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dSdteqn[a_] =
  -  $\beta S[0, a] II[0, a] - \mu S S[0, a] - (D[S[t, a], t] /. t \rightarrow 0) - D[S[0, a], a]$ 
dEEdteqn[a_] =  $(1 - \epsilon) \beta S[0, a] II[0, a] - \delta EE[0, a] -$ 
   $\mu E EE[0, a] - (D[EE[t, a], t] /. t \rightarrow 0) - D[EE[0, a], a]$ 
dIIdteqn[a_] =  $\epsilon \beta S[0, a] II[0, a] + \delta EE[0, a] - \mu I II[0, a] -$ 
   $(D[II[t, a], t] /. t \rightarrow 0) - D[II[0, a], a]$ 

-  $\mu S S[0, a] - \beta II[0, a] S[0, a] - S^{(0,1)}[0, a] - S^{(1,0)}[0, a]$ 

-  $\delta EE[0, a] - \mu E EE[0, a] + \beta (1 - \epsilon) II[0, a] S[0, a] - EE^{(0,1)}[0, a] - EE^{(1,0)}[0, a]$ 

 $\delta EE[0, a] - \mu I II[0, a] + \beta \epsilon II[0, a] S[0, a] - II^{(0,1)}[0, a] - II^{(1,0)}[0, a]$ 

yEeqn[t_, a_] = kE EE[t, a] - yE[t, a]
yIeqn[t_, a_] = kI II[t, a] - yI[t, a]
Seqn[t_, a_] = S[t, a] + EE[t, a] + II[t, a] - Ntot[t, a]

kE EE[t, a] - yE[t, a]

kI II[t, a] - yI[t, a]

EE[t, a] + II[t, a] - Ntot[t, a] + S[t, a]

yEmap[t_, a_] = Solve[yEeqn[t, a] == 0, EE[t, a]][[1]]
yImap[t_, a_] = Solve[yIeqn[t, a] == 0, II[t, a]][[1]]
Smap[t_, a_] = Solve[Seqn[t, a] == 0, S[t, a]][[1]]

 $\left\{ EE[t, a] \rightarrow \frac{yE[t, a]}{kE} \right\}$ 

 $\left\{ II[t, a] \rightarrow \frac{yI[t, a]}{kI} \right\}$ 

 $\{ S[t, a] \rightarrow -EE[t, a] - II[t, a] + Ntot[t, a] \}$ 

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dSdteqn2[a_] =
  dSdteqn[a] /. (Smap[t, a] /. t -> 0) /. (D[Smap[t, a], t] /. t -> 0) /. (D[Smap[t, a], a] /.
    t -> 0) /. (yEmap[t, a] /. t -> 0) /. (D[yEmap[t, a], t] /. t -> 0) /.
    (D[yEmap[t, a], a] /. t -> 0) /. (yImap[t, a] /. t -> 0) /.
    (D[yImap[t, a], t] /. t -> 0) /. (D[yImap[t, a], a] /. t -> 0)
dEEdteqn2[a_] = dEEdteqn[a] /. (Smap[t, a] /. t -> 0) /. (D[Smap[t, a], t] /. t -> 0) /.
  (D[Smap[t, a], a] /. t -> 0) /. (yEmap[t, a] /. t -> 0) /.
  (D[yEmap[t, a], t] /. t -> 0) /. (D[yEmap[t, a], a] /. t -> 0) /.
  (yImap[t, a] /. t -> 0) /. (D[yImap[t, a], t] /. t -> 0) /. (D[yImap[t, a], a] /. t -> 0)
dIIdteqn2[a_] = dIIdteqn[a] /. (Smap[t, a] /. t -> 0) /. (D[Smap[t, a], t] /. t -> 0) /.
  (D[Smap[t, a], a] /. t -> 0) /. (yEmap[t, a] /. t -> 0) /.
  (D[yEmap[t, a], t] /. t -> 0) /. (D[yEmap[t, a], a] /. t -> 0) /.
  (yImap[t, a] /. t -> 0) /. (D[yImap[t, a], t] /. t -> 0) /. (D[yImap[t, a], a] /. t -> 0)

