# Optimisation of Multiple Control Strategies on a Dengue Fever Model.

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#### Introduction

This example explores the optimal control on a dengue fever model using multiple combinations of NPIs strategies. The model and strategies tested match those presentented by Asamoah et al. (2021). The model takes into account 2 populations: human and female mosquito or vector population. The human population is divided into 5 compartments: susceptible S\_h, infected (symptomatic) I\_h, carrier (asymptomatic) I\_hA, partially immune P, and recovered R\_h. While the mosquito population is formed by 2 compartments: susceptible S\_v and infected I\_v.

The model is described by the following differential equations:

$$\begin{split} N_h(t) &= S_h(t) + I_h(t) + I_h A(t) + P(t) + R_h(t), \\ N_v(t) &= S_v(t) + I_v(t), \\ \lambda_h(t) &= \frac{(1 - u_1(t))b\beta_1}{N_h(t)} I_v(t), \\ \lambda_h 1(t) &= \frac{(1 - u_1(t))b\beta_2}{N_h(t)} I_v(t), \\ \lambda_v(t) &= \frac{b\beta_3}{N_h(t)} (I_h(t) + I_h A(t)). \end{split}$$

$$\begin{split} \frac{dS_h}{dt} &= \mu_h N_h - \lambda_h S_h - S_h u_2 - \mu_h S_h, \\ \frac{dI_h}{dt} &= \psi \lambda_h S_h + \omega \lambda_h 1P - (\mu_h + u_3 + \gamma_h) I_h, \\ \frac{dI_h A}{dt} &= (1 - \psi) \lambda_h S_h + (1 - \omega) \lambda_h 1P - (\mu_h + \gamma_h) I_h A, \\ \frac{dP}{dt} &= u_2 S_h + \rho u_3 I_h + \phi \gamma_h (I_h + I_h A) - \lambda_h 1P - \mu_h P, \\ \frac{dR_h}{dt} &= (1 - \rho) u_3 I_h + (1 - \phi) \gamma_h (I_h + I_h A) - \mu_h R_h, \\ \frac{dS_v}{dt} &= \mu_v N_v (1 - u_4) - \lambda_v S_v - \mu_v S_v - r_0 u_4 S_v, \\ \frac{dI_v}{dt} &= \lambda_v S_v - \mu_v I_v - r_0 u_4 I_v. \end{split}$$

