A collection of algorithms for large data and regression analysis

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Load core libraries

```
library(glmnet)
library(foreach)
library(care)
library(crossval)
library(fdrtool)
library(randomForest)
library(mboost)
library(e1071)
```

Section 1

1.1 - Linear analysis

1.1.1 - Linear regression

1.1.2 - OLS regression

```
OLS = solve(t(x1)%*%x1)%*%t(x1)%*%y # OLS regression
# t() - transposes the matrix
# solve() - invers the matrix
```

```
# %*% - matrix multiplication

x2 = cbind(x$sex, x$age, x$bmi, x$glu, x$map, x$ltg)

COV = solve(cov(x2))%*%cov(x2,y) # sample covariance based estimate

# x2 - a given matrix as x1 without intercept
# solve() - invers the matrix
# cov() - covariance
# %*% - matrix multiplication
```

1.2 - General Linear Prediction

1.2.1 - Linear Prediction

1.2.2 - Logistic Prediction

Inverse logit function $(logit^{-1}(eta) = exp(eta)/(exp(eta) + 1))$ to transform the linear predictor $(eta = x\beta)$ back to a probability which ranges between 0 and 1.

1.3 - Linear mixed model

```
load("E:/Imperial College London/Term 2/AR/Week1/exam.London")
dim(exam) #returns the dimensions of the dataset
# row x columns
# 3935 observations
# 10 covariates
# The standard linear model treats all observations independently and disregards
# potential group structures
fixed_effects = lm(normexam~standLRT+as.factor(school),
                   data=exam)
# fixed effects linear regression
# as.factor() - introducing covariates, (i.e grouped covariates such as number of schools)
library(nlme) # random effect package
random_intercept = lme(normexam~standLRT,
                       random = ~1|school,
                       data=exam)
# random = ~1/school - states that each school has a random intercept
summary(random intercept) # extract StdDev for intercept and residual
std_intercept = 0.3071927
std residual = 0.7535887
correlation_coefficient = std_intercept^2 / (std_intercept^2+std_residual^2)
# intraclass correlation coefficient
random_slope = lme(fixed=normexam~standLRT,
                   random = ~ 1 + standLRT | school,
                   data = exam)
# random = ~ 1 + standLRT | school - states that each slope is random depending on school
random_intercept_covariates= lme(normexam~
                                   standLRT + schavg + schgend,
                                 random = ~ 1 | school,
                                 data = exam)
# addition of covariates to model
anova(random_intercept, random_slope)
anova(random_intercept, random_intercept_covariates)
# anova() - anova test comparing models
```

Section 2

2.1 - Large data analysis

```
set.seed(694208008135)
load("E:/Imperial College London/Term 2/AR/Week2/data_epigenetic_clock_control")
#load data
y = control_mice$y_control # extract control (age)
hist(y, breaks=50)
# hist() - plots histogram
# breaks - sets x-scale
x = control_mice$x_control # extract number of mice and methylation site
dim(x) #returns the dimensions of the dataset
# row x columns
# 409 observations
# 3663 covariates
linear_regression = lm(y-x[,1])
summary(linear_regression)$coefficients
# summary can be used for indexing p-values of coefficients
p_{vec} = rep(NA, dim(x)[2])
# create empty vector of NA of size dim(x)[2] to store covariates
# dim(x)[2] OR ncol(x) can be used
for(i in 1:ncol(x)){
  p_vec[i]=summary(lm(y~x[,i]))$coefficients[2,4]
# for-loop which iterates through every covariate and extracts the respective
# coefficient, iteration only through univariate regression
sites_ranked = cbind(colnames(x), p_vec) # combining names and p-values
sites_ordered = sites_ranked[order(p_vec, decreasing = F),]
# ordering ranked sites by p-values in decreasing order
head(sites_ordered, 10) # showing top 10 sites ordered by decreasing values
```

2.2 - Multiple testing adjustment

```
library(qvalue)
hist(p_vec, breaks=50) # plots distribution of p-values
```

2.2.1 - Bonferroni correction

```
# bonferroni correction with p-values below 0.05
# most conservative
bonferroni = p.adjust(p_vec, method="bonferroni")
table(bonferroni<0.05)
output[which(bonferroni<0.05),]</pre>
```

2.2.2 - Benjamini-Hochberg FDR correction

```
# Benjamini-Hochberg FDR correction with p-values below 0.05
# most conservative FDR
bh = p.adjust(p_vec, method="BH")
table(bh<0.05)
output[which(bh<0.05),]</pre>
```

