

# **Module 7**

## **Designing Relational Schemas**

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### **95.305**

#### **Objectives**

- **Learn what makes one relational schema a better design than another**
- **Learn what functional dependencies are**
- **Learn normalization techniques for decomposing relational schemas**
- **Reference:  
Elmasri & Navathe, Chapter 12**

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## 95.305

### Topics

- **Design Guide-lines for Relational Schemas**
  - Functional Dependencies
  - Normal Forms
  - 2nd and 3rd Normal Forms
  - Boyce-Codd Normal Form

### What we have looked at so far

- So far we have designed relational schemas by
  - common sense
  - mappings from E-R diagrams
- Is there a more formal theory of what makes one grouping of attributes a “good” relation

## Simple Guide-lines for Good Relational Design

- based on:

- meaning of attributes
- minimizing redundancy
- minimizing null values in tuples
- not allowing spurious tuples

These objective can conflict with one another

## Attribute Meaning

- Attributes are generally grouped because together they convey some meaning
- For example the attributes may all convey information about an employee of a company
- If attributes are well organized with respect to their meaning, the relations are easier to understand, and the design easier to do.

## An example Company Schema

EMPLOYEE FK  
 ENAME SSN BDATE ADDRESS DNUMBER

DEPARTMENT FK  
 DNAME DNUMBER DMGRSSN DLOCATIONS

DEPT\_LOCATIONS  
DNUMBER DLOCATION

FK PROJECT FK  
 PNAME PNUMBER PLOCATION DNUM

WORKS\_ON  
SSN PNUMBER HOURS  
FK FK

### Is this a good shema?

It is in the sense that each table seems easy understand

The groupings make sense: EMPLOYEE, DEPARTMENT, PROJECT are distinct entities

DEPT\_LOCATIONS, WORKS\_ON are relationships

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## Design Guide-line

- **Guide-line #1**
- **Design schemas so that it's easy to explain its meaning.**
- **Don't combine attributes from different entities in the same relation**

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## What about these relations

EMP\_DEPT

ENAME SSN BDATE ADDRESS DNUMBER DNAME DMGRSSN

EMP\_PROJ

SSN PNUMBER HOURS ENAME PNAME PLOCATION

**These schemas appear easy enough to understand**

**But, they violate guide-line #1**

**So why is this bad?**

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## Redundancy

EMP\_DEPT

ENAME SSN BDATE ADDRESS DNUMBER DNAME DMGRSSN

**The DNAME, DMGRSSN will be repeated  
in the table for each employee of the department**

EMP_DEPT						
FNAME	<u>SSN</u>	BDATE	ADDRESS	DNUMBER	DNAME	DMGRSSN
Smith, John	123456789	9-Jan-55	731 Fondern	5	Research	333445555
Wong, Franklin	333445555	8-Dec-45	638 Voss	5	Research	333445555
Zelaya, Alicia	999887777	19-Jul-58	3321 Castle	4	Admin	987654321
Wallace, Jennifer	987654321	20-Jun-31	291 Berry	4	Admin	987654321
Narayan, Ramesh	666884444	15-Sep-52	975 Fire Oak	5	Research	333445555
English, Joyce	453453453	31-Jul-62	5631 Rice	5	Research	333445555
Jabber, Ahmad	987987987	29-Mar-59	980 Dallas	4	Admin	987654321
Borg, James	888665555	10-Nov-27	450 Stone	1	HeadQuarters	888665555

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## Update Anomalies

- Relations that violate Guide-line #1 will exhibit update anomalies
- Anomalies:
  - insertion anomalies
  - deletion anomalies
  - modification anomalies

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## Insertion Anomalies

EMP_DEPT						
FNAME	SSN	BDATE	ADDRESS	DNUMBER	DNAME	DMGRSSN
Smith, John	123456789	9-Jan-55	731 Fondern	5	Research	333445555
Wong, Franklin	333445555	8-Dec-45	638 Voss	5	Research	333445555
Zelaya, Alicia	999887777	19-Jul-58	3321 Castle	4	Admin	987654321
Wallace, Jennifer	987654321	20-Jun-31	291 Berry	4	Administration	987654321
Narayan, Ramesh	666884444	15-Sep-52	975 Fire Oak	5	Research	333445555
English, Joyce	453453453	31-Jul-62	5631 Rice	5	Res.	333445555
Jabber, Ahmad	987987987	29-Mar-59	980 Dallas	4	Admin	987654321
Borg, James	888665555	10-Nov-27	450 Stone	1	HeadQuarters	888665555

