

Slalom in complex time

Dealing with the imaginary position of a quantum orbit

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Quantum orbits provide a clear physical picture that underlies the Strong-Field Approximation and gives intuition for its results. One of their weirder features is the presence of an imaginary component of the position, which emerges naturally in first-principles approaches. Normally ignorable, this imaginary component will dominate if the electron returns to the ion, and it can force changes to the usual contour in the complex time plane. We explain why this happens and how to navigate the complex plane when it does. As a result, known and new Low Energy Structures emerge naturally.

What do imaginary positions do to the potential?

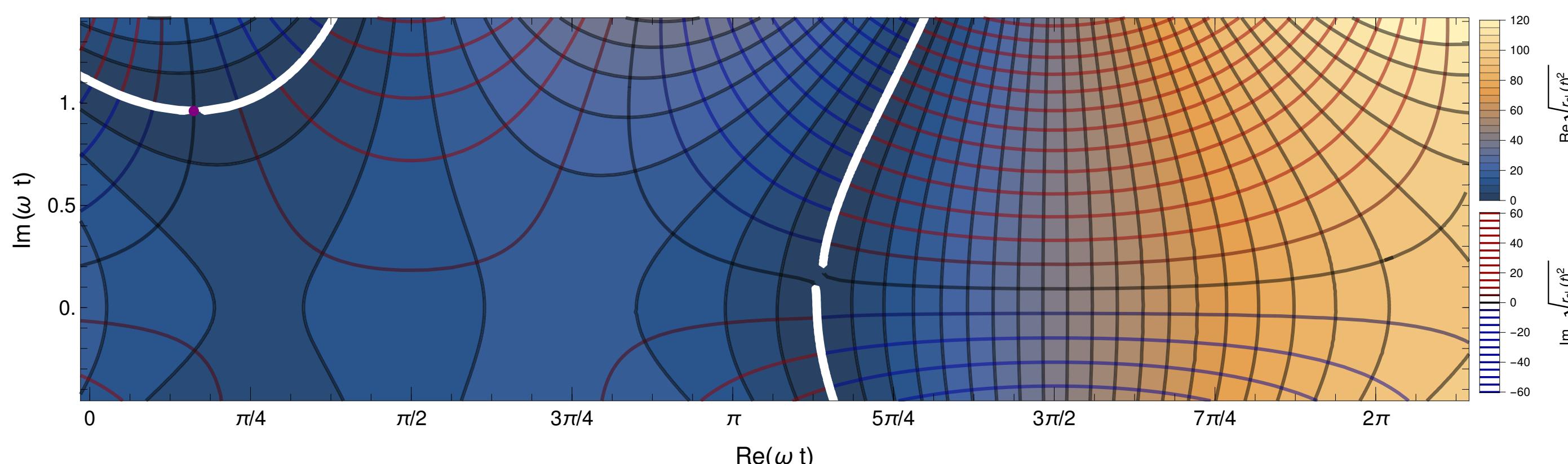
The Coulomb potential $U(r)$ must be analytically continued to the complex plane, as

$$U(\mathbf{r}_{cl}(t)) = \frac{-1}{\sqrt{\mathbf{r}_{cl}(t)^2}}.$$

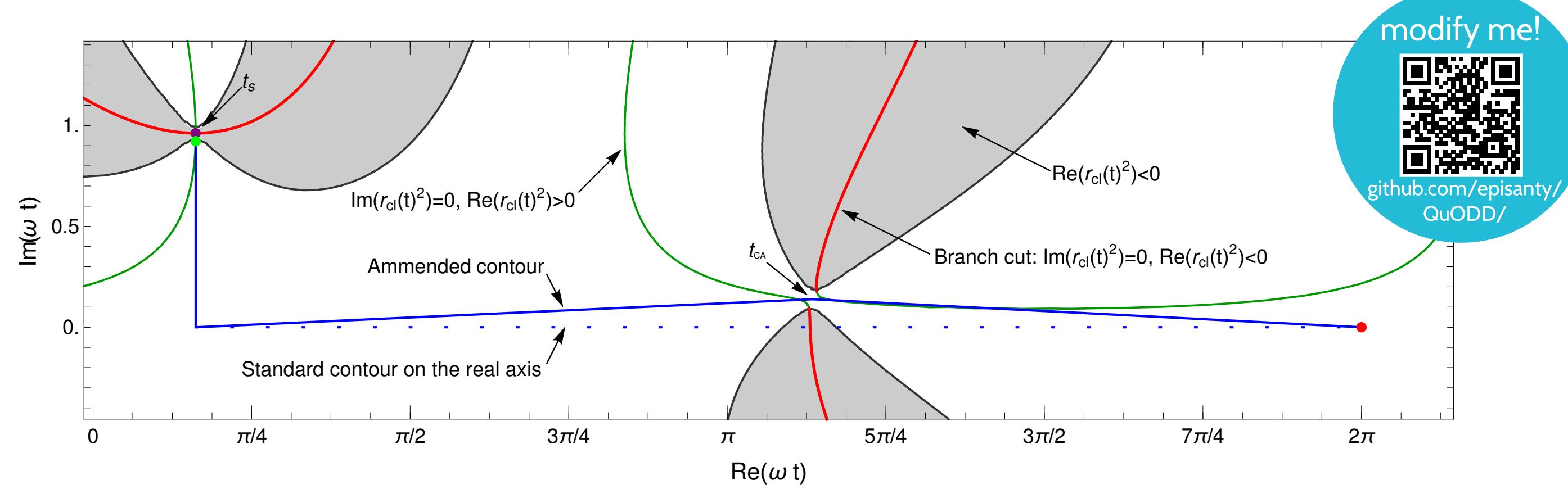
If $\mathbf{r}_{cl}(t)^2$ becomes negative, then the square root discontin-