#### Libraries

```
using JuMP
using Ipopt
using OrdinaryDiffEq
using Plots
using Measures
using MAT
```

#### **Functions**

#### Model

```
function dengue_ode!(du,u,p,t)
     (S_h, I_h, I_hA, P, R_h, S_v, I_v) = u
     (\beta 1, \beta 2, \beta 3, \rho, \psi, \gamma_h, \omega, \mu_h, \mu_v, \phi, r_0, u1, u2, u3, u4) = p
    N h = S h + I h + I hA + P + R h
    N_v = S_v + I_v
    \lambda_h = ((1 - u1) * b * \beta1 / N_h) * I_v
    \lambda_h1 = ((1 - u1) * b * \beta2 / N_h) * I_v
    \lambda_v = (b * \beta 3 / N_h) * (I_h + I_hA)
    @inbounds begin
         du[1] = \mu_h * N_h - \lambda_h * S_h - S_h * u^2 - \mu_h * S_h
                                                                                                              # dS h
         du[2] = \psi * \lambda_h * S_h + \omega * \lambda_h 1 * P - (\mu_h + u 3 + \gamma_h) * I_h
                                                                                                              # dI h
         du[3] = (1 - \psi) * \lambda h * S h + (1 - \omega) * \lambda h1 * P - (\mu h + \gamma h) * I hA
                                                                                                              # dI h
         du[4] = u2 * S_h + \rho * u3 * I_h + \phi * \gamma_h * (I_h + I_hA) - \lambda_h1 * P - \mu_h * P # dP/d
         du[5] = (1 - \rho) * u3 * I_h + (1 - \phi) * \gamma_h * (I_h + I_hA) - \mu_h * R_h
                                                                                                              # dR h
```

```
du[7] = \lambda_v * S_v - \mu_v * I_v - r_0 * u4 * I_v
                                                                                         # dI_v
    end
    nothing
end:
To create plots
function create combined plot(ts, u opts, S h opts, I h opts, I hA opts, P opts, R h opts, S v
    color_palette = [:red, :blue, :green, :magenta, :purple, :cyan, :orange, :yellow];
    linestyle_palette = [:solid, :dash, :dot, :dashdot, :dashdotdot, :dash, :dot, :dashdot];
    combined_plot = plot(layout = (3, 2), dpi=300, size=(1200,1500),
                            left_margin=10mm, right_margin=10mm, top_margin=10mm, bottom_margi
        # First subplot:
        for (i, I_h_opt) in enumerate(I_h_opts)
            plot!(combined_plot[1, 1], ts, I_h_opt,
                label="Scenario $(scenario_label[i])",
                linewidth=3, color=color_palette[i], thickness_scaling=1,
                xlim=(0, ts[end]),
                ylim=(0, 600),
                xtickfontsize=12, ytickfontsize=12,
                xguidefontsize=14, yguidefontsize=14,
                legendfontsize=12, legend=:topright)
        xlabel!(combined_plot[1, 1], "Time (days)")
        ylabel!(combined_plot[1, 1], "Symptomatic humans, I_h")
        title!(combined_plot[1, 1], "Symptomatic humans, I_h")
        # Second subplot:
        for (i, I_hA_opt) in enumerate(I_hA_opts)
            plot!(combined_plot[1, 2], ts, I_hA_opt,
                label="Scenario $(scenario_label[i])",
                linewidth=3, color=color_palette[i], thickness_scaling=1,
                xlim=(0, ts[end]+1),
                ylim=(0, 800),
                xtickfontsize=12, ytickfontsize=12,
                xguidefontsize=14, yguidefontsize=14,
                legendfontsize=12, legend=:topright)
        xlabel!(combined_plot[1, 2], "Time (days)")
        ylabel!(combined_plot[1, 2], "Asymptomatic humans, I_hA")
        title!(combined_plot[1, 2], "Asymptomatic humans, I_hA")
        # Third subplot:
```

 $du[6] = \mu_v * N_v * (1 - u4) - \lambda_v * S_v - \mu_v * S_v - r_0 * u4 * S_v$ 