**How do we insert a new employee who is not assigned to a department? (null-fill the dept. fields)**

**How do we enter a new dept. which does not have any employees? (cannot null fill a key attribute)**

**How do we know redundant information is consistently entered? (e.g. Admin vs. Administration)**

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## Deletion Anomalies

EMP_DEPT						
FNAME	<u>SSN</u>	BDATE	ADDRESS	DNUMBER	DNAME	DMGRSSN
Smith, John	123456789	9-Jan-55	731 Fondern	5	Research	333445555
Wong, Franklin	333445555	8-Dec-45	638 Voss	5	Research	333445555
Zelaya, Alicia	999887777	19-Jul-58	3321 Castle	4	Admin	987654321
Wallace, Jennifer	987654321	20-Jun-31	291 Berry	4	Administration	987654321
Narayan, Ramesh	666884444	15-Sep-52	975 Fire Oak	5	Research	333445555
English, Joyce	453453453	31-Jul-62	5631 Rice	5	Res.	333445555
Jabber, Ahmad	987987987	29-Mar-59	980 Dallas	4	Admin	987654321
Borg, James	888665555	10-Nov-27	450 Stone	1	HeadQuarters	888665555

**If we delete the last employee of a department, we must also delete the department**

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## Modification Anomalies

EMP_DEPT						
FNAME	<u>SSN</u>	BDATE	ADDRESS	DNUMBER	DNAME	DMGRSSN
Smith, John	123456789	9-Jan-55	731 Fondern	5	Research	999887777
Wong, Franklin	333445555	8-Dec-45	638 Voss	5	Research	999887777
Zelaya, Alicia	999887777	19-Jul-58	3321 Castle	4	Admin	987654321
Wallace, Jennifer	987654321	20-Jun-31	291 Berry	4	Admin	987654321
Narayan, Ramesh	666884444	15-Sep-52	975 Fire Oak	5	Research	333445555
English, Joyce	453453453	31-Jul-62	5631 Rice	5	Research	333445555
Jabber, Ahmad	987987987	29-Mar-59	980 Dallas	4	Admin	987654321
Borg, James	888665555	10-Nov-27	450 Stone	1	HeadQuarters	888665555

**If we change the manager of dept. #5, we must know to change all occurrences of this information, or else the database will be inconsistent (see table)**

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## Design Guide-line

- **Design Guide-line #2**
- **Design the relational schemas so that no insertion, deletion, or modification anomalies occur in the relations**
- **If any anomalies are present, they must be noted clearly so that updating programs can operate correctly**

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## Reasons for breaking Guide-lines

EMPLOYEE FK  
ENAME SSN BDATE ADDRESS DNUMBER

DEPARTMENT FK  
DNAME DNUMBER DMGRSSN

EMP\_DEPT  
ENAME SSN BDATE ADDRESS DNUMBER DNAME DMGRSSN

- **EMPLOYEE, DEPARTMENT are better for base relations**
- **EMP\_DEPT may be suitable for a view**
- **EMP\_DEPT may result in better performance and warrant keeping redundancies**

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## NULL Values

- “Fat” relations are subject to many NULL values
- e.g. if attributes apply only to some tuples
- NULL values cause problems in understanding, joins, and aggregate functions
- NULL values can be difficult to interpret
  - attribute does not apply
  - attribute value is unknown
  - attribute value is absent

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## Design Guide-line

- Design Guide-line #3
- Try to avoid relation attributes whose values can be NULL
- If NULLs are unavoidable, try to ensure they only apply to exceptional cases
- e.g. if only 10% of employees have an office, it may be inappropriate to include an OFFICE\_NUM attribute in the EMPLOYEE table
- Instead, create an EMP\_OFFICE relation

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## Spurious Tuples

EMP\_PROJ

<u>SSN</u>	PNUMBER	HOURS	ENAME	PNAME	PLOCATION
------------	---------	-------	-------	-------	-----------

EMP\_LOC

<u>ENAME</u>	<u>PLOCATION</u>
--------------	------------------

EMP\_PROJ1

<u>SSN</u>	<u>PNUMBER</u>	HOURS	PNAME	PLOCATION
------------	----------------	-------	-------	-----------

- Consider decomposing EMP\_PROJ into two separate relations EMP\_LOC and EMP\_PROJ1
- EMP\_LOC means employee name ENAME works on some project a location PLOCATION
- EMP\_PROJ1 means employee SSN works on project PNUMBER for HOURS at PLOCATION
- Is this a good decomposition?

