```
dSdteqn[t_] =
 m T[0] - β c II[t, 0] / (c + EE[t, 0] + II[t, 0]) - μS c  - (D[S[t, a], a] /. a → 0)
dEEdteqn[t_] = (1 - ϵ) β c II[t, 0] / (c + EE[t, 0] + II[t, 0]) -
  δ EE[t, 0] - μE EE[t, 0] - D[EE[t, 0], t] - (D[EE[t, a], a] /. a → 0)
dIIdteqn[t_] = ϵ β c II[t, 0] / (c + EE[t, 0] + II[t, 0]) + δ EE[t, 0] -
  μI II[t, 0] - D[II[t, 0], t] - (D[II[t, a], a] /. a → 0)
```

$$-c\,\mu S - \frac{c\,\beta\,II[t,0]}{c + EE[t,0] + II[t,0]} + m\,T[0] - S^{(0,1)}[t,0]$$

$$-\delta\,EE[t,0] - \mu E\,EE[t,0] + \frac{c\,\beta\,(1-\epsilon)\,II[t,0]}{c + EE[t,0] + II[t,0]} - EE^{(0,1)}[t,0] - EE^{(1,0)}[t,0]$$

$$\delta\,EE[t,0] - \mu I\,II[t,0] + \frac{c\,\beta\,\epsilon\,II[t,0]}{c + EE[t,0] + II[t,0]} - II^{(0,1)}[t,0] - II^{(1,0)}[t,0]$$

```
yEeqn[t_, a_] = kE EE[t, a] - yE[t, a]
yIeqn[t_, a_] = kI II[t, a] - yI[t, a]
```

$$kE\,EE[t,a] - yE[t,a]$$

$$kI\,II[t,a] - yI[t,a]$$

```
yEmap[t_, a_] = Solve[yEeqn[t, a] == 0, EE[t, a]][[1]]
yImap[t_, a_] = Solve[yIeqn[t, a] == 0, II[t, a]][[1]]
```

$$\left\{EE[t,a] \to \frac{yE[t,a]}{kE}\right\}$$

$$\left\{II[t,a] \to \frac{yI[t,a]}{kI}\right\}$$

```
dSdteqn2[t_] = dSdteqn[t] /. (yEmap[t, a] /. a → 0) /. (D[yEmap[t, a], t] /. a → 0) /.
      (D[yEmap[t, a], a] /. a → 0) /. (yImap[t, a] /. a → 0) /.
    (D[yImap[t, a], t] /. a → 0) /. (D[yImap[t, a], a] /. a → 0)
dEEdteqn2[t_] = dEEdteqn[t] /. (yEmap[t, a] /. a → 0) /. (D[yEmap[t, a], t] /. a → 0) /.
      (D[yEmap[t, a], a] /. a → 0) /. (yImap[t, a] /. a → 0) /.
    (D[yImap[t, a], t] /. a → 0) /. (D[yImap[t, a], a] /. a → 0)
dIIdteqn2[t_] = dIIdteqn[t] /. (yEmap[t, a] /. a → 0) /. (D[yEmap[t, a], t] /. a → 0) /.
      (D[yEmap[t, a], a] /. a → 0) /. (yImap[t, a] /. a → 0) /.
    (D[yImap[t, a], t] /. a → 0) /. (D[yImap[t, a], a] /. a → 0)
```

$$-c\,\mu S + m\,T[0] - \frac{c\,\beta\,yI[t,0]}{kI\left(c + \frac{yE[t,0]}{kE} + \frac{yI[t,0]}{kI}\right)} - S^{(0,1)}[t,0]$$

$$-\frac{\delta\,yE[t,0]}{kE} - \frac{\mu E\,yE[t,0]}{kE} + \frac{c\,\beta\,(1-\epsilon)\,yI[t,0]}{kI\left(c + \frac{yE[t,0]}{kE} + \frac{yI[t,0]}{kI}\right)} - \frac{yE^{(0,1)}[t,0]}{kE} - \frac{yE^{(1,0)}[t,0]}{kE}$$

$$\frac{\delta\,yE[t,0]}{kE} - \frac{\mu I\,yI[t,0]}{kI} + \frac{c\,\beta\,\epsilon\,yI[t,0]}{kI\left(c + \frac{yE[t,0]}{kE} + \frac{yI[t,0]}{kI}\right)} - \frac{yI^{(0,1)}[t,0]}{kI} - \frac{yI^{(1,0)}[t,0]}{kI}$$

```
Smap[t_] = Solve[dSdteqn2[t] == 0, S^(0,1)[t, 0]][[1]]
```

