

Slalom in complex time

Dealing with the imaginary position of a quantum orbit

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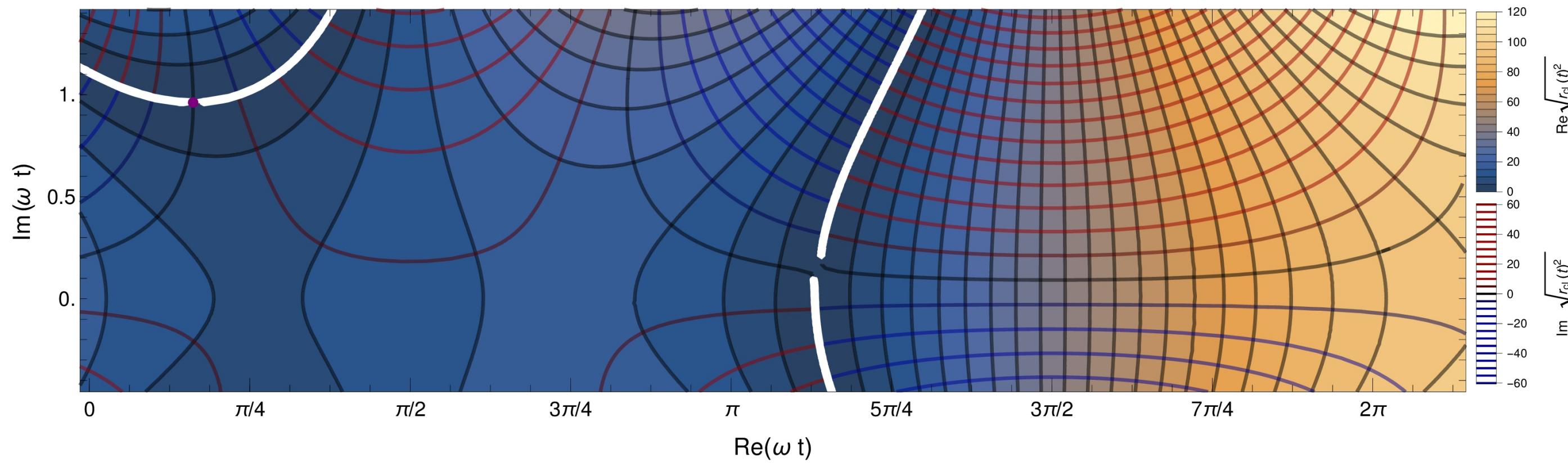
Quantum orbits provide a clear physical picture of electron dynamics driven by strong infrared laser fields: tunnelling out of the potential well, followed by oscillations in the field. One of their weirder features is the presence - in essentially every case - of an imaginary component of the position, as an inevitable footprint of the tunnelling event. Normally ignored, this imaginary component dominates when the electron returns to the ion and its real position is small. We explain what this means, and how to handle its consequences. As a result, known and new Low Energy Structures emerge naturally.

2 What do imaginary positions do to the potential?

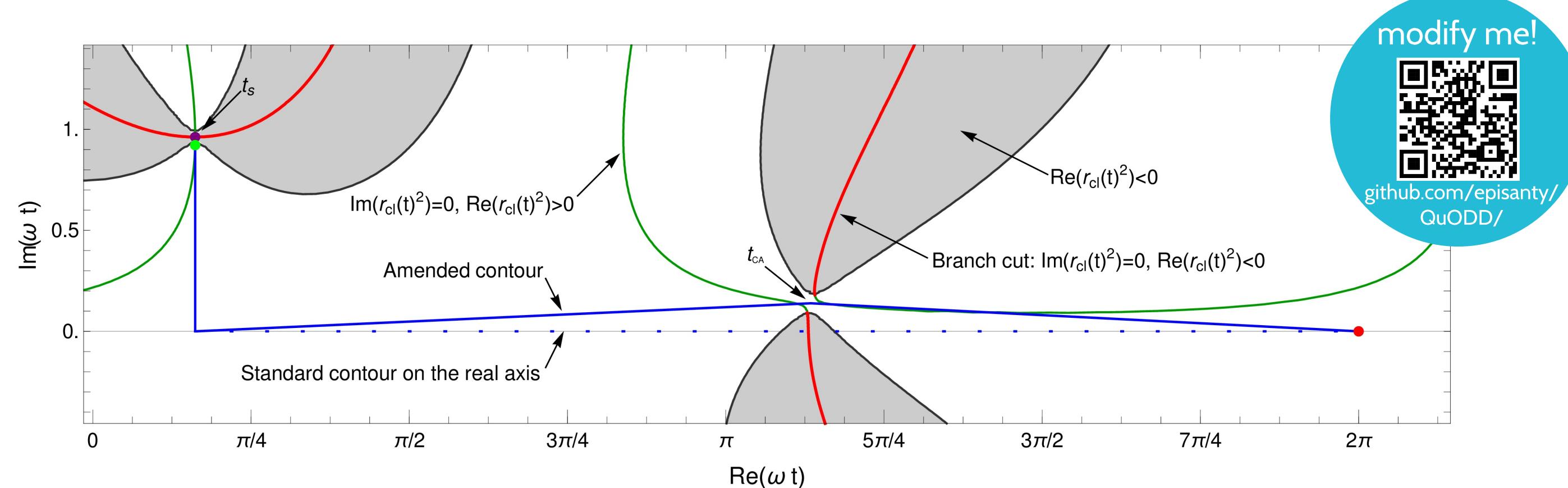
For complex trajectories, the Coulomb potential must be analytically continued to the complex plane, as