-μS (Ntot[0, a] -  $\frac{yE[0, a]}{kE}$  -  $\frac{yI[0, a]}{kI}$ ) -

$$\frac{\beta yI[0, a] \left( Ntot[0, a] - \frac{yE[0, a]}{kE} - \frac{yI[0, a]}{kI} \right)}{kI} - Ntot^{(0,1)}[0, a] + \frac{yE^{(0,1)}[0, a]}{kE} +$$


$$\frac{yI^{(0,1)}[0, a]}{kI} - Ntot^{(1,0)}[0, a] + \frac{yE^{(1,0)}[0, a]}{kE} + \frac{yI^{(1,0)}[0, a]}{kI}$$


$$- \frac{\delta yE[0, a]}{kE} - \frac{\mu E yE[0, a]}{kE} +$$


$$\frac{\beta (1 - \epsilon) yI[0, a] \left( Ntot[0, a] - \frac{yE[0, a]}{kE} - \frac{yI[0, a]}{kI} \right)}{kI} - \frac{yE^{(0,1)}[0, a]}{kE} - \frac{yE^{(1,0)}[0, a]}{kE}$$


$$\frac{\delta yE[0, a]}{kE} - \frac{\mu I yI[0, a]}{kI} +$$


$$\frac{\beta \epsilon yI[0, a] \left( Ntot[0, a] - \frac{yE[0, a]}{kE} - \frac{yI[0, a]}{kI} \right)}{kI} - \frac{yI^{(0,1)}[0, a]}{kI} - \frac{yI^{(1,0)}[0, a]}{kI}$$


IPOpish1[a_] = Denominator[Together[dSdteqn2[a]]] Together[dSdteqn2[a]]
IPOpish2[a_] = Denominator[Together[dEEdteqn2[a]]] Together[dEEdteqn2[a]]
IPOpish3[a_] = Denominator[Together[dIIdteqn2[a]]] Together[dIIdteqn2[a]]

-kE kI2 μS Ntot[0, a] + kI2 μS yE[0, a] + kE kI μS yI[0, a] - kE kI β Ntot[0, a] yI[0, a] +
  kI β yE[0, a] yI[0, a] + kE β yI[0, a]2 - kE kI2 Ntot(0,1)[0, a] + kI2 yE(0,1)[0, a] +
  kE kI yI(0,1)[0, a] - kE kI2 Ntot(1,0)[0, a] + kI2 yE(1,0)[0, a] + kE kI yI(1,0)[0, a]

-kI2 δ yE[0, a] - kI2 μE yE[0, a] + kE kI β Ntot[0, a] yI[0, a] -
  kE kI β ∈ Ntot[0, a] yI[0, a] - kI β yE[0, a] yI[0, a] + kI β ∈ yE[0, a] yI[0, a] -
  kE β yI[0, a]2 + kE β ∈ yI[0, a]2 - kI2 yE(0,1)[0, a] - kI2 yE(1,0)[0, a]

kI2 δ yE[0, a] - kE kI μI yI[0, a] + kE kI β ∈ Ntot[0, a] yI[0, a] -
  kI β ∈ yE[0, a] yI[0, a] - kE β ∈ yI[0, a]2 - kE kI yI(0,1)[0, a] - kE kI yI(1,0)[0, a]