uously changes sign. This is a branch cut which should not be integrated over.

The solution is to see $U(\mathbf{r}_{cl}(t))$ as a complex function in its own right with branch cuts which the contour should navigate directly, and which can cross the real axis.



Contour plot of $\sqrt{\mathbf{r}_{cl}(t)^2}$ on the complex time plane. The white lines are branch cuts where the imaginary part discontinuously changes sign.

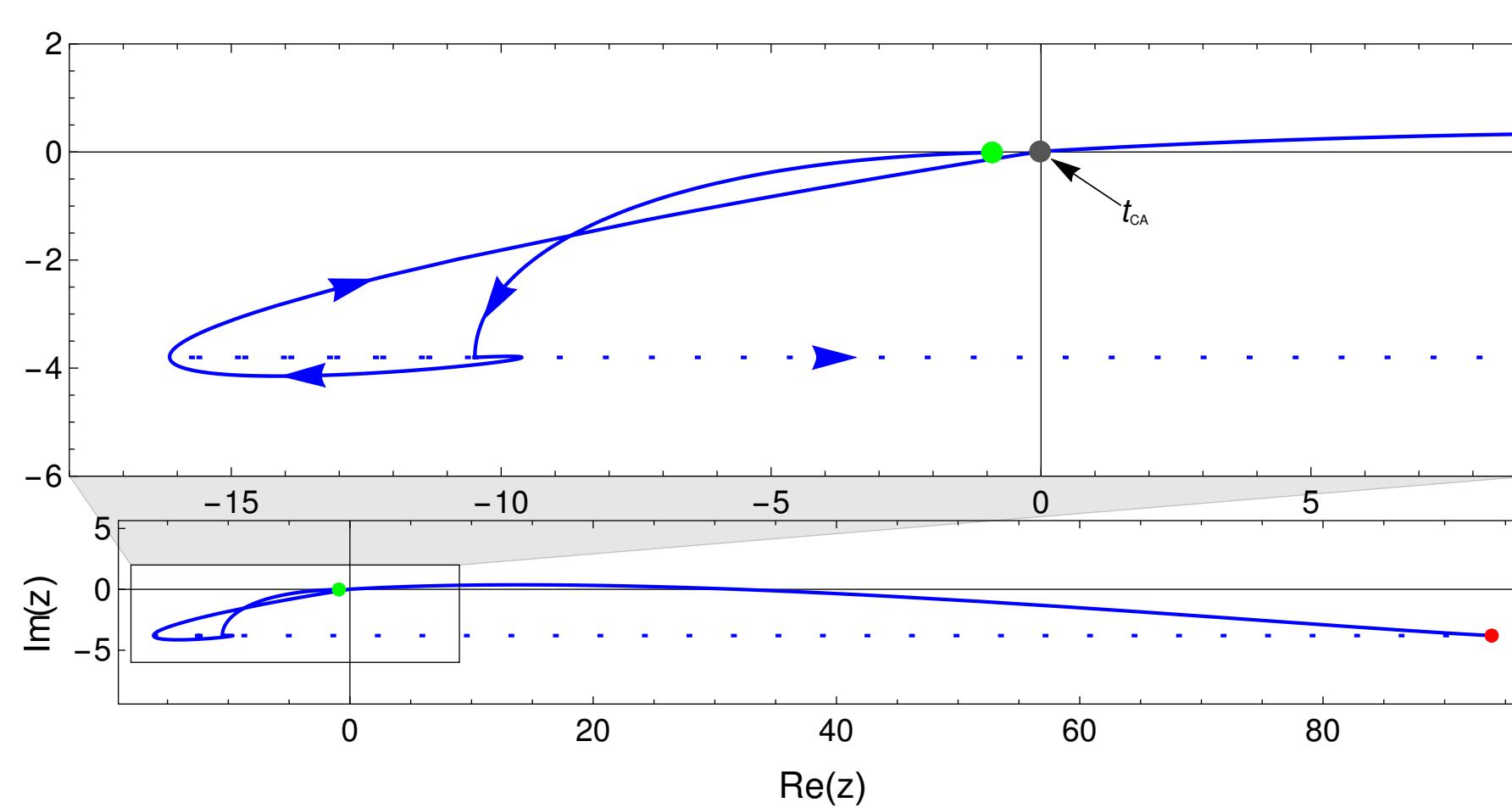


Sketch of the graphic above. Red lines are branch cuts of $\sqrt{\mathbf{r}_{cl}(t)^2}$, where $\mathbf{r}_{cl}(t)^2$ is real and negative. Green lines are the positive real axis and at gray regions $\text{Re}(\mathbf{r}_{cl}(t)^2) < 0$. The