$$U(\mathbf{r}_{cl}(t)) = \frac{-1}{\sqrt{\mathbf{r}_{cl}(t)^2}}.$$

If $\mathbf{r}_{cl}(t)^2$ becomes negative, then the square root discontinuously changes sign. This is a branch cut which should be avoided, but what if it crosses the real time axis? The solution is to see $U(\mathbf{r}_{cl}(t))$ as a complex function, and deform the path into complex time to avoid the branch cuts.



Contour plot of $\sqrt{\mathbf{r}_{cl}(t)^2}$ on the complex time plane. The white lines are branch cuts where the imaginary part discontinuously changes sign.

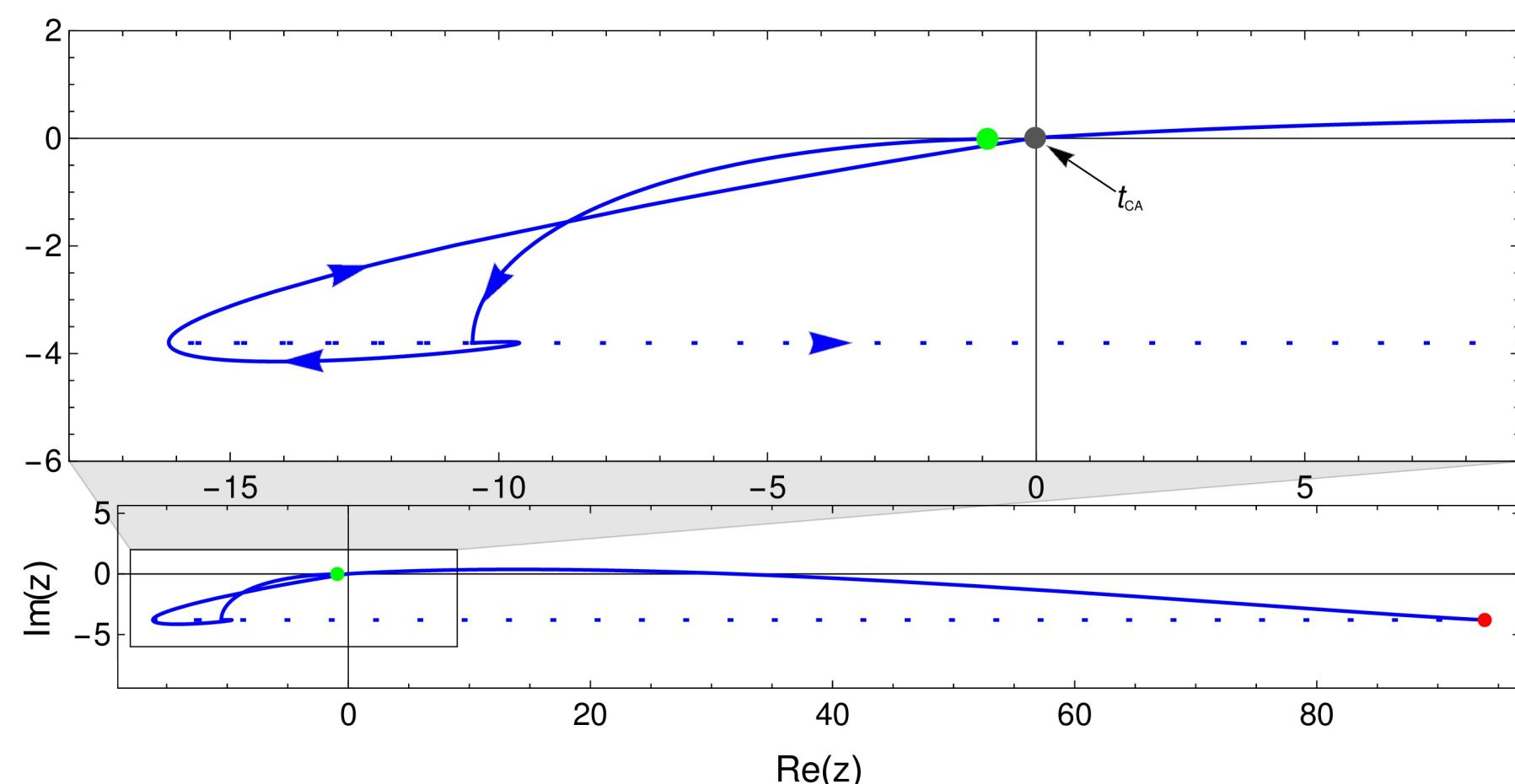


Sketch of the graphic above. Red lines are branch cuts of $\sqrt{\mathbf{r}_{cl}(t)^2}$, where $\mathbf{r}_{cl}(t)^2$ is real and negative. Green lines are the positive real axis and