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Monos1 = Sort[MonomialList[IPOPish1[a], {yI[0, a], yE[0, a],
  (D[yI[t, a], t] /. t → 0), (D[yI[t, a], a] /. t → 0), (D[yI[t, a], t, a] /. t → 0),
  (D[yI[t, a], {t, 2}] /. t → 0), (D[yI[t, a], {a, 2}] /. t → 0),
  (D[yE[t, a], t] /. t → 0), (D[yE[t, a], a] /. t → 0), (D[yE[t, a], t, a] /. t → 0),
  (D[yE[t, a], {t, 2}] /. t → 0), (D[yE[t, a], {a, 2}] /. t → 0)}]]
Monos2 = Sort[MonomialList[IPOPish2[a], {yI[0, a], yE[0, a],
  (D[yI[t, a], t] /. t → 0), (D[yI[t, a], a] /. t → 0), (D[yI[t, a], t, a] /. t → 0),
  (D[yI[t, a], {t, 2}] /. t → 0), (D[yI[t, a], {a, 2}] /. t → 0),
  (D[yE[t, a], t] /. t → 0), (D[yE[t, a], a] /. t → 0), (D[yE[t, a], t, a] /. t → 0),
  (D[yE[t, a], {t, 2}] /. t → 0), (D[yE[t, a], {a, 2}] /. t → 0)}]]
Monos3 = Sort[MonomialList[IPOPish3[a], {yI[0, a], yE[0, a],
  (D[yI[t, a], t] /. t → 0), (D[yI[t, a], a] /. t → 0), (D[yI[t, a], t, a] /. t → 0),
  (D[yI[t, a], {t, 2}] /. t → 0), (D[yI[t, a], {a, 2}] /. t → 0),
  (D[yE[t, a], t] /. t → 0), (D[yE[t, a], a] /. t → 0), (D[yE[t, a], t, a] /. t → 0),
  (D[yE[t, a], {t, 2}] /. t → 0), (D[yE[t, a], {a, 2}] /. t → 0)}]]

{kI2  $\mu$  S yE[0, a], (kE kI  $\mu$  S - kE kI  $\beta$  Ntot[0, a]) yI[0, a],
 kI  $\beta$  yE[0, a] yI[0, a], kE  $\beta$  yI[0, a]2, kI2 yE(0,1)[0, a], kE kI yI(0,1)[0, a],
 -kE kI2  $\mu$  S Ntot[0, a] - kE kI2 Ntot(0,1)[0, a] - kE kI2 Ntot(1,0)[0, a],
 kI2 yE(1,0)[0, a], kE kI yI(1,0)[0, a]}

{(-kI2  $\delta$  - kI2  $\mu$  E) yE[0, a],
 (kE kI  $\beta$  Ntot[0, a] - kE kI  $\beta \in$  Ntot[0, a]) yI[0, a], (-kI  $\beta$  + kI  $\beta \in$ ) yE[0, a] yI[0, a],
 (-kE  $\beta$  + kE  $\beta \in$ ) yI[0, a]2, -kI2 yE(0,1)[0, a], -kI2 yE(1,0)[0, a]}

