, a] → -EE[t, a] - II[t, a] + Ntot[t, a]}

```
dSdteqn2[a_] =
 dSdteqn[a] /. (Smap[t, a] /. t → 0) /. (D[Smap[t, a], t] /. t → 0) /. (D[Smap[t, a], a] /.
        t → 0) /. (yEmap[t, a] /. t → 0) /. (D[yEmap[t, a], t] /. t → 0) /.
     (D[yEmap[t, a], a] /. t → 0) /. (yImap[t, a] /. t → 0) /.
    (D[yImap[t, a], t] /. t → 0) /. (D[yImap[t, a], a] /. t → 0)
dEEdteqn2[a_] = dEEdteqn[a] /. (Smap[t, a] /. t → 0) /. (D[Smap[t, a], t] /. t → 0) /.
        (D[Smap[t, a], a] /. t → 0) /. (yEmap[t, a] /. t → 0) /.
      (D[yEmap[t, a], t] /. t → 0) /. (D[yEmap[t, a], a] /. t → 0) /.
     (yImap[t, a] /. t → 0) /. (D[yImap[t, a], t] /. t → 0) /. (D[yImap[t, a], a] /. t → 0)
dIIdteqn2[a_] = dIIdteqn[a] /. (Smap[t, a] /. t → 0) /. (D[Smap[t, a], t] /. t → 0) /.
        (D[Smap[t, a], a] /. t → 0) /. (yEmap[t, a] /. t → 0) /.
      (D[yEmap[t, a], t] /. t → 0) /. (D[yEmap[t, a], a] /. t → 0) /.
     (yImap[t, a] /. t → 0) /. (D[yImap[t, a], t] /. t → 0) /. (D[yImap[t, a], a] /. t → 0)
```

$$-\mu S\left(\text{Ntot}[0, a] - \frac{yE[0, a]}{kE} - \frac{yI[0, a]}{kI}\right) -$$

$$\frac{\beta\, yI[0, a]\left(\text{Ntot}[0, a] - \frac{yE[0,a]}{kE} - \frac{yI[0,a]}{kI}\right)}{kI\,\text{Ntot}[0, a]} - \text{Ntot}^{(0,1)}[0, a] + \frac{yE^{(0,1)}[0, a]}{kE} +$$

$$\frac{yI^{(0,1)}[0, a]}{kI} - \text{Ntot}^{(1,0)}[0, a] + \frac{yE^{(1,0)}[0, a]}{kE} + \frac{yI^{(1,0)}[0, a]}{kI}$$

$$-\frac{\delta\, yE[0, a]}{kE} - \frac{\mu E\, yE[0, a]}{kE} +$$

$$\frac{\beta\,(1-\epsilon)\, yI[0, a]\left(\text{Ntot}[0, a] - \frac{yE[0,a]}{kE} - \frac{yI[0,a]}{kI}\right)}{kI\,\text{Ntot}[0, a]} - \frac{yE^{(0,1)}[0, a]}{kE} - \frac{yE^{(1,0)}[0, a]}{kE}$$

$$\frac{\delta\, yE[0, a]}{kE} - \frac{\mu I\, yI[0, a]}{kI} +$$

$$\frac{\beta\,\epsilon\, yI[0, a]\left(\text{Ntot}[0, a] - \frac{yE[0,a]}{kE} - \frac{yI[0,a]}{kI}\right)}{kI\,\text{Ntot}[0, a]} - \frac{yI^{(0,1)}[0, a]}{kI} - \frac{yI^{(1,0)}[0, a]}{kI}$$

```
IPOPish1[a_] = Denominator[Together[dSdteqn2[a]]] Together[dSdteqn2[a]]
IPOPish2[a_] = Denominator[Together[dEEdteqn2[a]]] Together[dEEdteqn2[a]]
IPOPish3[a_] = Denominator[Together[dIIdteqn2[a]]] Together[dIIdteqn2[a]]
```

$-kE\,kI^2\,\mu S\,\text{Ntot}[0, a]^2 + kI^2\,\mu S\,\text{Ntot}[0, a]\,yE[0, a] - kE\,kI\,\beta\,\text{Ntot}[0, a]\,yI[0, a] +$
$\quad kE\,kI\,\mu S\,\text{Ntot}[0, a]\,yI[0, a] + kI\,\beta\,yE[0, a]\,yI[0, a] + kE\,\beta\,yI[0, a]^2 -$
$\quad kE\,kI^2\,\text{Ntot}[0, a]\,\text{Ntot}^{(0,1)}[0, a] + kI^2\,\text{Ntot}[0, a]\,yE^{(0,1)}[0, a] +$
$\quad kE\,kI\,\text{Ntot}[0, a]\,yI^{(0,1)}[0, a] - kE\,kI^2\,\text{Ntot}[0, a]\,\text{Ntot}^{(1,0)}[0, a] +$
$\quad kI^2\,\text{Ntot}[0, a]\,yE^{(1,0)}[0, a] + kE\,kI\,\text{Ntot}[0, a]\,yI^{(1,0)}[0, a]$

