```
dSdteqn[t_, a_] = - β S[t, a] II[t, a] - μS S[t, a] - D[S[t, a], t] - D[S[t, a], a]
dEEdteqn[t_, a_] =
 (1 - ϵ) β S[t, a] II[t, a] - δ EE[t, a] - μE EE[t, a] - D[EE[t, a], t] - D[EE[t, a], a]
dIIdteqn[t_, a_] = ϵ β S[t, a] II[t, a] + δ EE[t, a] -
  μI II[t, a] - D[II[t, a], t] - D[II[t, a], a]
```

$$-\mu S\, S[t, a] - \beta\, II[t, a]\, S[t, a] - S^{(0,1)}[t, a] - S^{(1,0)}[t, a]$$

$$-\delta\, EE[t, a] - \mu E\, EE[t, a] + \beta\, (1 - \epsilon)\, II[t, a]\, S[t, a] - EE^{(0,1)}[t, a] - EE^{(1,0)}[t, a]$$

$$\delta\, EE[t, a] - \mu I\, II[t, a] + \beta\, \epsilon\, II[t, a]\, S[t, a] - II^{(0,1)}[t, a] - II^{(1,0)}[t, a]$$

```
yEeqn[t_, a_] = kE EE[t, a] - yE[t, a]
yIeqn[t_, a_] = kI II[t, a] - yI[t, a]
```

$$kE\, EE[t, a] - yE[t, a]$$

$$kI\, II[t, a] - yI[t, a]$$

```
yEmap[t_, a_] = Solve[yEeqn[t, a] == 0, EE[t, a]][[1]]
yImap[t_, a_] = Solve[yIeqn[t, a] == 0, II[t, a]][[1]]
```

$$\left\{EE[t, a] \to \frac{yE[t, a]}{kE}\right\}$$

$$\left\{II[t, a] \to \frac{yI[t, a]}{kI}\right\}$$

```
dSdteqn2[t_, a_] =
 dSdteqn[t, a] /. yEmap[t, a] /. D[yEmap[t, a], t] /. D[yEmap[t, a], a] /. yImap[t, a] /.
   D[yImap[t, a], t] /. D[yImap[t, a], a]
dEEdteqn2[t_, a_] = dEEdteqn[t, a] /. yEmap[t, a] /. D[yEmap[t, a], t] /.
     D[yEmap[t, a], a] /. yImap[t, a] /. D[yImap[t, a], t] /. D[yImap[t, a], a]
dIIdteqn2[t_, a_] = dIIdteqn[t, a] /. yEmap[t, a] /. D[yEmap[t, a], t] /.
     D[yEmap[t, a], a] /. yImap[t, a] /. D[yImap[t, a], t] /. D[yImap[t, a], a]
```

$$-\mu S\, S[t, a] - \frac{\beta\, S[t, a]\, yI[t, a]}{kI} - S^{(0,1)}[t, a] - S^{(1,0)}[t, a]$$

$$-\frac{\delta\, yE[t, a]}{kE} - \frac{\mu E\, yE[t, a]}{kE} + \frac{\beta\, (1 - \epsilon)\, S[t, a]\, yI[t, a]}{kI} - \frac{yE^{(0,1)}[t, a]}{kE} - \frac{yE^{(1,0)}[t, a]}{kE}$$

$$\frac{\delta\, yE[t, a]}{kE} - \frac{\mu I\, yI[t, a]}{kI} + \frac{\beta\, \epsilon\, S[t, a]\, yI[t, a]}{kI} - \frac{yI^{(0,1)}[t, a]}{kI} - \frac{yI^{(1,0)}[t, a]}{kI}$$

```
Smap[t_, a_] = Solve[dIIdteqn2[t, a] == 0, S[t, a]][[1]]
```