# dS\_v

```
for (i, S_v_opt) in enumerate(S_v_opts)
    plot!(combined_plot[2, 1], ts, S_v_opt,
        label="Scenario $(scenario label[i])",
        linewidth=3, color=color_palette[i], thickness_scaling=1,
        xlim=(0, ts[end]),
        ylim=(0, 6500),
        xtickfontsize=12, ytickfontsize=12,
        xguidefontsize=14, yguidefontsize=14,
        legendfontsize=12,
        legend=:right)
end
xlabel!(combined_plot[2, 1], "Time (days)")
ylabel!(combined_plot[2, 1], "Susceptible mosquitoes, S_v")
title!(combined_plot[2, 1], "Susceptible mosquitoes, S_v")
# Fourth subplot:
for (i, I_v_opt) in enumerate(I_v_opts)
    plot!(combined_plot[2, 2], ts, I_v_opt,
        label="Scenario $(scenario_label[i])",
        linewidth=3, color=color palette[i], thickness scaling=1,
        xlim=(0, ts[end]),
        ylim=(0, 3500),
        xtickfontsize=12, ytickfontsize=12,
        xguidefontsize=14, yguidefontsize=14,
        legendfontsize=12, legend=:topright)
end
xlabel!(combined_plot[2, 2], "Time (days)")
ylabel!(combined_plot[2, 2], "Infected mosquitoes, I_v")
title!(combined_plot[2, 2], "Infected mosquitoes, I_v")
# Fifth subplot:
for (i, P_opt) in enumerate(P_opts)
    plot!(combined_plot[3, 1], ts, P_opt,
        label="Scenario $(scenario_label[i])",
        linewidth=3, color=color_palette[i], thickness_scaling=1,
        xlim=(0, ts[end]),
        ylim=(0, P_axis),
        xtickfontsize=12, ytickfontsize=12,
        xguidefontsize=14, yguidefontsize=14,
        legendfontsize=12, legend=(P_legend))
xlabel!(combined_plot[3, 1], "Time (days)")
ylabel!(combined_plot[3, 1], "Partially immune, P")
title!(combined_plot[3, 1], "Partially immune, P")
if !isempty(u_opts)
```

```
# u_opts = [u1_opt_1, u2_opt_1, u3_opt_1, u4_opt_1]
           for (i, u_opt) in enumerate(u_opts)
               plot!(combined_plot[3, 2], ts, u_opt, label="u$i", linewidth=3, color=color_pa
                   xlim=(0, ts[end]+1),
                   ylim=(0, 1.1),
                   xtickfontsize=12, ytickfontsize=12,
                   xguidefontsize=14, yguidefontsize=14,
                   legendfontsize=12, legend columns=4,
                   legend=:topright)
           end
           xlabel!(combined_plot[3, 2], "Time (days)")
           title!(combined_plot[3, 2], "Control Profiles Scenario $(scenario_label[2])")
       end
   display(combined_plot)
       # Save the plot if `save=true`
       filename = "OptControl_Dengue_$((scenario_label[2])).png"
       savefig(combined plot, filename)
       println("Plot saved as $filename")
   end
end;
To easily run the optimisation of different strategies
function run_optimization(u1_max, u2_max, u3_max, u4_max)
   # Model definition with Ipopt optimizer
   model = Model(Ipopt.Optimizer)
   set_optimizer_attribute(model, "max_iter", 10000)
   set_optimizer_attribute(model, "print_level", 5)
   set_optimizer_attribute(model, "tol", 1e-6)
   u1_init = 0
   u2 init = 0
   u3_init = 0
   u4_init = 0
   T = Int(tf/dt)
   @variable(model, S_h[1:(T+1)] >= 0) # Susceptible humans
                                       # Infected symptomatic
   @variable(model, I_h[1:(T+1)] >= 0)
   @variable(model, I_hA[1:(T+1)] >= 0) # Carriers asymptomatic
```

```
@variable(model, 0 <= u2[1:(T+1)] <= u2_max)</pre>
                                                                                                                                                                                                                                                                                                                                         # Vaccination
@variable(model, 0 <= u3[1:(T+1)] <= u3_max)</pre>
                                                                                                                                                                                                                                                                                                                                  # Treatment (prophylactics)
@variable(model, 0 \le u4[1:(T+1)] \le u4_max)
                                                                                                                                                                                                                                                                                                                                 # Insecticides
# Initial conditions
@constraints(model, begin
                          S_h[1] == S_h0
                         I_h[1] == I_h0
                         I_hA[1] == I_hA0
                         P[1] == P0
                         R h[1] == R h0
                         S_v[1] == S_v0
                         I_v[1] == I_v0
                         u1[1] == u1_init
                         u2[1] == u2_init
                         u3[1] == u3_init
                         u4[1] == u4_init
                          [t=(T+1)], u1[t] == u1[t-1]
                          [t=(T+1)], u2[t] == u2[t-1]
                          [t=(T+1)], u3[t] == u3[t-1]
                          [t=(T+1)], u4[t] == u4[t-1]
end);
# Population sizes and infection rates
@expressions(model, begin
                         N_h[t=1:T], (S_h[t] + I_h[t] + I_hA[t] + P[t] + R_h[t])
                         N_v[t=1:T], (S_v[t] + I_v[t])
                         lambda\_h[t=1:T]\,,\ ((1\ -\ u1[t])\ *\ b\ *\ beta\_1\ /\ N\_h[t])\ *\ I\_v[t]
                         lambda\_h1[t=1:T] \;,\;\; ((1 \;-\; u1[t]) \;\; * \;\; b \;\; * \;\; beta\_2 \;\; / \;\; N\_h[t]) \;\; * \;\; I\_v[t]
                          lambda_v[t=1:T], (b * beta_3 / N_h[t]) * (I_h[t] + I_hA[t])
end)
# ODEs
@constraints(model, begin
 [t=1:T] \;,\; S_h[t+1] \; == \; S_h[t] \; + \; (mu_h \; * \; N_h[t] \; - \; lambda_h[t] \; * \; S_h[t] \; - \; S_h[t] \; * \; u2[t] \; - \; mu_h[t] \; + \; u2[t] \; - \; mu_h[t] \; - \; u2[t] \; - \;
[t=1:T], I_h[t+1] == I_h[t] + (psi * lambda_h[t] * S_h[t] + omega * lambda_h1[t] * P[t] -
[t=1:T], I_hA[t+1] == I_hA[t] + ((1 - psi) * lambda_h[t] * S_h[t] + (1 - omega) * lambda_h[t] + (1 - omega) * 
[t=1:T], P[t+1] == P[t] + (u2[t] * S_h[t] + rho * u3[t] * I_h[t] + phi * gamma_h * (I_h[t] + phi) 
 [t=1:T], R_h[t+1] == R_h[t] + ((1 - rho) * u3[t] * I_h[t] + (1 - phi) * gamma_h * (I_h[t] + (1 - rho) * u3[t] + (1 - rho) 
 [t=1:T], S_v[t+1] == S_v[t] + (mu_v * N_v[t] * (1 - u4[t]) - lambda_v[t] * <math>S_v[t] - mu_v * M_v[t]
[t=1:T], \ I\_v[t+1] \ == \ I\_v[t] \ + \ (lambda\_v[t] \ * \ S\_v[t] \ - \ mu\_v \ * \ I\_v[t] \ - \ r\_0 \ * \ u4[t] \ * \ I\_v[t]
end);
```