2.2.3 - q-value FDR correction

```
# q-value FDR correction with p-values below 0.05
qobj = qvalue(p=p_vec)
qvalues = qobj$qvalues
table(qvalues<0.05)
output[which(qvalues<0.05),]
qobj$pi0 # extract Null variable</pre>
```

2.2.4 - Local FDR correction

```
# local FDR correction with p-values below 0.05
# leas conservative FDR

lfdr_out = fdrtool(x=p_vec, statistic="pvalue", verbose=FALSE)

table(lfdr_out$lfdr<0.2)
output[which(lfdr_out$lfdr<0.05),]

lfdr_out$param # extracts censored and eta values</pre>
```

Section 3

3.1 - Penalised regression

3.1.1 - Lasso regression

```
set.seed(694208008135)
load("E:/Imperial College London/Term 2/AR/Week2/data_epigenetic_clock_control")

y = control_mice$y_control # extract control (age)

hist(y, breaks=50)

# hist() - plots histogram
# breaks - sets x-scale
```

```
x = control_mice$x_control # extract number of mice and methylation site
is.matrix(x)
\# ensure x is a matrix
dim(x) #returns the dimensions of the dataset
# row x columns
# 409 observations
# 3663 covariates
# lasso cross validation optimising the mean squared error
lasso_cv = cv.glmnet(x,y,family="gaussian",alpha=1, type.measure="mse")
lasso_cv$lambda.min # extracts the lambda that minimises the mse
lasso_cv$lambda.1se # extracts the lambda that is the largest lambda
# (smallest model) that has a mse
# that is within 1 standard error of the minimum mse
# lasso regression with lambda min
lasso min = glmnet(x,y,family="gaussian",alpha=1,lambda=lasso cv$lambda.min)
sum(abs(lasso_min$beta)>0) # variables included in model
# lasso regression with lambda 1se
lasso_1se = glmnet(x,y,family="gaussian",alpha=1,lambda=lasso_cv$lambda.1se)
sum(abs(lasso 1se$beta)>0) # variables included in model
# regularisation parameter, cross-validation error and model size plots
par(mfrow=c(1,3))
plot(1:100, lasso_cv$lambda, main="1. Regularization parameter",
     xlab="Regularisation model", ylab="Lambda")
abline(v=which(lasso_cv$lambda == lasso_cv$lambda.min),col="red",lwd=2)
abline(v=which(lasso_cv$lambda == lasso_cv$lambda.1se),col="red",lty=2,lwd=2)
plot(1:100, lasso_cv$cvm, main="2. Cross-validation error",
     xlab="Regularisation model", ylab="Cross-validation error")
abline(v=which(lasso_cv$lambda == lasso_cv$lambda.min),col="red",lwd=2)
abline(v=which(lasso_cv$lambda == lasso_cv$lambda.1se),col="red",lty=2,lwd=2)
plot(1:100, as.vector(lasso_cv$nzero), main= "3. Model size",
     xlab="Regularisation model", ylab="Model size")
abline(v=which(lasso_cv$lambda == lasso_cv$lambda.min),col="red",lwd=2)
abline(v=which(lasso_cv$lambda == lasso_cv$lambda.1se),col="red",lty=2,lwd=2)
```

Figure 1 shows the regularisation parameter which decreases from around 1.4 to nearly zero. The first models have a strong regularisation while the last models only have very little regularisation. The vertical red line indicated the lambda with the minimum cv-error, and the dashed red line indicates the largest lambda (smallest model) within one standard error of the minimum cv-error.

Figure 2 shows the bias-variance trade-off of the test error. At first the cross-validation error reduces when reducing the regularisation parameter, but at some point there is a saturation. After reaching the minimum error it increases again when further reducing the regularisation parameter. There is a clear minimum of the cross-validation error at the 67th position of the lambda vector which allows us to define the optimal

regularisation parameter (lambda.min). The largest lambda (sparsest model) within 1 standard error is at position 41.