$$\left\{S[t, a] \to \frac{-kI\, \delta\, yE[t, a] + kE\, \mu I\, yI[t, a] + kE\, yI^{(0,1)}[t, a] + kE\, yI^{(1,0)}[t, a]}{kE\, \beta\, \epsilon\, yI[t, a]}\right\}$$

```
TeXForm[Simplify[(-kI δ yE[t, a] + kE μI yI[t, a] + kE yI^(0,1)[t, a] + kE yI^(1,0)[t, a])/(kE β ϵ yI[t, a])]]
```

```
\frac{\text{kE} \left(\text{yI}^{(0,1)}(t,a)+\text{yI}^{(1,0)}(t,a)+\text{$\mu $I}
   \text{yI}(t,a)\right)-\delta  \text{kI} \text{yE}(t,a)}{\beta  \text{kE} \epsilon
   \text{yI}(t,a)}
```

```
dSdteqn3[t_, a_] = dSdteqn2[t, a] /. Smap[t, a] /. D[Smap[t, a], t] /. D[Smap[t, a], a]
dEEdteqn3[t_, a_] =
 dEEdteqn2[t, a] /. Smap[t, a] /. D[Smap[t, a], t] /. D[Smap[t, a], a]
```

$$-\frac{-\text{kI}\,\delta\,\text{yE}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}[t,a]+\text{kE}\,\text{yI}^{(0,1)}[t,a]+\text{kE}\,\text{yI}^{(1,0)}[t,a]}{\text{kE}\,\text{kI}\,\epsilon}-$$

$$\frac{\mu\text{S}\left(-\text{kI}\,\delta\,\text{yE}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}[t,a]+\text{kE}\,\text{yI}^{(0,1)}[t,a]+\text{kE}\,\text{yI}^{(1,0)}[t,a]\right)}{\text{kE}\,\beta\,\epsilon\,\text{yI}[t,a]}+$$

$$\frac{\text{yI}^{(0,1)}[t,a]\left(-\text{kI}\,\delta\,\text{yE}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}[t,a]+\text{kE}\,\text{yI}^{(0,1)}[t,a]+\text{kE}\,\text{yI}^{(1,0)}[t,a]\right)}{\text{kE}\,\beta\,\epsilon\,\text{yI}[t,a]^2}+$$

$$\frac{\text{yI}^{(1,0)}[t,a]\left(-\text{kI}\,\delta\,\text{yE}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}[t,a]+\text{kE}\,\text{yI}^{(0,1)}[t,a]+\text{kE}\,\text{yI}^{(1,0)}[t,a]\right)}{\text{kE}\,\beta\,\epsilon\,\text{yI}[t,a]^2}-$$

$$\frac{-\text{kI}\,\delta\,\text{yE}^{(0,1)}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}^{(0,1)}[t,a]+\text{kE}\,\text{yI}^{(0,2)}[t,a]+\text{kE}\,\text{yI}^{(1,1)}[t,a]}{\text{kE}\,\beta\,\epsilon\,\text{yI}[t,a]}-$$

$$\frac{-\text{kI}\,\delta\,\text{yE}^{(1,0)}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}^{(1,0)}[t,a]+\text{kE}\,\text{yI}^{(1,1)}[t,a]+\text{kE}\,\text{yI}^{(2,0)}[t,a]}{\text{kE}\,\beta\,\epsilon\,\text{yI}[t,a]}$$

$$-\frac{\delta\,\text{yE}[t,a]}{\text{kE}}-\frac{\mu\text{E}\,\text{yE}[t,a]}{\text{kE}}-\frac{\text{yE}^{(0,1)}[t,a]}{\text{kE}}-\frac{\text{yE}^{(1,0)}[t,a]}{\text{kE}}+\frac{1}{\text{kE}\,\text{kI}\,\epsilon}$$
$$(1-\epsilon)\left(-\text{kI}\,\delta\,\text{yE}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}[t,a]+\text{kE}\,\text{yI}^{(0,1)}[t,a]+\text{kE}\,\text{yI}^{(1,0)}[t,a]\right)$$

```
IPOPish1[t_, a_] = Denominator[Together[dSdteqn3[t, a]]] Together[dSdteqn3[t, a]]
IPOPish2[t_, a_] = Denominator[Together[dEEdteqn3[t, a]]] Together[dEEdteqn3[t, a]]
```