$$\left\{S^{(0,1)}[t,\,0] \to -c\,\mu S + m\,T[0] - \frac{c\,\beta\,yI[t,\,0]}{kI\left(c + \frac{yE[t,0]}{kE} + \frac{yI[t,0]}{kI}\right)}\right\}$$

```
dEEdteqn3[t_] = dEEdteqn2[t] /. Smap[t]
dIIdteqn3[t_] = dIIdteqn2[t] /. Smap[t]
```

$$-\frac{\delta\,yE[t,\,0]}{kE} - \frac{\mu E\,yE[t,\,0]}{kE} + \frac{c\,\beta\,(1-\epsilon)\,yI[t,\,0]}{kI\left(c + \frac{yE[t,0]}{kE} + \frac{yI[t,0]}{kI}\right)} - \frac{yE^{(0,1)}[t,\,0]}{kE} - \frac{yE^{(1,0)}[t,\,0]}{kE}$$

$$\frac{\delta\,yE[t,\,0]}{kE} - \frac{\mu I\,yI[t,\,0]}{kI} + \frac{c\,\beta\,\epsilon\,yI[t,\,0]}{kI\left(c + \frac{yE[t,0]}{kE} + \frac{yI[t,0]}{kI}\right)} - \frac{yI^{(0,1)}[t,\,0]}{kI} - \frac{yI^{(1,0)}[t,\,0]}{kI}$$

```
IPOPish1[t_] = Denominator[Together[dEEdteqn3[t]]] Together[dEEdteqn3[t]]
IPOPish2[t_] = Denominator[Together[dIIdteqn3[t]]] Together[dIIdteqn3[t]]
```

$$\begin{aligned}&-c\,kE\,kI\,\delta\,yE[t,\,0] - c\,kE\,kI\,\mu E\,yE[t,\,0] - kI\,\delta\,yE[t,\,0]^2 - kI\,\mu E\,yE[t,\,0]^2 + \\ &\ c\,kE^2\,\beta\,yI[t,\,0] - c\,kE^2\,\beta\,\epsilon\,yI[t,\,0] - kE\,\delta\,yE[t,\,0]\,yI[t,\,0] - kE\,\mu E\,yE[t,\,0]\,yI[t,\,0] - \\ &\ c\,kE\,kI\,yE^{(0,1)}[t,\,0] - kI\,yE[t,\,0]\,yE^{(0,1)}[t,\,0] - kE\,yI[t,\,0]\,yE^{(0,1)}[t,\,0] - \\ &\ c\,kE\,kI\,yE^{(1,0)}[t,\,0] - kI\,yE[t,\,0]\,yE^{(1,0)}[t,\,0] - kE\,yI[t,\,0]\,yE^{(1,0)}[t,\,0]\end{aligned}$$

$$\begin{aligned}&c\,kE\,kI^2\,\delta\,yE[t,\,0] + kI^2\,\delta\,yE[t,\,0]^2 + c\,kE^2\,kI\,\beta\,\epsilon\,yI[t,\,0] - c\,kE^2\,kI\,\mu I\,yI[t,\,0] + \\ &\ kE\,kI\,\delta\,yE[t,\,0]\,yI[t,\,0] - kE\,kI\,\mu I\,yE[t,\,0]\,yI[t,\,0] - kE^2\,\mu I\,yI[t,\,0]^2 - \\ &\ c\,kE^2\,kI\,yI^{(0,1)}[t,\,0] - kE\,kI\,yE[t,\,0]\,yI^{(0,1)}[t,\,0] - kE^2\,yI[t,\,0]\,yI^{(0,1)}[t,\,0] - \\ &\ c\,kE^2\,kI\,yI^{(1,0)}[t,\,0] - kE\,kI\,yE[t,\,0]\,yI^{(1,0)}[t,\,0] - kE^2\,yI[t,\,0]\,yI^{(1,0)}[t,\,0]\end{aligned}$$

```
Monos1 = Sort[MonomialList[IPOPish1[t], {yI[t, 0], yE[t, 0], (D[yI[t, a], t] /. a → 0),
    (D[yI[t, a], a] /. a → 0), (D[yE[t, a], t] /. a → 0), (D[yE[t, a], a] /. a → 0)}]]
Monos2 = Sort[MonomialList[IPOPish2[t], {yI[t, 0], yE[t, 0], (D[yI[t, a], t] /. a → 0),
    (D[yI[t, a], a] /. a → 0), (D[yE[t, a], t] /. a → 0), (D[yE[t, a], a] /. a → 0)}]]
```