# Infected mosquitoes

# Treated bednet

@variable(model, I v[1:(T+1)] >= 0)

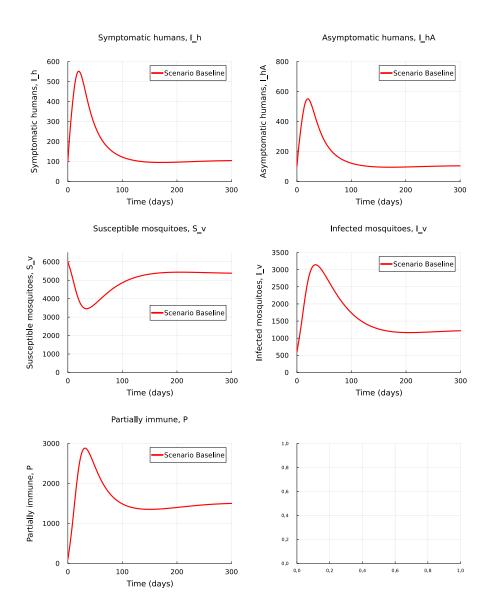
@variable(model, 0 <= u1[1:(T+1)] <= u1 max)</pre>

#### Running the model without interventions

#### **Parameters**

```
beta_1 = 0.75; #Transmission prob from I_v to S_h
beta_2 = 0.375; #Transmission prob from I_h to S_v
beta_3 = 0.75; #Transmission prb from I_v to P
b = 0.5;
                #Avg bitting rate per mosquito per person
rho = 0.01;  #Proportion of treated individuals with partial immunity
psi = 0.4;  #Proportion of incidence rate from S_h to I_h
gamma_h = 0.3288330; #Disease related death rate of humans
omega = 0.54; #Proportion of incidence rate from P to I_h
mu_h = 0.0045; #Natural mortality rate and recruitment rate of humans
mu_v = 0.0323; #Natural mortality rate and recruitment rate of vector
phi = 0.48;  #Proportion of natural Recovery
r_0 = 0.005; #Enhance death rate
u_1 = 0.0;
               #Treated bednet
u_2 = 0.0;
               #Vaccination
u_3 = 0.0;
               #Treatment (prophylactics)
u_4 = 0.0;
                #Insecticides
Time and initial conditions
t0 = 0.0;
tf = 300.0;
dt = 0.5;
ts = collect(0:dt:tf);
```

```
S_h0 = 10000;
I_h0 = 100;
I hA0 = 500;
P0 = 100;
R_h0 = 1000;
S_v0 = 6000;
I_v0 = 600;
\delta = 0.001;
alg = Tsit5();
u0 = [S_h0, I_h0, I_hA0, P0, R_h0, S_v0, I_v0];
prob_base = ODEProblem(dengue_ode!, u0, (t0, tf), params)
sol_base = solve(prob_base, alg, saveat=ts);
Extract data:
t_base = sol_base.t;
S_h_baseline = sol_base[1, :];
I_h_baseline = sol_base[2, :];
I_hA_baseline = sol_base[3, :];
P_baseline = sol_base[4, :];
R_h_baseline = sol_base[5, :];
S_v_baseline = sol_base[6, :];
I_v_baseline = sol_base[7, :];
Plot baseline
Scenario_test = create_combined_plot(t_base,
[S_h_baseline],
[I_h_baseline],
[I_h_baseline],
[P_baseline],
[R_h_baseline],
[S_v_baseline],
[I_v_baseline],
["Baseline"], save=false)
```