$$\begin{aligned}&\text{kI}^2\,\delta\,\mu\text{S}\,\text{yE}[t,a]\,\text{yI}[t,a]-\text{kE}\,\text{kI}\,\mu\text{I}\,\mu\text{S}\,\text{yI}[t,a]^2+\text{kI}\,\beta\,\delta\,\text{yE}[t,a]\,\text{yI}[t,a]^2-\\&\text{kE}\,\beta\,\mu\text{I}\,\text{yI}[t,a]^3+\text{kI}^2\,\delta\,\text{yI}[t,a]\,\text{yE}^{(0,1)}[t,a]-\text{kI}^2\,\delta\,\text{yE}[t,a]\,\text{yI}^{(0,1)}[t,a]-\\&\text{kE}\,\text{kI}\,\mu\text{S}\,\text{yI}[t,a]\,\text{yI}^{(0,1)}[t,a]-\text{kE}\,\beta\,\text{yI}[t,a]^2\,\text{yI}^{(0,1)}[t,a]+\\&\text{kE}\,\text{kI}\,\text{yI}^{(0,1)}[t,a]^2-\text{kE}\,\text{kI}\,\text{yI}[t,a]\,\text{yI}^{(0,2)}[t,a]+\text{kI}^2\,\delta\,\text{yI}[t,a]\,\text{yE}^{(1,0)}[t,a]-\\&\text{kI}^2\,\delta\,\text{yE}[t,a]\,\text{yI}^{(1,0)}[t,a]-\text{kE}\,\text{kI}\,\mu\text{S}\,\text{yI}[t,a]\,\text{yI}^{(1,0)}[t,a]-\\&\text{kE}\,\beta\,\text{yI}[t,a]^2\,\text{yI}^{(1,0)}[t,a]+2\,\text{kE}\,\text{kI}\,\text{yI}^{(0,1)}[t,a]\,\text{yI}^{(1,0)}[t,a]+\\&\text{kE}\,\text{kI}\,\text{yI}^{(1,0)}[t,a]^2-2\,\text{kE}\,\text{kI}\,\text{yI}[t,a]\,\text{yI}^{(1,1)}[t,a]-\text{kE}\,\text{kI}\,\text{yI}[t,a]\,\text{yI}^{(2,0)}[t,a]\end{aligned}$$

$$\begin{aligned}&-\text{kI}\,\delta\,\text{yE}[t,a]-\text{kI}\,\epsilon\,\mu\text{E}\,\text{yE}[t,a]+\text{kE}\,\mu\text{I}\,\text{yI}[t,a]-\text{kE}\,\epsilon\,\mu\text{I}\,\text{yI}[t,a]-\text{kI}\,\epsilon\,\text{yE}^{(0,1)}[t,a]+\\&\text{kE}\,\text{yI}^{(0,1)}[t,a]-\text{kE}\,\epsilon\,\text{yI}^{(0,1)}[t,a]-\text{kI}\,\epsilon\,\text{yE}^{(1,0)}[t,a]+\text{kE}\,\text{yI}^{(1,0)}[t,a]-\text{kE}\,\epsilon\,\text{yI}^{(1,0)}[t,a]\end{aligned}$$

```
TeXForm[IPOPish2[t, a]]
```

```
-\text{kE} \epsilon  \text{yI}^{(0,1)}(t,a)-\text{kE} \epsilon  \text{yI}^{(1,0)}(t,a)+\te
   \text{yI}^{(1,0)}(t,a)+\text{kE} \text{$\mu $I} \text{yI}(t,a)-\text{kE} \text{$\mu $I}
   \text{yE}^{(0,1)}(t,a)-\text{kI} \epsilon  \text{yE}^{(1,0)}(t,a)+\delta  (-\text{kI})
   \epsilon  \text{yE}(t,a)
```

```
Monos1 = Sort[MonomialList[IPOPish1[t, a], {yI[t, a], D[yI[t, a], t], D[yI[t, a], a],
    D[yI[t, a], t, a], D[yI[t, a], {t, 2}], D[yI[t, a], {a, 2}], D[yE[t, a], t],
    D[yE[t, a], a], D[yE[t, a], t, a], D[yE[t, a], {t, 2}], D[yE[t, a], {a, 2}]}]]
Monos2 = Sort[MonomialList[IPOPish2[t, a], {yI[t, a], D[yI[t, a], t], D[yI[t, a], a],
    D[yI[t, a], t, a], D[yI[t, a], {t, 2}], D[yI[t, a], {a, 2}], D[yE[t, a], t],
    D[yE[t, a], a], D[yE[t, a], t, a], D[yE[t, a], {t, 2}], D[yE[t, a], {a, 2}]}]]
```