at gray regions $\text{Re}(\mathbf{r}_{cl}(t)^2) < 0$. The standard contour is shown dashed and crosses a branch cut. Instead, the contour should pass the closest-approach time t_{CA} .

6 Trading imaginary time for a real position

Changing the time-plane contour to pass through the closest-approach time t_{CA} has the effect of minimizing the imaginary part of the trajectory at the crucial times when the real part is small. When the potential $U(\mathbf{r})$ is most sensitive to imaginary positions, one needs to compromise with a slightly imaginary time.

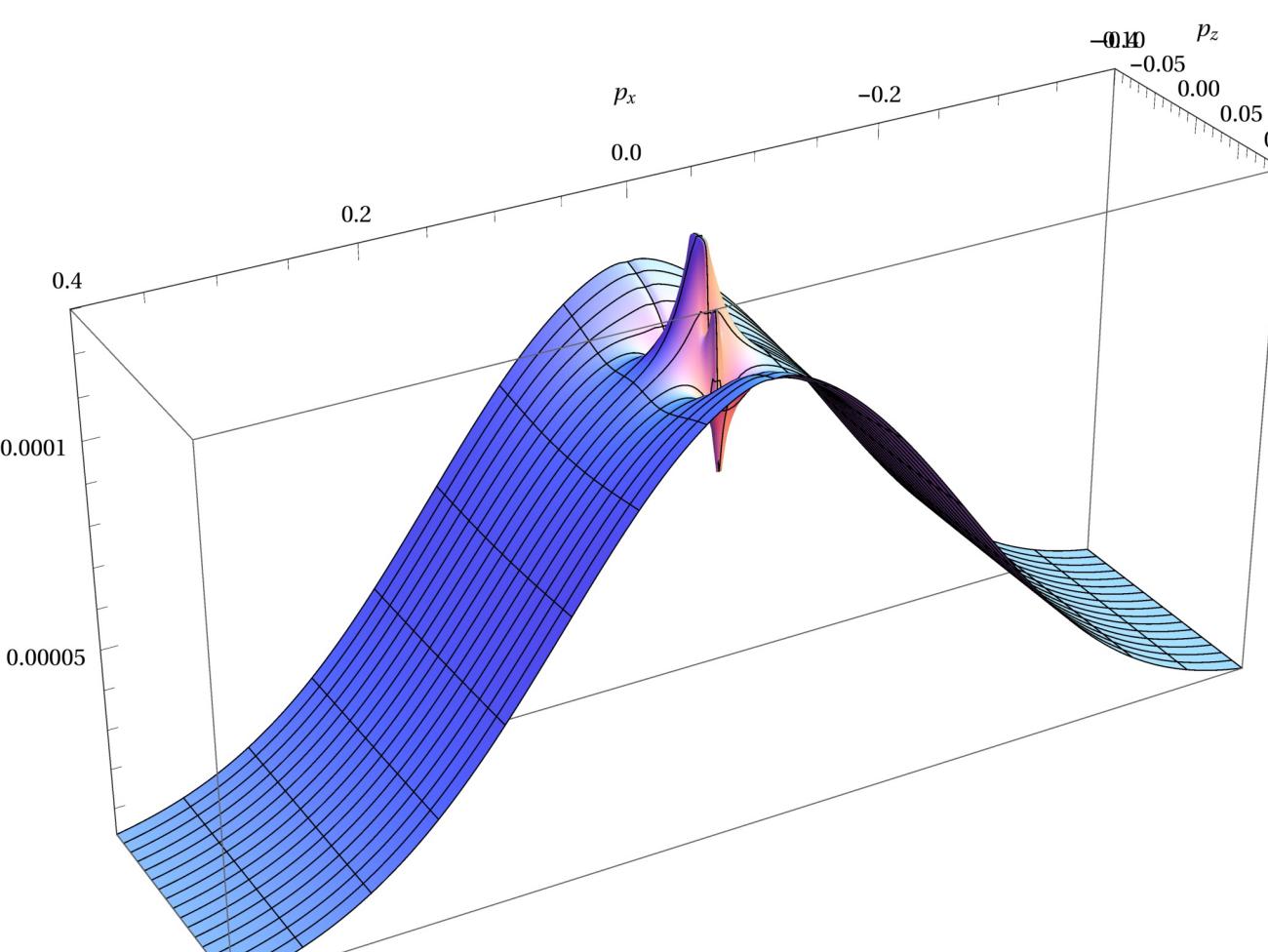
Trajectories on the complex z plane corresponding to the time contours above. The dashed contour over the real axis has a large imaginary position when it passes the ion.