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## EMP\_PROJ Decomposition

EMP_PROJ					
SSN	PNUMBER	HOURS	ENAME	PNAME	PLOCATION
123456789	1	32.5	Smith, John	X	Bellaire
123456789	2	7.5	Smith, John	Y	Sugarland
666884444	3	40	Narayan, Ramesh	Z	Houston
453453453	1	20	English, Joyce	X	Bellaire
453453453	2	20	English, Joyce	Y	Sugarland
...	...	...	...	...	...

EMP_PROJ1				
SSN	PNUMBER	HOURS	PNAME	PLOCATION
123456789	1	32.5	X	Bellaire
123456789	2	7.5	Y	Sugarland
666884444	3	40	Z	Houston
453453453	1	20	X	Bellaire
453453453	2	20	Y	Sugarland
...	...	...	...	...

EMP_LOC	
ENAME	PLOCATION
Smith, John	Bellaire
Smith, John	Sugarland
Narayan, Ramesh	Houston
English, Joyce	Bellaire
English, Joyce	Sugarland
...	...

**Is this a good decomposition?**

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## Attempt to recover EMP-PROJ info with a JOIN

EMP_PROJ1				
SSN	PNUMBER	HOURS	PNAME	PLOCATION
123456789	1	32.5	X	Bellaire
123456789	2	7.5	Y	Sugarland
666884444	3	40	Z	Houston
453453453	1	20	X	Bellaire
453453453	2	20	Y	Sugarland
...	...	...	...	...

EMP_LOC	
ENAME	PLOCATION
Smith, John	Bellaire
Smith, John	Sugarland
Narayan, Ramesh	Houston
English, Joyce	Bellaire
English, Joyce	Sugarland
...	...

### Natural Join

**Spurious  
Tuples**

SSN	PNUMBER	HOURS	PNAME	PLOCATION	ENAME
123456789	1	32.5	X	Bellaire	Smith, John
123456789	1	32.5	X	Bellaire	English, Joyce
123456789	2	7.5	Y	Sugarland	Smith, John
123456789	2	7.5	Y	Sugarland	English, Joyce
666884444	3	40	Z	Houston	Narayan, Ramesesh
453453453	1	20	X	Bellaire	English, Joyce
453453453	1	20	X	Bellaire	Smith, John
453453453	2	20	Y	Sugarland	English, Joyce
453453453	2	20	Y	Sugarland	Smith, John
...	...	...	...	...	...

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## Spurious Tuples

- Result of the Join produced spurious tuples, information which does not really exist
- Could not recover the information that was present in EMP\_PROJ relation
- Problem: natural join criteria (equality) was on an attribute which is neither:
  - a primary key in either relation
  - a foreign key in one referencing the other

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## Design Guideline

- **Design Guideline #4**
  - **Design relation schemas so that they can be joined with equality conditions on attributes which are either:**
    - primary keys
    - foreign keys
- in a way that ensures no spurious tuples**

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## 95.305

### Topics

- **Design Guide-line for Relational Schemas**
- **Functional Dependencies**
- **Normal Forms**
- **2nd and 3rd Normal Forms**
- **Boyce-Codd Normal Form**

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## Formalizing the Guide-lines

- **Functional Dependencies** are a first step to formalizing the Design Guide-lines
- They are a very important concept in the design of relational schemas
- They lead to the creation of Normal Forms which avoid the problems we have discussed thus far

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## Functional Dependencies

- A Functional Dependency is a constraint on two sets of relation attributes
- Consider relation  $R=\{A1, A2, ..., An\}$
- Given two attribute subsets  $X$  and  $Y$  of  $R$

The functional dependency  $X \rightarrow Y$  means

given any two tuples  $t1$  and  $t2$  from  $r(R)$ ,  
if  $t1[X] = t2[X]$ , then  $t1[Y] = t2[Y]$

- We say that functionally  $X$  determines  $Y$

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### e.g. Functional Dependency

EMP\_DEPT

ENAME	<u>SSN</u>	BDATE	ADDRESS	DNUMBER	DNAME	DMGRSSN
-------	------------	-------	---------	---------	-------	---------



DNUMBER  $\rightarrow$  {DNAME, DMGRSSN}

- **Interpretation:** if two tuples of EMP\_DEPT have the same DNUMBER value, they also have the same DNAME and DMGRSSN values
- **Imposing a functional dependency on a relation schema limits the tuples which would be considered legal**