$$\left\{kI^2\,\delta\,\mu S\,yE[t,a]\,yI[t,a],\ (-kE\,kI\,\mu I\,\mu S+kI\,\beta\,\delta\,yE[t,a])\,yI[t,a]^2,\ -kE\,\beta\,\mu I\,yI[t,a]^3,\right.$$
$$kI^2\,\delta\,yI[t,a]\,yE^{(0,1)}[t,a],\ -kI^2\,\delta\,yE[t,a]\,yI^{(0,1)}[t,a],\ -kE\,kI\,\mu S\,yI[t,a]\,yI^{(0,1)}[t,a],$$
$$-kE\,\beta\,yI[t,a]^2\,yI^{(0,1)}[t,a],\ kE\,kI\,yI^{(0,1)}[t,a]^2,\ -kE\,kI\,yI[t,a]\,yI^{(0,2)}[t,a],$$
$$kI^2\,\delta\,yI[t,a]\,yE^{(1,0)}[t,a],\ -kI^2\,\delta\,yE[t,a]\,yI^{(1,0)}[t,a],\ -kE\,kI\,\mu S\,yI[t,a]\,yI^{(1,0)}[t,a],$$
$$-kE\,\beta\,yI[t,a]^2\,yI^{(1,0)}[t,a],\ 2\,kE\,kI\,yI^{(0,1)}[t,a]\,yI^{(1,0)}[t,a],$$
$$\left.kE\,kI\,yI^{(1,0)}[t,a]^2,\ -2\,kE\,kI\,yI[t,a]\,yI^{(1,1)}[t,a],\ -kE\,kI\,yI[t,a]\,yI^{(2,0)}[t,a]\right\}$$

$$\left\{-kI\,\delta\,yE[t,a]-kI\,\epsilon\,\mu E\,yE[t,a],\ (kE\,\mu I-kE\,\epsilon\,\mu I)\,yI[t,a],\ -kI\,\epsilon\,yE^{(0,1)}[t,a],\right.$$
$$\left.(kE-kE\,\epsilon)\,yI^{(0,1)}[t,a],\ -kI\,\epsilon\,yE^{(1,0)}[t,a],\ (kE-kE\,\epsilon)\,yI^{(1,0)}[t,a]\right\}$$

```
Last[Monos1]
```

$$-kE\,kI\,yI[t,a]\,yI^{(2,0)}[t,a]$$

```
MonicMonos1 =
 Monos1 / (Last[Monos1] /. {yI[t, a] → 1, D[yI[t, a], t] → 1, D[yI[t, a], a] → 1,
     D[yI[t, a], t, a] → 1, D[yI[t, a], {t, 2}] → 1, D[yI[t, a], {a, 2}] → 1,
     yE[t, a] → 1, D[yE[t, a], t] → 1, D[yE[t, a], a] → 1,
     D[yE[t, a], t, a] → 1, D[yE[t, a], {t, 2}] → 1, D[yE[t, a], {a, 2}] → 1})
MonicMonos2 = Monos2 / (Last[Monos2] /. {yI[t, a] → 1, D[yI[t, a], t] → 1,
     D[yI[t, a], a] → 1, D[yI[t, a], t, a] → 1, D[yI[t, a], {t, 2}] → 1,
     D[yI[t, a], {a, 2}] → 1, yE[t, a] → 1, D[yE[t, a], t] → 1, D[yE[t, a], a] → 1,
     D[yE[t, a], t, a] → 1, D[yE[t, a], {t, 2}] → 1, D[yE[t, a], {a, 2}] → 1})
```