7 Photoelectron spectra

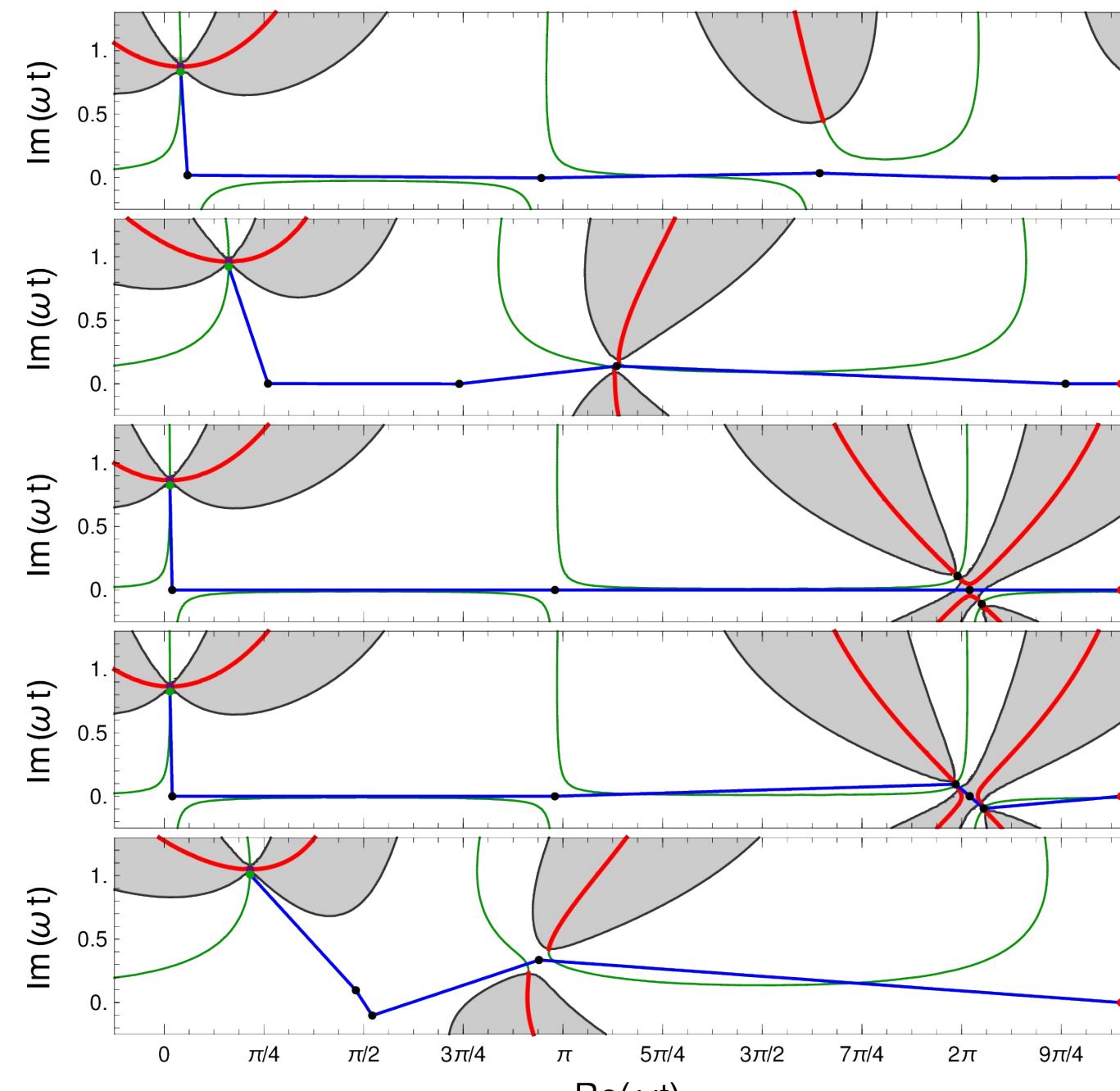
Soft recollisions cause cusps with abrupt cut-offs, as the change in contour brings in closer approaches to the ion with destructive amplitudes.

These cusps qualitatively match the transverse profile shown in high-resolution experiments [Pullen et al., J Phys B **47** 204010].



8 Branch cut slalom

Hopping through t_{CA} s, when they're out of the barrier, gives a perfect slalom score.