standard contour is shown dashed and crosses a branch cut. Instead, the contour should pass the closest-approach time t_{CA} .

Trading imaginary time for a real position

Changing the time-plane contour to pass through the closest-approach time t_{CA} has the effect of minimizing the imaginary part of the trajectory at the crucial times when the real part is small. When the potential $U(r)$ is most sensitive to imaginary positions, one needs to compromise with a slightly imaginary time.

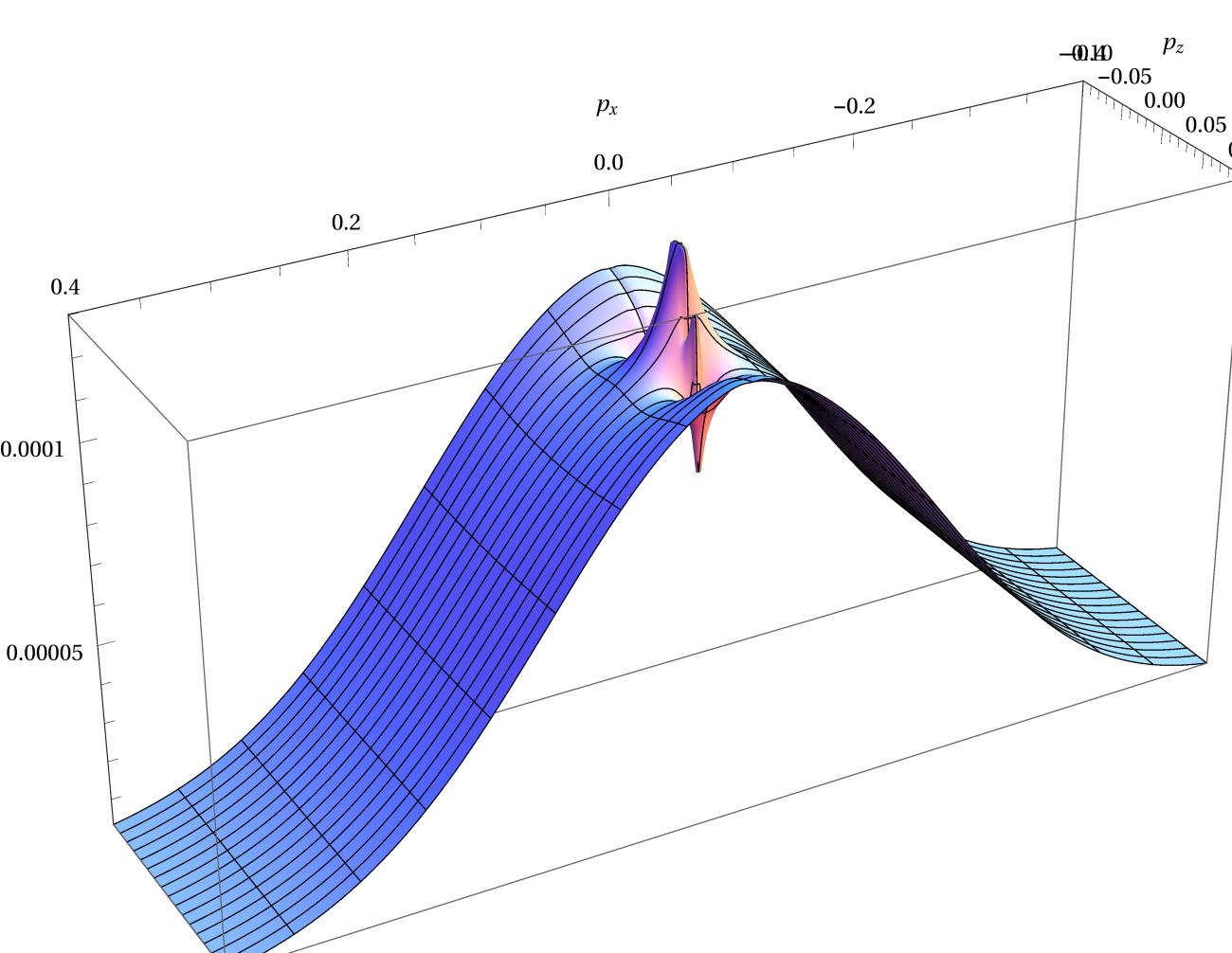
Trajectories on the complex z plane corresponding to the time contours above. The dashed contour over the real axis has a large imaginary position when it passes the ion.



