In[∘]:=
```
dSdteqn[t_] = - β c II[t, 0] - μS c - (D[S[t, a], a] /. a → 0)
dEEdteqn[t_] = (1 - ϵ) β c II[t, 0] - δ EE[t, 0] -
  μE EE[t, 0] - D[EE[t, 0], t] - (D[EE[t, a], a] /. a → 0)
dIIdteqn[t_] = ϵ β c II[t, 0] + δ EE[t, 0] - μI II[t, 0] -
  D[II[t, 0], t] - (D[II[t, a], a] /. a → 0)
```

Out[∘]= $-c\,\mu S - c\,\beta\,II[t,0] - S^{(0,1)}[t,0]$

Out[∘]= $-\delta\,EE[t,0] - \mu E\,EE[t,0] + c\,\beta\,(1-\epsilon)\,II[t,0] - EE^{(0,1)}[t,0] - EE^{(1,0)}[t,0]$

Out[∘]= $\delta\,EE[t,0] + c\,\beta\,\epsilon\,II[t,0] - \mu I\,II[t,0] - II^{(0,1)}[t,0] - II^{(1,0)}[t,0]$

In[∘]:=
```
yEeqn[t_, a_] = kE EE[t, a] - yE[t, a]
yIeqn[t_, a_] = kI (ϵ β S[t, a] × II[t, a] + δ EE[t, a]) - yI[t, a]
```

Out[∘]= $kE\,EE[t,a] - yE[t,a]$

Out[∘]= $kI\,(\delta\,EE[t,a] + \beta\,\epsilon\,II[t,a] \times S[t,a]) - yI[t,a]$

In[∘]:=
```
yEmap[t_, a_] = Solve[yEeqn[t, a] == 0, EE[t, a]][[1]]
yImap[t_, a_] = Solve[yIeqn[t, a] == 0, II[t, a]][[1]]
```

Out[∘]= $\left\{EE[t,a] \to \frac{yE[t,a]}{kE}\right\}$

Out[∘]= $\left\{II[t,a] \to \frac{-kI\,\delta\,EE[t,a] + yI[t,a]}{kI\,\beta\,\epsilon\,S[t,a]}\right\}$

In[∘]:=
```
dSdteqn2[t_] =
 dSdteqn[t] /. (yImap[t, a] /. a → 0) /. (D[yImap[t, a], t] /. a → 0) /. (D[yImap[t, a],
      a] /. a → 0) /. (yEmap[t, a] /. a → 0) /. (D[yEmap[t, a], t] /. a → 0) /.
   (D[yEmap[t, a], a] /. a → 0) /. (S[t, 0] → c) /. (S^(1,0)[t, 0] → 0)
dEEdteqn2[t_] = dEEdteqn[t] /. (yImap[t, a] /. a → 0) /. (D[yImap[t, a], t] /. a → 0) /.
    (D[yImap[t, a], a] /. a → 0) /. (yEmap[t, a] /. a → 0) /.
   (D[yEmap[t, a], t] /. a → 0) /. (D[yEmap[t, a], a] /. a → 0) /.
  (S[t, 0] → c) /. (S^(1,0)[t, 0] → 0)
dIIdteqn2[t_] = dIIdteqn[t] /. (yImap[t, a] /. a → 0) /. (D[yImap[t, a], t] /. a → 0) /.
    (D[yImap[t, a], a] /. a → 0) /. (yEmap[t, a] /. a → 0) /.
   (D[yEmap[t, a], t] /. a → 0) /. (D[yEmap[t, a], a] /. a → 0) /.
  (S[t, 0] → c) /. (S^(1,0)[t, 0] → 0)
```

Out[∘]=
$$-c\,\mu S-\frac{-\frac{kI\,\delta\,yE[t,0]}{kE}+yI[t,0]}{kI\,\epsilon}-S^{(0,1)}[t,0]$$