$$\left\{-\frac{kI\,\delta\,\mu S\,yE[t,a]\,yI[t,a]}{kE},\ -\frac{(-kE\,kI\,\mu I\,\mu S+kI\,\beta\,\delta\,yE[t,a])\,yI[t,a]^2}{kE\,kI},\right.$$
$$\frac{\beta\,\mu I\,yI[t,a]^3}{kI},\ -\frac{kI\,\delta\,yI[t,a]\,yE^{(0,1)}[t,a]}{kE},\ \frac{kI\,\delta\,yE[t,a]\,yI^{(0,1)}[t,a]}{kE},$$
$$\mu S\,yI[t,a]\,yI^{(0,1)}[t,a],\ \frac{\beta\,yI[t,a]^2\,yI^{(0,1)}[t,a]}{kI},\ -yI^{(0,1)}[t,a]^2,$$
$$yI[t,a]\,yI^{(0,2)}[t,a],\ -\frac{kI\,\delta\,yI[t,a]\,yE^{(1,0)}[t,a]}{kE},\ \frac{kI\,\delta\,yE[t,a]\,yI^{(1,0)}[t,a]}{kE},$$
$$\mu S\,yI[t,a]\,yI^{(1,0)}[t,a],\ \frac{\beta\,yI[t,a]^2\,yI^{(1,0)}[t,a]}{kI},\ -2\,yI^{(0,1)}[t,a]\,yI^{(1,0)}[t,a],$$
$$\left.-yI^{(1,0)}[t,a]^2,\ 2\,yI[t,a]\,yI^{(1,1)}[t,a],\ yI[t,a]\,yI^{(2,0)}[t,a]\right\}$$

$$\left\{\frac{-kI\,\delta\,yE[t,a]-kI\,\epsilon\,\mu E\,yE[t,a]}{kE-kE\,\epsilon},\ \frac{(kE\,\mu I-kE\,\epsilon\,\mu I)\,yI[t,a]}{kE-kE\,\epsilon},\right.$$
$$\left.-\frac{kI\,\epsilon\,yE^{(0,1)}[t,a]}{kE-kE\,\epsilon},\ yI^{(0,1)}[t,a],\ -\frac{kI\,\epsilon\,yE^{(1,0)}[t,a]}{kE-kE\,\epsilon},\ yI^{(1,0)}[t,a]\right\}$$

```
Coeffs1 = MonicMonos1 /. {yI[t, a] → 1, D[yI[t, a], t] → 1,
    D[yI[t, a], a] → 1, D[yI[t, a], t, a] → 1, D[yI[t, a], {t, 2}] → 1,
    D[yI[t, a], {a, 2}] → 1, yE[t, a] → 1, D[yE[t, a], t] → 1, D[yE[t, a], a] → 1,
    D[yE[t, a], t, a] → 1, D[yE[t, a], {t, 2}] → 1, D[yE[t, a], {a, 2}] → 1}
Coeffs2 = MonicMonos2 /. {yI[t, a] → 1, D[yI[t, a], t] → 1, D[yI[t, a], a] → 1,
    D[yI[t, a], t, a] → 1, D[yI[t, a], {t, 2}] → 1, D[yI[t, a], {a, 2}] → 1,
    yE[t, a] → 1, D[yE[t, a], t] → 1, D[yE[t, a], a] → 1,
    D[yE[t, a], t, a] → 1, D[yE[t, a], {t, 2}] → 1, D[yE[t, a], {a, 2}] → 1}
```