Photoelectron spectra

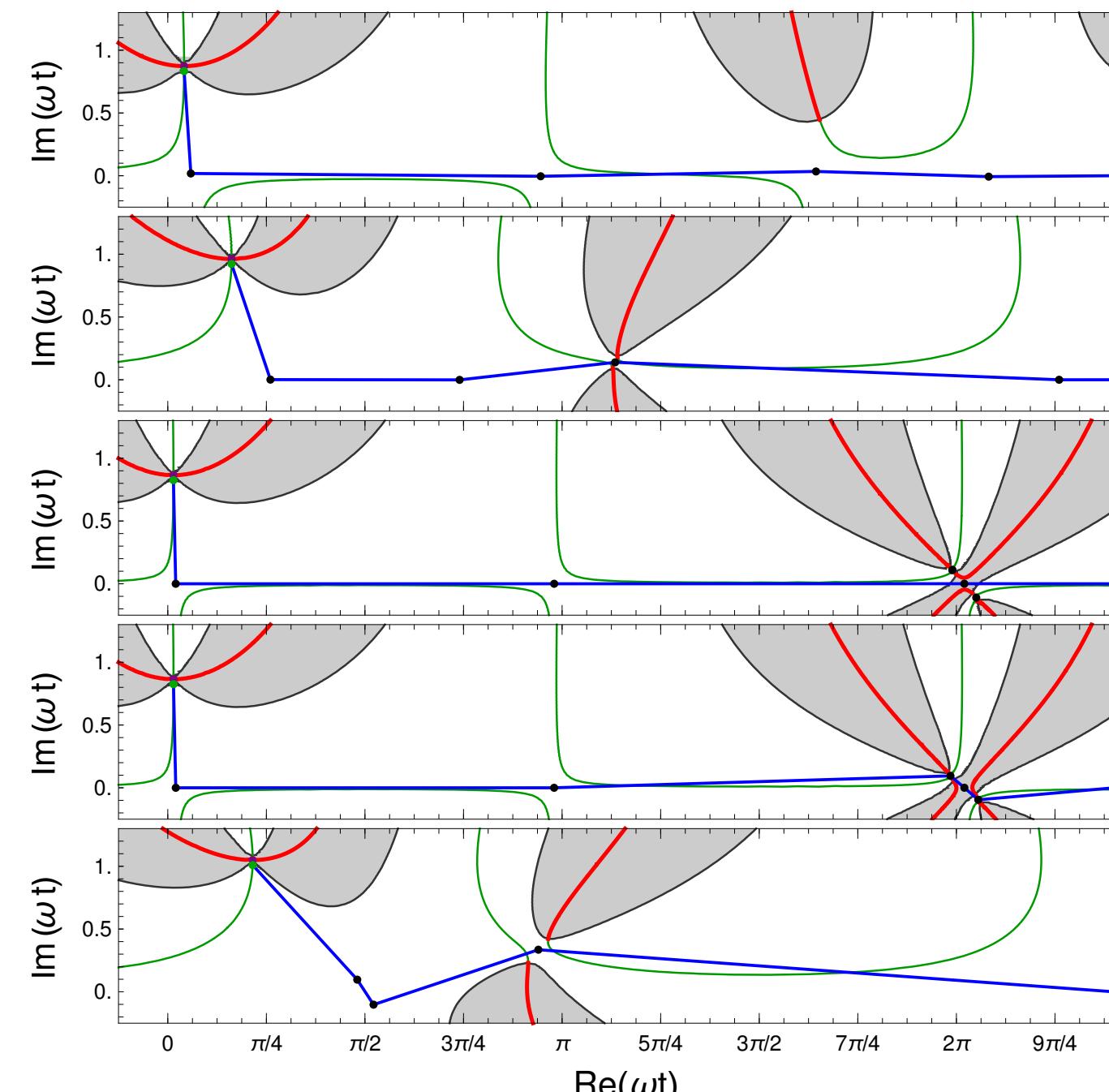
Soft recollisions cause cusps with abrupt cut-offs, as the change in contour brings in closer approaches to the ion with destructive amplitudes.

These cusps qualitatively match the transverse profile shown in high-resolution experiments [Pullen et al., J Phys B **47** 204010].