1 Why imaginary positions?

Electron tunnelling induced by the laser field $\mathbf{F} = \frac{d\mathbf{A}}{dt}$ can be described using trajectory language,

$$\mathbf{r}_{cl}(t) = \int_{t_s}^t \mathbf{v}(\tau) d\tau = \int_{t_s}^t (\mathbf{p} + \mathbf{A}(\tau)) d\tau \quad (1)$$

but for the kinetic energy $\mathbf{v}^2/2$ to be negative under the barrier, time needs to be complex-valued. The electron enters the barrier at a complex time t_s , and it is detected at a large time T . For the trajectory (1) any path joining t_s and T is acceptable. The conventional path goes down to the real axis and then continues along it: $t_s \rightarrow \text{Re}(t_s) \rightarrow T$.

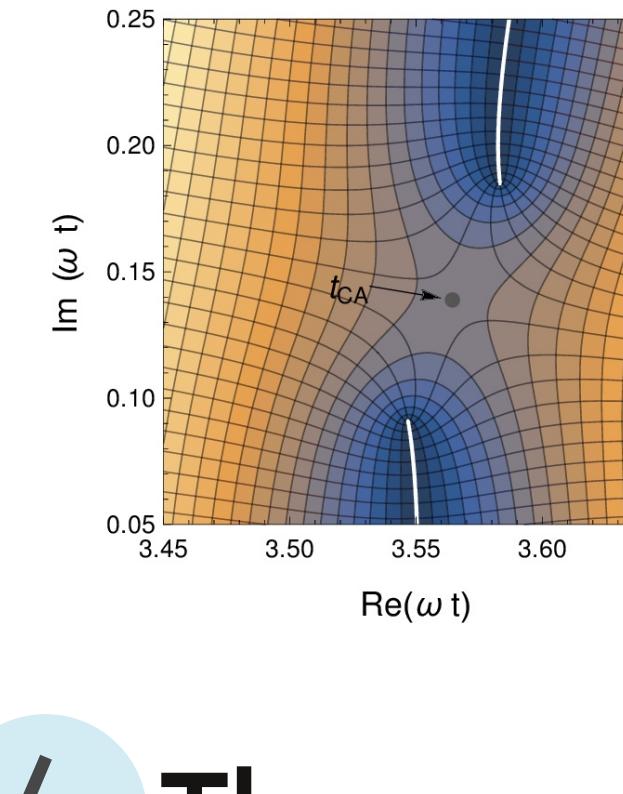
In general this trajectory will pick up an imaginary component as one integrates down to the real axis. This imaginary part is constant along the real axis but it does affect the values of the potential.

The trajectory can then be used to calculate the photo-electron ionization amplitude as

$$a(\mathbf{p}) \propto e^{i\mathbf{p}_0 t_s} e^{-\frac{i}{2} \int_{t_s}^T (\mathbf{p} + \mathbf{A}(\tau))^2 d\tau} e^{-i \int_{t_s}^T U(\mathbf{r}_{cl}(\tau)) d\tau} \quad (2)$$

The interaction of the photoelectron with its parent ion is included as the integral of the potential $U(\mathbf{r})$ along the trajectory $\mathbf{r}_{cl}(t)$.

3 How do I avoid the branch cuts?



Branch cuts typically come in pairs with a 'slalom gate' between them. This gate must always contain a saddle point in the middle to match the contour geometries of the branch cuts.