Figure 3 shows the model complexity. With strong regularisation few variables are included into the model. When reducing the regularisation, the model size increases. The model with the lambda that miminises the MSE includes 301 variables, while the sparsest model within 1 standard error includes 158 variables.*

3.1.2 - Ridge regression

```
# ridge cross validation optimising the mean squared error

ridge_cv = cv.glmnet(x,y,family="gaussian",alpha=0, type.measure="mse")
ridge_cv$lambda.min # extracts the lambda that minimises the mse
ridge_cv$lambda.1se # extracts the lambda that is the largest lambda
# (smallest model) that has a mse
# that is within 1 standard error of the minimum mse

# lasso regression with lambda min

lasso_min = glmnet(x,y,family="gaussian",alpha=0,lambda=ridge_cv$lambda.min)
sum(abs(lasso_min$beta)>0) # variables included in model

# lasso regression with lambda 1se

lasso_lse = glmnet(x,y,family="gaussian",alpha=0,lambda=ridge_cv$lambda.1se)
sum(abs(lasso_1se$beta)>0) # variables included in model

# Ridge regression does not perform variable selection,
# all regression coefficients are unequal to zero and thus included into the model.
```

3.1.3 - Elastic-net regression

3.2 - Generalised Prediction rule

```
# compute squared error risk (MSE)
 out = mean( (ynew - test.y)^2 )
 return(out)
# rdige with K-fol cross-validation with B segments (i.e for lambda = 1se)
ridge_cvk = crossval(prediction_rule, x, y, lambda=ridge_cv$lambda.1se, alpha=0,
                         K=5, B=20, verbose=FALSE)
lasso_cvk = crossval(prediction_rule, x, y, lambda=lasso_cv$lambda.1se, alpha=1,
                        K=5, B=20, verbose=FALSE)
elasticnet_cvk = crossval(prediction_rule, x, y, lambda=elasticnet_cv$lambda.1se,
                         alpha=elasticnet cv$alpha, K=5, B=20, verbose=FALSE)
#table for comparison of mse and se
table.out = rbind(c(ridge_cv$stat, ridge_cv$stat.se),
                  c(lasso_cv$stat, lasso_cv$stat.se),
                  c(elasticnet_cv$stat, elasticnet_cv$stat.se))
rownames(table.out) = c("ridge", "lasso", "elastic net")
colnames(table.out) = c("mse", "se")
table.out
```

3.3 - Shrinkage t-score

```
load("E:/Imperial College London/Term 2/AR/Week3/JAMA2011_breast_cancer")
y = data_bc$rcb
table(y)
x = data_bc$x
dim(x)
library(corpcor)
library(st)
sample.var = var.shrink(x,lambda=0) # sample variance with no shrinkage
shrink.var = var.shrink(x) # shrinkage variance with unspecified lambda
# boxplot template for comparing variances
boxplot(cbind(sort(sample.var)[1:1000],
              sort(shrink.var)[1:1000]),
       ylim=c(0,0.25),
names=c("sample variance", "shrinkage variance"))
shrink_t = shrinkt.stat(X=as.matrix(x), L=as.factor(y)) # shrinkage t-score
# for speed X and L may be defined outside of function
# multiple testing correction on t-statistic
FDR_shrinkage = fdrtool(shrinkt, statistic = "normal", verbose =FALSE)
```

```
sum(FDR_shrinkage$lfdr<0.2)
# statistic = "normal" - fit a Normal-distribution to the t-score</pre>
```

Section 4

4.1 - Predicting treatment (classification)

4.1.1 - Diagonal discriminant analysis

```
load("E:/Imperial College London/Term 2/AR/Week3/JAMA2011_breast_cancer")
library(sda)
library(pROC)
library(PRROC)
y = as.factor(data_bc$rcb)
table(y) # 0 = excellent response, 1 = lesser response
x = as.matrix(data_bc$x)
dim(x)
# ranking features using sda function
DDA_ranking = sda.ranking(x, y, diagonal = TRUE)
DDA_variables = sum(DDA_ranking[,"lfdr"]<0.2)</pre>
# factors passing <0.2 threshold and IDs
selected_variables = DDA_ranking[,"idx"][1:DDA_variables] # gene indexing
# build prediction rule for dda
DDA_rule = sda(x[, selected_variables, drop = FALSE], y, diagonal = TRUE)
DDA_predicted = predict(DDA_rule, x[, selected_variables, drop = FALSE],
                        verbose = FALSE)
# sensitivity (TPR)/specificity (TNR) based on confusion matrix
cM = confusionMatrix(as.character(y),
                   as.character(DDA_predicted$class),
                   negative="0")
# FP, TP, TN, FN
confusion_dataframe = data.frame(c(cM[[2]],cM[[4]]),c(cM[[1]],cM[[3]]))
colnames(confusion_dataframe)=c("Positive", "Negative")
row.names(confusion_dataframe)=c("Positive", "Negative")
knitr::kable(confusion_dataframe, "pipe")
TPR = cM[2]/(cM[2]+cM[4]) # true positive rate = TP/TP+FN
TNR = cM[3]/(cM[1]+cM[3]) # true negative rate = TN/FP+TN
```

```
head(DDA_predicted$posterior) # case/ no case
ROC = roc(y, DDA_predicted$posterior[,2])
plot(ROC) # ROC
ROC$auc # AUC
# compute the area under the curve (AUC)
# of the receiver operating characteristic (ROC)
```