Branch cut slalom

Hopping through t_{CA} s, when they're out of the barrier, gives a perfect slalom score.



Why imaginary positions?

To extend the SFA to include the Coulomb interaction between the ion and a continuum electron, one would like an expression which includes its effect as a phase:

$$a(\mathbf{p}) = R(\mathbf{p}) e^{i p_z t_s} e^{-\frac{1}{2} \int_{t_s}^T (\mathbf{p} + \mathbf{A}(\tau))^2 d\tau} e^{-i \int_{t_s}^T U(\mathbf{r}_{cl}(\tau)) d\tau} \quad (1)$$

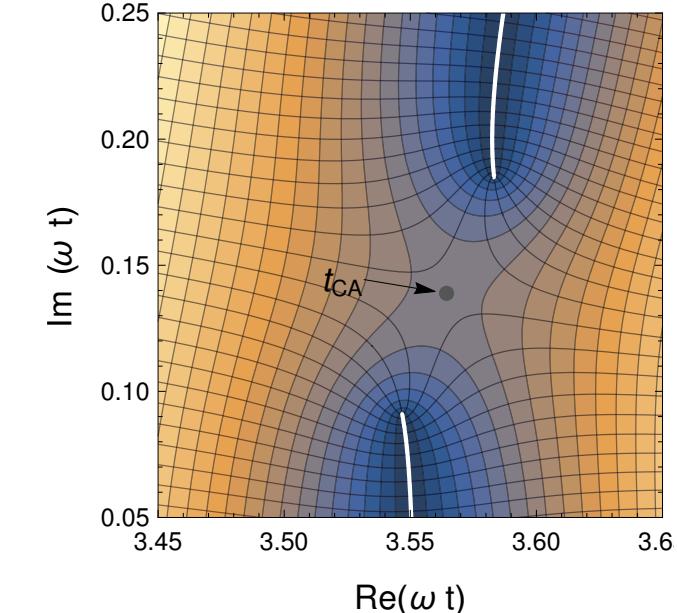
The Coulomb phase is best incorporated directly into the continuum wavefunction, which can be done with eikonal-Volkov states. The ion is treated separately using analytical R -matrix techniques.

The quantum orbit picture is obtained via a saddle-point argument, giving expression (1) with a trajectory

$$\mathbf{r}_{cl}(t) = \int_{t_s}^t \mathbf{p} + \mathbf{A}(\tau) dt$$

which is real at the entrance of the barrier, at the complex ionization time t_s . In general this trajectory will pick up an imaginary component as one integrates down to the real axis. This imaginary part is constant along the real axis but it does affect the values of the potential.

How do I avoid the branch cuts?



Branch cuts typically come in pairs with a 'slalom gate' between them. This gate must always contain a saddle point in the middle to match the contour geometries of the branch cuts.