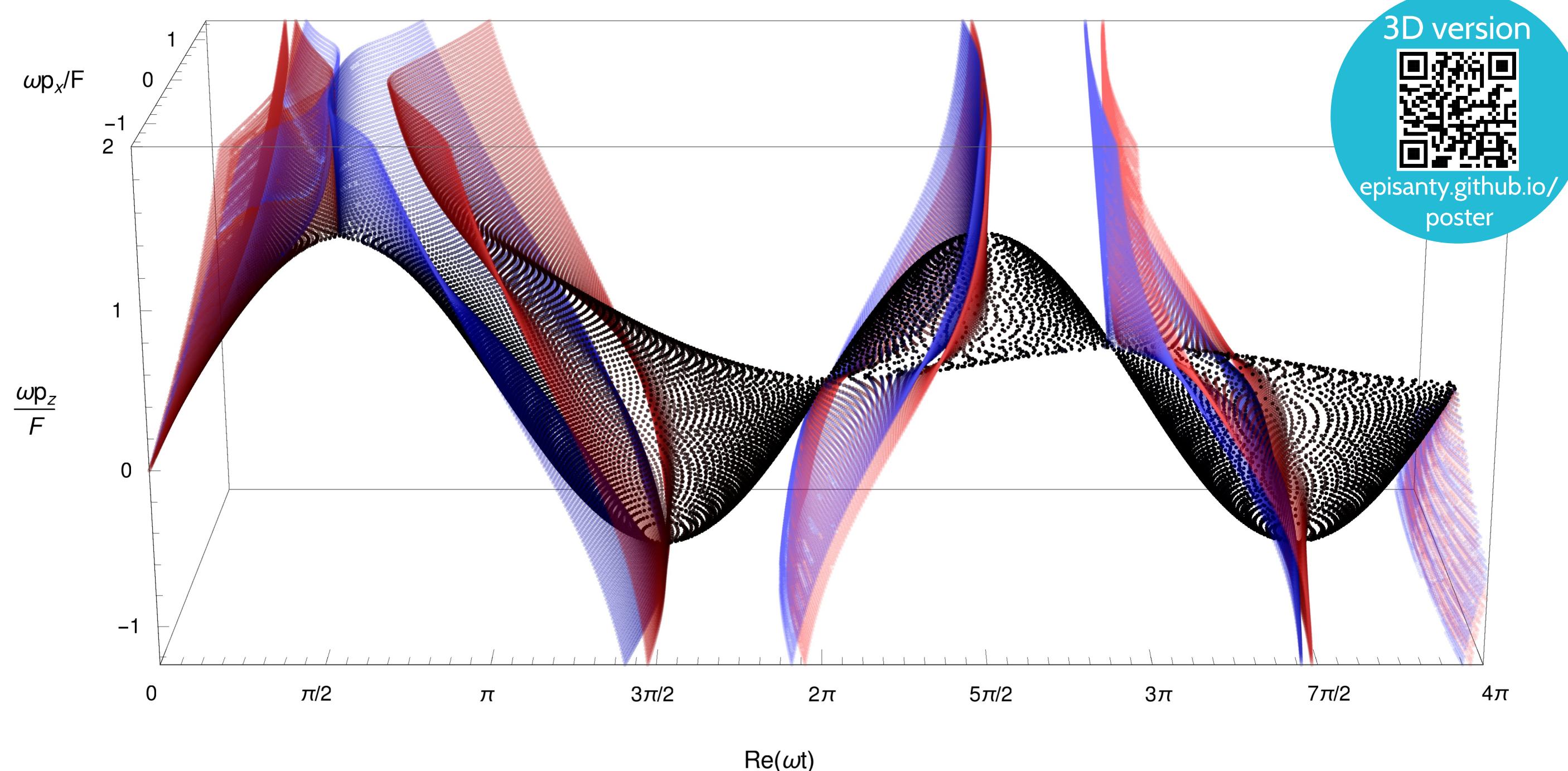
This is a saddle point of $\sqrt{\mathbf{r}_{cl}(t)^2}$ and therefore of $\mathbf{r}_{cl}(t)^2$, and it satisfies the equation

$$\frac{d}{dt} \mathbf{r}_{cl}(t)^2 \Big|_{t_{CA}} = \mathbf{r}_{cl}(t_{CA}) \cdot \mathbf{v}(t_{CA}) = 0. \quad (3)$$

This is a closest-approach equation: the (complex) position is orthogonal to the velocity.

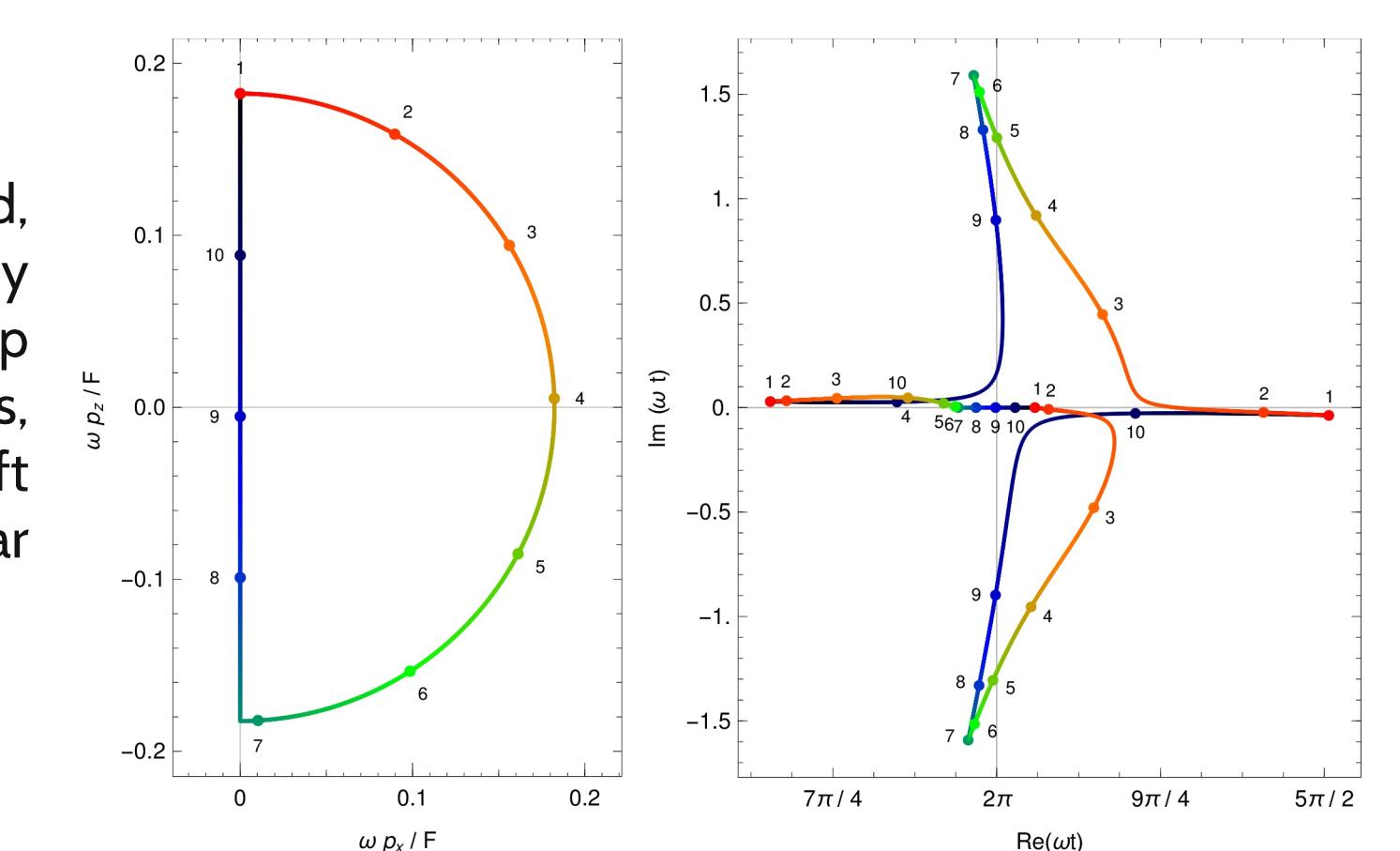
4 The geometry of closest-approach times