$$\begin{aligned}&\{(-c\,kE\,kI\,\delta - c\,kE\,kI\,\mu E)\,yE[t,\,0],\ (-kI\,\delta - kI\,\mu E)\,yE[t,\,0]^2,\\ &\ \left(c\,kE^2\,\beta - c\,kE^2\,\beta\,\epsilon\right)\,yI[t,\,0],\ (-kE\,\delta - kE\,\mu E)\,yE[t,\,0]\,yI[t,\,0],\\ &\ -c\,kE\,kI\,yE^{(0,1)}[t,\,0],\ -kI\,yE[t,\,0]\,yE^{(0,1)}[t,\,0],\ -kE\,yI[t,\,0]\,yE^{(0,1)}[t,\,0],\\ &\ -c\,kE\,kI\,yE^{(1,0)}[t,\,0],\ -kI\,yE[t,\,0]\,yE^{(1,0)}[t,\,0],\ -kE\,yI[t,\,0]\,yE^{(1,0)}[t,\,0]\}\end{aligned}$$

$$\begin{aligned}&\{c\,kE\,kI^2\,\delta\,yE[t,\,0],\ kI^2\,\delta\,yE[t,\,0]^2,\ \left(c\,kE^2\,kI\,\beta\,\epsilon - c\,kE^2\,kI\,\mu I\right)\,yI[t,\,0],\\ &\ (kE\,kI\,\delta - kE\,kI\,\mu I)\,yE[t,\,0]\,yI[t,\,0],\ -kE^2\,\mu I\,yI[t,\,0]^2,\ -c\,kE^2\,kI\,yI^{(0,1)}[t,\,0],\\ &\ -kE\,kI\,yE[t,\,0]\,yI^{(0,1)}[t,\,0],\ -kE^2\,yI[t,\,0]\,yI^{(0,1)}[t,\,0],\\ &\ -c\,kE^2\,kI\,yI^{(1,0)}[t,\,0],\ -kE\,kI\,yE[t,\,0]\,yI^{(1,0)}[t,\,0],\ -kE^2\,yI[t,\,0]\,yI^{(1,0)}[t,\,0]\}\end{aligned}$$

```
MonicMonos1 =
 Monos1 / (Last[Monos1] /. {yI[t, 0] → 1, yE[t, 0] → 1, (D[yI[t, a], t] /. a → 0) → 1,
      (D[yI[t, a], a] /. a → 0) → 1, (D[yE[t, a], t] /. a → 0) → 1,
      (D[yE[t, a], a] /. a → 0) → 1})
MonicMonos2 = Monos2 / (Last[Monos2] /. {yI[t, 0] → 1, yE[t, 0] → 1,
      (D[yI[t, a], t] /. a → 0) → 1, (D[yI[t, a], a] /. a → 0) → 1,
      (D[yE[t, a], t] /. a → 0) → 1, (D[yE[t, a], a] /. a → 0) → 1})
```

$$\left\{-\frac{(-c\,kE\,kI\,\delta - c\,kE\,kI\,\mu E)\,yE[t,0]}{kE}, -\frac{(-kI\,\delta - kI\,\mu E)\,yE[t,0]^2}{kE},\right.$$
$$-\frac{\left(c\,kE^2\,\beta - c\,kE^2\,\beta\in\right)\,yI[t,0]}{kE}, -\frac{(-kE\,\delta - kE\,\mu E)\,yE[t,0]\,yI[t,0]}{kE},$$
$$c\,kI\,yE^{(0,1)}[t,0], \frac{kI\,yE[t,0]\,yE^{(0,1)}[t,0]}{kE}, yI[t,0]\,yE^{(0,1)}[t,0],$$
$$\left.c\,kI\,yE^{(1,0)}[t,0], \frac{kI\,yE[t,0]\,yE^{(1,0)}[t,0]}{kE}, yI[t,0]\,yE^{(1,0)}[t,0]\right\}$$