Out[∘]=
$$-\frac{\delta\,yE[t,0]}{kE}-\frac{\mu E\,yE[t,0]}{kE}+\frac{(1-\epsilon)\left(-\frac{kI\,\delta\,yE[t,0]}{kE}+yI[t,0]\right)}{kI\,\epsilon}-\frac{yE^{(0,1)}[t,0]}{kE}-\frac{yE^{(1,0)}[t,0]}{kE}$$

Out[∘]=
$$\frac{\delta\,yE[t,0]}{kE}+\frac{-\frac{kI\,\delta\,yE[t,0]}{kE}+yI[t,0]}{kI}-\frac{\mu I\left(-\frac{kI\,\delta\,yE[t,0]}{kE}+yI[t,0]\right)}{c\,kI\,\beta\,\epsilon}+\frac{\left(-\frac{kI\,\delta\,yE[t,0]}{kE}+yI[t,0]\right)S^{(0,1)}[t,0]}{c^2\,kI\,\beta\,\epsilon}-\frac{-\frac{kI\,\delta\,yE^{(0,1)}[t,0]}{kE}+yI^{(0,1)}[t,0]}{c\,kI\,\beta\,\epsilon}-\frac{-\frac{kI\,\delta\,yE^{(1,0)}[t,0]}{kE}+yI^{(1,0)}[t,0]}{c\,kI\,\beta\,\epsilon}$$

In[∘]:=
```
Smap[t_] = Solve[dSdteqn2[t] == 0, S^(0,1)[t, 0]][[1]]
```

Out[∘]=
$$\left\{S^{(0,1)}[t,0]\to\frac{-c\,kE\,kI\,\epsilon\,\mu S+kI\,\delta\,yE[t,0]-kE\,yI[t,0]}{kE\,kI\,\epsilon}\right\}$$

In[∘]:=
```
dEEdteqn3[t_] = dEEdteqn2[t] /. Smap[t]
dIIdteqn3[t_] = dIIdteqn2[t] /. Smap[t]
```

Out[∘]=
$$-\frac{\delta\, yE[t,0]}{kE}-\frac{\mu E\, yE[t,0]}{kE}+\frac{(1-\epsilon)\left(-\frac{kI\,\delta\, yE[t,0]}{kE}+yI[t,0]\right)}{kI\,\epsilon}-\frac{yE^{(0,1)}[t,0]}{kE}-\frac{yE^{(1,0)}[t,0]}{kE}$$

Out[∘]=
$$\frac{\delta\, yE[t,0]}{kE}+\frac{-\frac{kI\,\delta\, yE[t,0]}{kE}+yI[t,0]}{kI}-\frac{\mu I\left(-\frac{kI\,\delta\, yE[t,0]}{kE}+yI[t,0]\right)}{c\,kI\,\beta\,\epsilon}+\frac{\left(-\frac{kI\,\delta\, yE[t,0]}{kE}+yI[t,0]\right)\left(-c\,kE\,kI\,\epsilon\,\mu S+kI\,\delta\, yE[t,0]-kE\, yI[t,0]\right)}{c^2\,kE\,kI^2\,\beta\,\epsilon^2}-\frac{-\frac{kI\,\delta\, yE^{(0,1)}[t,0]}{kE}+yI^{(0,1)}[t,0]}{c\,kI\,\beta\,\epsilon}-\frac{-\frac{kI\,\delta\, yE^{(1,0)}[t,0]}{kE}+yI^{(1,0)}[t,0]}{c\,kI\,\beta\,\epsilon}$$

In[∘]:=
```
IPOPish1[t_] = Denominator[Together[dEEdteqn3[t]]] × Together[dEEdteqn3[t]]
IPOPish2[t_] = Denominator[Together[dIIdteqn3[t]]] × Together[dIIdteqn3[t]]
```

Out[∘]=
$$-kI\,\delta\, yE[t,0]-kI\,\epsilon\,\mu E\, yE[t,0]+kE\, yI[t,0]-kE\,\epsilon\, yI[t,0]-kI\,\epsilon\, yE^{(0,1)}[t,0]-kI\,\epsilon\, yE^{(1,0)}[t,0]$$