Closest-approach times as defined by Eq. (3) have a rich geometry. They form a multiple-sheet surface which wraps around the corresponding classical surface. Black points are mostly real, and red (blue) points have positive (negative) imaginary parts.

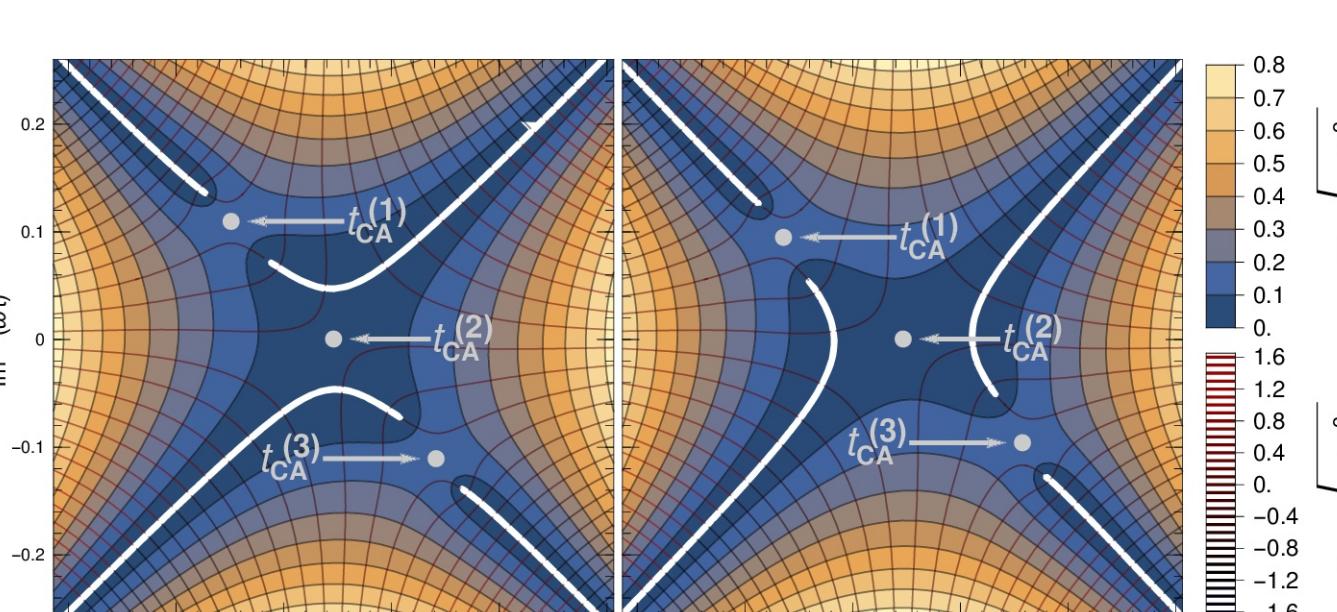


5 Soft recollisions

The multiple leaves of this surface are interconnected, which means that (3) has multiple roots but they cannot be consistently indexed. If one traces out a loop in momentum space and returns to the starting points, the t_{CA} s will swap labels. This happens at soft recollisions where the electron's velocity is small near the ion.