$$\left\{-\frac{c\,kI^2\,\delta\,yE[t,0]}{kE}, -\frac{kI^2\,\delta\,yE[t,0]^2}{kE^2}, -\frac{\left(c\,kE^2\,kI\,\beta\in - c\,kE^2\,kI\,\mu I\right)\,yI[t,0]}{kE^2},\right.$$
$$-\frac{(kE\,kI\,\delta - kE\,kI\,\mu I)\,yE[t,0]\,yI[t,0]}{kE^2}, \mu I\,yI[t,0]^2,$$
$$c\,kI\,yI^{(0,1)}[t,0], \frac{kI\,yE[t,0]\,yI^{(0,1)}[t,0]}{kE}, yI[t,0]\,yI^{(0,1)}[t,0],$$
$$\left.c\,kI\,yI^{(1,0)}[t,0], \frac{kI\,yE[t,0]\,yI^{(1,0)}[t,0]}{kE}, yI[t,0]\,yI^{(1,0)}[t,0]\right\}$$

```
Coeffs1 = MonicMonos1 /.
   {yI[t, 0] → 1, yE[t, 0] → 1, (D[yI[t, a], t] /. a → 0) → 1, (D[yI[t, a], a] /. a → 0) → 1,
    (D[yE[t, a], t] /. a → 0) → 1, (D[yE[t, a], a] /. a → 0) → 1, T[0] → 1}
Coeffs2 = MonicMonos2 /. {yI[t, 0] → 1, yE[t, 0] → 1,
    (D[yI[t, a], t] /. a → 0) → 1, (D[yI[t, a], a] /. a → 0) → 1,
    (D[yE[t, a], t] /. a → 0) → 1, (D[yE[t, a], a] /. a → 0) → 1, T[0] → 1}
```

$$\left\{-\frac{-c\,kE\,kI\,\delta - c\,kE\,kI\,\mu E}{kE}, -\frac{-kI\,\delta - kI\,\mu E}{kE},\right.$$
$$\left.-\frac{c\,kE^2\,\beta - c\,kE^2\,\beta\in}{kE}, -\frac{-kE\,\delta - kE\,\mu E}{kE}, c\,kI, \frac{kI}{kE}, 1, c\,kI, \frac{kI}{kE}, 1\right\}$$

$$\left\{-\frac{c\,kI^2\,\delta}{kE}, -\frac{kI^2\,\delta}{kE^2}, -\frac{c\,kE^2\,kI\,\beta\in - c\,kE^2\,kI\,\mu I}{kE^2},\right.$$
$$\left.-\frac{kE\,kI\,\delta - kE\,kI\,\mu I}{kE^2}, \mu I, c\,kI, \frac{kI}{kE}, 1, c\,kI, \frac{kI}{kE}, 1\right\}$$

```
Coeffs = Union[Coeffs1, Coeffs2]
```

$$\left\{1,\ c\,kI,\ \frac{kI}{kE},\ -\frac{kI^2\,\delta}{kE^2},\ -\frac{c\,kI^2\,\delta}{kE},\ -\frac{c\,kE^2\,\beta - c\,kE^2\,\beta\,\epsilon}{kE},\ -\frac{-kE\,\delta - kE\,\mu E}{kE},\ -\frac{-kI\,\delta - kI\,\mu E}{kE},\right.$$
$$\left.-\frac{-c\,kE\,kI\,\delta - c\,kE\,kI\,\mu E}{kE},\ \mu I,\ -\frac{kE\,kI\,\delta - kE\,kI\,\mu I}{kE^2},\ -\frac{c\,kE^2\,kI\,\beta\,\epsilon - c\,kE^2\,kI\,\mu I}{kE^2}\right\}$$

```
xCoeffs = Coeffs /. {β → a1, δ → a2, ϵ → a3, μE → a5, c → a7, kE → a8, kI → a9, m → a10}
```

$$\left\{1,\ a7\,a9,\ \frac{a9}{a8},\ -\frac{a2\,a9^2}{a8^2},\ -\frac{a2\,a7\,a9^2}{a8},\ -\frac{a1\,a7\,a8^2 - a1\,a3\,a7\,a8^2}{a8},\right.$$
$$-\frac{-a2\,a8 - a5\,a8}{a8},\ -\frac{-a2\,a9 - a5\,a9}{a8},\ -\frac{-a2\,a7\,a8\,a9 - a5\,a7\,a8\,a9}{a8},$$
$$\left.\mu I,\ -\frac{a2\,a8\,a9 - a8\,a9\,\mu I}{a8^2},\ -\frac{a1\,a3\,a7\,a8^2\,a9 - a7\,a8^2\,a9\,\mu I}{a8^2}\right\}$$

```
Solve[Coeffs == xCoeffs, {β, δ, ϵ, μE, c, kE, kI, m}]
```

```
MessageTemplate[Solve, svars, Equations may not give solutions for all "solve" variables.,
  2, 50, 2, 33627909590358880545, Local]
```

$$\left\{\left\{\beta \to a1,\ \delta \to a2,\ \epsilon \to a3,\ \mu E \to a5,\ c \to \frac{a7\,a9}{kI},\ kE \to \frac{a8\,kI}{a9}\right\}\right\}$$