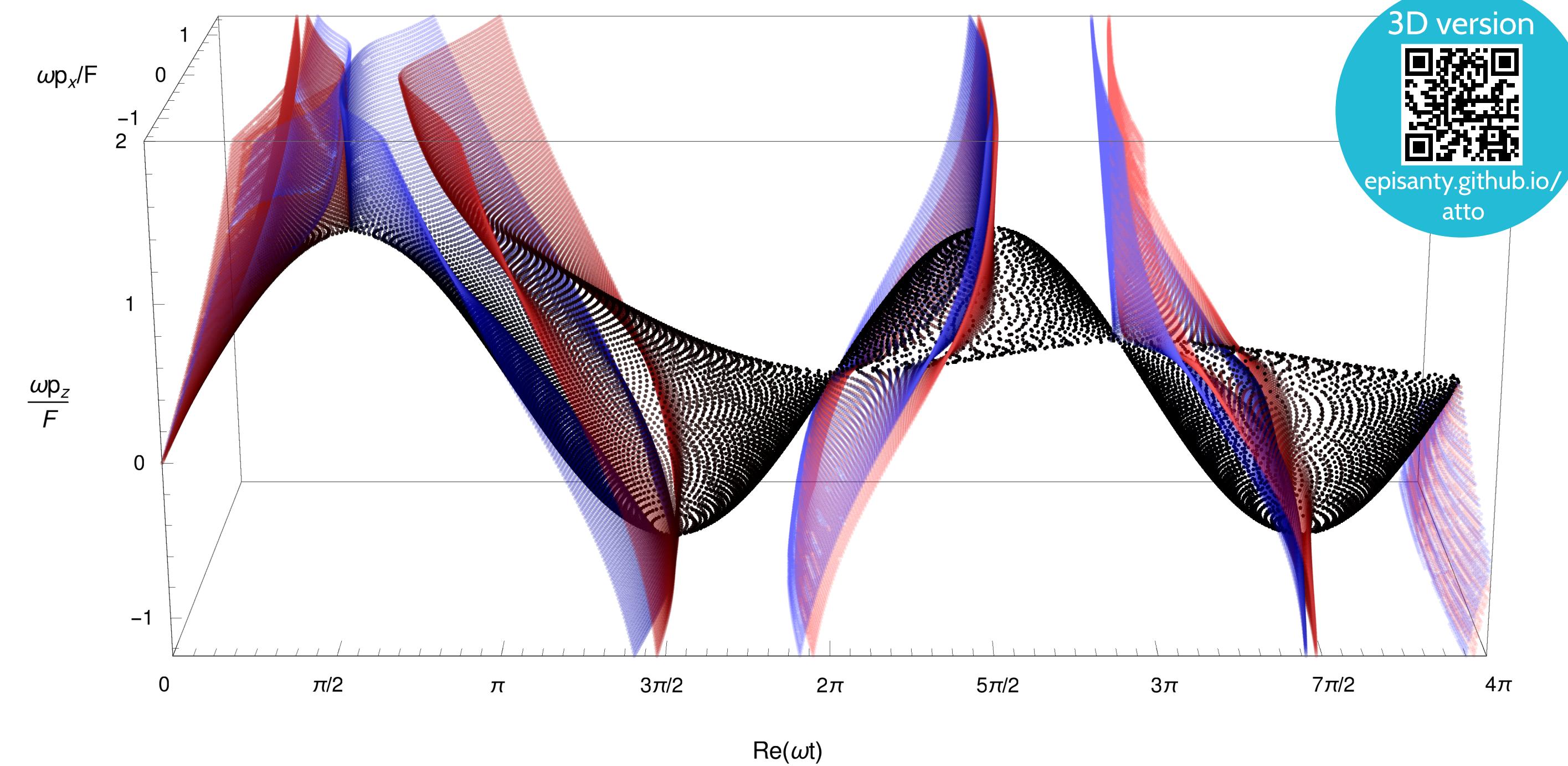
This is a saddle point of $\sqrt{\mathbf{r}_{cl}(t)^2}$ and therefore of $\mathbf{r}_{cl}(t)^2$, and it satisfies the equation

$$\frac{d}{dt} \mathbf{r}_{cl}(t)^2 \Big|_{t_{CA}} = \mathbf{r}_{cl}(t_{CA}) \cdot \mathbf{v}(t_{CA}) = 0. \quad (2)$$

This is a closest-approach equation: the (complex) position is orthogonal to the velocity.