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### Functional Dependencies

- Observations
- if X is a candidate key of R, then  
 $X \rightarrow Y$  hold for any subset Y of R
- $X \rightarrow Y$  in R does NOT imply  $Y \rightarrow X$

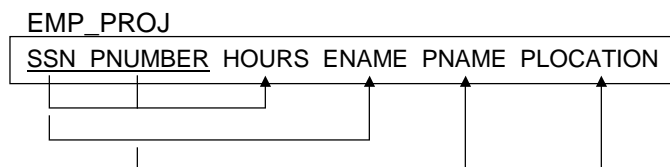
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## Functional Dependencies

- Functional Dependencies depend on the meaning of the attributes (what the designer meant them to represent)
- Functional Dependencies further describe the relation schema by placing constraints on its attributes which must hold at all times
- Functional Dependencies cannot be inferred from the table data, they must be explicitly stated by designer (based on meaning of relations)

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## example Functional Dependencies



SSN -> ENAME  
PNUMBER -> {PNAME, PLOCATION}  
{SSN, PNUMBER} -> HOURS

- Note diagrammatic representation on schema

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## Dependencies cannot be inferred

TEACH		
TEACHER	COURSE	TEXT
Smith	Data Structures	Bartram
Smith	Data Management	Al-Nour
Hall	Compilers	Hoffman
Brown	Data Structures	Augenthaler

- **Does TEXT -> COURSE?**  
-we cannot tell from the data it might be, but probably not
- **Does COURSE -> TEXT?**  
-No, we can tell this from the single counter example

## Dependency Inference

- **Given a set of functional dependencies we can often infer others**
- **e.g. if**  
 $F = \{ \text{SSN} \rightarrow \{\text{ENAME}, \text{BDATE}, \text{ADDRESS}, \text{DNUMBER}\}, \text{DNUMBER} \rightarrow \{\text{DNAME}, \text{DMGRSSN}\} \}$
- **Then the following is also true**  
 $\text{SSN} \rightarrow \{\text{DNAME}, \text{DMGRSSN}\},$   
 $\text{SSN} \rightarrow \text{SSN},$   
 $\text{DNUMBER} \rightarrow \text{DNAME}$
- **The designer will specify dependencies that are semantically obvious, but the database must also check for inferred dependencies**



## Closure of a dependency set

- **e.g. if**  
 $F = \{ \text{SSN} \rightarrow \{\text{ENAME}, \text{BDATE}, \text{ADDRESS}, \text{DNUMBER}\}, \text{DNUMBER} \rightarrow \{\text{DNAME}, \text{DMGRSSN}\} \}$
- **Then the following are also true**  
 $\text{SSN} \rightarrow \{\text{DNAME}, \text{DMGRSSN}\},$   
 $\text{SSN} \rightarrow \text{SSN},$   
 $\text{DNUMBER} \rightarrow \text{DNAME}$
- **the closure,  $F^+$ , of a dependency set  $F$  is the set of all dependencies which can be inferred from  $F$**

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## Inference Rules

- (1) if  $Y$  is a subset of  $X$ , then  $X \rightarrow Y$  (reflexive)
- (2) if  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  (augmentation)
- (3) if  $X \rightarrow Y$ ,  $Y \rightarrow Z$ , then  $X \rightarrow Z$  (transitive)
- (4) if  $X \rightarrow YZ$ , then  $X \rightarrow Y$  (decomposition)
- (5) if  $X \rightarrow Y$ ,  $X \rightarrow Z$ , then  $X \rightarrow YZ$  (union)
- (6) if  $X \rightarrow Y$ ,  $WY \rightarrow Z$ , then  $WX \rightarrow Z$  (pseudotransitive)

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## Inference Rules

- (1) if Y is a subset of X, then  $X \rightarrow Y$  (reflexive)
- (2) if  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  (augmentation)
- (3) if  $X \rightarrow Y$ ,  $Y \rightarrow Z$ , then  $X \rightarrow Z$  (transitive)

Rules 1-3 are sound and complete, known as **Armstrong's inference rules**, or **Armstrong's Axioms**

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## Proof of Decomposition Rule

- (4) if  $X \rightarrow YZ$ , then  $X \rightarrow Y$  (decomposition)

**Proof:**

- (a)  $X \rightarrow YZ$  ( given )
- (b)  $YZ \rightarrow Y$  (reflexive )
- (c)  $X \rightarrow Y$  (transitive )