{kI2  $\delta$  yE[0, a], (-kE kI  $\mu$  I + kE kI  $\beta \in$  Ntot[0, a]) yI[0, a],
 -kI  $\beta \in$  yE[0, a] yI[0, a], -kE  $\beta \in$  yI[0, a]2, -kE kI yI(0,1)[0, a], -kE kI yI(1,0)[0, a]}

```

**MonicMonos1 =**

**Monos1** / (Last[Monos1] /. {yI[0, a] → 1, yE[0, a] → 1, (D[yI[t, a], t] /. t → 0) → 1,  
 (D[yI[t, a], a] /. t → 0) → 1, (D[yI[t, a], t, a] /. t → 0) → 1,  
 (D[yI[t, a], {t, 2}] /. t → 0) → 1, (D[yI[t, a], {a, 2}] /. t → 0) → 1,  
 (D[yE[t, a], t] /. t → 0) → 1, (D[yE[t, a], a] /. t → 0) → 1,  
 (D[yE[t, a], t, a] /. t → 0) → 1, (D[yE[t, a], {t, 2}] /. t → 0) → 1,  
 (D[yE[t, a], {a, 2}] /. t → 0) → 1}))

**MonicMonos2 = Monos2** / (Last[Monos2] /. {yI[0, a] → 1, yE[0, a] → 1,  
 (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,  
 (D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,  
 (D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,  
 (D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,  
 (D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1}))

**MonicMonos3 = Monos3** / (Last[Monos3] /. {yI[0, a] → 1, yE[0, a] → 1,  
 (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,  
 (D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,  
 (D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,  
 (D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,  
 (D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1}))

$$\begin{aligned}
 & \left\{ \frac{kI \mu S yE[0, a]}{kE}, \frac{(kE kI \mu S - kE kI \beta N_{tot}[0, a]) yI[0, a]}{kE kI}, \right. \\
 & \frac{\beta yE[0, a] yI[0, a]}{kE}, \frac{\beta yI[0, a]^2}{kI}, \frac{kI yE^{(0,1)}[0, a]}{kE}, yI^{(0,1)}[0, a], \frac{1}{kE kI} \\
 & \left( -kE kI^2 \mu S N_{tot}[0, a] - kE kI^2 N_{tot}^{(0,1)}[0, a] - kE kI^2 N_{tot}^{(1,0)}[0, a] \right), \\
 & \left. \frac{kI yE^{(1,0)}[0, a]}{kE}, yI^{(1,0)}[0, a] \right\} \\
 & \left\{ -\frac{(-kI^2 \delta - kI^2 \mu E) yE[0, a]}{kI^2}, \right. \\
 & -\frac{(kE kI \beta N_{tot}[0, a] - kE kI \beta \in N_{tot}[0, a]) yI[0, a]}{kI^2}, -\frac{(-kI \beta + kI \beta \in) yE[0, a] yI[0, a]}{kI^2}, \\
 & \left. -\frac{(-kE \beta + kE \beta \in) yI[0, a]^2}{kI^2}, yE^{(0,1)}[0, a], yE^{(1,0)}[0, a] \right\} \\
 & \left\{ -\frac{kI \delta yE[0, a]}{kE}, -\frac{(-kE kI \mu I + kE kI \beta \in N_{tot}[0, a]) yI[0, a]}{kE kI}, \right. \\
 & \left. \frac{\beta \in yE[0, a] yI[0, a]}{kE}, \frac{\beta \in yI[0, a]^2}{kI}, yI^{(0,1)}[0, a], yI^{(1,0)}[0, a] \right\}
 \end{aligned}$$

**Coeffs1 = MonicMonos1 /.**

**{yI[0, a] → 1, yE[0, a] → 1, (D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,**  
**(D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,**  
**(D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,**  
**(D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,**  
**(D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1,**  
**Ntot[0, a] → 1, D[Ntot[0, a], a] → 1, (D[Ntot[t, a], t] /. t → 0) → 1}**

**Coeffs2 = Simplify[MonicMonos2 /. {yI[0, a] → 1, yE[0, a] → 1,**

**(D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,**  
**(D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,**  
**(D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,**  