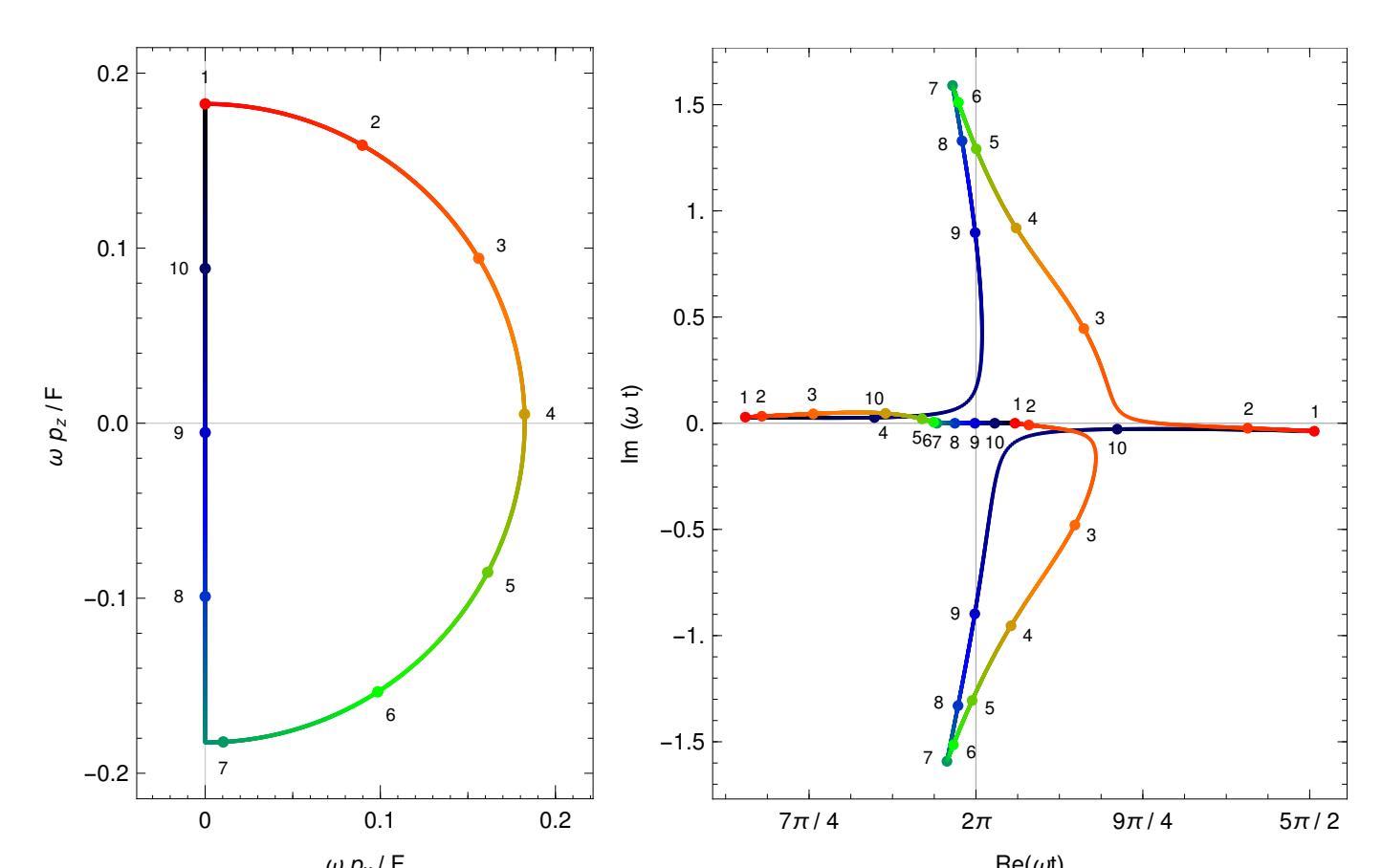
The geometry of closest-approach times

Closest-approach times as defined by Eq. (2) have a rich geometry. They form a multiple-sheet surface which wraps around the corresponding classical surface. Black points are mostly real, and red (blue) points have positive (negative) imaginary parts.

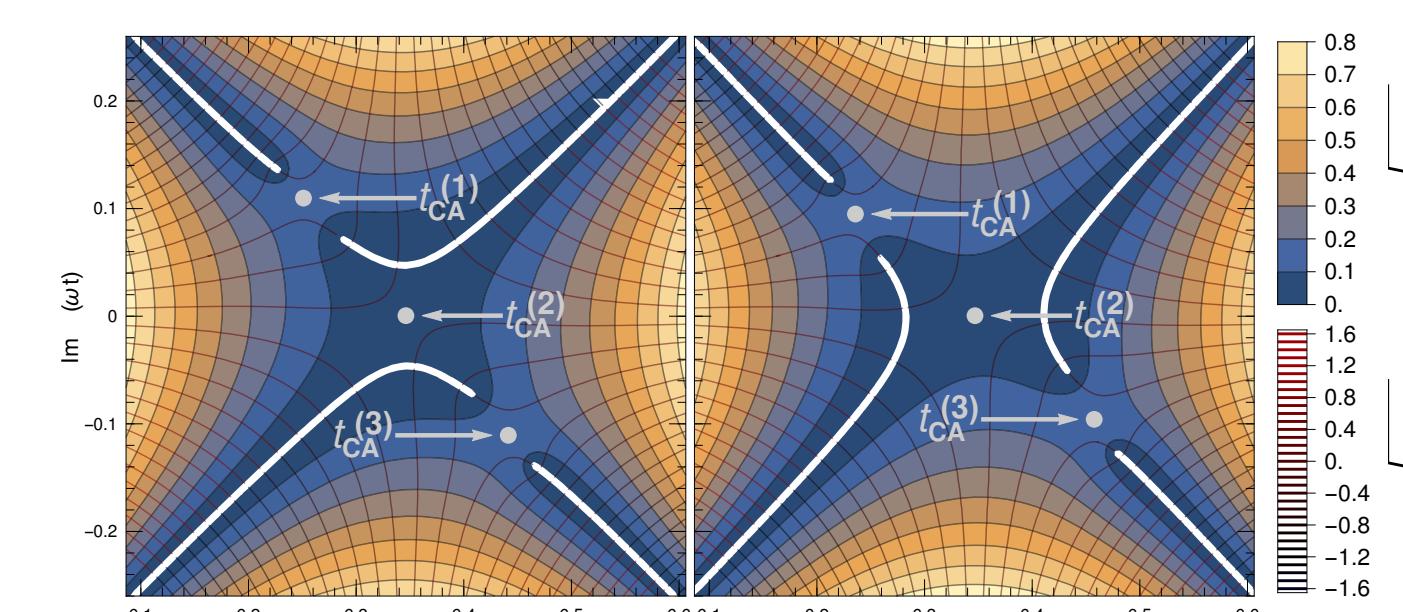


Soft recollisions

The multiple leaves of this surface are interconnected, which means that (2) has multiple roots but they cannot be consistently indexed. If one traces out a loop in momentum space and returns to the starting points, the t_{CA} s will swap labels. This happens at soft recollisions where the electron's velocity is small near the ion.