Out[∘]=
$$c\,kE\,kI^2\,\delta\,\epsilon\,\mu I\, yE[t,0]+c\,kE\,kI^2\,\delta\,\epsilon\,\mu S\, yE[t,0]-kI^2\,\delta^2\, yE[t,0]^2+c^2\,kE^2\,kI\,\beta\,\epsilon^2\, yI[t,0]-c\,kE^2\,kI\,\epsilon\,\mu I\, yI[t,0]-c\,kE^2\,kI\,\epsilon\,\mu S\, yI[t,0]+2\,kE\,kI\,\delta\, yE[t,0]\times yI[t,0]-kE^2\, yI[t,0]^2+c\,kE\,kI^2\,\delta\,\epsilon\, yE^{(0,1)}[t,0]-c\,kE^2\,kI\,\epsilon\, yI^{(0,1)}[t,0]+c\,kE\,kI^2\,\delta\,\epsilon\, yE^{(1,0)}[t,0]-c\,kE^2\,kI\,\epsilon\, yI^{(1,0)}[t,0]$$

In[∘]:=
```
Monos1 = Sort[MonomialList[IPOPish1[t], {yI[t, 0], yE[t, 0], (D[yI[t, a], t] /. a → 0),
    (D[yI[t, a], a] /. a → 0), (D[yE[t, a], t] /. a → 0), (D[yE[t, a], a] /. a → 0)}]]
Monos2 = Sort[MonomialList[IPOPish2[t], {yI[t, 0], yE[t, 0], (D[yI[t, a], t] /. a → 0),
    (D[yI[t, a], a] /. a → 0), (D[yE[t, a], t] /. a → 0), (D[yE[t, a], a] /. a → 0)}]]
```

Out[∘]=
$$\left\{(-kI\,\delta-kI\,\epsilon\,\mu E)\, yE[t,0],\ (kE-kE\,\epsilon)\, yI[t,0],\ -kI\,\epsilon\, yE^{(0,1)}[t,0],\ -kI\,\epsilon\, yE^{(1,0)}[t,0]\right\}$$

Out[∘]=
$$\left\{\left(c\,kE\,kI^2\,\delta\,\epsilon\,\mu I+c\,kE\,kI^2\,\delta\,\epsilon\,\mu S\right) yE[t,0],\ -kI^2\,\delta^2\, yE[t,0]^2,\ \left(c^2\,kE^2\,kI\,\beta\,\epsilon^2-c\,kE^2\,kI\,\epsilon\,\mu I-c\,kE^2\,kI\,\epsilon\,\mu S\right) yI[t,0],\ 2\,kE\,kI\,\delta\, yE[t,0]\times yI[t,0],\ -kE^2\, yI[t,0]^2,\ c\,kE\,kI^2\,\delta\,\epsilon\, yE^{(0,1)}[t,0],\ -c\,kE^2\,kI\,\epsilon\, yI^{(0,1)}[t,0],\ c\,kE\,kI^2\,\delta\,\epsilon\, yE^{(1,0)}[t,0],\ -c\,kE^2\,kI\,\epsilon\, yI^{(1,0)}[t,0]\right\}$$

In[•]:=
```
MonicMonos1 =
 Monos1 / (Last[Monos1] /. {yI[t, 0] → 1, yE[t, 0] → 1, (D[yI[t, a], t] /. a → 0) → 1,
     (D[yI[t, a], a] /. a → 0) → 1, (D[yE[t, a], t] /. a → 0) → 1,
     (D[yE[t, a], a] /. a → 0) → 1})
MonicMonos2 = Monos2 / (Last[Monos2] /. {yI[t, 0] → 1, yE[t, 0] → 1,
     (D[yI[t, a], t] /. a → 0) → 1, (D[yI[t, a], a] /. a → 0) → 1,
     (D[yE[t, a], t] /. a → 0) → 1, (D[yE[t, a], a] /. a → 0) → 1})
```