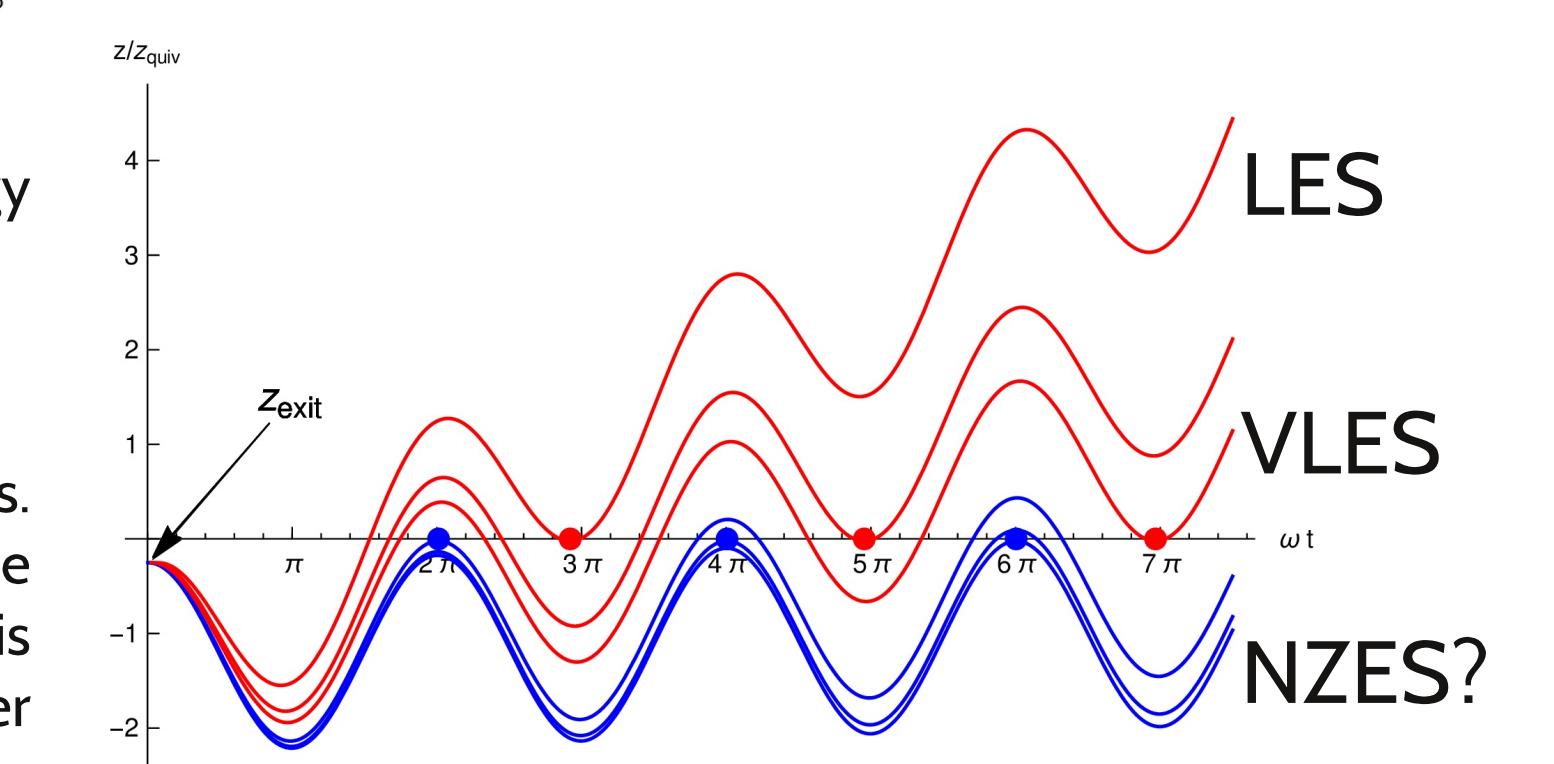
At soft recollisions, the topology of the branch cuts changes abruptly, forcing a change in the appropriate integration contour. This causes a sudden change from enhancement to suppression of ionization. The contour should pass t_{CA} s only when they are outside of the tunnelling barrier: when $\frac{1}{2}\mathbf{v}(t_{CA})^2 > 0$.



Soft recollisions occur in two series at different energy ranges, and with different scaling:

$$p_z^{\text{sr}} \sim \frac{F}{\omega(n+1)\pi} \quad (n \text{ even}), \quad \text{and} \quad p_z^{\text{sr}} \sim \frac{\omega \kappa^2}{F(n+1)2\pi} \quad (n \text{ odd}).$$

The former is responsible for Low-Energy Structures. The latter, at lower energy, should contribute to the recently-found Near-Zero Energy Structures. This scaling suggests higher- l_p targets should have clearer NZESs.



Acknowledgments

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Read the paper

E Pisanty and M Ivanov. Slalom in complex time: emergence of low-energy structures in tunnel ionization via complex time contours. arXiv:1507.00011 (2015).