$-kI^2\,\delta\,\text{Ntot}[0, a]\,yE[0, a] - kI^2\,\mu E\,\text{Ntot}[0, a]\,yE[0, a] + kE\,kI\,\beta\,\text{Ntot}[0, a]\,yI[0, a] -$
$\quad kE\,kI\,\beta\,\epsilon\,\text{Ntot}[0, a]\,yI[0, a] - kI\,\beta\,yE[0, a]\,yI[0, a] + kI\,\beta\,\epsilon\,yE[0, a]\,yI[0, a] -$
$\quad kE\,\beta\,yI[0, a]^2 + kE\,\beta\,\epsilon\,yI[0, a]^2 - kI^2\,\text{Ntot}[0, a]\,yE^{(0,1)}[0, a] - kI^2\,\text{Ntot}[0, a]\,yE^{(1,0)}[0, a]$

$kI^2\,\delta\,\text{Ntot}[0, a]\,yE[0, a] + kE\,kI\,\beta\,\epsilon\,\text{Ntot}[0, a]\,yI[0, a] -$
$\quad kE\,kI\,\mu I\,\text{Ntot}[0, a]\,yI[0, a] - kI\,\beta\,\epsilon\,yE[0, a]\,yI[0, a] - kE\,\beta\,\epsilon\,yI[0, a]^2 -$
$\quad kE\,kI\,\text{Ntot}[0, a]\,yI^{(0,1)}[0, a] - kE\,kI\,\text{Ntot}[0, a]\,yI^{(1,0)}[0, a]$

```
Monos1 = Sort[MonomialList[IPOPish1[a], {yI[0, a], yE[0, a],
     (D[yI[t, a], t] /. t → 0), (D[yI[t, a], a] /. t → 0), (D[yI[t, a], t, a] /. t → 0),
     (D[yI[t, a], {t, 2}] /. t → 0), (D[yI[t, a], {a, 2}] /. t → 0),
     (D[yE[t, a], t] /. t → 0), (D[yE[t, a], a] /. t → 0), (D[yE[t, a], t, a] /. t → 0),
     (D[yE[t, a], {t, 2}] /. t → 0), (D[yE[t, a], {a, 2}] /. t → 0)}]]
Monos2 = Sort[MonomialList[IPOPish2[a], {yI[0, a], yE[0, a],
     (D[yI[t, a], t] /. t → 0), (D[yI[t, a], a] /. t → 0), (D[yI[t, a], t, a] /. t → 0),
     (D[yI[t, a], {t, 2}] /. t → 0), (D[yI[t, a], {a, 2}] /. t → 0),
     (D[yE[t, a], t] /. t → 0), (D[yE[t, a], a] /. t → 0), (D[yE[t, a], t, a] /. t → 0),
     (D[yE[t, a], {t, 2}] /. t → 0), (D[yE[t, a], {a, 2}] /. t → 0)}]]
Monos3 = Sort[MonomialList[IPOPish3[a], {yI[0, a], yE[0, a],
     (D[yI[t, a], t] /. t → 0), (D[yI[t, a], a] /. t → 0), (D[yI[t, a], t, a] /. t → 0),
     (D[yI[t, a], {t, 2}] /. t → 0), (D[yI[t, a], {a, 2}] /. t → 0),
     (D[yE[t, a], t] /. t → 0), (D[yE[t, a], a] /. t → 0), (D[yE[t, a], t, a] /. t → 0),
     (D[yE[t, a], {t, 2}] /. t → 0), (D[yE[t, a], {a, 2}] /. t → 0)}]]
```

$$\left\{ kI^2 \, \mu S \, Ntot[0, a] \, yE[0, a], \; (-kE \, kI \, \beta \, Ntot[0, a] + kE \, kI \, \mu S \, Ntot[0, a]) \, yI[0, a], \right.$$
$$kI \, \beta \, yE[0, a] \, yI[0, a], \; kE \, \beta \, yI[0, a]^2, \; kI^2 \, Ntot[0, a] \, yE^{(0,1)}[0, a],$$
$$kE \, kI \, Ntot[0, a] \, yI^{(0,1)}[0, a], \; -kE \, kI^2 \, \mu S \, Ntot[0, a]^2 -$$
$$kE \, kI^2 \, Ntot[0, a] \, Ntot^{(0,1)}[0, a] - kE \, kI^2 \, Ntot[0, a] \, Ntot^{(1,0)}[0, a],$$
$$\left. kI^2 \, Ntot[0, a] \, yE^{(1,0)}[0, a], \; kE \, kI \, Ntot[0, a] \, yI^{(1,0)}[0, a] \right\}$$