Out[•]=
$$\left\{-\frac{(-kI\,\delta - kI\,\epsilon\,\mu E)\,yE[t,0]}{kI\,\epsilon},\ -\frac{(kE - kE\,\epsilon)\,yI[t,0]}{kI\,\epsilon},\ yE^{(0,1)}[t,0],\ yE^{(1,0)}[t,0]\right\}$$

Out[•]=
$$\left\{-\frac{\left(c\,kE\,kI^2\,\delta\,\epsilon\,\mu I + c\,kE\,kI^2\,\delta\,\epsilon\,\mu S\right)yE[t,0]}{c\,kE^2\,kI\,\epsilon},\ \frac{kI\,\delta^2\,yE[t,0]^2}{c\,kE^2\,\epsilon},\right.$$
$$-\frac{\left(c^2\,kE^2\,kI\,\beta\,\epsilon^2 - c\,kE^2\,kI\,\epsilon\,\mu I - c\,kE^2\,kI\,\epsilon\,\mu S\right)yI[t,0]}{c\,kE^2\,kI\,\epsilon},\ -\frac{2\,\delta\,yE[t,0]\times yI[t,0]}{c\,kE\,\epsilon},$$
$$\left.\frac{yI[t,0]^2}{c\,kI\,\epsilon},\ -\frac{kI\,\delta\,yE^{(0,1)}[t,0]}{kE},\ yI^{(0,1)}[t,0],\ -\frac{kI\,\delta\,yE^{(1,0)}[t,0]}{kE},\ yI^{(1,0)}[t,0]\right\}$$

In[•]:=
```
Coeffs1 = MonicMonos1 /.
  {yI[t, 0] → 1, yE[t, 0] → 1, (D[yI[t, a], t] /. a → 0) → 1, (D[yI[t, a], a] /. a → 0) → 1,
   (D[yE[t, a], t] /. a → 0) → 1, (D[yE[t, a], a] /. a → 0) → 1}
Coeffs2 = MonicMonos2 /. {yI[t, 0] → 1, yE[t, 0] → 1,
   (D[yI[t, a], t] /. a → 0) → 1, (D[yI[t, a], a] /. a → 0) → 1,
   (D[yE[t, a], t] /. a → 0) → 1, (D[yE[t, a], a] /. a → 0) → 1}
```

Out[•]=
$$\left\{-\frac{-kI\,\delta - kI\,\epsilon\,\mu E}{kI\,\epsilon},\ -\frac{kE - kE\,\epsilon}{kI\,\epsilon},\ 1,\ 1\right\}$$

Out[•]=
$$\left\{-\frac{c\,kE\,kI^2\,\delta\,\epsilon\,\mu I + c\,kE\,kI^2\,\delta\,\epsilon\,\mu S}{c\,kE^2\,kI\,\epsilon},\ \frac{kI\,\delta^2}{c\,kE^2\,\epsilon},\right.$$
$$\left.-\frac{c^2\,kE^2\,kI\,\beta\,\epsilon^2 - c\,kE^2\,kI\,\epsilon\,\mu I - c\,kE^2\,kI\,\epsilon\,\mu S}{c\,kE^2\,kI\,\epsilon},\ -\frac{2\,\delta}{c\,kE\,\epsilon},\ \frac{1}{c\,kI\,\epsilon},\ -\frac{kI\,\delta}{kE},\ 1,\ -\frac{kI\,\delta}{kE},\ 1\right\}$$