## **Strategies**

```
# Weights for the objective function C_1 = 5; C_2 = 5; C_3 = 5; D_1 = 16.62;
```

```
D_2 = 2.5;
D_3 = 5;
D_4 = 16.62;
silent = true;
```

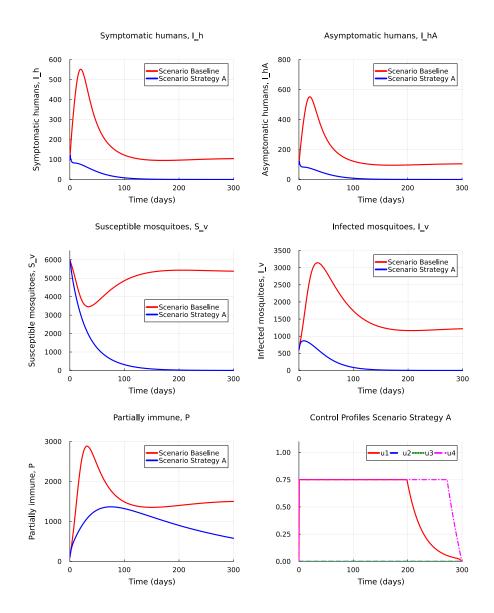
#### Strategy A

This strategy combines the use of treated bednets and insectisides

```
outputA_values = open("OptDengue_StratA_log.txt", "w") do file
    redirect_stdout(file) do
        return run_optimization(0.75, 0, 0, 0.75)
    end
end

# Now extract values from output_values
u1_opt_A, u2_opt_A, u3_opt_A, u4_opt_A, S_h_opt_A, I_h_opt_A, I_hA_opt_A, P_opt_A, R_h_opt_A,
    Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl:
    @ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
    FWarning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl:
    @ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
    FWarning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl:
    @ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
    FWarning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl:
    @ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
    FWarning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl:
    @ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
```

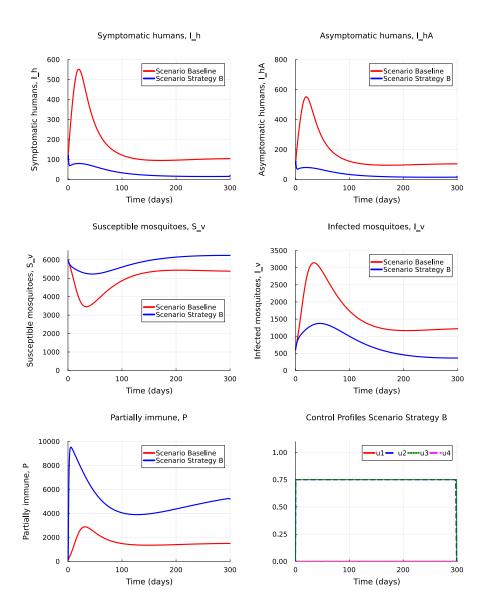
Plot optimal results obtained from Strategy A vs Baseline



#### Strategy B

This strategy combines the use of treatment (prophylactics) and vaccination

```
outputB_values = open("OptDengue_StratB_log.txt", "w") do file
    redirect_stdout(file) do
        return run_optimization(0, 0.75, 0.75, 0)
    end
end
```



### Strategy C

This strategy combines the use of treated bednets, treatment (prophylactics) and insectisides

```
outputC_values = open("OptDengue_StratC_log.txt", "w") do file
  redirect_stdout(file) do
     return run_optimization(0.75, 0, 0.75, 0.75)
  end
```