$$\left\{ \left(-kI^2 \, \delta \, Ntot[0, a] - kI^2 \, \mu E \, Ntot[0, a] \right) yE[0, a], \right.$$
$$(kE \, kI \, \beta \, Ntot[0, a] - kE \, kI \, \beta \in Ntot[0, a]) \, yI[0, a], \; (-kI \, \beta + kI \, \beta \in) \, yE[0, a] \, yI[0, a],$$
$$\left. (-kE \, \beta + kE \, \beta \in) \, yI[0, a]^2, \; -kI^2 \, Ntot[0, a] \, yE^{(0,1)}[0, a], \; -kI^2 \, Ntot[0, a] \, yE^{(1,0)}[0, a] \right\}$$

$$\left\{ kI^2 \, \delta \, Ntot[0, a] \, yE[0, a], \; (kE \, kI \, \beta \in Ntot[0, a] - kE \, kI \, \mu I \, Ntot[0, a]) \, yI[0, a], \right.$$
$$-kI \, \beta \in yE[0, a] \, yI[0, a], \; -kE \, \beta \in yI[0, a]^2,$$
$$\left. -kE \, kI \, Ntot[0, a] \, yI^{(0,1)}[0, a], \; -kE \, kI \, Ntot[0, a] \, yI^{(1,0)}[0, a] \right\}$$

```
MonicMonos1 =
 Monos1 / (Last[Monos1] /. {yI[0, a] → 1, yE[0, a] → 1, (D[yI[t, a], t] /. t → 0) → 1,
      (D[yI[t, a], a] /. t → 0) → 1, (D[yI[t, a], t, a] /. t → 0) → 1,
      (D[yI[t, a], {t, 2}] /. t → 0) → 1, (D[yI[t, a], {a, 2}] /. t → 0) → 1,
      (D[yE[t, a], t] /. t → 0) → 1, (D[yE[t, a], a] /. t → 0) → 1,
      (D[yE[t, a], t, a] /. t → 0) → 1, (D[yE[t, a], {t, 2}] /. t → 0) → 1,
      (D[yE[t, a], {a, 2}] /. t → 0) → 1})
MonicMonos2 = Monos2 / (Last[Monos2] /. {yI[0, a] → 1, yE[0, a] → 1,
      (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,
      (D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,
      (D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,
      (D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,
      (D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1})
MonicMonos3 = Monos3 / (Last[Monos3] /. {yI[0, a] → 1, yE[0, a] → 1,
      (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,
      (D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,
      (D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,
      (D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,
      (D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1})
```

$$\left\{ \frac{kI\,\mu S\,yE[0, a]}{kE}, \frac{(-kE\,kI\,\beta\,Ntot[0, a] + kE\,kI\,\mu S\,Ntot[0, a])\,yI[0, a]}{kE\,kI\,Ntot[0, a]}, \right.$$

$$\frac{\beta\,yE[0, a]\,yI[0, a]}{kE\,Ntot[0, a]}, \frac{\beta\,yI[0, a]^2}{kI\,Ntot[0, a]}, \frac{kI\,yE^{(0,1)}[0, a]}{kE}, yI^{(0,1)}[0, a],$$

$$\left(-kE\,kI^2\,\mu S\,Ntot[0, a]^2 - kE\,kI^2\,Ntot[0, a]\,Ntot^{(0,1)}[0, a] - \right.$$

$$\left. kE\,kI^2\,Ntot[0, a]\,Ntot^{(1,0)}[0, a]\right) \Big/ (kE\,kI\,Ntot[0, a]), \frac{kI\,yE^{(1,0)}[0, a]}{kE}, yI^{(1,0)}[0, a]\right\}$$

$$\left\{ -\frac{\left(-kI^2\,\delta\,Ntot[0, a] - kI^2\,\mu E\,Ntot[0, a]\right)\,yE[0, a]}{kI^2\,Ntot[0, a]}, \right.$$

$$-\frac{(kE\,kI\,\beta\,Ntot[0, a] - kE\,kI\,\beta\,\epsilon\,Ntot[0, a])\,yI[0, a]}{kI^2\,Ntot[0, a]}, -\frac{(-kI\,\beta + kI\,\beta\,\epsilon)\,yE[0, a]\,yI[0, a]}{kI^2\,Ntot[0, a]},$$

$$-\frac{(-kE\,\beta + kE\,\beta\,\epsilon)\,yI[0, a]^2}{kI^2\,Ntot[0, a]}, yE^{(0,1)}[0, a], yE^{(1,0)}[0, a]\right\}$$