4.1.2 - Linear discriminant analysis

```
# linear discriminant analysis
LDA_ranking = sda.ranking(x, y, diagonal = FALSE) # ranking features using sda function
LDA_variables = sum(LDA_ranking[,"lfdr"]<0.2)</pre>
# factors passing <0.2 threshold and IDs
selected_variables = LDA_ranking[,"idx"][1:LDA_variables] # gene indexing
# build prediction rule for dda
LDA_rule = sda(x[, selected_variables, drop = FALSE], y, diagonal = FALSE)
LDA_predicted = predict(LDA_rule, x[, selected_variables, drop = FALSE],
                        verbose = FALSE)
# sensitivity (TPR)/specificity (TNR) based on confusion matrix
cM = confusionMatrix(as.character(y),
                   as.character(LDA_predicted$class),
                   negative="0")
# FP, TP, TN, FN
confusion_dataframe = data.frame(c(cM[[2]],cM[[4]]),c(cM[[1]],cM[[3]]))
colnames(confusion_dataframe)=c("Positive", "Negative")
row.names(confusion_dataframe)=c("Positive", "Negative")
knitr::kable(confusion_dataframe, "pipe")
TPR = cM[2]/(cM[2]+cM[4]) # true positive rate = TP/TP+FN
TNR = cM[3]/(cM[1]+cM[3]) # true negative rate = TN/FP+TN
head(LDA_predicted$posterior) # case/ no case
ROC = roc(y, LDA_predicted$posterior[,2])
plot(ROC) # ROC
ROC$auc # AUC
# compute the area under the curve (AUC)
# of the receiver operating characteristic (ROC)
```

4.1.3 - Support vector machine regression

```
# support vector machine
```

```
SVM_rule = svm(x,y) # build svm train rule
SVM_predicted = predict(SVM_rule, x, decision.values = FALSE) # build prediction rule
SVM_predicted_DV = attr(SVM_predicted, "decision.values")
SVM_predicted = predict(SVM_rule, x)
# sensitivity (TPR)/specificity (TNR) based on confusion matrix
cM = confusionMatrix(as.character(y),
                   as.character(SVM_predicted),
                   negative="0")
# FP, TP, TN, FN
confusion_dataframe = data.frame(c(cM[[2]],cM[[4]]),c(cM[[1]],cM[[3]]))
colnames(confusion_dataframe)=c("Positive", "Negative")
row.names(confusion_dataframe)=c("Positive", "Negative")
knitr::kable(confusion_dataframe, "pipe")
TPR = cM[2]/(cM[2]+cM[4]) # true positive rate = TP/TP+FN
TNR = cM[3]/(cM[1]+cM[3]) # true negative rate = TN/FP+TN
head(SVM_predicted$posterior) # case/ no case
ROC = roc(y, SVM_predicted$posterior[,2])
plot(ROC) # ROC
ROC$auc # AUC
# compute the area under the curve (AUC)
# of the receiver operating characteristic (ROC)
```

4.1.4 - Random forest regression

CAUTION: It is NOT good practise to evaluate a prediction rule on the same dataset that was used to establish the prediction rule, results may be misleading due to overfitting.