#### end

```
# Now extract values from output_values
u1_opt_C, u2_opt_C, u3_opt_C, u4_opt_C, S_h_opt_C, I_h_opt_C, I_hA_opt_C, P_opt_C, R_h_opt_C,

_ Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl.

@ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185

_ Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl.

@ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185

_ Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl.

@ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185

_ Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl.

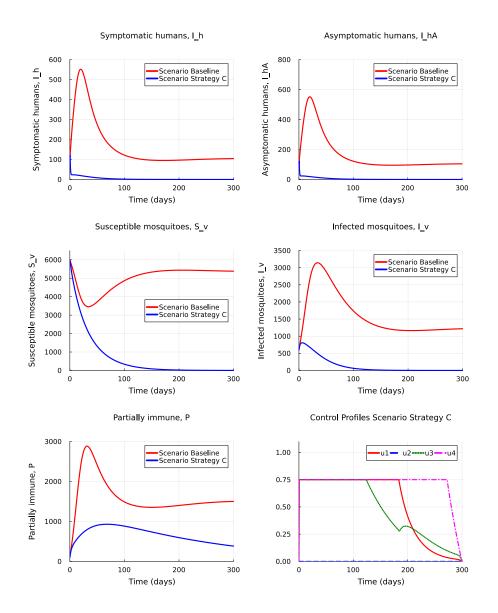
@ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185

Plot optimal results obtained from Strategy C vs Baseline

Scenario_C = create_combined_plot(t_base,
```

[I\_v\_baseline, I\_v\_opt\_C],

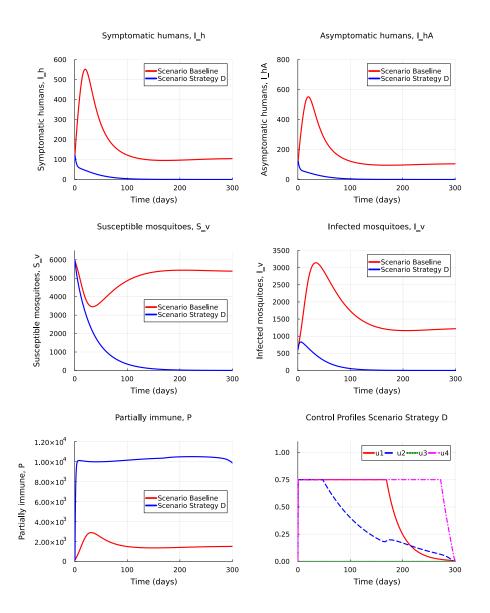
["Baseline", "Strategy C"], save=false)



### Strategy D

This strategy combines the use of treated bednets, insectisides and vaccination

```
outputD_values = open("OptDengue_StratD_log.txt", "w") do file
    redirect_stdout(file) do
        return run_optimization(0.75, 0.75, 0, 0.75)
    end
end
```



### Strategy E

This strategy combines the use of the four control interventions treated bednets, insectisides, treatment and vaccination

```
outputE_values = open("OptDengue_StratE_log.txt", "w") do file
    redirect_stdout(file) do
        return run_optimization(0.75, 0.75, 0.75, 0.75)
    end
```

#### end

```
# Now extract values from output values
u1_opt_E, u2_opt_E, u3_opt_E, u4_opt_E, S_h_opt_E, I_h_opt_E, I_hA_opt_E, P_opt_E, R_h_opt_E,
√ Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl
L@ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
Γ Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl
^{f L} @ JuMP.Containers \sim/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
r Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl
L@ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
r Warning: Axis contains one element: 601. If intended, you can safely ignore this warning. To expl
L@ JuMP.Containers ~/.julia/packages/JuMP/xlp0s/src/Containers/DenseAxisArray.jl:185
Plot optimal results obtained from Strategy E vs Baseline
Scenario_E = create_combined_plot(t_base,
```

```
[u1_opt_E, u2_opt_E, u3_opt_E, u4_opt_E],
[S_h_baseline, S_h_opt_E],
[I_h_baseline, I_h_opt_E],
[I h baseline, I h opt E],
[P_baseline, P_opt_E],
[R_h_baseline, R_h_opt_E],
[S_v_baseline, S_v_opt_E],
[I_v_baseline, I_v_opt_E],
["Baseline", "Strategy E"], P_axis=12000, P_legend=:right, save=false)
```