In[•]:=
```
Coeffs = Union[Coeffs1, Coeffs2]
```

Out[•]=
$$\left\{1,\ -\frac{kI\,\delta}{kE},\ \frac{1}{c\,kI\,\epsilon},\ -\frac{2\,\delta}{c\,kE\,\epsilon},\ \frac{kI\,\delta^2}{c\,kE^2\,\epsilon},\ -\frac{kE - kE\,\epsilon}{kI\,\epsilon},\ -\frac{-kI\,\delta - kI\,\epsilon\,\mu E}{kI\,\epsilon},\right.$$
$$\left.-\frac{c^2\,kE^2\,kI\,\beta\,\epsilon^2 - c\,kE^2\,kI\,\epsilon\,\mu I - c\,kE^2\,kI\,\epsilon\,\mu S}{c\,kE^2\,kI\,\epsilon},\ -\frac{c\,kE\,kI^2\,\delta\,\epsilon\,\mu I + c\,kE\,kI^2\,\delta\,\epsilon\,\mu S}{c\,kE^2\,kI\,\epsilon}\right\}$$

In[∘]:=
```
xCoeffs =
 Coeffs /. {β → a1, δ → a2, ϵ → a3, μS → a4, μE → a5, μI → a6, c → a7, kE → a8, kI → a9}
```

Out[∘]=
$$\left\{1, -\frac{a2\ a9}{a8}, \frac{1}{a3\ a7\ a9}, -\frac{2\ a2}{a3\ a7\ a8}, \frac{a2^2\ a9}{a3\ a7\ a8^2}, -\frac{a8 - a3\ a8}{a3\ a9}, -\frac{-a2\ a9 - a3\ a5\ a9}{a3\ a9}, -\frac{-a3\ a4\ a7\ a8^2\ a9 - a3\ a6\ a7\ a8^2\ a9 + a1\ a3^2\ a7^2\ a8^2\ a9}{a3\ a7\ a8^2\ a9}, -\frac{a2\ a3\ a4\ a7\ a8\ a9^2 + a2\ a3\ a6\ a7\ a8\ a9^2}{a3\ a7\ a8^2\ a9}\right\}$$

In[∘]:=
```
Solve[Coeffs == xCoeffs, {β, δ, ϵ, μS, μE, μI, c, kE, kI}]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[∘]=
$$\left\{\left\{\beta \to \frac{a1\ a3\ kE\ (-1+\epsilon)}{(-1+a3)\ a8\ \epsilon}, \delta \to \frac{a2\ (-1+a3)\ \epsilon}{a3\ (-1+\epsilon)}, \mu E \to \frac{-a2\ a3 - a3\ a5 + a2\ \epsilon + a3\ a5\ \epsilon}{a3\ (-1+\epsilon)}, \mu I \to a4 + a6 - \mu S, c \to \frac{(-1+a3)\ a7\ a8}{kE\ (-1+\epsilon)}, kI \to \frac{a3\ a9\ kE\ (-1+\epsilon)}{(-1+a3)\ a8\ \epsilon}\right\}\right\}$$

In[∘]:=

```
(* extra junk *)
```

In[∘]:=
```
GroebnerBasis[Coeffs - xCoeffs, {β, δ, ϵ, μS, μE, μI, c, kE}]
```

Out[∘]=
$$\left\{-a3\ a7 + c - \frac{a7\ a8}{kE} + \frac{a3\ a7\ a8}{kE}, -\frac{a8}{c} + \frac{a8}{a3\ c} - \frac{kE}{a3\ a7} + \frac{kE}{c}, \frac{1}{a7\ a8} - \frac{a3}{a8\ c} - \frac{1}{c\ kE} + \frac{a3}{c\ kE}, -\frac{1}{a3\ a7\ a8} + \frac{1}{a8\ c} - \frac{1}{c\ kE} + \frac{1}{a3\ c\ kE}, -a6 + \mu I, -a2 - a5 + \frac{a2\ kE}{a8} + \mu E, -a4 + \mu S, -\frac{a3\ a7}{c} + \epsilon, -1 + \frac{a8}{kE} - \frac{a8}{a3\ kE} + \frac{1}{\epsilon}, -\frac{a2\ kE}{a8} + \delta, -a1 + \beta, -\frac{1}{a1} + \frac{1}{\beta}\right\}$$