$$\left\{ -\frac{kI\,\delta\,yE[0, a]}{kE}, -\frac{(kE\,kI\,\beta\,\epsilon\,Ntot[0, a] - kE\,kI\,\mu I\,Ntot[0, a])\,yI[0, a]}{kE\,kI\,Ntot[0, a]}, \right.$$

$$\frac{\beta\,\epsilon\,yE[0, a]\,yI[0, a]}{kE\,Ntot[0, a]}, \frac{\beta\,\epsilon\,yI[0, a]^2}{kI\,Ntot[0, a]}, yI^{(0,1)}[0, a], yI^{(1,0)}[0, a]\right\}$$

```
Coeffs1 = MonicMonos1 /.
  {yI[0, a] → 1, yE[0, a] → 1, (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,
   (D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,
   (D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,
   (D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,
   (D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1,
   Ntot[0, a] → 1, D[Ntot[0, a], a] → 1, (D[Ntot[t, a], t] /. t → 0) → 1}
Coeffs2 = Simplify[MonicMonos2 /. {yI[0, a] → 1, yE[0, a] → 1,
   (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,
   (D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,
   (D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,
   (D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,
   (D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1,
   Ntot[0, a] → 1, D[Ntot[0, a], a] → 1, (D[Ntot[t, a], t] /. t → 0) → 1}]
Coeffs3 = Simplify[MonicMonos3 /. {yI[0, a] → 1, yE[0, a] → 1,
   (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,
   (D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,
   (D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,
   (D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,
   (D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1,
   Ntot[0, a] → 1, D[Ntot[0, a], a] → 1, (D[Ntot[t, a], t] /. t → 0) → 1}]
```

$$\left\{ \frac{kI\,\mu S}{kE}, \; \frac{-kE\,kI\,\beta + kE\,kI\,\mu S}{kE\,kI}, \; \frac{\beta}{kE}, \; \frac{\beta}{kI}, \; \frac{kI}{kE}, \; 1, \; \frac{-2\,kE\,kI^2 - kE\,kI^2\,\mu S}{kE\,kI}, \; \frac{kI}{kE}, \; 1 \right\}$$

$$\left\{ \delta + \mu E, \; \frac{kE\,\beta\,(-1+\epsilon)}{kI}, \; \frac{\beta - \beta\,\epsilon}{kI}, \; -\frac{kE\,\beta\,(-1+\epsilon)}{kI^2}, \; 1, \; 1 \right\}$$

$$\left\{ -\frac{kI\,\delta}{kE}, \; -\beta\,\epsilon + \mu I, \; \frac{\beta\,\epsilon}{kE}, \; \frac{\beta\,\epsilon}{kI}, \; 1, \; 1 \right\}$$

```
Coeffs = Simplify[Union[Coeffs1, Coeffs2, Coeffs3]]
```

$$\left\{ 1, \; \frac{kI}{kE}, \; \frac{\beta}{kE}, \; \frac{\beta}{kI}, \; -\frac{kI\,\delta}{kE}, \; -\frac{kE\,\beta\,(-1+\epsilon)}{kI^2}, \; \frac{kE\,\beta\,(-1+\epsilon)}{kI}, \right.$$
$$\left. \frac{\beta\,\epsilon}{kE}, \; \frac{\beta\,\epsilon}{kI}, \; \frac{\beta - \beta\,\epsilon}{kI}, \; \delta + \mu E, \; -\beta\,\epsilon + \mu I, \; \frac{kI\,\mu S}{kE}, \; -\beta + \mu S, \; -kI\,(2 + \mu S) \right\}$$

```
xCoeffs =
 Coeffs /. {β → a1, δ → a2, ϵ → a3, μS → a4, μE → a5, μI → a6, c → a7, kE → a8, kI → a9}
```

$$\left\{ 1, \; \frac{a9}{a8}, \; \frac{a1}{a8}, \; \frac{a1}{a9}, \; -\frac{a2\,a9}{a8}, \; -\frac{a1\,(-1+a3)\,a8}{a9^2}, \; \frac{a1\,(-1+a3)\,a8}{a9}, \; \frac{a1\,a3}{a8}, \right.$$
$$\left. \frac{a1\,a3}{a9}, \; \frac{a1-a1\,a3}{a9}, \; a2+a5, \; -a1\,a3+a6, \; \frac{a4\,a9}{a8}, \; -a1+a4, \; -(2+a4)\,a9 \right\}$$

```
Solve[Coeffs == xCoeffs, {β, δ, ϵ, μS, μE, μI, c, kE, kI}]
```

MessageTemplate[Solve , svars , Equations may not give solutions for all "solve" variables. ,

  2 , 53 , 1 , 33 627 900 467 745 714 090 , Local]

$$\{\{\beta \to a1, \; \delta \to a2, \; \epsilon \to a3, \; \mu S \to a4, \; \mu E \to a5, \; \mu I \to a6, \; kE \to a8, \; kI \to a9\}\}$$