**(D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,**  
**(D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1,**  
**Ntot[0, a] → 1, D[Ntot[0, a], a] → 1, (D[Ntot[t, a], t] /. t → 0) → 1}]**

**Coeffs3 = Simplify[MonicMonos3 /. {yI[0, a] → 1, yE[0, a] → 1,**

**(D[yI[t, a], t] /. t → 0) → 1, (D[yI[t, a], a] /. t → 0) → 1,**  
**(D[yI[t, a], t, a] /. t → 0) → 1, (D[yI[t, a], {t, 2}] /. t → 0) → 1,**  
**(D[yI[t, a], {a, 2}] /. t → 0) → 1, (D[yE[t, a], t] /. t → 0) → 1,**  
**(D[yE[t, a], a] /. t → 0) → 1, (D[yE[t, a], t, a] /. t → 0) → 1,**  
**(D[yE[t, a], {t, 2}] /. t → 0) → 1, (D[yE[t, a], {a, 2}] /. t → 0) → 1,**  
**Ntot[0, a] → 1, D[Ntot[0, a], a] → 1, (D[Ntot[t, a], t] /. t → 0) → 1}]**

**{**  $\frac{kI \mu S}{kE}$ ,  $\frac{-kE kI \beta + kE kI \mu S}{kE kI}$ ,  $\frac{\beta}{kE}$ ,  $\frac{\beta}{kI}$ ,  $\frac{kI}{kE}$ ,  $1$ ,  $\frac{-2 kE kI^2 - kE kI^2 \mu S}{kE kI}$ ,  $\frac{kI}{kE}$ ,  $1$  **}**

**{**  $\delta + \mu E$ ,  $\frac{kE \beta (-1 + \epsilon)}{kI}$ ,  $\frac{\beta - \beta \epsilon}{kI}$ ,  $-\frac{kE \beta (-1 + \epsilon)}{kI^2}$ ,  $1$ ,  $1$  **}**

**{**  $-\frac{kI \delta}{kE}$ ,  $-\beta \epsilon + \mu I$ ,  $\frac{\beta \epsilon}{kE}$ ,  $\frac{\beta \epsilon}{kI}$ ,  $1$ ,  $1$  **}**

**Coeffs = Simplify[Union[Coeffs1, Coeffs2, Coeffs3]]**

**{**  $1$ ,  $\frac{kI}{kE}$ ,  $\frac{\beta}{kE}$ ,  $\frac{\beta}{kI}$ ,  $-\frac{kI \delta}{kE}$ ,  $-\frac{kE \beta (-1 + \epsilon)}{kI^2}$ ,  $\frac{kE \beta (-1 + \epsilon)}{kI}$ ,  
 $\frac{\beta \epsilon}{kE}$ ,  $\frac{\beta \epsilon}{kI}$ ,  $\frac{\beta - \beta \epsilon}{kI}$ ,  $\delta + \mu E$ ,  $-\beta \epsilon + \mu I$ ,  $\frac{kI \mu S}{kE}$ ,  $-\beta + \mu S$ ,  $-kI (2 + \mu S)$  **}**

**TeXForm[Coeffs]**

**\left\{1,\frac{\text{kI}}{\text{kE}},\frac{\beta}{\text{kE}},\frac{\beta}{\text{kI}},-\frac{\text{kI} \delta}{\text{kE}},-\frac{\text{kE} \beta (-1 + \epsilon)}{\text{kI}^2},\frac{\text{kE} \beta (-1 + \epsilon)}{\text{kI}},\right.**  
**\frac{\beta \epsilon}{\text{kE}},\frac{\beta \epsilon}{\text{kI}},\frac{\beta - \beta \epsilon}{\text{kI}},\delta + \mu E,-\beta \epsilon + \mu I,\frac{\text{kI} \mu S}{\text{kE}},-\beta + \mu S,-\text{kI} (2 + \mu S)\}**

**xCoeffs =**

**Coeffs /. {β → a1, δ → a2, ε → a3, μS → a4, μE → a5, μI → a6, c → a7, kE → a8, kI → a9}**

**{**  $1$ ,  $\frac{a9}{a8}$ ,  $\frac{a1}{a8}$ ,  $\frac{a1}{a9}$ ,  $-\frac{a2 a9}{a8}$ ,  $-\frac{a1 (-1 + a3) a8}{a9^2}$ ,  $\frac{a1 (-1 + a3) a8}{a9}$ ,  $\frac{a1 a3}{a8}$ ,  
 $\frac{a1 a3}{a9}$ ,  $\frac{a1 - a1 a3}{a9}$ ,  $a2 + a5$ ,  $-a1 a3 + a6$ ,  $\frac{a4 a9}{a8}$ ,  $-a1 + a4$ ,  $-(2 + a4) a9$  **}**

```
Solve[Coeffs == xCoeffs, { $\beta$ ,  $\delta$ ,  $\epsilon$ ,  $\mu_S$ ,  $\mu_E$ ,  $\mu_I$ , c, kE, kI}]
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MessageTemplate[Solve, svars, Equations may not give solutions for all "solve" variables. ,  
2, 52, 2, 33627877287655448522, Local]
```

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{ { $\beta \rightarrow a1$ ,  $\delta \rightarrow a2$ ,  $\epsilon \rightarrow a3$ ,  $\mu_S \rightarrow a4$ ,  $\mu_E \rightarrow a5$ ,  $\mu_I \rightarrow a6$ , kE  $\rightarrow a8$ , kI  $\rightarrow a9$ } }
```