4.2 - Evaluating prediction performance using cross-validation

```
prediction_function = function(Xtrain, Ytrain, Xtest, Ytest, method)
  if (method=="dda" | method =="lda") {
    if (method=="dda") { # diagonal discriminant analysis
     diagonal=TRUE
    else if (method=="lda") { # linear discriminant analysis
      diagonal==FALSE
   }
   DA_ranking = sda.ranking(Xtrain, Ytrain, verbose=FALSE, diagonal=diagonal, fdr=TRUE)
   ranked_variables = sum(DA_ranking[,"lfdr"]<0.2)</pre>
    selected_variables = DA_ranking[,"idx"][1:ranked_variables]
    # fit and predict
   DA_fit = sda(Xtrain[, selected_variables, drop=FALSE], Ytrain, diagonal=diagonal, verbose=FALSE)
   DA_predict = predict(DA_fit, Xtest[, selected_variables, drop=FALSE], verbose=FALSE)$class
   # count false and true positives/negatives
   negative = levels(Ytrain)[1] # negatives or baseline is the good response class
    cm = confusionMatrix(Ytest, DA_predict, negative=negative)
  }
  else if (method=="svm"){
    # fit
   SVM_fit=svm(Xtrain, Ytrain)
    #predict
   SVM_predict=predict(SVM_fit, Xtest)
    # count false and true positives/negatives
   negative = levels(Ytest)[1] # negatives are the good response class
    cm = confusionMatrix(Ytest, SVM_predict, negative=negative)
  }
  else if (method=="randomforest"){
   random_forest_fit = randomForest(y=Ytrain, x=Xtrain)
    #predict
   random_forest_predict = predict(random_forest_fit, Xtest, type="response")
```

```
# count false and true positives/negatives
negative = levels(Ytest)[1] # negatives are the good response class
cm = confusionMatrix(Ytest, random_forest_predict, negative=negative)
}
else{
    print("Please provide appropriate method, 'dda', 'lda', 'svm', 'randomforest' ")
}
return(cm)
}

# k-fold crossvalidation with B repetitions to evaluate prediction performance
TPR = rep(0,4)
TNR = rep(0,4)
K=5
B=20
```

4.2.1 - Evaluating prediction performance of DDA cross-validation

4.2.2 - Evaluating prediction performance of LDA cross-validation

4.2.3 - Evaluating prediction performance of SVM cross-validation

4.2.4 - Evaluating prediction performance of random forest cross-validation

```
TNR[4] = randomforest_cv$stat[3]/(randomforest_cv$stat[1]+randomforest_cv$stat[3])

# final results
# true positive rate (sensitivity)
TPR
# true negative rate (specificity)
TNR
```

Section 5

5.1 - Non-parametric prediction using random forests

```
load("E:/Imperial College London/Term 2/AR/Week3/data_epigenetic_clock_control")
y = control_mice$y_control
x = control_mice$x_control
dim(x)
# build random forest with 100 trees
random_forest = randomForest(x = x, y = y, ntree = 100, importance = TRUE)
importance = random_forest$importance # extract random forest importance
importance_order = importance[order(importance[,2], decreasing = TRUE),] [1:10,]
# decreasing = TRUE - rank the top 10 most important values
varImpPlot(random_forest) # visualise importance from random forest
# bagging random forest
bagging = randomForest(x, y, ntree = 100, importance = TRUE, mtry = ncol(x))
# mtry = ncol(x) - predictors to split tree, <math>ncol(x) indicates baggin
# function to compare performance for different training lengths of random forest
random_forest_prediction = function(train.x, train.y, test.x, test.y, ntree){
    #fit the model and build a prediction rule
   random_forest_fit = randomForest(train.x, train.y, ntree = ntree)
    #predict the new observation based on the test data and the prediction rule
   random_forest_predicted = predict(random_forest_fit , test.x)
    # compute squared error risk (MSE)
   out = mean( (random_forest_predicted - test.y)^2 )
   return( out )
}
# comparing prediction performance
# k-fold crossvalidation with B repetitions to evaluate prediction performance
```

```
B=10 # ideally 100-1000 repetitions
randomforest_cv_nt10 = crossval(random_forest_prediction, x, y, ntree = 10,
                                K, B, verbose = FALSE)
randomforest_cv_nt10$stat
randomforest_cv_nt10$stat.se
randomforest_cv_nt100 = crossval(random_forest_prediction, x, y, ntree=100,
                                   K=5, B=10, verbose=FALSE)
randomforest_cv_nt100$stat
randomforest_cv_nt100$stat.se
# It is recommended to use at least 500 or more trees.
# Random forest vs regularised regression - prediction performance
table.out = rbind(c(ridge_cvk$stat, ridge_cvk$stat.se), c(lasso_cvk$stat, lasso_cvk$stat.se),
c(elasticnet_cvk$stat, elasticnet_cvk$stat.se), c(randomforest_cv_nt10$stat, randomforest_cv_nt10$stat
rownames(table.out) = c("ridge", "lasso", "elastic net", "randomforest_cv_nt10", "randomforest_cv_nt100
colnames(table.out) = c("mse", "se")
table.out
```

5.2 - Non-parametric prediction, multiple approaches