Université d'Ottawa . University of Ottawa

Faculté de génie
École d'ingénierie
et de technologie de l'information

Faculty of Engineering
School of information Technology
and Engineering

CSI 3105
FINAL EXAMINATION

Duration: 3 hours
December. 1998

Professor: Dr. S.C. Boyd

NAME (please print): _____

STUDENT NUMBER: _____

NOTE:
1. This exam is a closed book exam, and no calculators are allowed.
2. All questions are to be answered on the question paper in the space provide.
3. A list of formulas which may be useful are given on the last page.
4. The total mark for this exam is 100 with mark allocation as shown below, and beside each question.

PROBLEM	MARKS	MARKS OBTAINED
1	15	
2	20	
3	15	
4	10	
5	15	
6	25	
TOTAL	100	

Good Luck! / Bonne Chance!

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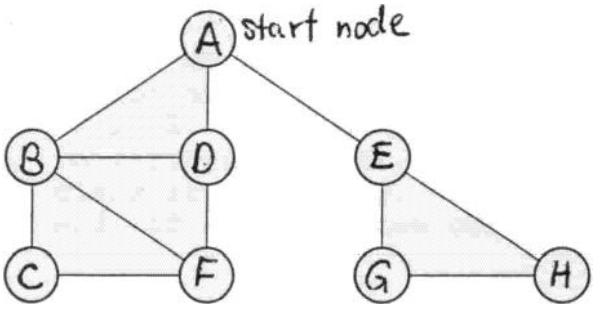
1. [15 marks total]

Consider the graph shown below, and its adjacency list.



A	B, D, E
B	A, D, F, C
C	B, F
D	A, B, F
E	A, G, H
F	D, B, C
G	E, H
H	G, E

a) [10 marks]

Using the algorithm for Depth First Search (DFS) as discussed in class (and the implementation discussed in class), show the order the nodes in G would be visited by the algorithm by numbering the nodes in that order on the graph above.

b) [5 marks]

What is the complexity of DFS in general for a graph with n nodes and m edges? (You do not need to explain your answer, just state the complexity).

2. [20 marks total]
Suppose you are given an array L which contains N integers. We wish to check if L is a permutation of the numbers 1, 2, 3, ..., N (i.e. check if L contains each of the numbers 1, 2, 3, ..., N exactly once, in any order). For example, for N = 5, L=[2, 1, 3, 5, 4] is a permutation, and so is L = [1, 2, 3, 4, 5] but of course L = [1, 2, 3, 4, 6] is not.

a) [15 marks]
Design an algorithm which checks if L represents a permutation of 1, 2, ..., N or not. You should explain your algorithm CLEARLY in structured English, and you may make use of any algorithms from class if they help. You will be marked on the clarity, efficiency and (of course!) the correctness of your algorithm.

b) [5 marks]

Do a complexity analysis of your algorithm from a). Be sure to state your input size, what you are counting, and what inputs give the worst case.

3] [15 marks total]
Consider the dynamic programming algorithm for solving the knapsack problem, as described in class.

a) [10 marks]
Write the recursive formula we used for calculating entry P[I,J] of the dynamic programming table, and explain what P[I,J] represents in the table.

b) [5 marks]
Suppose you are given the following solution table for a knapsack problem with 5 items (here I only show the values for I and N, where I stands for include, and N stands for not include), use this table to find the items which should be packed for maximum profit. On the table CIRCLE the entries which you used in calculating your answer.

Item	Weight \ Knapsack Size	0	1	2	3	4	5	6	7	8	9	10	11
1	1	N	I	I	I	I	I	I	I	I	I	I	I
2	2	N	N	I	I	I	I	I	I	I	I	I	I
3	5	N	N	N	N	N	I	I	I	I	I	I	I
4	6	N	N	N	N	N	N	I	I	I	I	I	I
5	7	N	N	N	N	N	N	N	I	I	I	I	N

4. [10 marks]

Consider the problem where you are trying to pack files onto disk segments which all have fixed size 1. You have sizes for all of the files, and these sizes are all ≤ 1. You wish to pack the files onto the disk segments in such a way that you use the minimum number of disk segments.

a) Suggest a scheme for providing a lower bound for this problem, and prove that it does indeed provide a lower bound in general. Your lower bound should be good enough to prove that 4 segments would be the minimum for packing files with the following sizes:

.8, .3, .2, .7, .2, .1, .3, ● 3, .1, .5, .1.

b) Give an example for which your lower bound scheme fails (i.e. an example for which there is a gap between the lower bound and the minimum number of segments required).

5.[15 marks total:

'Suppose you are given a directed graph stored in an adjacency list structure where for each node v, the out-neighbours of v are listed. You wish to check if a given node k has the same number of in-neighbours as out-neighbours.

a) [10 marks]
Describe CLEARLY in structured English how you could do this. You will be marked on the clarity, efficiency and correctness of you algorithm.

b) [5 marks]
Do a complexity analysis of your algorithm from part a)

6. [25 marks total

The following questions are short-answer questions, and you are not required to show your work. They are worth 5 marks each.

a) Consider the following algorithm which finds XN, where X≥1 and N≥0.

```
Procedure POWER(X,N:integer; Answer:integer)

var Temp: integer;

begin
   If N=0 then
     Answer: = 1
   Else
     If (N mod 2)=0 then begin
        POWER(X,(N div 2), Temp);
        Answer:= Temp * Temp;
        end
     else begin
        POWER(X,( (N-1) div 2), Temp);
        Answer:= Temp*Temp*X;
     end;
end;
```

Do a worst-case analysis for POWER (i.e find an exact formula for W(N), not a recursive one). You do not need to show your work. In your analysis use N as your input size, and count the number of calls made to POWER (including the original call). For example, for N=4, the number of calls would be 4.

b) Suppose you have a decision problem A which you wish to show is NP-complete. State what you must show about problem A in order to show that it is NP-complete.

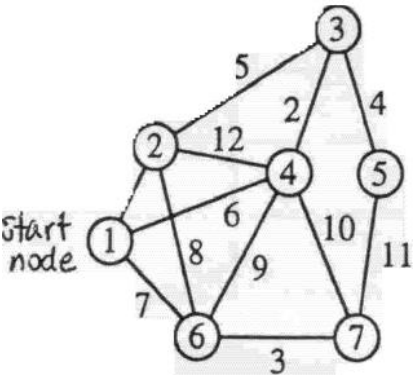
c) Suppose you are given a decision tree for a sorting algorithm called SORT for any array of length 5. Describe how you could find the worst case for inputs of size 5 (i.e. W(5)) from the tree.

d) Suppose I have an algorithm A for which the worst-case analysis for inputs of size n is as follows:

$W(n) = 2W(\lceil n/2 \rceil) + n + 1$
$W(1) = 0.$

What is the complexity of algorithm A? Once again, you do not need to show your work (just state your answer).

e) Using Prim's algorithm (as discussed in class) find a MST for the graph below. Show your MST on the graph, state its weight, and list the edges in your MST in the order they were added by Prim's algorithm.



FORMULAS WHICH MAY BE HELPFUL:

$$\sum_{i=1}^{n} i^2 = \frac{2n^3+3n^2+n}{6} \qquad \sum_{i=0}^{k} 2^i = 2^{k+1} \qquad \sum_{i=0}^{k} \frac{1}{2^i} = 2 - \frac{1}{2^k}$$

$$\sum_{i=1}^{k} i2^i = (k-1)2^{k+1}+2 \qquad \sum_{i=2}^{n} \frac{1}{i} = \ln n \qquad \sum_{i=1}^{n} \lg i \geq n \lg n - 1.5n$$