$$\left\{-\frac{kI\,\delta\,\mu S}{kE},\ -\frac{kI\,\beta\,\delta-kE\,kI\,\mu I\,\mu S}{kE\,kI},\ \frac{\beta\,\mu I}{kI},\ -\frac{kI\,\delta}{kE},\right.$$
$$\left.\frac{kI\,\delta}{kE},\ \mu S,\ \frac{\beta}{kI},\ -1,\ 1,\ -\frac{kI\,\delta}{kE},\ \frac{kI\,\delta}{kE},\ \mu S,\ \frac{\beta}{kI},\ -2,\ -1,\ 2,\ 1\right\}$$

$$\left\{\frac{-kI\,\delta-kI\,\epsilon\,\mu E}{kE-kE\,\epsilon},\ \frac{kE\,\mu I-kE\,\epsilon\,\mu I}{kE-kE\,\epsilon},\ -\frac{kI\,\epsilon}{kE-kE\,\epsilon},\ 1,\ -\frac{kI\,\epsilon}{kE-kE\,\epsilon},\ 1\right\}$$

```
Coeffs = Simplify[Union[Coeffs1, Coeffs2]]
```

$$\left\{-2,\ -1,\ 1,\ 2,\ \frac{\beta}{kI},\ -\frac{kI\,\delta}{kE},\ \frac{kI\,\delta}{kE},\ \frac{kI\,\epsilon}{kE\,(-1+\epsilon)},\right.$$
$$\left.\frac{kI\,(\delta+\epsilon\,\mu E)}{kE\,(-1+\epsilon)},\ \frac{\beta\,\mu I}{kI},\ \mu I,\ \mu S,\ -\frac{kI\,\delta\,\mu S}{kE},\ -\frac{\beta\,\delta}{kE}+\mu I\,\mu S\right\}$$

```
TeXForm[{kI ϵ/(kE (-1 + ϵ)), kI (δ + ϵ μE)/(kE (-1 + ϵ)), β μI/kI, μI, μS, -(kI δ μS)/kE, -(β δ)/kE + μI μS}]
```

```
\left\{\frac{\text{kI} \epsilon }{\text{kE} (\epsilon -1)},\frac{\text{kI} (\delta +\text{
   -1)},\frac{\beta  \text{$\mu $I}}{\text{kI}},\text{$\mu $I},\text{$\mu $S},-\frac{\delt
   $S}}{\text{kE}},\text{$\mu $I} \text{$\mu $S}-\frac{\beta  \delta }{\text{kE}}\right\}
```

```
xCoeffs =
 Coeffs /. {β → a1, δ → a2, ϵ → a3, μS → a4, μE → a5, μI → a6, c → a7, kE → a8, kI → a9}
```

$$\left\{-2,\ -1,\ 1,\ 2,\ \frac{a1}{a9},\ -\frac{a2\,a9}{a8},\ \frac{a2\,a9}{a8},\ \frac{a3\,a9}{(-1+a3)\,a8},\right.$$
$$\left.\frac{(a2+a3\,a5)\,a9}{(-1+a3)\,a8},\ \frac{a1\,a6}{a9},\ a6,\ a4,\ -\frac{a2\,a4\,a9}{a8},\ a4\,a6-\frac{a1\,a2}{a8}\right\}$$

```
Solve[Coeffs == xCoeffs, {β, δ, ϵ, μS, μE, μI, c, kE, kI}]
```

MessageTemplate[Solve, svars, Equations may not give solutions for all "solve" variables., 2, 56, 2, 33627853190062898696, Local]

$$\left\{\left\{\beta\to\frac{a1\,kI}{a9},\ \delta\to\frac{a2\,a9\,kE}{a8\,kI},\ \epsilon\to\frac{a3\,a9\,kE}{a3\,a9\,kE+a8\,kI-a3\,a8\,kI},\right.\right.$$
$$\left.\left.\mu S\to a4,\ \mu E\to\frac{-a2\,a9\,kE+a2\,a8\,kI+a5\,a8\,kI}{a8\,kI},\ \mu I\to a6\right\}\right\}$$