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### Proof of Union Rule

(5) if  $X \rightarrow Y$ ,  $X \rightarrow Z$ , then  $X \rightarrow YZ$  (union)

**Proof:**

- (a)  $X \rightarrow Y$ ,  $X \rightarrow Z$  ( given )
- (b)  $X \rightarrow XY$  (augmentation, note  $xx \rightarrow y \Rightarrow x \rightarrow y$  )
- (c)  $XY \rightarrow YZ$  (augmentation )
- (c)  $X \rightarrow YZ$  (transitive )

### Proof of Pseudo-Transitive Rule

(6) if  $X \rightarrow Y$ ,  $WY \rightarrow Z$ , then  $WX \rightarrow Z$  (pseudotransitive))

**Proof:**

- (a)  $X \rightarrow Y$ ,  $WY \rightarrow Z$  ( given )
- (b)  $WX \rightarrow WY$  (augmentation )
- (c)  $WX \rightarrow Z$  (transitive )

## Closure of an Attribute

- The closure,  $A^+$ , of an attribute  $A$ , is the set of all attributes functionally determined by  $A$
- e.g. given  
 $F = \{ \text{SSN} \rightarrow \text{ENAME}, \text{PNUMBER} \rightarrow \{\text{PNAME}, \text{PLOCATION}\}, \{\text{SSN}, \text{PNUMBER}\} \rightarrow \text{HOURS} \}$
- We can calculate the following attribute closures  
 $\{\text{SSN}\}^+ = \{\text{SSN}, \text{ENAME}\}$   
 $\{\text{PNUMBER}\}^+ = \{\text{PNUMBER}, \text{PNAME}, \text{PLOCATION}\}$   
 $\{\text{SSN}, \text{PNUMBER}\}^+ = \{\text{SSN}, \text{PNUMBER}, \text{ENAME}, \text{PNAME}, \text{PLOCATION}, \text{HOURS}\}$

**EXERCISE:** You should be able to do this using the inference rules

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## Computing Attribute Closure

- Let  $X$  be an attribute set such that  $X \rightarrow W$  is an functional dependency in  $F$
- You can compute  $X^+$  as follows

```
X+ := X;
repeat
    oldX+ := X+;
    for each Y → Z in F do
        {if Y is a subset of X+
         then X+ := X+ union Z }
until (oldX+ = X+)
```

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### Computing an Attributes Closure (example)

- $F = \{ \text{SSN} \rightarrow \text{ENAME}, \text{PNUMBER} \rightarrow \{\text{PNAME}, \text{PLOCATION}\}, \{\text{SSN}, \text{PNUMBER}\} \rightarrow \text{HOURS} \}$
- **compute**  $\{\text{SSN}, \text{PNUMBER}\}^+$   
 $x^+ := \{\text{SSN}, \text{PNUMBER}\}$   
 $x^+ := \{\text{SSN}, \text{PNUMBER}, \text{HOURS}\}$  since  $\{\text{SSN}, \text{PNUMBER}\} \rightarrow \text{HOURS}$   
 $x^+ := \{\text{SSN}, \text{PNUMBER}, \text{HOURS}, \text{ENAME}\}$  since  $\text{SSN} \rightarrow \text{ENAME}$   
 $x^+ := \{\text{SSN}, \text{PNUMBER}, \text{HOURS}, \text{ENAME}, \text{PNAME}, \text{PLOCATION}\}$  since  $\text{PNUMBER} \rightarrow \{\text{PNAME}, \text{PLOCATION}\}$   
return  $x^+$

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### Example Exercise

- **Given**  
 $R = (A, B, C, G, H, I)$   
 $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
- **Is  $A \rightarrow H$  logically implied by  $F$ ?**

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### Example Exercise

- Given  
 $R = (A, B, C, G, H, I)$   
 $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$

- Is  $A \rightarrow H$  logically implied by  $F$ ?

- YES  
 $A \rightarrow B$  (given)  
 $B \rightarrow H$  (given)  
 $A \rightarrow H$  (transitive)

### Example Exercise

- Given  
 $R = (A, B, C, G, H, I)$   
 $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$

- What is  $A^+$  ?

### Example Exercise

- Given  
     $R = (A, B, C, G, H, I)$   
     $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
- What is  $A^+$  ?
- $A^+ := A$   
     $A^+ := AB$  (since  $A \rightarrow B$ )  
     $A^+ := ABC$  (since  $A \rightarrow C$ )  
     $A^+ := ABCH$  (since  $B \rightarrow H$ )
- so  $A^+ = ABCH$