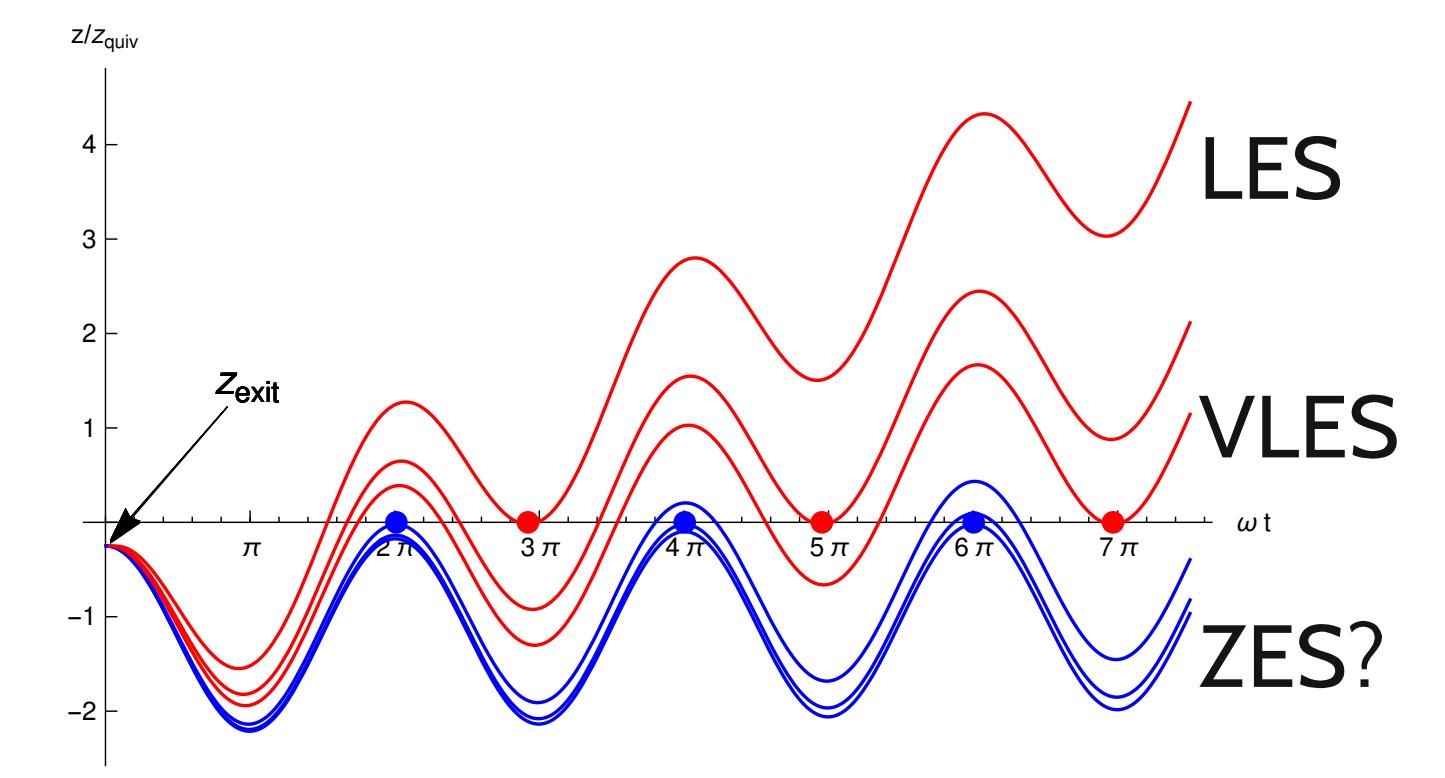
At soft recollisions, the topology of the branch cuts changes abruptly, forcing a change in the appropriate integration contour. This causes a sudden change from enhancement to suppression of ionization. The contour should pass t_{CA} s only when they are outside of the tunnelling barrier: when $\frac{1}{2}\mathbf{v}(t_{CA})^2 > 0$.



Soft recollisions occur in two series at different energy ranges, and with different scaling:

$$p_z^{\text{sr}} \sim \frac{F}{\omega(n+1)\pi} \quad (n \text{ even}), \quad \text{and} \quad p_z^{\text{sr}} \sim \frac{\omega}{F(n+1)2\pi} \quad (n \text{ odd}).$$

The former is responsible for Low-Energy Structures. The latter, at lower energy, should contribute to the recently-found Zero-Energy Structures. This scaling suggests higher- p_z targets should have clearer ZESs.



Acknowledgments

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Read the paper

